Paper:

Analysis and Design of a New Micro Jerk Sensor with Viscous Coupling

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A new method for jerk detection (derivative of acceleration) is proposed. By using a 2 degrees of freedom (DOF) model with viscous coupling, we measure jerk directly as a position change in mass without differential circuit. Analysis by numerical formulas show the principle of the proposed jerk detection and simulation results show suitable parameters such as viscosity, mass ratio, and spring coefficient ratio for jerk detection.
We also propose a micro jerk sensor based on the micro electro mechanical system (MEMS). Results of analysis show that the microstructure is suitable for the proposed viscous coupling.

Keywords: jerk, sensor, viscous, coupling, mems

1. Introduction

Jerk is a physical unit meaning the derivative of acceleration over time. Jerk is not well known and little studied. Jerk detection is very useful for accurately controlling of bodies in motion, such as precise control of numerical control (NC) machines and motor controls for trains. Using jerk improves the control response and avoids excessive input of acceleration to an object.

Figure 1 shows an example of acceleration measurement. When a force was applied to an object at $t = 0$, acceleration increased. In many cases, we want to predict acceleration for control. In the figure, acceleration reaches to G_{t2} at $t = t_2$. Alternatively, we estimate prospective acceleration $G_{t2'}$ at $t = t_2$ instead of measuring acceleration G_{t2} directly.

$$G_{t2'} = G_{t1} + J_{t1}(t_2 - t_1) \quad \ldots\ldots\ldots\ldots (1)$$

where :
G_{t1} : measured acceleration at $t = t_1$
G_{t2} : measured acceleration at $t = t_2$
$G_{t2'}$: estimated acceleration at $t = t_2$
J_{t1} : measured jerk at $t = t_1$

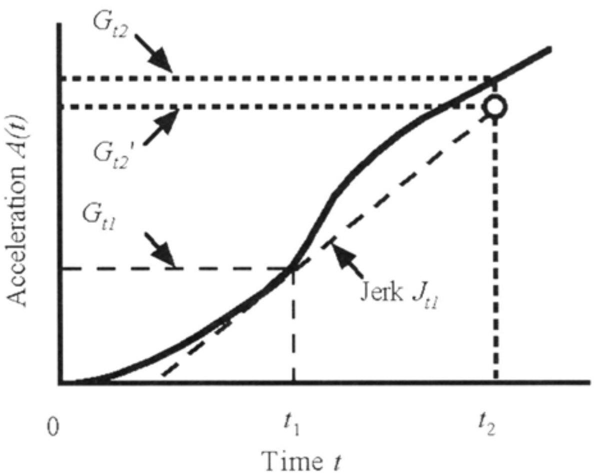


Fig. 1. Prediction of acceleration by jerk.

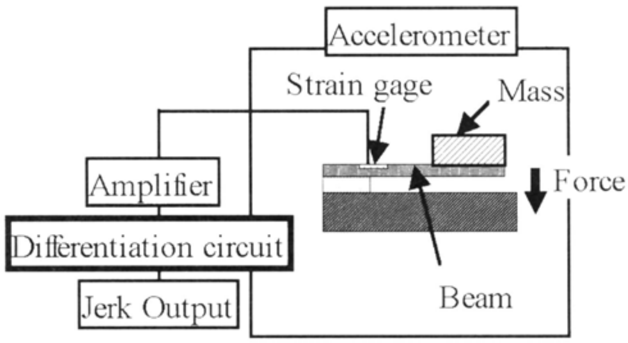


Fig. 2. Conventional measurement.

Eq.1 means that prospective acceleration $G_{t2'}$ is obtained with present acceleration G_{t1} and jerk J_{t1}.

Jerk was conventionally obtained by measuring acceleration and differentiating the measured acceleration signal with a differentiation circuit (**Fig.2**).

Because, in conventional measurement, noise affects the measured acceleration signal, large errors occur when the acceleration signal differentiated (**Fig.3**). A highly responsive accelerometer signal, for example,

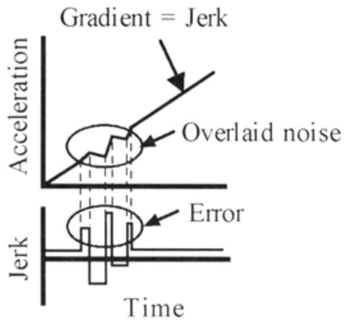


Fig. 3. Problem of conventional jerk detection method.

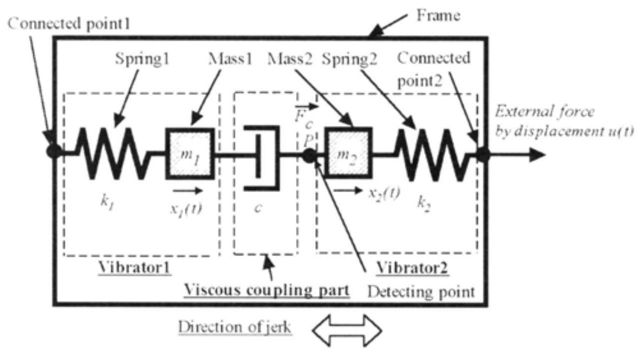


Fig. 4. Two degrees of freedom sensor model for jerk detection.

includes much noise and differentiating the signal makes a large error (Eq. 2), thus requiring other measurement.

We propose detecting jerk with a 2 DOF model and a sensor suitable for MEMS [1].

Accelerometer output = Accelerometer signal + Overlaid noise

$$\frac{d}{dt}V(t)=\frac{d}{dt}V_{Acc}(t)+\frac{d}{dt}V_{noise}(t) \quad \ldots\ldots \quad (2)$$

2. Basic Principle of Jerk Detection

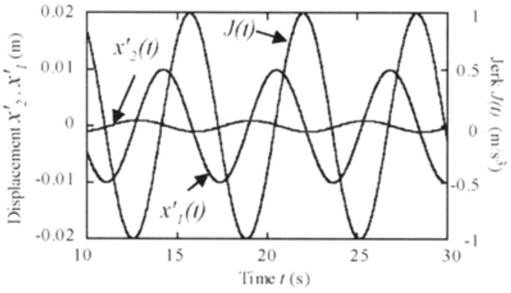
Figure 4 shows the 2 DOF model [2] for jerk detection Using 2 vibrators, vibrator 1 and vibrator 2. Vibrators have mass m_1 and m_2 and spring coefficient k_1 and k_2. Positions of mass1 and mass2 are x_1 and x_2.

Vibrators are connected by a viscous coupling shown as viscous coefficient c. The principle to detect jerk is as follows:

Motion equations of the 2 DOF model in **Fig.4** are derived as Eqs.3 and 4.

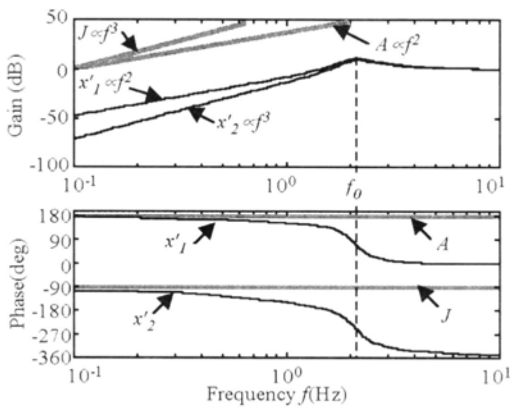
$$m_1 \cdot \ddot{x}_1(t) = k_1 \cdot (u(t) - x_1(t)) + c \cdot (\dot{x}_2(t) - \dot{x}_1(t)) \quad (3)$$

$$m_2 \cdot \ddot{x}_2(t) = k_2 \cdot (u(t) - x_2(t)) + c \cdot (\dot{x}_1(t) - \dot{x}_2(t)) \quad (4)$$



k_1=1(N/m), m_1=0.01(kg), c=0.1(Ns/m),
k_2=1(N/m), m_2=0.0001(kg), u_0=1(m), ω=1(rad/s)

Fig. 5. Calculated sine response (macro model).



k_1=1(N/m), m_1=0.01(kg), c=0.1(Ns/m),
k_2=1(N/m), m_2=0.0001(kg)

Fig. 6. Transfer function x_1'/u and x_2'/u of sensor model (macro model).

where
m_1, m_2: Mass of vibrator1 and vibrator2
k_1, k_2: Stiffness of spring1 and spring2
c: Viscous coefficient
$x_1(t)$, $x_2(t)$: Position of mass1 and mass2
F_c: Viscous force in viscous coupling part
$J(t)$: Jerk
$u(t)$: Sinusoidal displacement by external force
u_0: Amplitude of the sinusoidal displacement

External force input is applied to a frame of the model as sinusoidal wave displacement $u(t)$.

Relative positions $x_1'(t)$, $x_2'(t)$ between mass1, mass2 and the frame are defined as Eqs.5 and 6.

$$x_1'(t)=x_1(t)-u(t) \quad \ldots\ldots\ldots\ldots \quad (5)$$

$$x_2'(t)=x_2(t)-u(t) \quad \ldots\ldots\ldots\ldots \quad (6)$$

When mass1 is much larger than mass2 and spring coefficient k_2 is larger than k_1, Eq.7 is derived.

$$\frac{k_1}{m_1} << \frac{k_2}{m_2} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

If absolute relative position $|x_2'|$ is much smaller than $|x_1'|$ (Eq.8) and absolute relative velocity $|\dot{x}_2'|$ is much smaller than $|\dot{x}_1'|$ (Eq.9), Eq.3 is converted to Eq.10.

$$|x_1'| >> |x_2'| \quad \ldots\ldots\ldots\ldots\ldots (8)$$

$$|\dot{x}_1'| >> |\dot{x}_2'| \quad \ldots\ldots\ldots\ldots\ldots (9)$$

$$m_1 \cdot \ddot{x}_1'(t) + c \cdot \dot{x}_1'(t) + k_1 \cdot x_1'(t) = -m_1 \cdot \ddot{u}(t) \,. \quad (10)$$

The solution of Eq.10 is derived as Eqs.11 and 12 with sinusoidal displacement $u(t)$ in Eq.13, where ω_1 is the first resonant frequency of vibrator 1 in Eq.14 and ζ_1 is the damping ratio in Eq.15.

$$x_1'(t) = \frac{\left(\frac{\omega}{\omega_1}\right)^2}{\sqrt{\left(1-\left(\frac{\omega}{\omega_1}\right)^2\right)^2 + 2\zeta_1\left(\frac{\omega}{\omega_1}\right)^2}} \cdot u_0 \cdot Sin(\omega t - \phi)$$

$$\ldots\ldots\ldots\ldots\ldots (11)$$

$$\phi = \tan^{-1}\frac{2\zeta_1\left(\frac{\omega}{\omega_1}\right)}{1-\left(\frac{\omega}{\omega_1}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots (12)$$

$$u(t) = u_0 \cdot Sin(\omega t) \quad \ldots\ldots\ldots\ldots\ldots (13)$$

$$\omega_1 = \sqrt{\frac{k_1}{m_1}} \quad \ldots\ldots\ldots\ldots\ldots (14)$$

$$\zeta_1 = \frac{c}{\sqrt{4m_1 \cdot k_1}} \quad \ldots\ldots\ldots\ldots\ldots (15)$$

In $m_2 \approx 0$, and Eqs.8 and 9, Eq.4 becomes Eq.16.

$$x_2'(t) = \frac{c}{k_2} \cdot \dot{x}_1'(t) \quad \ldots\ldots\ldots\ldots\ldots (16)$$

By differentiating the $x_1'(t)$ in condition of $\omega / \omega_1 << 1$, Eq.17 is derived from Eq.11.

$$\dot{x}_1'(t) = \left(\frac{\omega}{\omega_1}\right)^2 \cdot u_0 \cdot \omega \cdot Cos(\omega t - \phi) \quad \ldots\ldots (17)$$

When sinusoidal displacement $u(t)$ is given by Eq.13, the first derivative of position for is velocity (Eq.18) and the second derivative is acceleration (Eq.19). The third derivative is jerk (Eq.20).

$$\frac{d}{dt}u(t) = u_0\omega Cos(\omega t) \quad \ldots\ldots\ldots\ldots\ldots (18)$$

$$\frac{d^2}{dt^2}u(t) = -u_0\omega^2 Sin(\omega t) \quad \ldots\ldots\ldots\ldots\ldots (19)$$

$$\frac{d^3}{dt^3}u(t) = -u_0\omega^3 Cos(\omega t) \quad \ldots\ldots\ldots\ldots\ldots (20)$$

With Eq.20, Eq.17 becomes Eq.21.

$$\dot{x}_1'(t) = -\left(\frac{1}{\omega_1}\right)^2 \cdot J(t) \quad \ldots\ldots\ldots\ldots\ldots (21)$$

Relative position $x_2'(t)$ is expressed by Eq.22 with Eqs.14, 16, and 21.

$$x_2'(t) = -\frac{c}{k_2} \cdot \frac{m_1}{k_1} \cdot J(t) \quad \ldots\ldots\ldots\ldots\ldots (22)$$

As a result, jerk J(t) is derived as follows:

$$J(t) = -\frac{k_1 \cdot k_2}{c \cdot m_1} \cdot x_2'(t) \quad \ldots\ldots\ldots\ldots\ldots (23)$$

Eq.23 shows jerk $J(t)$ in proportion to the measured relative position between mass2 and the frame.

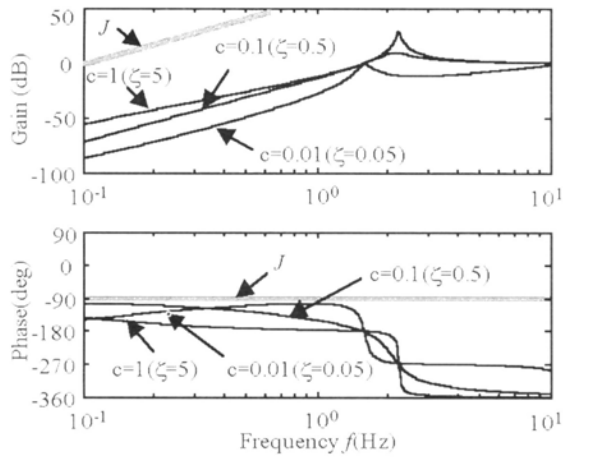
The principle of jerk detection is summarized as follows: When the frame undergoes sinusoidal displacement $u(t)$, mass1 has relative velocity $\dot{x}_1'(t)$ at a local coordinate in the frame. Assuming that m_2 is much smaller than m_1 and stiffness k_2 is larger than k_1, external force on vibrator 2 is negligible and relative velocity between mass1 and mass2 equals $\dot{x}_1'(t)$. Velocity $\dot{x}_1'(t)$ equals jerk (Eq.21). Viscous force F_c in viscous coupling is generated by $\dot{x}_1'(t)$. Viscous force F_c makes mass2 move and the position of mass2 is balanced by viscous force F_c and spring force. Relative displacement $x_2'(t)$ between $u(t)$ and mass2 is linear to velocity $\dot{x}_1'(t)$.

Jerk is detected by measuring relative position $x_2'(t)$ directly with no differentiation circuit.

3. Simulation

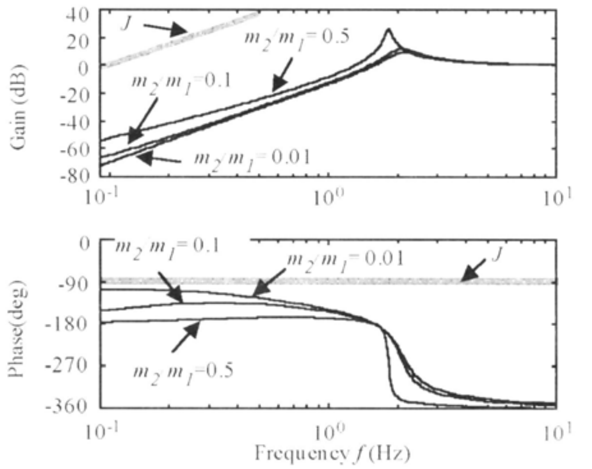
3.1. Sine Response as Time

Calculated sine responses of the jerk detection model are shown in **Fig.5**.



k_1=1(N/m), m_1=0.01(kg), k_2=1(N/m), m_2=0.0001(kg)

Fig. 7. Transfer function x_2'/u with some viscosity (macro model).



k_1=1(N/m), m_1=0.01(kg), k_2=1(N/m), c=0.1(Ns/m)

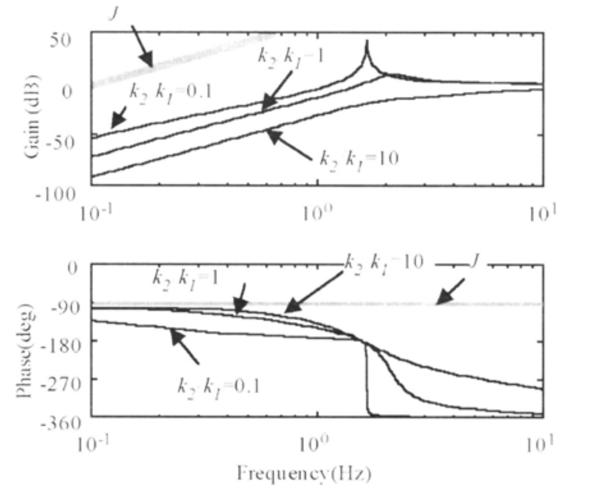
Fig. 8. Mass ratio dependence (macro model).

External force is applied to the frame (**Fig.4**) as sinusoidal displacement $u(t)$. Macro sizes are used as parameters for analysis. Input $u(t)$ is given as a position, so applied jerk $J(t)$ is given as the differential coefficient of the third order of $u(t)$ (Eq.20).

Figure 5 shows the relationship between relative position $x'_1(t)$, $x'_2(t)$ and jerk $J(t)$. There is a phase difference of $\pi/2$ between $J(t)$ and $x_1'(t)$, because the velocity of mass1 is proportion to jerk $J(t)$ (Eq.21). There is a phase difference between $x_1'(t)$ and $x_2'(t)$, because mass2 moves by viscous force F_c generated from velocity $\dot{x}_1'(t)$ (Eq.16).

3.2. Transfer Function of 2 DOF Model

Fig.6 shows the transfer function of the jerk detection model. The gain of $x_1'(t)$ increases in proportion to the square of frequency f, and the gain of $x_2'(t)$ increases in proportion to the cube of f. Theses results mean that $x_1'(t)$ stands for acceleration and $x_2'(t)$ indicates jerk. In low frequency below f_0, a first resonant frequency of the 2 DOF model, the phase lag of $x_2'(t)$ and jerk is the same -90 deg. As frequency f increases, the phase lag of $x_2'(t)$ differs from jerk, meaning that resonant frequency f_0 of the device is larger than the jerk input frequency.



k_1=1(N/m), m_1=0.01(kg), m_2=0.0001(kg), c=0.1(Ns/m)

Fig. 9. Spring coefficient ratio dependence (macro model).

3.3. Viscosity Dependence

The influence of viscous coefficient c was also evaluated. **Fig.7** shows the transfer function of the jerk detection model for viscous coefficient c. When we consider the influence of dumping, it is suitable to use damping ratio ζ instead of damping coefficient c. The relationship between damping ratio ζ and viscous coefficient c is shown in Eq. 15.

If damping ratio ζ =0.5, the behavior of gains is in proportion to jerk. If ζ = 5 and ζ = 0.05, the behavior of gain differs from that of ζ = 0.5. When damping ratio ζ is too large, mass 1 and mass 2 move in the same phase because of overdamping. When damping ratio ζ is 0.05, as the connection between vibrator 1 and vibrator 2 is very weak, viscous coupling between both vibrators is less than inertia applied to mass 2. In both overdamping and lower damping, the model cannot detect jerk. These analytical results show that the suitable damping ratio is ζ = 0.5 for jerk detection for the model.

3.4. Mass Ratio Dependence

In Eq.16, m_2 should be much smaller than m_1 to neglect inertial force for mass2. In this section, we analyze the relationship between mass ratio m_2 / m_1 and characteristics of the model by transfer function x_2'/u.

The variable is m_2. Other parameters are constant. **Fig.8** shows m_2 / m_1 = 0.1 or 0.01, where the behavior of gain is in proportion to jerk.

When mass ratio is m_2 / m_1 = 0.5, the slope of gain is

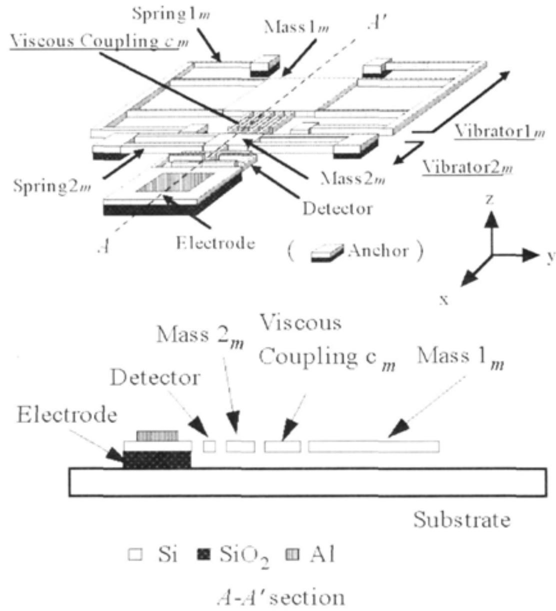
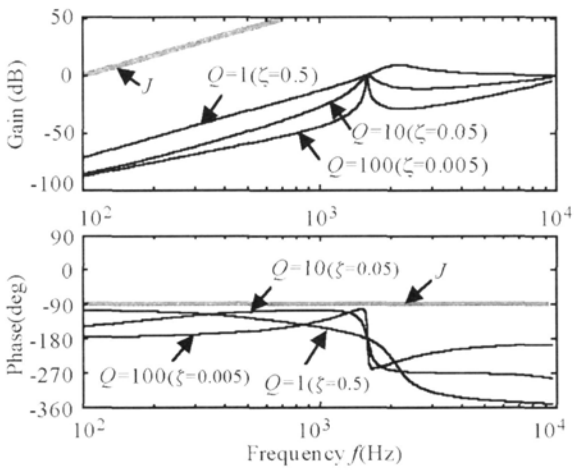


Fig. 10. Micro jerk sensor.



k_1=0.1(N/m), m_1=10^{-9}(kg), k_2=0.1(N/m), m_2=10^{-11}(kg)

Fig. 11. Transfer function x_2'/u of micro jerk sensor (micro model).

less than that of jerk. The behavior of the phase shows that when ratio m_2 / m_1 is small, phase lag is nearly -90 deg. When ratio m_2 / m_1 is large, phase lag approaches -180 deg, meaning that when the ratio is large, the model cannot neglect inertial force to mass2. These results show suitable mass ratio m_2 / m_1 for jerk detection is less than 0.01.

3.5. Spring Coefficient Ratio Dependence

Spring coefficient ratio k_2/k_1 is a dominant factor in characteristics of the model. **Fig.9** shows the relationship between spring coefficient ratio k_2 / k_1 and transfer function x_2'/u of the model.

The variable is k_2. Other parameters are constant. When ratio k_2 /k_1 is 1 or 10, the behavior of gain is in proportion to jerk. If ratio k_2 /k_1 is 0.1, the slope of gain is less than that of the jerk J. This results means that k_2 should be larger than k_1 to avoid detecting acceleration.

From the point of response, the resonant frequency of vibrator 2 should be higher than that of vibrator 1 (Eq.7).

4. MEMS Model for Jerk Sensor

Fig.10 shows the jerk sensor based on MEMS . Most parts float from a wafer except anchors. Mass 1_m is suspended by spring 1_m. Vibrator 1_m consists of mass1_m and spring 1_m, which moves by acceleration toward the x axis. Viscous coupling consists of many comb fingers with air gaps. The structure is realized by silicon on insulator (SOI) technology.

$$\zeta = \frac{1}{2Q} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (27)$$

The position of vibrator 2_m is measured by variation of capacitance between fingers at the detector.

The viscosity at the air gap is described with quality factor Q. The relationship between damping ratio ζ and quality factor Q is shown in Eq.27. Parameters for analysis are set as a micro size model. Results show that for ζ=0.5 (Q=1), gain is in proportion to jerk (**Fig.11**). Because the air gap is small, there is a large viscous force between fingers due to air [3]. This means that the microstructure is suitable for jerk detection with viscous coupling.

Interestingly, suitable damping ratios are the same for both macro and micro models. This suggests that we can design and analysis of the jerk sensor by damping ratio ζ regardless of sensor size.

5. Conclusions

(1) We propose a new way to detect jerk with a 2 DOF model. Using viscous coupling to connect 2 vibrators, the model measures jerk directly without a differential circuit.
(2) Analysis of the model with formulas and numerical simulation proved that the displacement of mass 2 indicates jerk.
(3) Suitable model parameters for jerk detection were obtained by studying dependence on viscosity, mass ratio, and the spring coefficient ratio.
(4) An analysis of the jerk sensor based on MEMS showed that the microstructure is suitable for proposed jerk detection with viscous coupling.

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