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THE SYMMETRICAL TRIAXIAL SEISMOMETER—ITS DESIGN FOR APPLICATION TO LONG-PERIOD SEISMOMETRY

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ABSTRACT

The symmetrical triaxial seismometer is one in which the orthogonal directions of response form equal angles of about 55 degrees with the vertical. The symmetrical design facilitates matching the response of the three elements, and in the long-period configuration minimizes some problems encountered from variations in the natural period caused by local tilting of the earth. A compact design suited to installation in a cased hole is feasible, which in turn permits more widespread application and allows reduction of localized, non-propagating surface noise and tilts.

Introduction

From the beginnings of seismometry, all seismograph designs have had their inception either in earlier successful instruments or in altered requirements of the art. The symmetrical triaxial design patented by Melton (1965) is no exception, and the instrument to be described here appears to find its greatest usefulness as a tool for observation of weak seismic waves of periods from 10 to 100 seconds, although it is not limited to this range. For the detection of seismic waves from great distances, many seismologists have resorted to seismograph installations in tunnels or mines to avoid man-made or other disturbances associated with the Earth's surface, so it has become logical to design an instrument which can be operated in a drilled hole of moderate diameter and depth.

As will appear in subsequent discussions, logical seismograph design is based on the requirement for observation of seismic signals from distant discrete events, such as earthquakes, and these signals are separable to some degree by filtering from a continuous ambient background of noise. The electromagnetic seismometer connected to the d'Arsonval galvanometer, as first described by Prince Boris Borisovich Golitsyn (Galitzin) (1903) is a logical choice for a filter network in which the response can be controlled by selection of the mechanical elements. Applying this line of reasoning to the problem of sensing waves of periods greater than 10 seconds, we conclude that our seismometer, an inertial device in this case, should have a natural period of 10 seconds or more; and the galvanometer (or any substitute) should also have a response at the longer periods. Actually, a long-period galvanometer acts somewhat like a fluxmeter, integrating the higher order of electrical response corresponding to acceleration of the seismometer mass. Wenner (1929) has offered a very complete analysis of seismometer-galvanometer systems. With the advent of sophisticated electronic systems, the use of a variable capacitance transducer has merited consideration; but on a long-period inertial seismometer, the tidal effects may be devastating. Tidal acceleration has a 2-hour component which varies between 1×10^{-7}g for the neap tide to 3×10^{-7}g for the spring tide, peak-to-peak. If we let Δx be the distance

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the seismometer mass M moves, relative to the frame, with a change of force ΔF, we have

$$\Delta F = M\Delta g = 4\pi^2 M\Delta x/T^2$$

Where T is the seismometer natural period,

so that
$$\Delta x = \frac{T^2}{4\pi^2}\Delta g$$

For a seismometer with 20-second period at neap tide,

$$\Delta x = \frac{20^2}{4\pi^2} \times 10^{-7} \times 9.8 \cong 1.0 \times 10^{-5} \text{ meters.}$$

At 1000 times magnification, the seismograph recording would then display a departure from zero of 5 to 15 millimeters every 6 hours, and much greater magnification would be impractical unless the tidal accelerations were either filtered out or neutralized in some manner. These same arguments apply to any type of displacement-sensing transducer.

Today, the best known suspension system for long-period vertical component seismometers is that of La Coste (1934). As La Coste and Romberg (1942) showed in their patent application, this suspension configuration is not limited to one spring position, or to the conventional form of the long-period vertical component seismometer. La Coste suspensions are used in the long-period symmetrical triaxial seismometer somewhat unconventionally, in that they are leveled and adjusted to an extremely long period. The actual period control is delegated to a specialized hinge.

The Melton concept of an "Angular Composite Seismometer," first discussed with associates on 7 and 8 March 1960, was supported by Advanced Research Project Agency (ARPA) funds in April, 1960. After early experiments showed promise (Hamilton and Stephens—1961), a patent application was filed. The patent was granted in December, 1965.

Choices of the Symmetrical Triaxial Configuration

Space does not permit discussion and illustration of the several preliminary designs and of the efforts of various individuals to discover, through trial and error, the most practical design for manufacture. It must suffice to describe only the two arrangements which were built after the decision had been reached that the instrument must fit within a 10-inch (25.4 cm) circle, so that it could be lowered into a reasonable-sized cased hole. The first arrangement chosen is shown in Figure 1, and has the advantage of not requiring special provisions to accommodate a very tall instrument. The designer was told to use 20-kilogram masses. McMillan (1964) describes the design and testing of this instrument, which employed remote-controlled "trimmer" masses to adjust period and centering, as shown in Figure 2. Two conclusions resulted from this design. The first was that the mass-pivot relation of each element was such that only about half of the 20-kilogram mass could be made effective in generating energy proportional to displacement with reference to the frame. The second conclusion was that remote control and adjustment of such an instrument was not only practical, but actually made it easier to attain proper adjustment.

The alternate configuration is that wherein the three individual seismometer elements are positioned one above the other, but arranged so that the projections of their sensitive axes onto the horizontal plane make azimuthal angles of 120 degrees. It results in a very tall instrument, as shown in Figure 12. However, this was the configuration finally chosen for production. The arguments for this symmetrical triaxial arrangement and for the choice of a down-hole design are now developed in more detail.

Advantages of the Symmetrical Triaxial Arrangement

If a seismometer is thought of as a device for sensing the motion of a volume within a small region, the motion of this volume, or mass, can be described in terms of six parameters and time. The six parameters are usually divided into three orthogonal for translation and three for rotation, and it is usual in seismometry to ignore those of rotation within the region of the seismometer. Historically and conventionally, the

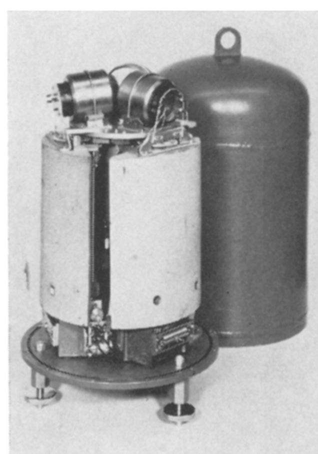

Fig. 1. Compact model of long-period triaxial seismometer, designed with remote controls for period and mass centering.

translational parameters have been north-south, east-west, and vertical, but modern computational procedures, analog or digital, are easily devised to rotate axes as desired, and detailed analysis of earthquakes often makes use of horizontal rotation to separate wave types. Thus today there remains little reason to follow earlier practice if another scheme is advantageous in terms of design and installation.

If we look at the usual design of a long-period horizontal-component seismometer, we see that, if the hinge or hinges do not exert any force to return the mass to its neutral, quiescent position, that position is determined solely by gravity. Further, only the vertical gravitational potential gradient produces this restoring force and determines the period, as it does in any ordinary pendulum. The triaxial design, however, makes use of a spring in a La Coste suspension to counter the gravitational field. If the La Coste suspension is to be *exact*, in the sense that it produces no force restoring the mass to a neutral position—in other words that it allows an infinite period—and if the *only* control of the period is to be through some other element, we must make provision to level the seismometer at all times. Otherwise, the period will change with tilt, as stated by La Coste.

Figure 3 represents the suspension with the mass supported on a boom hinged to a "vertical" mast. In this suspension, a and b, the distances from the main hinge at 0 to each end of the spring, are equal, a condition which simplifies the analysis. We can make a further simplification by assuming that the spring assembly is so designed and adjusted that the tension exerted is exactly proportional to the distance between A and B. This represents the so-called "zero length" spring of the La Coste suspension. Even when the mast is not exactly vertical, this suspension system can be made to balance at some particular angle between the boom and the mast by adjustment of the

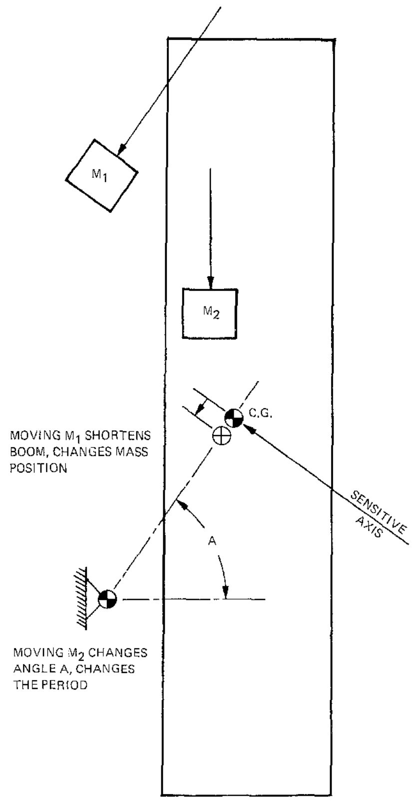

FIG. 2. Inertial mass relationships in each element of long-period triaxial seismometer of figure 1.

spring rate or the weight of the mass. Call this angle α_0, the value of α at balance. The natural period of the suspension under these conditions is determined by the angle of tilt of the mast, δ in figure 3, and any restoring torques produced by the hinges of flexures which hold the assembly together. If we change the tilt angle δ, the period will change and so will α_0, the angle α at balance. If, as a practical matter, we should desire to keep α_0 constant, say at 35.3°, we must adjust either the spring rate or the weight of the mass whenever we change δ. This is also true in the case of a conventional La Coste horizontal boom seismometer.

In this new seismometer, we elect to adjust the spring rate so that the La Coste spring torque and the torque produced by gravity acting on the mass are equal when δ is as close as possible to zero.

The period is controlled through the use of a specialized main hinge at 0. This hinge has been called a "triflexure" hinge for convenience, although two of its flexure ribbons have been split so there is a total of 5 flexures. It will be described later in more detail.

An equation for the natural period of this La Coste suspension can be derived from analysis of the change in torques produced by the La Coste spring, the several hinges and the weight of the suspended mass, as the system is perturbed. If we assume that the system is brought to balance at the desired α_0 for a particular mass M, we can write that the natural period is given by:

$$T = 2\pi r \sqrt{\frac{1}{gd \cos \alpha_0 - gd \cos (\alpha_0 + \delta) + k/M}}$$

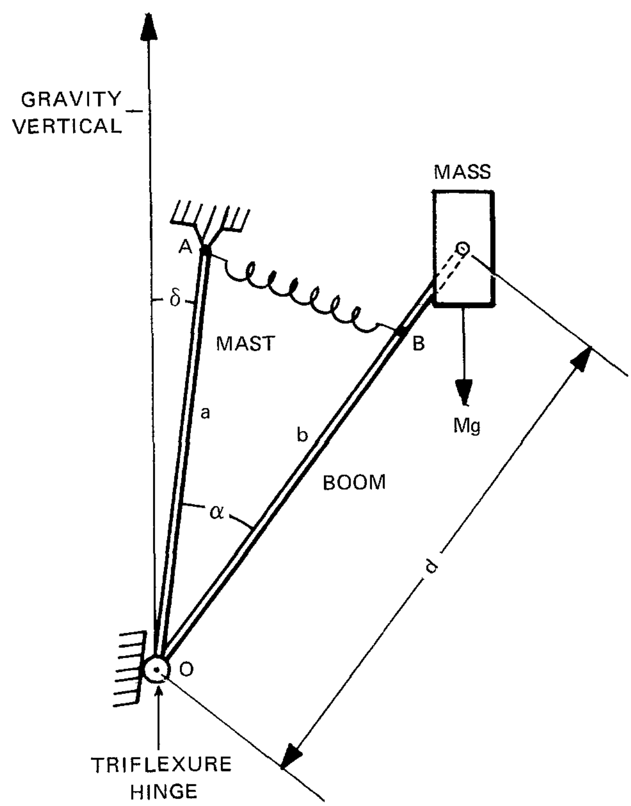

FIG. 3. La Coste suspension as employed in symmetrical triaxial seismometer. δ represents the angle of tilt.

where: r = radius of gyration about the main hinge at 0
g = acceleration of gravity
d = distance from 0 to the center of gravity of mass and boom
k = combined torque constant of all hinges
α_0 = angle between mast and boom at balance
δ = angle of tilt, taken positive clockwise in figure 3
M = total inertial mass

From damping and free period measurements, we know that for this seismometer

$$Mr^2 = 0.634 \text{ kilogram meters}^2$$

The total inertial mass is 10 kilograms, so that $r = 0.252$ meters. $\alpha_0 = 35.3°$. If we assume $k = 0$, we can calculate the period as a function of δ. However, if δ is very small and k is finite and positive, the period will be determined in the main by k. But note that the balance position depends on the local acceleration of gravity. Conversely, once the seismometer has been adjusted for the acceleration of gravity, with δ very small (ideally zero), its level can be determined by sensing the position of the mass relative to its stops. The practical mechanisms will be described later.

It is important to note that any forces applied directly to the inertial mass are the equivalent of, and cannot be distinguished from, accelerations of the seismometer frame as far as the electrical output is concerned. Thus, if a seismometer delivers an

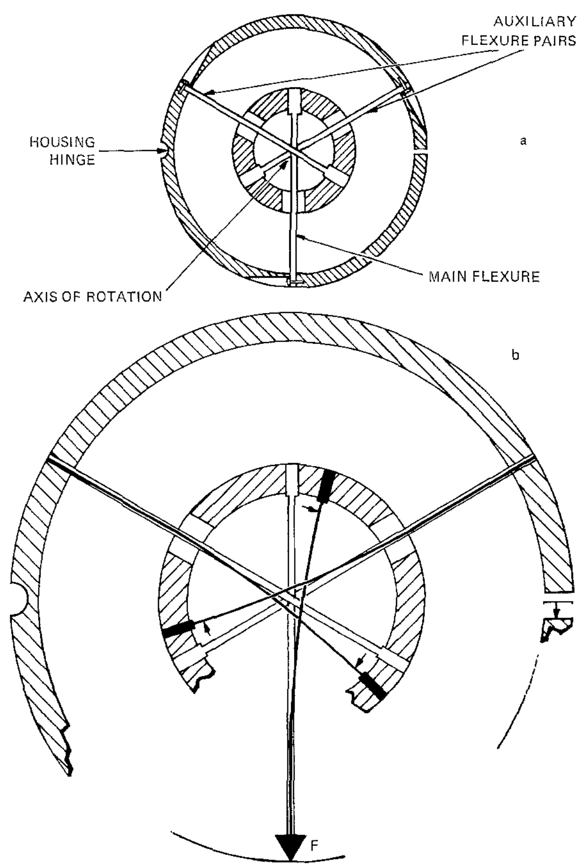

FIG. 4. a. Basic configuration of triflexure hinge. b. Introduction of force couple as inner member is rotated. When the flexures do not cross at the central axis, the force F tends to produce further rotation.

output when its frame is moved horizontally, it must also deliver an output when its frame is rotated about the center of gravity of its inertial mass in the vertical plane passing through its sensitive direction, because such a rotation applies a small vertical component of gravity. The net result is that the output caused by tilt appears the same as one which could be obtained by two successive integrations, with respect to time, of some horizontal displacement of the frame, and therefore, the electrical output decreases at the rate of 12 dB per octave with increase of period of tilt. Thus very long-term tilts will not generate much electrical voltage output from an electromagnetic transducer. In general, the only deleterious effect of tilt occurs when it is sufficient to move the mass to a position where the transducer is outside of its linear range.

However, as the shorter period tilts give ac outputs differing only in phase from short-period translations, there is no way of distinguishing which of the effects is sensed by a seismometer having only translational degrees of freedom.

The design of the triflexure hinge is covered in a two-part article by Weinstein (1965). The general principles involved are illustrated in simplified sketches, Figures 4a and 4b. Figure 4a shows the general construction consisting of concentric inner and outer rings, which are connected by one main and four auxiliary flexure ribbons in tension, *clamped* at their ends so they are thin beams in effect. The outer ring is clamped to the seismometer frame; the inner ring carries the boom which bears the inertial mass. The auxiliary flexures are paired and their individual widths are half that of the main flexure so that stresses per unit cross-section are equal in all flexures. Initially, the three flexures are straight, and they cross at the center of rotation. Figure 4b shows the

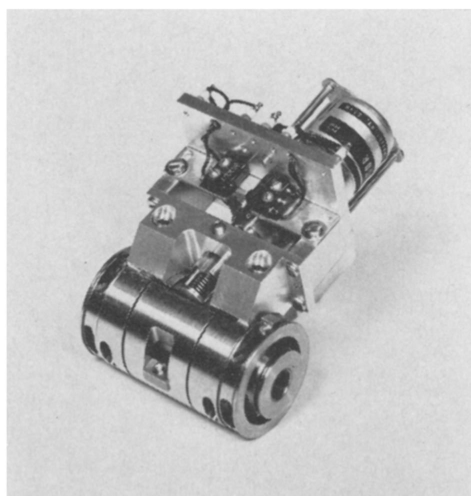

FIG. 5. Tri-flexure hinge with motor drive for tension adjustment.

flexure positions (as heavy lines) when the inner ring rotates within the outer ring. As the inner ring rotates, the mutual crossing points of the flexures separate. A couple results, tending to rotate the inner ring further. This couple is resisted by the stiffness of the flexures as beams, and in practice, the tension force F is adjusted, by a motor, gear, cam, and spring combination, until the concentric ring assembly produces a very small torque toward the original equilibrium position. It is this restoring torque which controls the period of the seismometer, normally to include periods from 10 to 25 seconds with the standard cam and spring. More precisely, the period of the seismometer is controlled by the total restoring torque of all hinges used in its construction; and the ability to produce controlled "centering" torques with the triflexure hinge permits adjustment of the period even if the other hinges in the seismometer produce "decentering" torques. Figure 5 shows the triflexure assembly with its motor drive which changes the tension on the main flexure ribbon to adjust the natural period of the seismometer.

Leveling tables for the "sensitive" and "cross" axes are sketched in Figure 6. The sensitive axis is the one in the plane determined by the boom direction and the vertical; the cross axis is in the plane perpendicular to the sensitive axis plane. The level of the cross axis table is sensed by mercury-contact switches, which close appropriate circuits to stepping motors that re-level the table as necessary. Overshoot does not occur because there is a "dead zone" in which neither switch closes. The seismometer element is insensitive to precise level in this plane.

Level in the sensitive axis plane is much more critical and in practice affects the centering of the mass. When the mass is exactly centered, the triflexure hinge exerts no centering force. A two-stage system has been devised. Switches with very low closure force (10 grams) are positioned at the limits of mass travel. Closure of one of

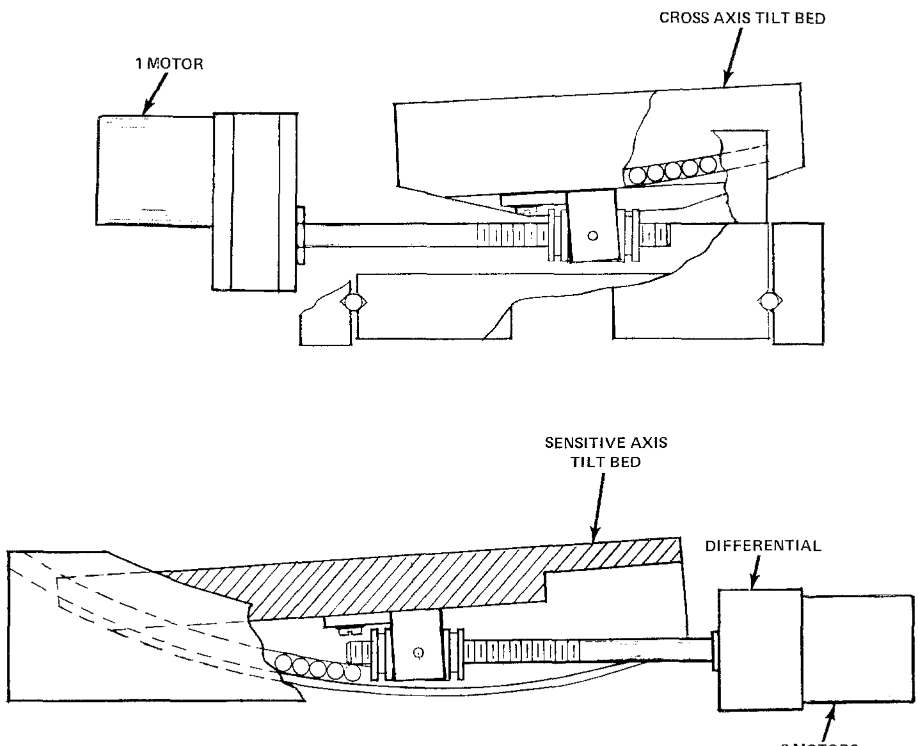

Fig. 6. Leveling mechanisms for one module of triaxial seismometer.

these switches causes a pulse rate of about 60 times a second to operate a stepping motor to drive the leveling table. As the mass exerts less and less force on the switch, the switch finally reacts to push the mass away. Then, as the switch opens, the pulse rate is slowed and comes under control of photocells which sense the centering of the mass. There are two motors for the centering operation, and their shafts are connected through a differential to the leveling table. One photocell determines the rate of pulses to one of the motors; the other photocell, the pulse rate to the other motor. Thus the relative speeds of the motors, determined by relative pulse rates, combine to center the mass, and when the pulse rates are the same, the mass remains centered although the motors may be turning.

Justification of the Down-Hole Design

Although dimensional changes and resulting instabilities of a seismometer can be minimized by careful design, changes in its environment can have even greater effects. In the long-period horizontal-component, tilt of the concrete pier on which it rests may even drive the mass against its stops. For years this effect was suspected; but often the seismometer itself was blamed, because the magnitude of pier tilt was too small for measurement except by the seismometer itself. For example, a tilt of 0.0001 radian or about 2 seconds of arc results in a displacement of 100 microns, an off-scale record at a magnification of 1000 or more. On the other hand, many thousands of man-hours over the years have been expended in the planning and construction of stable environments, usually concrete vaults where caves or mines were not available.

Whalen (1963) has discussed some measurements of the effects of earth loading near a concrete pier, as well as other noise sources such as wind noise, atmospheric pressure changes, and temperature variations. The instruments discussed by Whalen

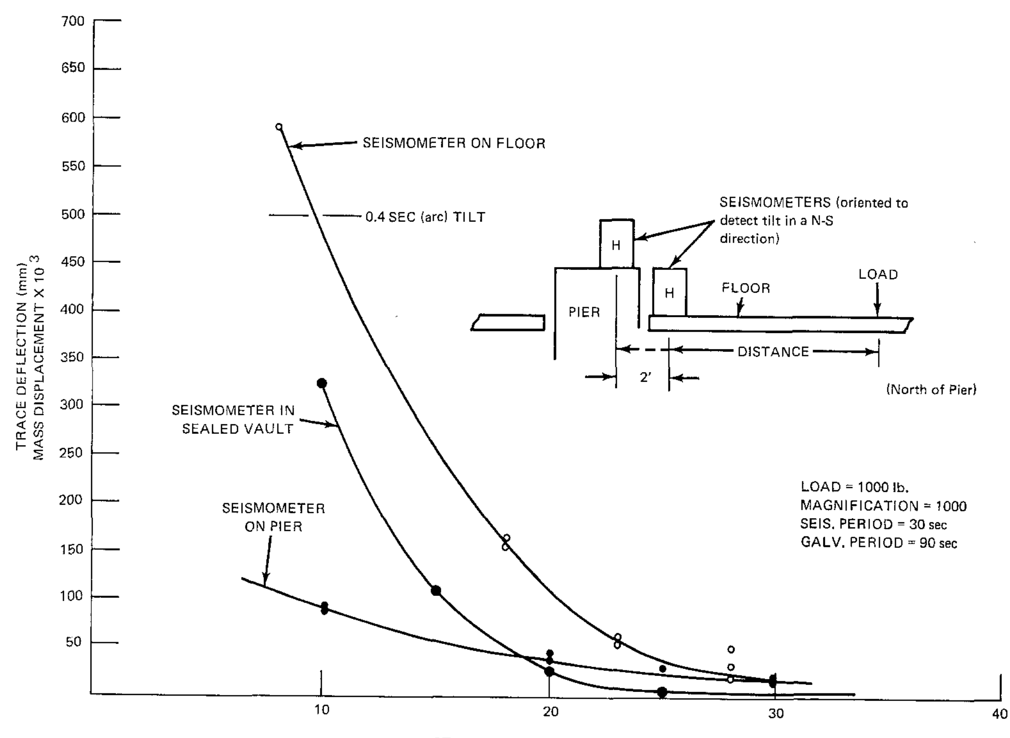

FIG. 7. Tilt induced in conventional long-period horixontal seismometer by load on floor or earth near seismometer pier.

had natural periods on the order of 20 seconds, and Figure 7 reproduces a figure from his report showing how earth loading can affect the horizontal-component instrument. Later, Milam (1965) reported tests in a mine at Las Cruces, New Mexico, one of the quietest seismological sites ever found in this country. Additional evidence of the effect of earth tilt, with seismograph magnifications on the order of 200,000 at a period of 25 seconds, was reported in the course of the "Long Range Seismic Measurements Program" (LRSM) undertaken as a result of the report of the Panel on Seismic Improvement (1959). Details of investigations during this program are covered in Geotech Technical Report 66-82, *Long-Period Seismograph Installation, La Paz, Bolivia,* dated 15 September 1966.

Some feeling for the problems of stabilizing long-period seismographs, and a further justification for placing them well below the surface of the earth can be reasoned by considering the magnitude of gravitational forces which are produced simply by the *proximity* of heavy masses. For instance, a 100 kilogram mass, separated 1 meter horizontally from a horizontal-component seismometer, will attract its suspended mass by the equivalent of 6.8×10^{-10} radians of tilt. With a magnification of 100,000 and a natural period of 20 seconds, the record deflection would be 6.8 mm. For the triaxial element having the same period, the deflection would be this amount multiplied by the sine of 55°, or about 5.5 mm. In this general context, a long-period seismograph is a gravitational-field sensing device, although in practice the effects are not observed because the changes do not occur quickly enough for them to be within the response passband of the electrical amplifying system.

Factors Determining the Magnitude of the Suspended Mass

The limiting sensitivity of inertial-type seismic detectors has been discussed by Wolf (1942), Byrne (1961), and Johnson and Matheson (1962). Curves useful to the designer appear in Geotech Technical Report 65-98 (1965). These curves are based on work by Matheson and Gilbert (1966), which discusses the frequency distribution of Brownian motion in seismographs. However, fixing the magnitude of the suspended mass depends also on how and where the seismograph is to be used, the sensitivity desired in terms of magnification, and most important of all, the magnitude of earth noise and other disturbances at all frequencies within the passband being investigated.

Melton (1966) has discussed the problem in a report written to guide engineering development of long-period seismographs in general. At the time of writing the 1966 report, this author was deeply concerned because some rather simple calculations with reasonable assumptions of earth noise seemed to indicate that a design with only 10 kilograms of suspended mass might result in producing an instrument whose sensitivity would be limited by Brownian motion at the longer periods. On the other hand, use of a mass of much more than 10 kilograms would probably result in a very unwieldy instrument, and there were competent people who felt that the use of 5 kilograms or even less would give a completely adequate device at much less cost. The problem becomes even more complex when practical bandwidths and response curves are considered, when it is noted that there is a dearth of measurements of earth noise at the longer periods, and when it is realized that only the long-period seismograph itself can measure this noise and that there is no practical basis for separating "internal" from "external" noise on a record when only one seismometer is used.

In 1965 a major effort, reported by Trott (1965), was made to find the actual limit of sensitivity of a seismograph experimentally by operating it under conditions such that the inherent instrumental noise of Brownian motion could be separated from that caused by earth movement. The experimental work was performed in the abandoned Bennett Mine, near Las Cruces, New Mexico. In this location, about 200 feet from the surface, it was found that, with a magnification of 130,000 at 25 seconds, the vertical component of earth noise was limited to a 1-millimeter record excursion over the passband of the seismograph, which is shown in Figure 8. At a period of 100 seconds, the magnification would be about 15 per cent of 130,000, slightly less than 20,000, so that a 1-millimeter recorded excursion would represent an earth displacement of about 50 millimicrons. This same investigation showed the earth noise at 100 seconds to be about an order of magnitude greater than at 25 seconds.

In the experiments to separate instrument noise from earth noise, two vertical component seismometers were adjusted to match their characteristics, and two galvanometer-input ("phototube") amplifiers (PTA), with long-period galvanometers (about 110 sec), were similarly matched. Additional gain was provided with other

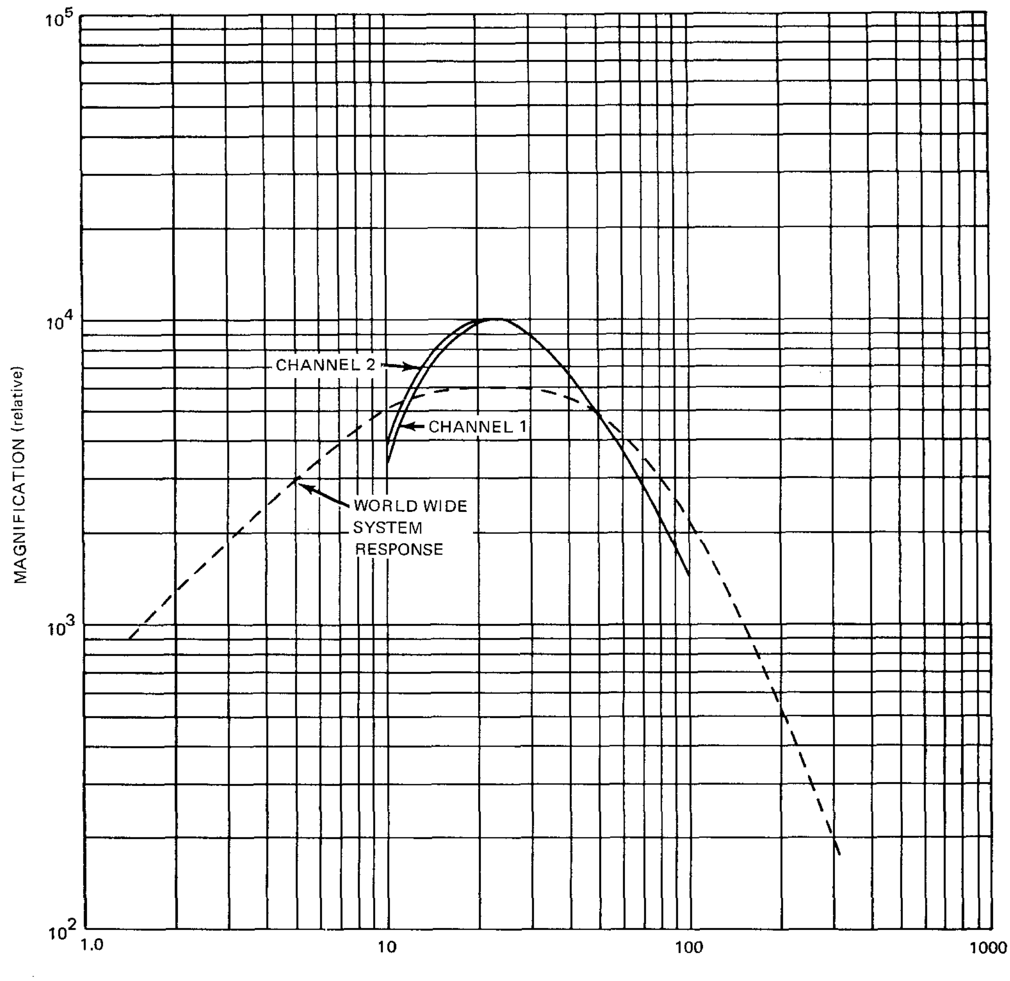

FIG. 8. Relative amplitude versus period responses of the vertical long-period channels used for cancellation tests and seismic noise recording. Dashed curve represents approximately the World-Wide System (USC&GS) response.

amplifiers at high-level signal points where their noise would not affect the experiment. Recordings were made of each amplified signal and of the difference of the two signals, all at appropriate levels. The gains of the individual channels were adjusted to match as closely as possible, the intent being to have the earth noise output signals cancel one another, leaving as a residue the incoherent noise.

The experimental work yielded amplitude spectra of earth noise within this seismograph passband at this specific location. A smoothed approximation of these spectra, modified to remove the influence of a known earthquake, is given in Figure 9. Each point on this curve represents the amplitude of the noise which would be observed through a filter having a passband of one-millihertz. That is, at 100 seconds, the periods viewed lie between 95.2 and 105.3 seconds; at 50 seconds, between 48.8 and 51.3 seconds. The dotted curve on this same figure is derived from one published by

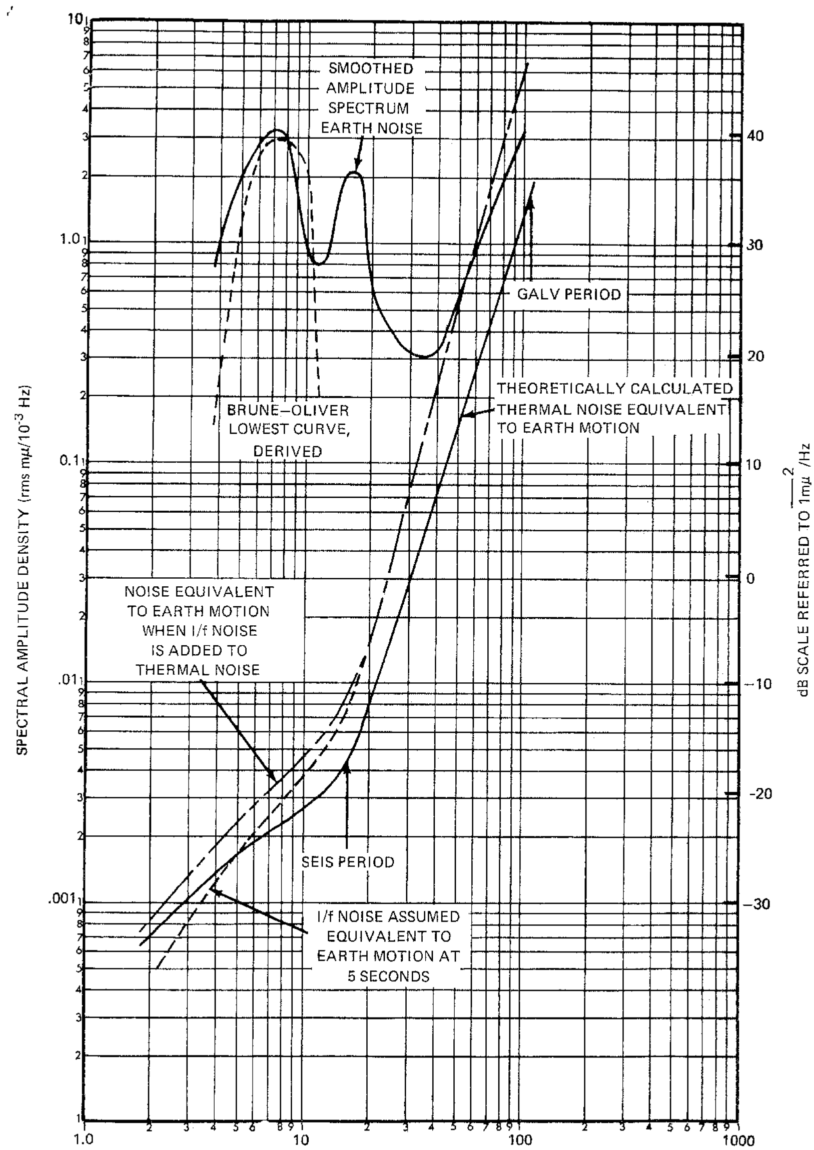

FIG. 9. Amplitude spectrum of earth noise, as found at Las Cruces, New Mexico, compared with Brune-Oliver minimum noise curve, and with curves representing thermal and $1/f$ noise, referenced to earth motion.

Brune and Oliver (1959). To make the necessary conversion, we have used the Brune-Oliver lowest curve, itself an estimated curve faired in by those authors on the basis of other data on noise at higher levels. We have assumed that the visual analysis of frequency data, on which these curves were based, tends to be about that of a filter with one-third octave bandwidth, that peak-to-peak amplitudes were recorded, and that amplitudes had a Gaussian distribution. Peak-to-peak values were divided by 4 to give rms values. To say that the agreement is surprising, we think is still an understatement because the difference in analysis techniques is so great. Yet the agreement is at least logical and gratifying.

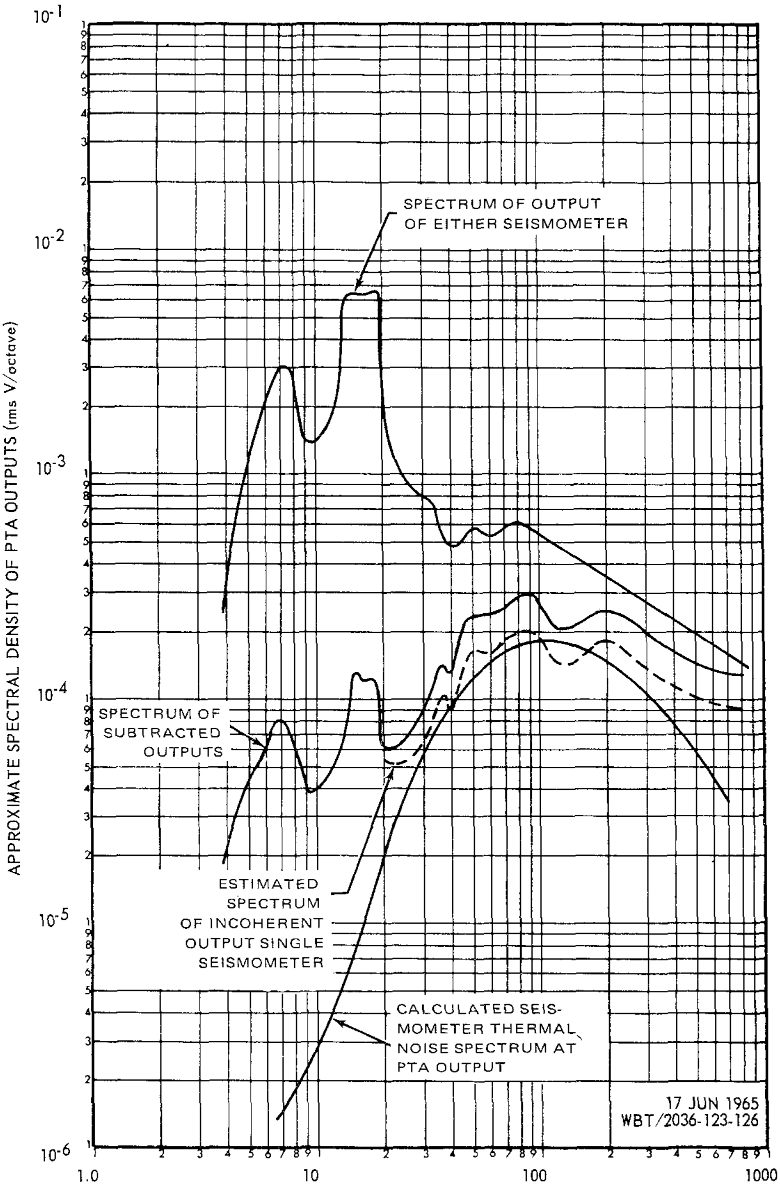

FIG. 10. Comparison of amplitude spectra obtained at Las Cruces from single vertical component seismometer, difference of outputs of two seismometers, estimated incoherent noise and calculated thermal noise of seismometer as observed at PTA output (after Trott, 1966).

On the other hand, the experimental work at Las Cruces yielded an estimate of the distribution of instrument noise at the longer periods. Figure 10 shows three curves from experimental data and one theoretical curve plotted on the basis of rms volts per octave. The top curve represents the smoothed spectrum which includes earth noise as recorded by either seismograph. The next lower curve gives the spectrum of the voltage output difference between the two seismographs, and represents the incoherent noise contributed by both seismographs plus any earth motion not completely cancelled because of mismatch of the two instruments. Trott reports that the precision of amplitude response matching (cancellation) of the two seismographs was good between 20 and 100 seconds, and within about 5 per cent at periods from 10 to 15 seconds.

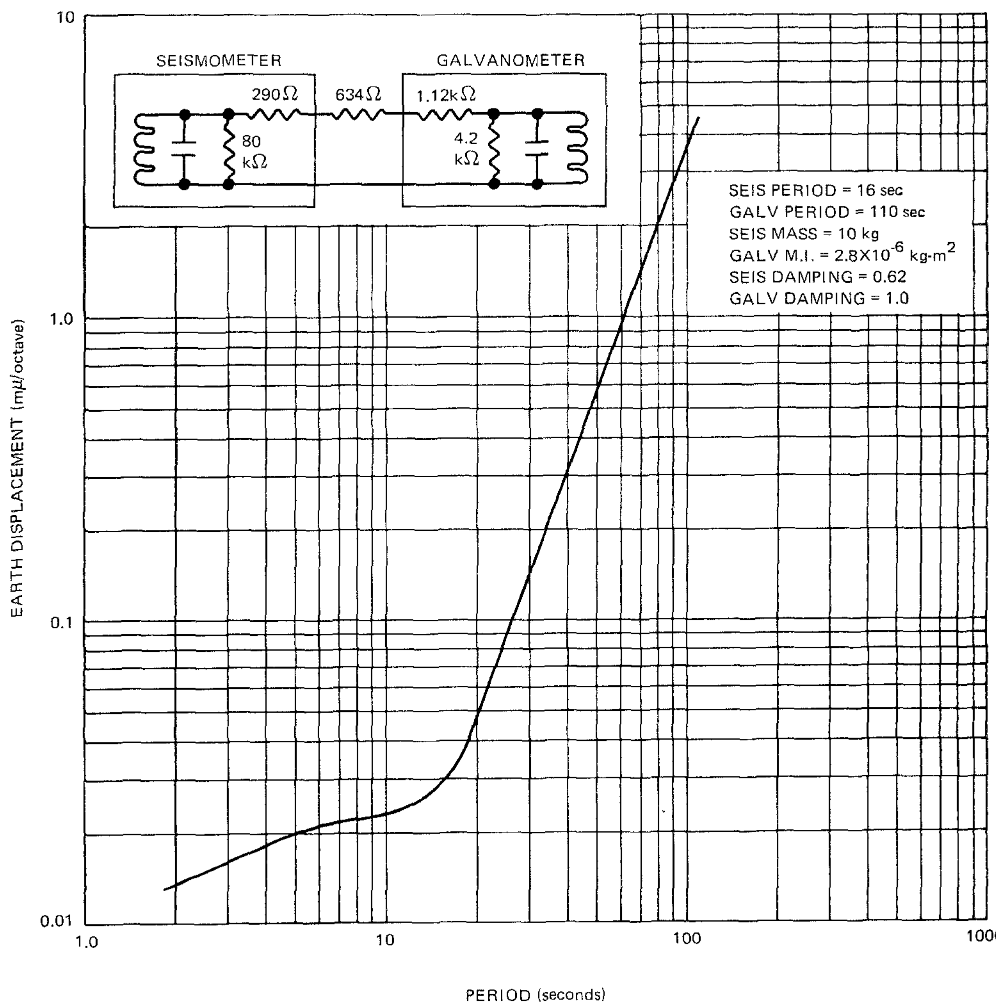

FIG. 11. Earth displacement equivalent to thermal noise in a long-period system with galvanometer registration, plotted on an octave basis.

(See Figure 15.). The dotted curve of Figure 10 is an estimate of the *individual* seismograph noise output, obtained by reducing the ordinates of the difference curve, and dividing them by $\sqrt{n}$, where n is 2 (instruments). The curve labeled "Calculated seismometer thermal noise spectrum at PTA output" was obtained through use of the computer program of Matheson and Gilbert (1966). (This is NBS OMNITAB Program 11-5-64.) Appropriate modification was made for the filter response used. PTA refers to a galvanometer-input amplifier which used phototubes to sense the deflection of a long-period (ca 110 sec) galvanometer. This is a valid curve because a galvanometer does not have "one-over-f" noise, as will be later defined. It does, however, assume that the phototube-amplifying-system noise is lower than the seismometer thermal noise as seen by the galvanometer.

The same general OMNITAB program has also been used to calculate the earth displacement equivalent to the thermal noise of a long-period seismometer-galvanometer system, shown in Figure 11 in terms of millimicrons per octave. A portion of this curve, converted to millimicrons per millihertz, appears on Figure 9 for comparison with earth noise.

Figure 9 thus permits comparison of the limiting sensitivity of one particular long-period seismometer-galvanometer system to expected earth motion at a very quiet site. The example illustrated here is for a 10-kilogram mass seismometer adjusted to a natural period of 16 seconds and direct-coupled to a galvanometer of 110-second natural period. The seismometer was damped to 0.63 times critical, the galvanometer critically damped. These dampings were attained with only a series resistor in the seismometer-galvanometer loop. By reference to Figure 9, we observe that, below 20 seconds, earth noise is many times instrument noise but above 40 seconds the earth noise is only about 3 to 4 times that corresponding to thermal agitation for the system examined. Increasing the period of the seismometer will improve this ratio somewhat. However, the ratio is not great at these longer periods, and it is calculated for a system wherein no noise is added during some electronic amplification process.

In almost any practical seismograph system an attenuator with a minimum insertion loss of 6 dB would be connected between the seismometer and any galvanometer or amplifier. This reduces the amplitude ratio of earth noise to thermal noise by a factor of 2. Noise added by an amplifying system can easily wipe out the remaining advantage in the system at the longer periods. Of course, this reasoning applies only to exceptionally quiet sites, and only for single seismometer detection. Nevertheless, the instrument designer must consider the "worst" case when he lays out the design.

Additional Conditions Imposed when Amplifiers Supplement or Replace the Galvanometer

As suggested, active amplifying systems always add some noise, and consideration must be given to its magnitude relative to thermal agitation. The noise added by active amplifiers is called "flicker noise" and consists of transient disturbances separated in time by quiescent intervals. Its average power is said to vary inversely with frequency, or sometimes as the minus 1.25 exponent of the frequency, depending upon the authority (see *International Dictionary of Physics and Electronics*, also *Encyclopedic Dictionary of Electronics and Nuclear Engineering*). If it varies inversely as the first power of frequency, it is called one-over-f noise, and we use it here in that restricted sense. It follows that a system with $1/f$ noise has constant noise power per octave, or a plot rising 3 dB per octave with increase of period, when plotted in terms of a constant bandwidth (Hertz) scale. On the other hand, "white," unfiltered noise, such as thermally generated noise, measured in terms of constant bandwidth, gives a "flat" line plot versus frequency or period. In general, $1/f$ noise, or flicker noise, is characteristic of "active" devices—devices which introduce energy—such as vacuum tubes and semiconductors. Some granular resistors also show $1/f$ noise. Passive filters, including galvanometers, cannot introduce $1/f$ noise.

When thermal noise is passed through a given bandpass filter, the power level of this noise is reduced by just the losses of the filter and its spectral plot becomes "humped," with the total remaining energy being represented by the area under the spectral plot. When $1/f$ noise is passed through this same filter, its spectral plot is also humped, but the plot rises at an additional 3 dB per octave, with period, on a constant bandwidth scale. If both thermal and $1/f$ noise are passed through the filter, the resulting spectral plot of total noise will be above the thermal noise plot wherever these two classes of noise are added, and equal to the thermal noise plot where the $1/f$ noise is much less than thermal noise. Therefore, to plot the resulting noise for an active amplifying system, we must know the point in frequency or period at which $1/f$ noise equals thermal noise—the intersection of the respective curves.

Unfortunately, to perform a high confidence level statistical analysis of noise in any system, one should have a minimum of 1000 cycles of the lowest frequency of interest. It follows that analysis of noise at 50 seconds involves about 14 hours of recording (and digitizing) by some very stable system whose noise level is known to be lower than the system being analyzed, within the passband of interest. For our purpose, we believe it is more instructive to consider a hypothetical amplifying system having one-over-f noise and with the same passband, when coupled to the seismometer, as the seismometer-galvanometer system, usually referred to as seismometer and PTA.

Accordingly, in Figure 9 we show a dashed curve whose intersection with the thermal noise curve is assumed to be at a period of 5 seconds. Realistically, this represents a very good amplifier design with the best available technology. The long-and-short dashed curve is the sum of the $1/f$ and thermal noise. The care which must be taken in the design of the complete system is emphasized by consideration of this figure.

The Seismometer and its Subassemblies

The assembled triaxial seismometer, uncased but including a hole lock and stabilizer, is shown at the left in Figure 12. Its total weight is about 191 kilograms (420 lbs), slightly less than a short-period Benioff seismometer. Its external diameter is 25.4 cm (10 inches) and its height is about $2\frac{1}{2}$ meters (99 inches). It is designed for operating with an electrical system for controlling its period, leveling, and mass locking and release.

As identified by labels on the photograph, the elements, from top to bottom are:

Cable assembly
Stabilizer
Triaxial module No. 3
Triaxial module No. 2
Triaxial module No. 1
Hole lock.

The hole lock can be replaced with a simple base if the seismometer is to rest on the bottom of the cased hole.

One module of the seismometer is shown at the right in Figure 12, with a labeling of important components, including the leveling and mass-locking motors. This module weighs 48.5 kilograms (107 lbs), including its case, an epoxy-bonded fiberglass tube which with O rings provides a pressure-tight enclosure.

Figure 13 is a drawing of the sensor elements to show relationships of the vertical invar mast, the boom and attached mass, and the iso-elastic spring. Temperature compensation components are also indicated, and the drawing shows that the invar mast is not stressed by dimensional changes of the leveling frame because the lower end of the mast is free to slide. The leveling frame is not shown here, but part of it can be seen in Figure 12, just back of the nearest quartz tube. It moves within the outer framework, which consists of the upper and lower end-plates connected by four rods. The frame permits releveling of the sensor elements, in either sensitive or cross axis planes, from tilts of about $4\frac{1}{2}°$.

The electrical output of each sensor is available from two 500-ohm coils, which can be connected either in series or parallel. Feedback operation can be achieved through one coil if this would be useful. A low resistance "calibration" coil is also included

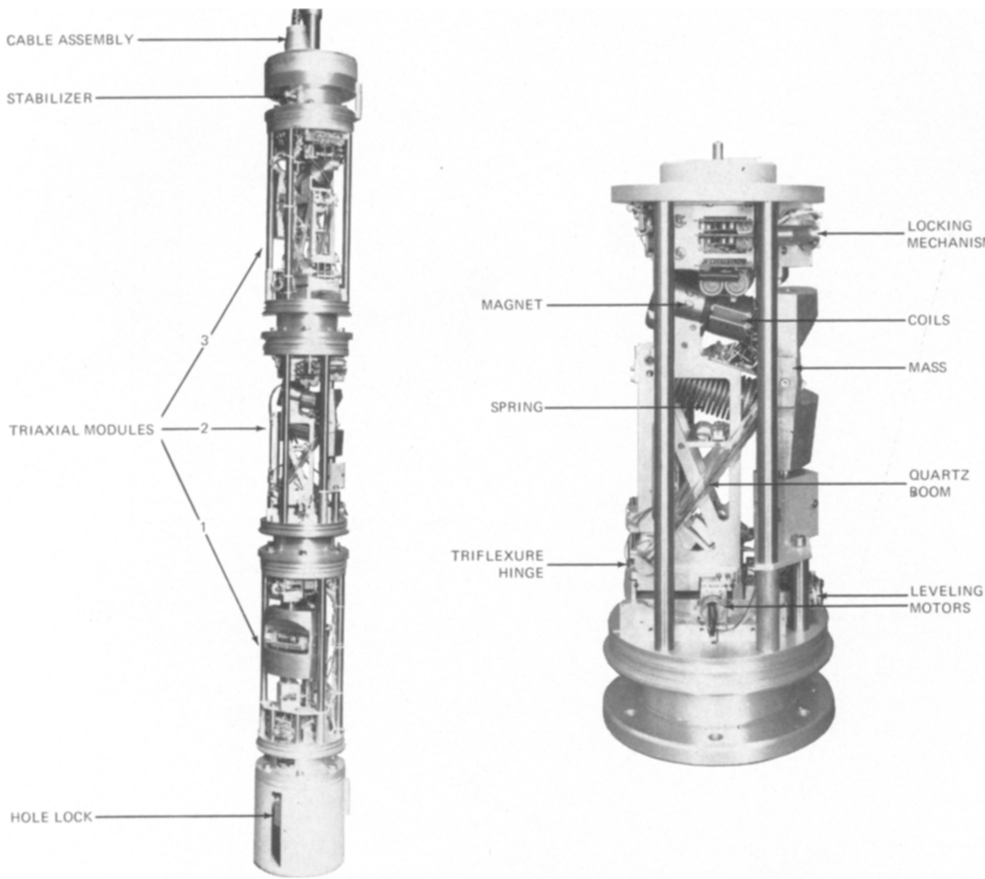

FIG. 12. Uncased Symmetrical Triaxial seismometer (left) and single module (right).

on the same coil form, as experience has shown that mutual coupling with the main coils is negligible at the periods for which this instrument is designed.

Choice of the Passband

Although under some conditions there may be a justification for building seismographs to magnify all earth vibrations equally within a given broad range of frequencies or periods, it is usually found that ambient earth noise tends to mask signals of interest, and that this same earth noise will occupy a large part of the available range of amplitudes on the recorder, or even of the digitizer if one is used. The obvious solution is to design the seismograph with a passband which will magnify the signals more at periods where the earth noise is low, attenuating all input energy at periods where the earth noise is high. Logical pursuit of this principle means that the seismograph passband should match the *inverse* spectrum of the ambient earth noise repre-

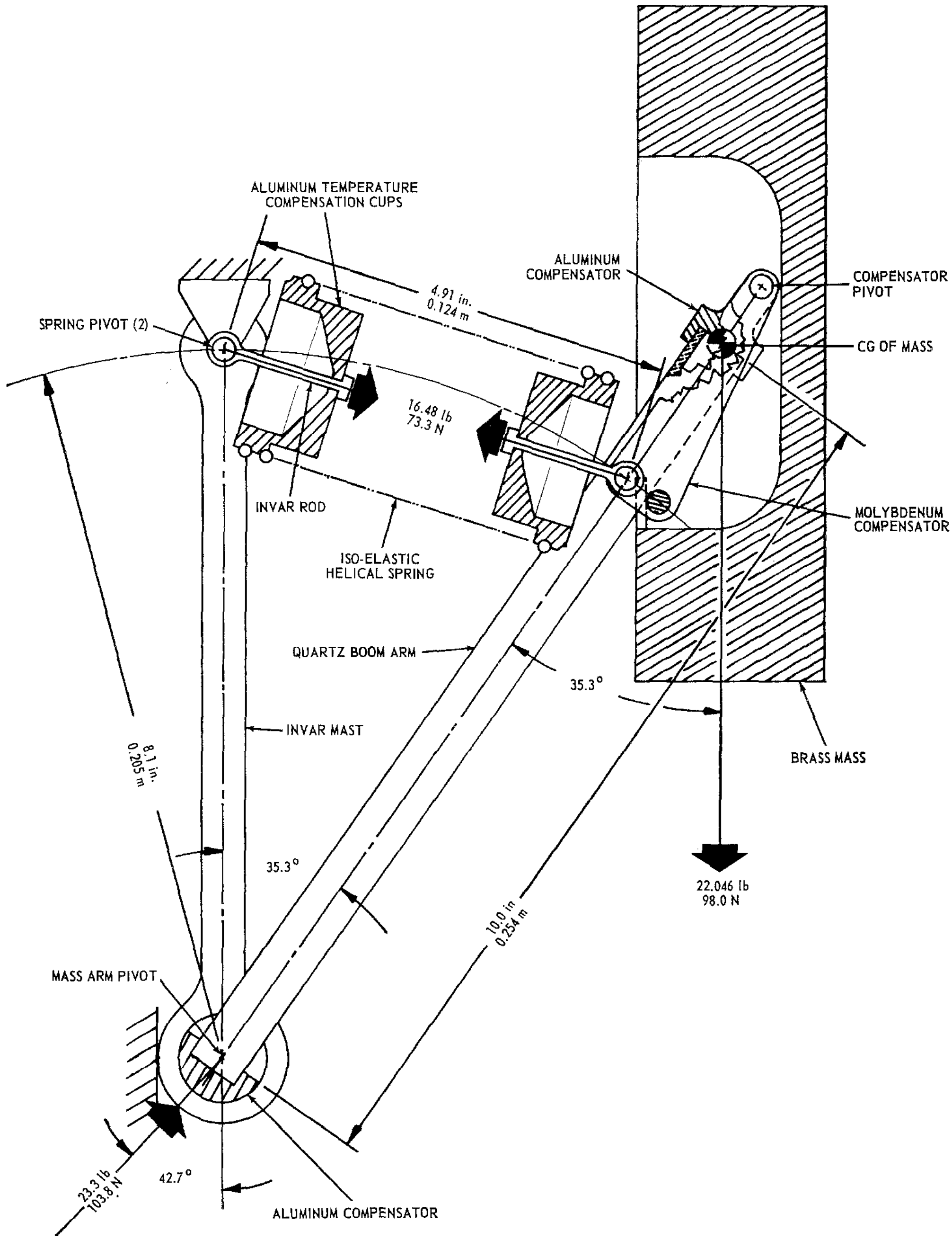

Fig. 13. Sensitive element components of the long-period triaxial seismometer. The heavy arrows represent static forces in pounds or Newtons.

sented by the upper curve in Figure 9. Accordingly, in Figure 14 we present the normalized inverse spectrum of earth noise, together with the measured response of a particular triaxial installation at the Uinta Basin Seismological Observatory (UBSO) near Vernal, Utah, and the approximate response of the World Wide Seismograph System (WWSS) long-period seismographs. The UBSO system uses additional electronic filtering to match the characteristics of conventional long-period seismographs installed there, so that valid comparisons may be made. This filtering is useful to attenuate earth noise at the shorter periods, but is clearly restrictive of permissible sensitivity at periods greater than about 40 seconds. It would seem that more response at

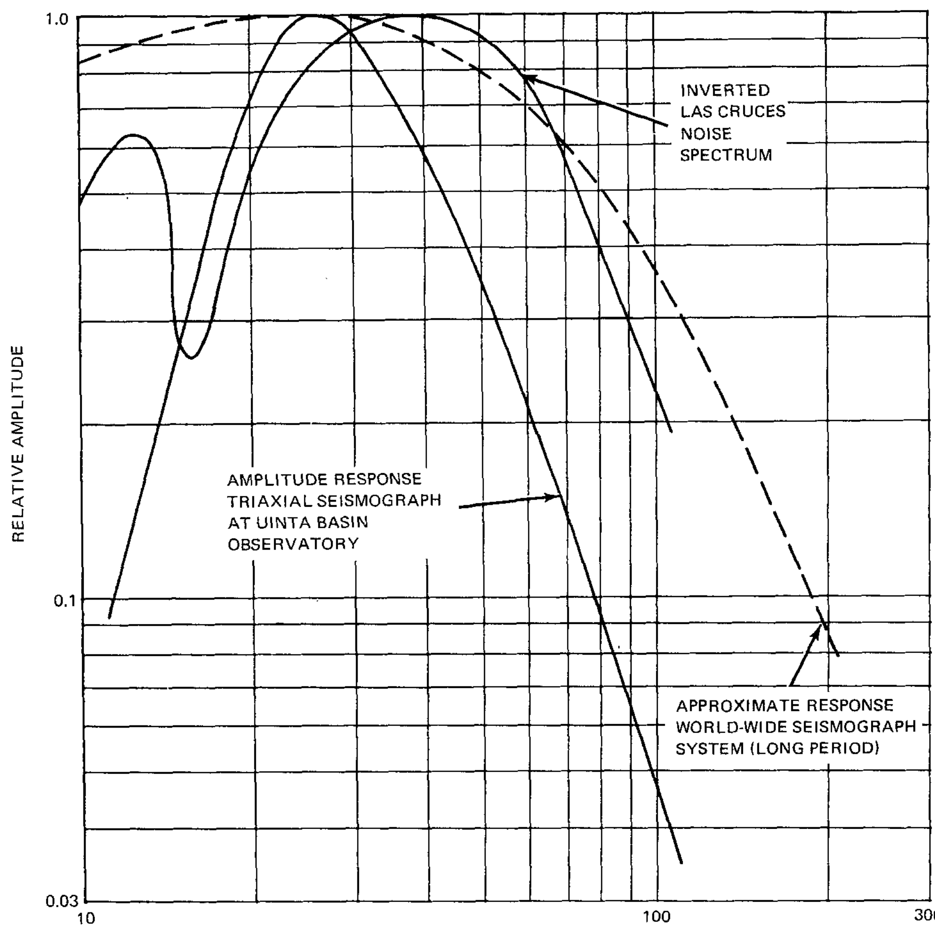

FIG. 14. Amplitude response of trixial seismograph installation used for comparison with conventional seismographs at Uinta Basin Seismological Observatory; here compared with the inverted Las Cruces noise spectrum and with the World-Wide Seismograph System response.

the longer periods could be tolerated at some observatories, if indeed the instruments can be isolated from surface noise.

As mentioned earlier in connection with the Benioff short-period seismometer, periods much longer than the natural period of the seismometer can be sensed if a long-period galvanometer is part of the amplifying system. In fact, either the seismometer or the galvanometer can be at, say, the high frequency (short-period) end of the passband. Of course, the entire band can be pushed toward longer periods if either or both seismometer and galvanometer natural periods are increased. When electronic amplifiers are used without a galvanometer input (phototube amplifier),

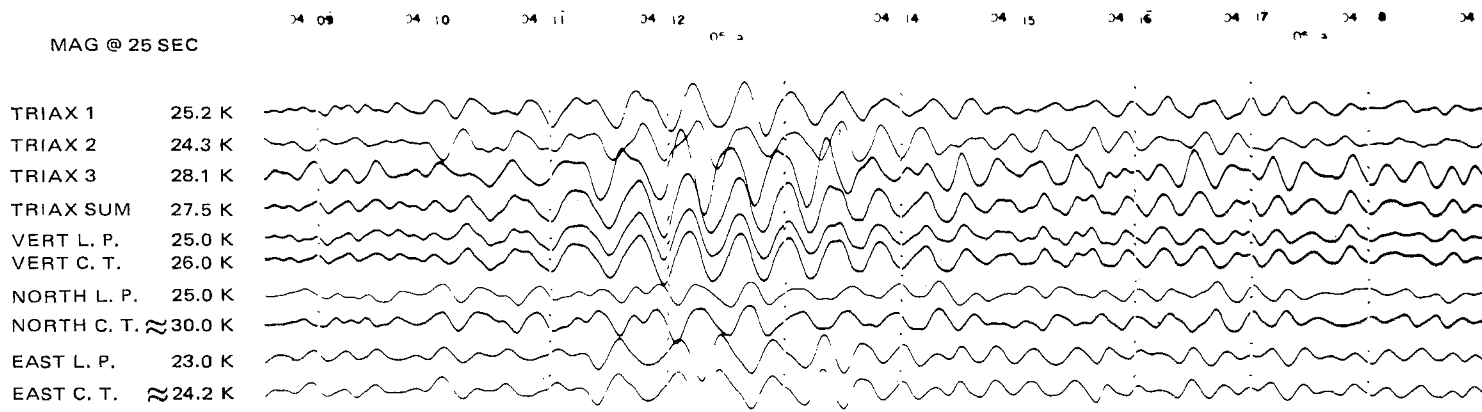

FIG. 15. Uinta Basin Observatory comparison test of trixial seismograph with conventional seismographs at that location. Traces 5 and 6, 7 and 8, 9 and 10 are paired to show similarity at all periods.

the problem of designing active electronic filters at these periods is severe, and the $1/f$ noise may be intolerable. For these reasons, there are unquestioned advantages to increasing the seismometer natural period. However, seismologists generally agree that achieving a *stable* seismometer at periods greater than 25 or 30 seconds requires very careful design, a good installation and adjustment in the case of the vertical component—and that in most environments such periods are impossible to attain with the horizontal component because of the localized tilt we have already discussed.

If we now assume that the local environment is so stable that only instrumental limitations need be considered, and if we desire the ultimate useful sensitivity in a passband centered at, say, 40 seconds period, there is merit in setting the seismometer period at about 30 seconds for use with a galvanometer, or even longer periods for use with an electronic (voltage) amplifier. The experimental module of the triaxial seismometer has been operated at a period of 40 seconds on a pier, after adjustment by remote control, and there is no fundamental reason why the complete seismometer (or any other seismometer with a La Coste suspension) cannot be operated at periods near this. But it is not reasonable to state categorically that all production modules of this new instrument, after shipment and under all expected field conditions, will equal the performance of the experimental module. What is hoped is that experience in production, assembly, test, and *installation* will justify the users in extending capabilities of the instrument as they feel the need.

Comparison of the Symmetrical Triaxial Response to that of Conventional Instruments

Figure 15 is a reproduction of a Develocorder record with output traces of the triaxial and conventional systems arranged for convenient comparison. The record was made at the Uinta Basin Observatory on 28 February 1968, with the triaxial seismometer locked into the casing at the 53-meter (175-feet) depth in a hole 61 meters (200 feet) deep. For comparison purposes, the triaxial outputs were run through coordinate transformers (C.T.), devices incorporating sine-cosine potentiometers and operational amplifiers. These devices mixed appropriate levels of voltage from triaxial channels to derive conventional "north" and "east" outputs. This was really unnecessary for the vertical component because the triaxial sum gives a vertical component, but was done for consistency. The magnifications shown are calculated for a 5 times enlargement of the 16 mm film, to be correct for full page reproduction in the *Bulletin* (K represents 1000). All seismometer periods were set at about 20 seconds.

Conclusions

From the studies, design experience, and testing up to the present, the following conclusions seem justified:

(1) Installation of a seismometer in a hole drilled and cased to a depth of 50 meters or more does much to alleviate problems associated with surface or near-surface vault installations.

(2) Remote control and adjustment of a long-period seismometer actually facilitates these procedures.

(3) All sensing elements of the symmetrical triaxial seismometer have the advantage inherent in the La Coste suspension, as distinguished from horizontal component sensors dependent on gravity for determining the natural period.

(4) Mass centering and period of each triaxial module are affected by tilt chiefly in only one plane, that of the "sensitive" axis.

(5) Control of seismometer period by adjustment of hinge torque is advantageous in that it provides a logical separation of period control from that of mass centering.

A discussion of the specifics of seismograph systems employing this seismometer is not pertinent here because of the many variations possible, depending on geographic location, availability of power, number used in the case of arrays, methods of recording, and ultimate use of the data. It is sufficient to state that manufacturing has been proceeding on the basis of the present design. It appears that, aside from modifying the coil impedances to work with different amplifiers, any alterations should be undertaken only after careful testing and review. Seismologists familiar with problems of long-period installations probably do not need this cautionary comment.

In this paper there has been very little discussion of operating the seismometer at very long natural periods, say 30 or 40 seconds, because all field testing has been designed to compare the instrument with conventional seismograph installations, most of which were in environments that limited the performance of the conventional systems. During such comparison, it is important to match passbands and to equalize magnification of the systems. Thus, there has been little opportunity to study the quality of data which can be made available through the ultimate capabilities of this new instrument.

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References

Brune, James N. and Jack Oliver (1959). The seismic noise of the Earth's surface, *Bull. Seism. Soc. Am.* **49**, 349–353.

Byrne, C. J. (1961). Instrument noise in seismometers, *Bull. Seism. Soc. Am.* **51**, 69–84.

Galitzin, B. (1903). Zur methodik der seismometrischen beobachtungen, *Isv. imp. Akad Nauk*, ser. 5, 19, xxx–xxxi. Lib. Cong. Call No. AS 262,.S34*.

Geotech Division (1965). Theoretical seismograph response and thermal noise curves, *Geotech Div., Tech. Rept. 65-98*, 168 p.

Geotech Division (1966). Long-period seismograph installation, La Paz, Bolivia, *Geotech Div., Tech. Rept. 66-60*, 20 p. Reprinted as appendix 3 to *Tech. Rept. 66-82*.

Hamilton, J. H. and Ernest Stephens, Jr. (1961). New developments in seismological instrumentation, paper presented before a colloquium on the detection of underground explosions, December 15, 1961, *Geotech Div., Tech. Rept. 62-1*, 35 p.

Johnson, Daniel P. and Harry Matheson (1962). An analysis of inertial seismometer-galvanometer combinations. *National Bureau of Standards Report 7454*.

La Coste, L. J. B. (1934). A new type long-period vertical seismograph. *Physics* **5**, 178–180.

La Coste, L. J. B. and A. Romberg (1942). Force measuring device, *U. S. Patent No. 2,293,437*.

Matheson, Harry and Walter Gilbert (1966). On the frequency distribution of Brownian motion. *Geotech Div., Tech. Rept. 66-90*, App. 2.

McMillan, R. E. (1964). Melton long-period borehole triaxial seismometer, Project VT/072, *Geotech Div., Tech. Rept. 64-89*, 32 p.

Melton, Ben S. (1965). Angular composite seismometer, *U. S. Patent No. 3,222,636*, filed March 1, 1963.

Melton, Ben S. (1966). The long-period seismograph—its usefulness and its development. *Geotech Div., Tech. Rept. 66-82*, 83 p.

* A free translation of portion of this reference follows on the next page.

Milam, W. C. (1965). Final report on study of the operation of the advanced long-period system, *Geotech Div., Tech. Rept. 65-93*, 125 p.

Panel on Seismic Improvement (1959). The need for fundamental research in seismology, U. S. Dept. of State.

Trott, Wayne (1965). Investigation of noise in long-period seismographs, *Geotech Div., Tech. Rept. 65-91*, 78 p.

Wenner, Frank (1929). A new seismometer equipped for electromagnetic damping and optical magnification (Theory, general design, and preliminary results) *Natl. Bur. Stand. J. Research* **2**, 963–999. (Out of print. Reproduced by Geotech, with permission of NBS).

Weinstein, Warren D. (1965). Flexure pivot bearings. *Machine Design*, **37**, 150–157 (June 10) and 136–145 (July 8).

Whalen, Joseph M. (1963). Instrumentation noise of long-period seismographs, paper presented at the 13th IUGG General Assembly at Berkeley, California, August 1963. *Geotech Div., Tech. Rept. 63-81*, 30 p.

Wolf, Alfred (1942). The limiting sensitivity of seismic detectors, *Geophysics*, **7**, 115–122.

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"A small modification to Davidson's device, (seismometer) supplying it with electromagnetic damping, was made by Academician Prince B. B. Golitsyn. Done in the instrument, certain changes will be possible, probably to eliminate completely the effect of displacements. Then Davidson's device will be completely adaptable for investigation of only variations of tilt. Academician Prince B. B. Golitsyn then reported that he submitted "Zur Methodik der seismometrischen Beobachtungen" (The Method of Seismometric Observations) for publication in the Bulletin of the Permanent Central Seismological Commission. This paper consists of two parts, Theoretical and Experimental. In the first part is discussed a problem of electromagnetic damping in seismic devices, in relation to the problem of application of the aperiodic galvanometer as registering apparatus. The second part contains a number of experimental recordings undertaken with the idea of checking the discussed theory. To this, B. B. Golitsyn added that, by the decision of the Seismological Commission, it was decided to apply as an experiment, recommended by him, this method of recording at the seismological station of Yuŕ ev (now Tartu, in Estonia—*author's note*) Astronomical Observatory.