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Exuberant interference: rainbows, tides, edges, (de)coherence. . .

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Young's pioneering studies of interference have led to fundamental developments in wave physics. Supernumerary rainbows were the first example of diffraction associated with caustics. Cotidal lines (connecting places where the tide is high at a given time) were the first example of wavefronts in the modern sense (pattern of phase contours $(\arg\psi_1 + \psi_2)$ of the superposition of waves ψ_1 and ψ_2, rather than the superposed patterns of the separate phases $\arg\psi_1$ and $\arg\psi_2$), and led to the discovery of phase singularities. Edge-diffracted waves extend the range of asymptotic methods applied to waves and continue to find diverse and unexpected applications. Young's understanding of the conditions for observing interference are now part of decoherence theory, which explains, for example, the emergence of the classical world from the quantum world.

Keywords: waves; interference; decoherence

1. Introduction

The principle of interference, namely the addition (superposition) of wave amplitudes, was known long before Thomas Young; for example, Newton was aware of it for mechanical waves. Young's originality lay in his insistence that the principle applies to light and provides a comprehensive framework for explaining a great variety of optical phenomena.

The discovery of interference was a complicated episode, and I will not pretend to apply a historian's perspective (for an instructive account, see Kipnis (1991)). Rather, my purpose here is to describe several of Young's many examples of interference, and show how they still resonate with our scientific preoccupations today, attesting to the continuing fertility of the great scientist's ideas.

As has often been pointed out, Young's term 'interference' is not the happiest way to refer to the process of inert addition, in which waves emerge from an encounter unaltered, without having interfered in the common-language sense. If we were reinventing terminology today, it would be more appropriate to reserve interference for the genuine interactions among nonlinear waves. Nevertheless, the term is so well established for the addition of linear waves that it would be perverse not to use it.

One contribution of 15 to a special Theme Issue 'Interference 200 years after Thomas Young's discoveries'.

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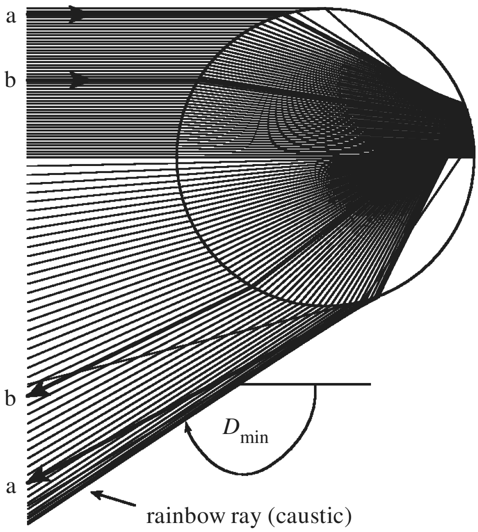

Figure 1. From a raindrop, two distinct rays (illustrated by thick lines a and b) emerge in each direction within the bow (i.e. $D > D_{\min}$).

2. Supernumerary rainbows

Under appropriate meteorological conditions (essentially showers where the raindrops are similar in size) several alternations of colour can be seen just inside the rainbow's main arc (Greenler 1980; Tricker 1970; Minnaert 1993). These are the supernumerary bows, not to be confused with the secondary rainbow, whose origin is quite different. It seems that although Newton was a very careful observer, and paid particular attention to phenomena that resisted his attempts to explain them, he never noticed supernumerary bows. They began to be mentioned in the literature only several decades after his death (Boyer 1959). And indeed it is impossible to understand supernumerary bows solely on the Newtonian picture of light rays.

Before Newton, Descartes (1637) (see also Boyer 1959; Lee & Fraser 2001) had explained the geometrical bow as an angular caustic, that is in terms of directional focusing of light rays deflected by a raindrop: the bow occurs where the deflection angle D, as a function of impact parameter, takes its minimum value $D_{\min}$ (close to 138°).

Young appreciated that supernumerary rainbows are interference fringes. For each colour, and in each direction inside the geometrical bow, two distinct rays emerge (figure 1). Young (1804) calculated the path difference P between the rays in each pair, in terms of the radius a of the drop, and estimated the angular positions of the maxima from the condition that P is an integer number of wavelengths λ. Operating the theory in reverse, he was able to estimate the sizes of raindrops in a distant shower. From a modern perspective, Young's insight is significant as the first example of wave interference associated with a geometrical caustic. Since caustics are the singularities of geometrical optics, this is a fundamental aspect of the physics of light.

Although Young was correct in interpreting supernumeraries in terms of interference, his theory was wrong in three respects. First, it predicted that the wave intensity would be infinite on the caustic, that is at the geometrical rainbow angle.

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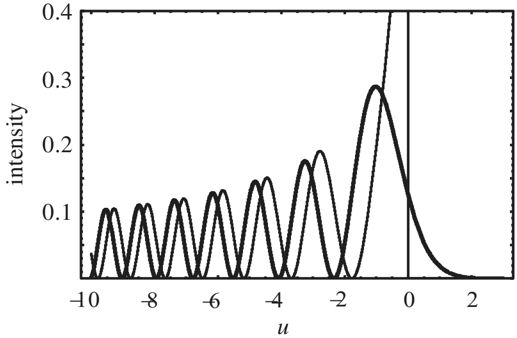

Figure 2. The Airy intensity $\mathrm{Ai}^2(u)$ across a rainbow (thick curve), and Young's approximation $\cos^2\{(2/3)(-u)^{3/2}\}/\pi\sqrt{-u}$ (thin curve).

Second, it predicted zero intensity on the dark side. Third, the rainbow maxima occur not when $P = n\lambda$ but close to $P = (n - 1/4)\lambda$.

Several decades later, the first two defects were remedied by Airy (1838). Using diffraction theory, Airy showed that the wave intensity near $D_{\min}$ is well approximated by

$$I(D) \propto \left(\frac{a}{\lambda}\right)^{213} \mathrm{Ai}^2\left((D_{\min} - D)\left(\frac{2\pi a}{3\lambda}\right)^{2/3} \frac{(n^2-1)^{1/2}}{(4-n^2)^{1/6}}\right), \tag{2.1}$$

where Ai denotes the eponymous Airy function,

$$\mathrm{Ai}(u) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}t \, \exp\{\mathrm{i}(\tfrac{1}{3}t^3 + ut)\}, \tag{2.2}$$

n is the refractive index (close to 4/3 for water), and Descartes' deflection angle is

$$D_{\min} = \pi + 2\sin^{-1}\left(\sqrt{\frac{4-n^2}{3}}\right) - 4\sin^{-1}\left(\frac{1}{n}\sqrt{\frac{4-n^2}{3}}\right). \tag{2.3}$$

On the bright inside of the geometrical bow, $D > D_{\min}$, so the argument u of Ai is negative, and the oscillations of Ai describe the supernumerary bows, that is, Young's interference fringes (figure 2). On the dark outside, $u > 0$ and Ai decays faster than exponentially, reflecting the absence of geometrical rays in this region of the sky. On the caustic, Ai(0) is finite, and Ai attains its maximum value just inside the caustic.

The third defect was remedied by Stokes (1847), who applied asymptotic analysis to Airy's integral and derived the extra $-\lambda/4$ term in the condition for the supernumerary oscillations. $\lambda/4$ corresponds to a phase advance of $\pi/2$, and Stokes's discovery was the precursor of many similar phases, for example, the Gouy phase associated with passage of light through a focus (Born & Wolf 1959; Siegman 1986), the extra '1/2' in the quantization condition for particles in a potential well (Berry & Mount 1972), and geometric phases (Shapere & Wilczek 1989).

The formula (2.1) applies only close to $D_{\min}$. However, a more sophisticated asymptotic approximation (Chester *et al.* 1957) enables the argument of Ai to be deformed to provide an accurate representation of the wave over a much wider angular range (Nussenzveig 1992). The same technique works not only for optical rainbows but also for angular caustics in the potential scattering of quantum particles (Ford & Wheeler 1959; Berry 1966).

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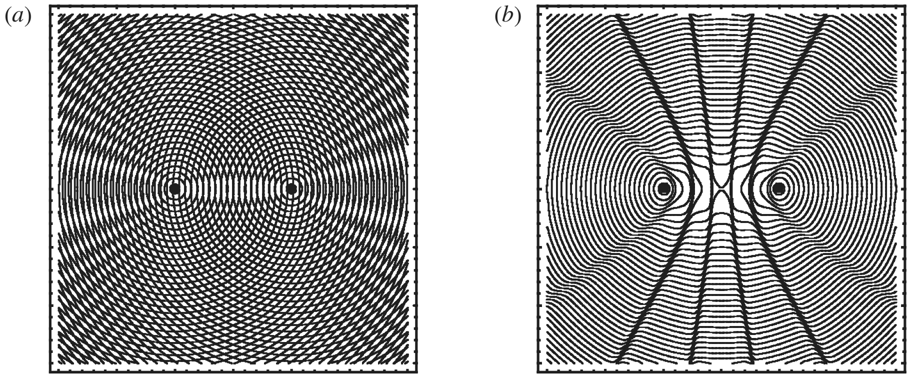

Figure 3. Waves ψ_+ and ψ_- from two point sources. (*a*) Superposition of wavefronts from ψ_+ and ψ_- separately; (*b*) wavefronts of the superposition, that is contours of phase $\arg(\psi_+ + \psi_-)$, at intervals of $\pi/4$.

The rainbow is an example of the simplest caustic, across which the number of geometrical rays changes by two. In the 1970s there emerged a complete mathematical description—catastrophe theory (Poston & Stewart 1978)—describing caustics involving more than two rays. Corresponding to each type of caustic is a wave pattern, generalizing Airy's integral. These 'diffraction catastrophes' have many applications in optics and throughout wave physics (Berry & Upstill 1980), and constitute a new class of mathematical special functions (Berry 2001*c*) with rich properties, forming part of the forthcoming Digital Library of Mathematical Functions (see http://dlmf.nist.gov).

3. Tides and wavefronts

Consistent with his all-embracing view of wave phenomena, Young regarded the tides as a wave encircling the Earth, of 12 h period and driven by the Moon and Sun. Before Young, Newton and Laplace had applied mechanical theory to the world's oceans, and had understood how tidal and astronomical periodicities are related (Cartwright 1999). Young's contribution was to seek to understand how the tides at different places are related, in the words of Whewell (1833), 'to account for their varieties and seeming anomalies'. To this end, he introduced the central concept of cotidal lines (it seems (Cartwright 1999) that the term was coined later, by Whewell). These are lines connecting places where the tide is high at a given time. In modern terminology, Young's cotidal lines are the wavefronts of the tide wave. To appreciate Young's originality, it is important to understand that there are two quite different sets of lines (or surfaces in three dimensions) that are commonly called wavefronts.

To illustrate these, consider two sources of circular waves. From each, rays radiate, and the circular normals to the rays are wavefronts of the first type. In regions reached by waves from both sources, the patterns of wavefronts overlap (figure 3*a*). The wavefronts of the second type are the contours of constant phase of the total wave, which look very different (figure 3*b*); in particular, they do not overlap. In mathematical terms, the phase pattern of $\psi_1 + \psi_2$ is not related in any simple way to the phase patterns of ψ_1 and ψ_2 separately: the wavefronts of the superposition are not the superposed wavefronts.

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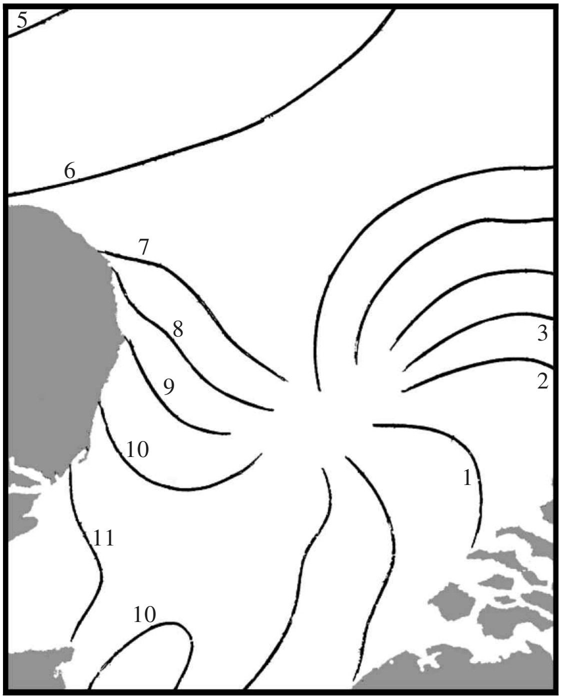

Figure 4. Amphidromic point (phase singularity) in the North Sea, where cotidal lines (labelled by time) meet (after Whewell 1836).

Young's cotidal lines capture the complexity of the patterns of the tides across the world. Their forms are determined by interference and diffraction of the tide wave by the boundaries of land masses. In modern terminology, Young's construction of cotidal lines amounts to regarding the tide height $h(\boldsymbol{r}, t)$ at any place $\boldsymbol{r} = (x, y)$ and time t as the real part of a complex wave, as follows. Let

$$h(\boldsymbol{r}, t) = \mathrm{Re}[\psi(\boldsymbol{r}) \exp(-\mathrm{i}\omega t)], \tag{3.1}$$

in which the complex function $\psi(\boldsymbol{r})$ has modulus ρ and phase χ, that is

$$\psi(\boldsymbol{r}) = \rho(\boldsymbol{r}) \exp(\mathrm{i}\chi(\boldsymbol{r})). \tag{3.2}$$

Then the cotidal lines for time t are given by

$$\chi(\boldsymbol{r}) = \omega t (\mathrm{mod}\, 2\pi). \tag{3.3}$$

It might seem perverse to think of the height of the tide, which is an essentially real wave, in terms of a complex function, but it is not perverse, for two reasons. First, because the contours of phase $\chi(\boldsymbol{r})$ give the pattern of cotidal lines; second, because the form (3.1) emphasizes that although the tide is a stationary wave, in the sense that the pattern of cotidal lines is fixed on the Earth's surface, it is not a standing wave, because the cotidal lines move: their labels change with time. In a standing wave (for example, the vibration of a membrane) the function ψ would be real, and the $\boldsymbol{r}$ and t dependences of the wave would separate. The complex position dependence in this description of the tides has the same origin as complex wave

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functions in quantum mechanics, namely the breaking of time-reversal symmetry: relative to the Earth, the Moon and Sun rotate in one direction (east to west), not the other.

Young did not possess the data necessary to draw the cotidal lines whose importance he identified. Thirty years later, Whewell (1833, 1836) made the first attempts, and immediately discovered a phenomenon that is now of central importance in wave physics: patterns of wavefronts can (and usually do) possess singularities. These are singularities of phase, from which wavefronts radiate like spokes of a wheel and where the modulus ρ vanishes. In the tides, Whewell called these phase singularities 'amphidromic points'; they are places where the tide is high at all times, that is, places of no tide. Whewell identified several amphidromic points in the North Sea; figure 4 shows one of them. The acceptance of Whewell's discovery was hindered by Airy's denial of the possibility of points of phase singularity (Airy 1845; Cartwright 1999), apparently based on a mathematical misunderstanding, but amphidromic points are now routinely observed (rather than being theoretically predicted) in all the world's oceans (Defant 1961).

Nowadays, phase singularities are being extensively studied experimentally and theoretically in optics (Soskin 1998; Soskin & Vasnetsov 2001; Vasnetsov & Staliunas 1999). They are places of perfect blackness, formed by complete destructive interference, and constituting the mathematically stable generalization of the dark interference fringe. In three dimensions, phase singularities are lines that can be curved (Nye & Berry 1974), and knotted and linked (Berry & Dennis 2001a, b). In quantum waves, phase singularities have been identified in scattering (Hirschfelder *et al*. 1974a, b; Hirschfelder & Tang 1976a, b), in the Aharonov–Bohm effect (Berry *et al*. 1980), and as vortices in superfluids and quantized flux lines in superconductors.

4. Edge diffraction

In attempting to understand Grimaldi's observation (Born & Wolf 1959; Kipnis 1991) that light hitting a sharp edge is deviated into the geometrical shadow, Newton (1730), interpreting the phenomenon in terms of rays, conjectured that an edge might deflect rays passing close to it, by exerting a force. To account for Grimaldi's observation of what we now know to be diffraction fringes, Newton further conjectured: 'Are not the rays of Light, in passing by the edges and sides of Bodies, bent several times backwards and forwards, with a motion like that of an Eel? And do not the three Fringes of Colour'd Light above-mentioned arise from three such bendings?' (There is nothing special about the number three; Grimaldi's experiments were with white light, so the higher-order fringes were blurred by decoherence (see § 5).)

Young, who (perhaps out of reverence for Newton) often presented his own ideas as natural developments of Newton's (Young 1802, 1804; Kipnis 1991), modified Newton's picture by regarding an edge as a secondary source of waves ('light... inflected... from... outlines of the object'), which then interfere with the primary incident and geometrically reflected waves to produce the patterns observed by Grimaldi. Surprisingly, both Newton and Young are correct, though it has taken three centuries to appreciate this completely.

Our present understanding rests on the exact solution by Sommerfeld (1896) of Maxwell's equations for electromagnetic waves incident on a half-plane. For appro-

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priate scalar components of the vector electric field, Sommerfeld's wave is

$$\psi(\boldsymbol{r}) = \exp(\tfrac{1}{4}\mathrm{i}\pi)[\exp(\mathrm{i}x)F(u_-(\boldsymbol{r})) + \alpha\exp(-\mathrm{i}x)F(u_+(\boldsymbol{r}))]$$
$$(\alpha = 1)\ \text{(Neumann)}, \quad (\alpha = -1)\ \text{(Dirichlet)}, \quad (\alpha = 0)\ \text{(black)}, \quad (4.1)$$

where distances are measured in units of $\lambda/2\pi$, α describes the physical nature of the screen, F denotes the Fresnel integral,

$$F(u) = \int_u^\infty \mathrm{d}t\ \exp(\mathrm{i}\pi t^2), \quad (4.2)$$

and the arguments $u_\pm(\boldsymbol{r})$ are given in polar coordinates r, ϕ by

$$u_-(\boldsymbol{r}) = -\sqrt{\frac{2r}{\pi}}\sin\tfrac{1}{2}\phi, \qquad u_+(\boldsymbol{r}) = \sqrt{\frac{2r}{\pi}}\cos\tfrac{1}{2}\phi. \quad (4.3)$$

Here the wave is incident from $x = -\infty$, and the screen extends from $(0, 0)$ to $(0, -\infty)$, with its two sides at $\phi = -\pi/2$ and $\phi = 3\pi/2$.

One way to interpret Sommerfeld's solution in terms of edge waves is based on writing (4.2) in the form

$$F(u) = \exp(\tfrac{1}{4}\mathrm{i}\pi)\Theta(-u) + \mathrm{sgn}(u)\int_{|u|}^\infty \mathrm{d}t\ \exp(\mathrm{i}\pi t^2). \quad (4.4)$$

The first term comes from the stationary point at $t = 0$, and represents the contribution from the geometrical-optics rays, discontinuous across the shadow boundary $u = 0$. The second term is associated with the endpoint $t = |u|$, and represents the edge waves; far from the shadow boundary, this contribution can be expressed as an asymptotic series in powers of $1/|u|$. More generally, diffraction from screens of arbitrary shape can be expressed as a sum of geometric and edge-wave contributions (Born & Wolf 1959), vindicating Young's insight.

A very accurate asymptotic theory of edge waves has been given by Lewis & Boersma (1969). Edge waves and the associated rays have been incorporated into a more general asymptotic (short-wave) theory of diffraction, also including waves scattered from corners and creeping along smooth surfaces (Keller 1962).

To vindicate Newton's insight, it is necessary to depict Sommerfeld's wave solution in a way that most closely corresponds to the rays of geometrical optics. This was done by Braunbek & Laukien (1952), who plotted the streamlines corresponding to ψ, namely, the current (energy flow) vector field,

$$\boldsymbol{j}(\boldsymbol{r}) = \mathrm{Im}\,\psi^*\nabla\psi = \rho^2\nabla\chi. \quad (4.5)$$

Streamlines are the normals to Young's wavefronts (that is, not the wavefronts of geometrical optics). The undulations of Newton's eel are clearly visible (figure 5*a*), justifying his interpretation of Grimaldi's observation. The associated intensity fringes are shown in figure 5*b*.

Newton was also correct in postulating a force to cause the undulations. This force was discovered by Madelung (1926), in the context of the 'hydrodynamic' formulation of quantum mechanics, though the ideas apply throughout wave physics. If wave equations are written in terms of ρ and $\boldsymbol{j}$ rather than ψ, the streamlines are influenced non-locally by a 'quantum potential' in addition to the Newtonian force. (Recently,

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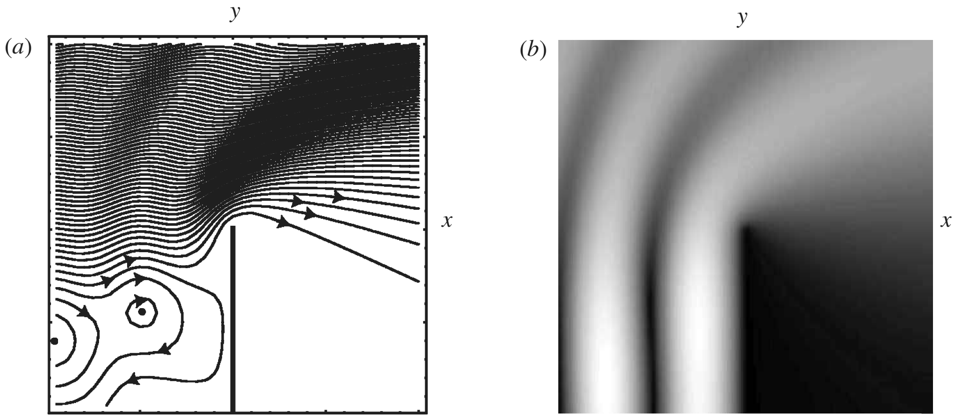

Figure 5. (*a*) Newton's eel: lines of current $\boldsymbol{j}$ (equation (4.5)) for Sommerfeld's edge-diffracted wave (4.1)–(4.3), for Dirichlet boundary conditions; the incident wave comes from the left. (*b*) Intensity $|\psi|^2$ corresponding to (*a*), calculated from equation (4.1), showing interference fringes; white corresponds to high intensity, black to zero intensity. Each image is two wavelengths square.

Madelung's formalism has become popular under the name 'Bohmian mechanics' (Holland 1993).)

Newton seems not to have anticipated what is clear from figure 5*a*, that the forces could be strong enough to cause the streamlines to close into loops, forming vortices in the current. Only the first two vortices are visible in figure 5*a*; there are infinitely many of them, caused by interference between the incident, reflected, and edge-diffracted waves. The vortices are alternative representations of the phase singularities discussed previously, and indeed their manifestations in light are often referred to as optical vortices.

There have been many applications of edge waves. An unexpected recent one is to give a new representation of the Aharonov–Bohm wavefunction in quantum mechanics (Aharonov & Bohm 1959). The single line of magnetic flux can be generated as the curl of a vector potential corresponding to a phase-changing half-plane, and the 'Cheshire cat' diffraction from this 'edge without a screen' reproduces the original Aharonov–Bohm wavefunction to high accuracy (Berry & Shelankov 1999). Another unexpected application gives a very accurate representation of the irregular (indeed, fractal) modes of unstable laser cavities as a sum over waves diffracted from the edges of the mirrors (Berry *et al.* 2001*b*; Berry 2001*b*).

5. Decoherence

Central to wave physics is the fact that the equation $1 + 1 = 2$ does not apply to intensities. Instead, we have addition of amplitudes, giving, for two waves with phases ϕ and $-\phi$, the intensity addition law

$$|\exp(\mathrm{i}\phi) + \exp(-\mathrm{i}\phi)|^2 = 4\cos^2\phi, \tag{5.1}$$

which can take any value between 0 and 4, depending on ϕ. The aim of Young's careful experiments was to provide convincing evidence that light was a wave phenomenon in this sense. The evidence consisted of interference fringes. It was necessary for Young

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to explain why, if his interpretation in terms of interference was correct, fringes are not evident in every optical observation.

The problem was an urgent one, since in many of Young's experiments the light sources were candle flames. Alternatively stated, Young had to address what we nowadays call the problem of coherence, or, in a phrase attributed to Rayleigh, why two candles are twice as bright as one. Replying to a critic, Young wrote:

> The reviewer has cursorily observed that... every surface opposed to the light of two candles would appear to be covered with fringes of colours. Let us suppose this assertion true—what will be the consequence? In all common cases the fringes will demonstrably be *invisible*; since, if we calculate the length and breadth of each fringe, we shall find that a hundred such fringes would not cover the point of a needle.

This is true, but misses the point that a more powerful reason for the unobservability of the fringes is their rapid motion, arising from their origin in independently vibrating atoms.

Nevertheless, there is evidence (Kipnis 1991) that

> Young possessed a considerable understanding of the concept of coherence, and... applied it correctly. Unfortunately, Young did not pass his knowledge of coherence on to his readers, since his presentation of this concept was neither lucid nor complete.

It was Fresnel who stated the conditions for interference clearly, and began the mathematization of the idea, later greatly extended by Rayleigh.

At the heart of the explanation of the loss of coherence—decoherence, as it is now called—is the observation that random and rapidly changing disturbances to the phases $\pm\phi$ in (5.1), from any of a variety of causes, will reduce the contrast of the fringes. The assumption of a Gaussian distribution of the disturbances (justified by the central-limit theorem in the common situation where the disturbances are the sum of many independent contributions), causing a phase uncertainty Δ, leads to the fundamental decoherence equation:

$$\begin{aligned}\langle 4\cos^2\phi\rangle_\Delta &= \frac{1}{\Delta\sqrt{\pi}}\int_{-\infty}^{\infty}\mathrm{d}\alpha\, 4\cos^2(\phi-\alpha)\exp\left(-\frac{\alpha^2}{2\Delta^2}\right)\\ &= 2[1+\cos(2\phi)\exp(-2\Delta^2)] \to 2 \quad \text{as } \Delta\to\infty.\end{aligned} \tag{5.2}$$

Decoherence is very effective; if $\Delta = \pi/2$, the fringe contrast is reduced by a factor $\exp(-2\Delta^2) = 0.007$, and if $\Delta = \pi$ the factor is 2.7×10^{-9}. An implication of (5.2) is that $1+1=2$ can be viewed as a geometrical-optics limit, induced by decoherence.

Interference fringes formed with white light are blurred by decoherence, for the obvious reason that the maxima for different spectral components occur at different places, so the fringes overlap. This can have unexpected consequences. For example, in the diffraction pattern associated with a cusped caustic (Pearcey 1946) the fringes parallel and perpendicular to the symmetry axis scale according to different powers of the wavelength (Berry & Upstill 1980), so the blurring effect of decoherence is much stronger in one direction than the other. In white-light cusp diffraction this causes a striking effect (Berry & Klein 1996*a*): the pattern is striated with long thin

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lines that are not present in any of the patterns corresponding to the individual spectral components.

In recent years, decoherence has moved to centre stage in physics, with the realization (Omnes 1997; Zurek & Paz 1994, 1995; Zurek 1998) that it is crucial to understanding the emergence of the classical world from the quantum world. The decoherence equation (5.2) might seem too humble to merit such a grandiose claim. Its non-triviality emerges as the classical limit is approached. When Planck's constant, $\hbar$, is small in comparison to classical quantities with the dimensions of action, interference fringes are very delicate (just as in the geometrical-optics limit of small wavelength), and become pathologically sensitive to decoherence.

One application is to the emergence of chaos. As originally conceived (Schuster 1988), chaos is a classical phenomenon (sensitive dependence on initial conditions). Its quantum counterpart is problematic, because in the bound systems where chaos might be expected the quantum spectrum governing the evolution of any observable (position, for example) is discrete. This is the quantum suppression of classical chaos. It is true that the time required for quantum mechanics to exert its influence increases as $\hbar$ decreases (as a consequence of the correspondence principle), but this occurs too slowly to explain the existence of classical chaos. The true reason why chaos emerges is that no system can be truly isolated (if it were, we could never know about it), and tiny uncontrolled influences from the environment destroy the delicate conspiracy of phases on which quantum mechanics depends (Berry 2001*a*). In other words, the quantum suppression of classical chaos is itself suppressed by decoherence.

An extreme example illustrating the effectiveness of decoherence is the chaotic tumbling of Saturn's satellite Hyperion under the joint influence of Saturn and its large moon Titan. Regarding Hyperion as a quantum rotator with about 10^{60} quanta of angular momentum $\hbar$, R. Fox (unpublished work) estimated the time it would take for quantum effects to suppress the chaos, if Hyperion were isolated. The answer is about 40 years. This surprisingly short time (a consequence of exponential stretching of classical phase space) bears no relation to reality, because Hyperion is not isolated, and a rough estimate (Berry 2001*a*) of the decoherence associated with the uncontrolled 'patter of photons' arriving from the Sun, which we use to observe Hyperion, leads to classicalization in a time of order 10^{-50} s. This time is far shorter than the time for chaos to emerge (about 100 days in the case of Hyperion), consistent with other arguments (Braun *et al.* 2001) demonstrating that decoherence is a universal route facilitating classical behaviour in large systems, irrespective of whether they are chaotic.

6. Coherence

Systems protected against decoherence—for example, by employing monochromatic collimated light, for which $\Delta \ll 1$ in (5.2)—can exhibit interference. Here are some extreme examples.

The first concerns diffraction gratings, or striated surfaces as Young called them. Although there is evidence (Kipnis 1991) that Young derived the elementary formula for the angles at which diffracted light emerges, the beginnings of the theory in its modern form, incorporating interference of waves from infinitely many sources, were given by Fraunhofer several decades later. These studies, appropriate to most of the practical applications of gratings (e.g. in spectroscopy), concern the far field. Only

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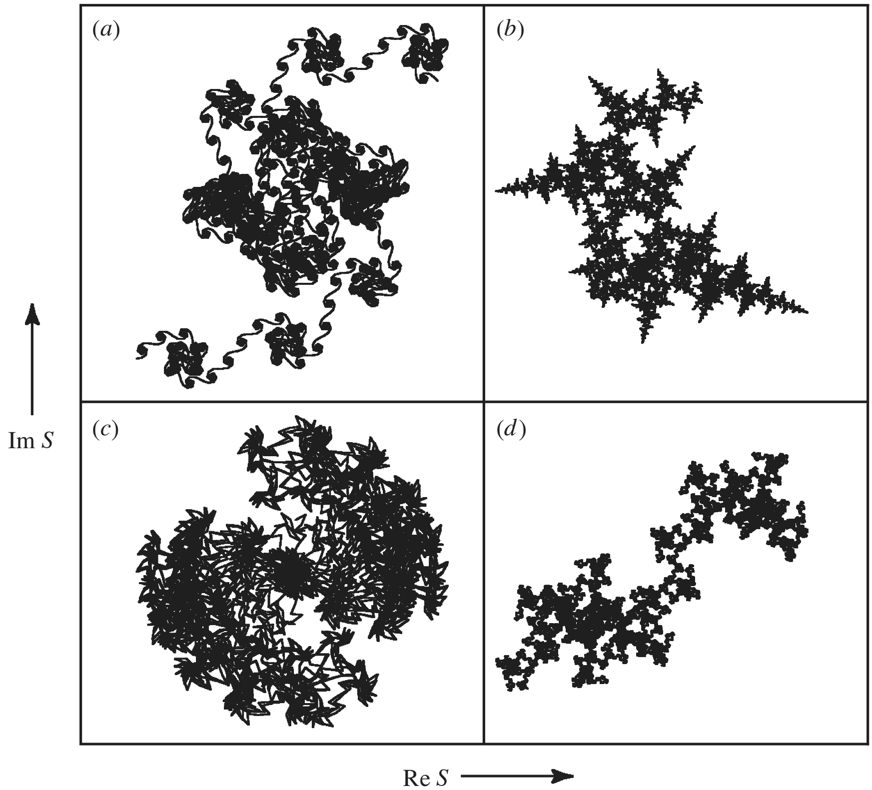

Figure 6. Extreme coherence: curlicues generated in the complex S-plane by $N = 10\,000$ terms of the incomplete irrational Gauss sums (6.1), representing the superposition of waves diffracted from N slits, for the indicated values of τ. (a) $\tau = \sqrt{101} - 10$; (b) $\tau = (\sqrt{5} - 1)/2$; (c) $\tau = 1/\pi$; (d) $\tau = 2^{-1/3}$.

recently have we begun to understand the much greater richness of structure in the near field.

In the simplest case, consider light with wavelength λ incident on a grating constructed from N narrow slits with spacing a. At a point P at a distance z from the grating, the amplitude of the light depends (in the paraxial approximation), on the sum

$$S(N, \tau) = \sum_{n=1}^{N} \exp(\mathrm{i}\pi\tau n^2), \tag{6.1}$$

where $\tau = a^2/(\lambda z)$. The sum of unit vectors is a superposition, embodying the coherent interference of waves from all the slits. As N increases for fixed τ, S forms intricate patterns of curlicues in the plane $\mathrm{Re}\, S$, $\mathrm{Im}\, S$ (according to the Oxford English Dictionary, a curlicue is 'a fantastic curl or twist'). As figure 6 illustrates, the patterns sensitively reflect the arithmetic nature of τ, as described for example by the continued fraction of τ. A comprehensive renormalization scheme (Berry & Goldberg 1988) relates the patterns for different τ and on different scales.

More easily observed are the diffraction patterns from a Ronchi grating (equal opaque and transparent bars), as a function of position rather than N. In the parax-

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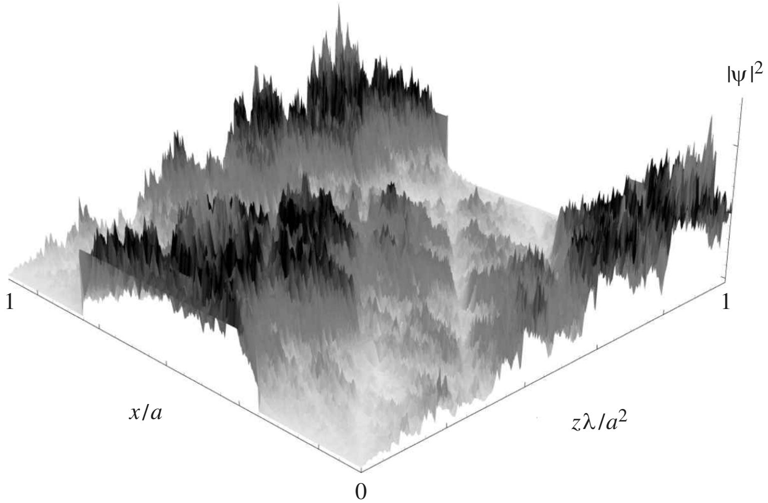

Figure 7. Intensity $|\psi|^2$ beyond a Ronchi grating with slit spacing a, coherently illuminated with monochromatic light of wavelength λ. The pattern is fractal both transversely (x) and longitudinally (z) (Berry & Klein 1996*b*; Berry *et al*. 2001*a*).

ial approximation, these patterns are periodic perpendicular to the grating as well as across it, with longitudinal period a^2/λ; this effect was discovered by Talbot (1836), an early disciple of Young. Within the (greatly elongated) unit cell, graphs of intensity are fractal curves (Berry & Klein 1996*b*), with different fractal dimensions in different directions (figure 7). The same mathematics governs the evolution, according to the Schrödinger equation, of waves describing the quantum state of a particle in a box, if the initial state possesses a discontinuity (Berry 1996), and the periodicity in time gives rise to quantum revivals (Averbukh & Perelman 1989; Yeazell & Stroud 1991; Nowak *et al*. 1997). In two dimensions, the Talbot and quantum patterns generate rich fractal 'carpets' (Berry *et al*. 2001*a*). These intricate wave structures, reflecting extreme coherence, are the physical embodiments of the Gauss sums of number theory (cf. (6.1)) (Apostol 1976).

Young generalized the concept of interference from mechanical waves (sound, tides) to light. Now interference has been observed not only for photons but for massive elementary particles (photons, electrons, neutrons, protons, etc.), and also for composite objects such as atoms and molecules. The most complicated structures for which interference has been created, in recent virtuoso experiments by Nairz *et al*. (2000), are fullerene molecules.

A natural next step would be the demonstration of interference in a beam of living creatures, or almost-alive entities such as viruses. This would be of interest from a fundamental standpoint, because coherence involves isolation from the environment, whereas life essentially involves the exchange of energy and information with an environment, and hence decoherence, provided the interaction could enable the interfering paths to be distinguished. One is tempted to envisage a complementarity between being alive and displaying quantum interference, in which a creature could suspend the interaction it needs to live (hold its breath, as it were) long enough for interference fringes to form with detectable contrast: partly alive, and partly quantum. However, Professor Anton Zeilinger has pointed out that the living creatures

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(e.g. bacteria), together with a supply of nutrients, etc., could be enclosed in sealed containers, and there seems no fundamental obstacle to achieving interference with a beam of such 'test tubes' of arbitrary size.

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