Complex Light

Jeff Secor, **Robert Alfano** and **Solyman Ashrafi** review the fundamental physics and the burgeoning applications of complex light beams.

physicsworld Discovery

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Dr Jeff Secor
Research Associate, Institute for Ultrafast Spectroscopy and Lasers, The City College of New York, New York, USA

Prof Robert Alfano
Distinguished Professor of Science and Engineering, Institute for Ultrafast Spectroscopy and Lasers, The City College of New York, New York, USA

Dr Solyman Ashrafi
Managing Partner, NxGen Partners, Dallas, Texas, USA

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Contents

Abstract

The emerging field of complex light—the study and application of custom light beams with tailored intensity, polarization or phase—is a focal point for fundamental breakthroughs in optical science. As this review will show, those advances in fundamental understanding, coupled with the latest developments in complex light generation, are translating into a range of diverse and cross-disciplinary applications that span microscopy, high-data-rate communications, optical trapping and quantum optics. We can expect more twists along the way, too, as researchers seek to manipulate and control the propagation speed of complex light beams, while others push the more exotic possibilities afforded by complex light in quantum-entanglement experiments.

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About the authors

Jeff Secor

Jeff Secor recently received his PhD and is currently a research associate in physics at the Institute for Ultrafast Spectroscopy and Lasers, The City College of New York. His interests are in fundamental physics and spectroscopy, with applications to photoactive nano compounds and solar-energy harvesting. He has worked with optical studies of natural photosynthetic systems, condensed matter and optoelectronics, and organic carbon dots. He recently began working in the field of complex light and its applications to physical spectroscopy. He is also an avid urban gardener.

Robert R Alfano

Prof Robert R Alfano PhD is Distinguished Professor of Science and Engineering at The City College of New York (CCNY) of the City University of New York. He has contributed greatly to the field of ultrafast laser science and is a pioneer in the application of light and photonic technologies to the study of biological, biomedical and condensed-matter systems. Some of his most significant achievements include the discovery and description of the supercontinuum, development of new tunable Cr^{3+}/Cr^{4+} lasers, advances in laser spectroscopic and optical imaging techniques, and studies of ultrafast pulse propagation in scattering media. He is a fellow of the APS, OSA, IEEE, New York Academy of Sciences, and an Alfred P Sloan fellow. He received his PhD in physics from New York University and spent eight years at GTE Labs (now Verizon) before joining CCNY. He received the OSA Charles Hard Townes award in 2008, SPIE Britton Chance Biomedical Optics award in 2012, the APS Arthur L Schawlow prize in laser science in 2013 and OSA 2016 Feld Award in Biophotonics.

Solyman Ashrafi

Solyman Ashrafi PhD is a managing partner at NxGen Partners and a technology entrepreneur with over 25 years of product development expertise. He has previously led the social messaging, digital media (music and video), financial services, advertising and rich communications services product teams at metroPCS. Here he was responsible for all product partnerships with telecom operators, internet companies, advertising agencies, video/music content providers, search companies, app developers and OTTs. Prior to metroPCS, he designed and built telecom networks for major operators in the US and Europe and held executive positions at Nortel, Ericsson, LCC and CSC. During his career, Solyman has raised money from multiple funds and launched a number of venture-backed companies in software, technology and cloud-based applications. Solyman was the stakeholder board member of an incubator for Nortel in 1997 and an accelerator for Ericsson in 1999 for their investment on new technology companies. He also worked closely with Roger Linquist (CEO and Founder) in the 2013 merger of metroPCS with T-Mobile. Solyman holds a PhD in applied physics, a MEE in communications engineering, an MSc in wave propagation, and a BEE degree in electrical engineering.

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Introduction

The traditional salient properties of light are its polarization, frequency and velocity, traveling at the universal upper speed limit *c*. A wealth of physical phenomena, from high-bandwidth fibre-optic communication to polarized sunglasses, photosynthesis in plants to the primary processes in vision, have been discovered and described using these traditional properties of light. It turns out, however, that light can be a lot more complex than previously considered. The emerging field of complex (or structured) light describes abstract topological properties of light—i.e., beams with customized intensity, polarization and phase—and it is these properties that are ushering in a renaissance in many areas of optical science and technology.

A structured light beam is a solution to Maxwell's equations that, generally, has a non-homogeneous spatial profile, but is still well defined in the phase, amplitude or polarization profiles. For instance, linearly polarized light is simple to consider, and just as simple to experimentally generate using conventional polarizers. The action of a linear polarizer on a light beam gives a homogenous polarization profile across the beam cross-section. One can imagine all points on the beam cross-section to have the polarization vector pointing in the same direction. And because light has two degrees of freedom, there are two orthogonal states of linear polarization ('vertical' and 'horizontal' are useful labels, but any two orthogonal directions could work) and all directions of polarization can in principle be described as combinations of linear and horizontal.

On the other hand, one particular form of complex light is that of radial polarization, which is a condition where the electric-field vector everywhere points outward (this would be homogenous in cylindrical co-ordinates and we'll come back to the ideas of co-ordinate systems in the next section). The electric-field vector in this case is not the same along a path that circles the beam profile (in contrast to the linearly polarized case). The horizontal and vertical directions give a simple basis set for homogenous light polarization, while new forms of radial and azimuthal polarization form an orthogonal set that can be used to describe light.

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Another popular group of complex light beams are those that carry orbital angular momentum. Classical and quantum physics deals with orbital angular momentum in a variety of situations, but its treatment within the realm of light-beam propagation is central to advances in the understanding and application of complex light. In 1936, Richard Beth demonstrated that circularly polarized light could transfer spin angular momentum to matter, thereby causing it to rotate [1]. Seventy years later, the same effect was observed with the orbital angular momentum of light.

Background

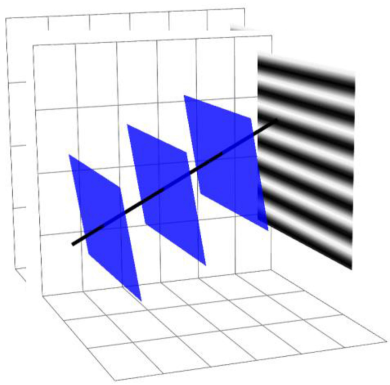
Propagating electromagnetic waves are best described by Maxwell's equations. Yet 150 years after their discovery there is more physics to be found within electromagnetic theory, and ideas borrowed from quantum mechanics have further helped to develop the theories of these new complex properties of light. There are many ways to solve Maxwell's equations for the electromagnetic field $E(r, t)$. For propagating waves, one can look for solutions that separate into a spatial component and an oscillation term, such that explicit solutions of Maxwell equations will impose conditions for the oscillating and the spatial terms.

The choice of co-ordinate system and approximations leads to different special-function solutions of propagating waves with unique symmetries (figure 1). The electromagnetic field can be described by considering the surfaces of constant phase. Symmetric surfaces in Cartesian co-ordinates are planes perpendicular to each co-ordinate direction. In spherical co-ordinates r, θ and ϕ, the co-ordinate symmetric surfaces are spheres, cones and circles, and a simple solution to Maxwell's equations in spherical co-ordinates is that of a spherical wave.

Cylindrical co-ordinates r, θ and z have co-ordinate symmetric surfaces of cylinders, planes and circles. The cavity geometry of a laser system is naturally described in cylindrical co-ordinates using the paraxial approximation, which simplifies the mathematical analysis for light beams that have wave vectors predominantly along the optical axis, or a small radial component, similar to the small-angle approximation used in lens analysis. Some common solutions to Maxwell's equations in cylindrical coordinates that are important to complex light are the Hermite–Gaussian, Laguerre–Gaussian and Bessel cavity modes[1].

The Laguerre–Gaussian modes are a particular solution to Maxwell's equation with an azimuthally varying phase term $e^{il\phi}$, for integer value of l describing the Laguerre–Gaussian mode and describing the amount of orbital angular momentum of the light field. The phase profile varies continuously around the optical axis, which leads to a helicoidal wave front with the screw axis along the optical axis. These are the picturesque 'donut' beam profiles with a dark central region of the

[1] Many complex beams are found in the mathematical solutions of the paraxial approximation, which considers geometry used in waveguides or optical transmission lines. They are named after the mathematical models that describe their properties. Hermite–Gaussian are rectangularly symmetric. Laguerre-Gaussian are made of combinations of Hermite–Gaussians. Bessel beams are circularly symmetric with the axis of symmetry along the propagation direction.



Plane waves in Cartesian co-ordinates

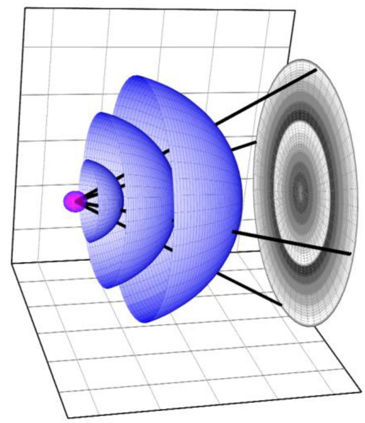
$$\Phi = \mathrm{k} \bullet \mathrm{r}$$

$$\bar{\nabla}\Phi = k_x\hat{\imath} + k_y\hat{\jmath} + k_z\hat{k}$$

The surfaces of constant phase are planes perpendicular to the propagation vector $k = k_x\hat{\imath} + k_y\hat{\jmath} + k_z\hat{k}$
The periodicity rate of the striped pattern describes the angle between the wave propagation direction and the cross-sectional plane of measurement.

Point-source emission with spherical co-ordinates

The isotropic emission means the wave vector is everywhere radial, so in this case $\bar{\nabla}\,\Phi = k_r\hat{r}$
and the surfaces of constant phase are spheres. The projection of the phase fronts onto a plane are a series of concentric rings.



Helical mode in cylindrical co-ordinates

The spatial phase of a helical mode with topological charge ***l*** is $\Phi = \mathrm{k} \bullet \mathrm{r} - \mathrm{i}l\varphi$. In the paraxial regime, with negligible radial wave vector, the surface of constant phase is a helicoid with surface normal to

$$\bar{\nabla}\,\Phi = k_z\hat{z} - \frac{l}{r}\hat{\varphi}$$

The cross-section of the wave has a phase that winds around the axis.

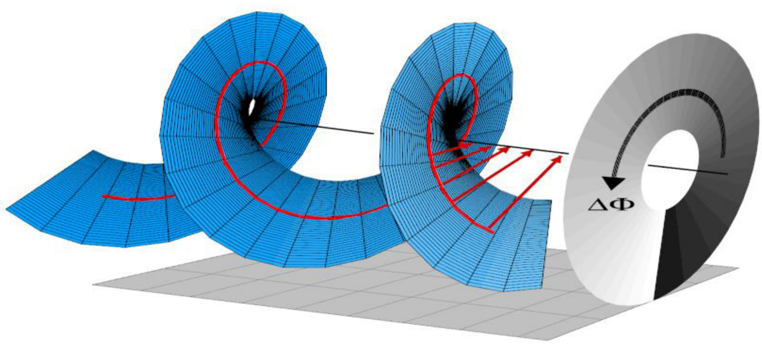


Figure 1. The wavefronts are surfaces of constant phase at fixed time, perpendicular to the direction of $\bar{\nabla}\Phi$, where Φ is the phase of the wave. Above are examples of wave fronts in different co-ordinate systems. The black and white patterns are the projections of the phase surface onto the XY-plane.

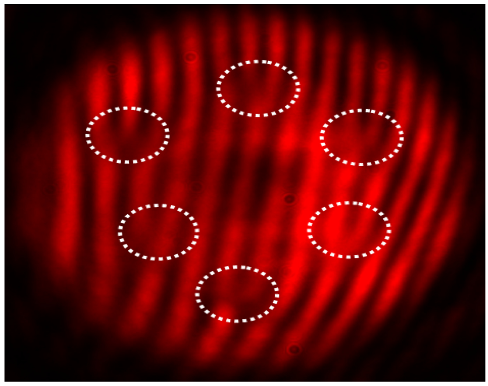


Figure 2. When a Laguerre–Gaussian beam combines with a Gaussian beam in an interferometer, the topological charges are evidenced by bifurcations in the interference lines circled in white. This beam has a total orbital angular momentum value of 6.

beam. The radial intensity of the Laguerre–Gaussian solution is polynomial in r, which modifies the radial intensity. But it is the azimuthally varying phase term that creates a rapidly varying spatial phase near the centre and a phase singularity at $r = 0$. These phase singularities with 'charge' of value l are what lead to the new topology as seen in their interference patterns (figure 2).

Some of the earliest demonstrations of structured light beams serve as an intuitive demonstration of their nature. In the early 1990s, for example, Les Allen and co-workers demonstrated the generation of Laguerre–Gaussian beams by transforming Hermite–Gaussian laser modes through a cylindrical mode converter [2]. The combination of orthogonal Hermite–Gaussian modes with phase offsets leads to a Laguerre–Gaussian beam with orbital angular momentum.

The astigmatism used to generate the helical mode can also be used to determine the helical properties of any beam under study. Since the propagation through an astigmatic optical system can convert Hermite–Gaussian modes to Laguerre–Gaussian reversibly, a simple method to decompose the beam and determine its orbital angular momentum content is to pass the beam through a biconvex spherical lens that is rotated about an axis perpendicular to the optical axis. The tilted lens introduces astigmatism into the focused light. As the beam goes through the focal plane, the Gouy phase[2] of the beam changes rapidly and the two orthogonal components interfere, resulting in a pattern from which the orbital angular momentum value can be counted from the number of fringes in the image.

Giovanni Milione and collaborators developed a more advanced approach to beam analysis by expanding the classical Poincaré sphere theory to include complex light [3]. The Poincaré sphere is an elegant mathematical description of the polarization state of light. The north and south poles represent opposite-handedness

[2] The Gouy phase is a remarkable physical property of light fields that is often overlooked, but important nonetheless. The Gouy phase shift is a phase shift that occurs through the focus of a light field. A striking and simple example is an inverted image through a lens, which is a phase shift of 180°.

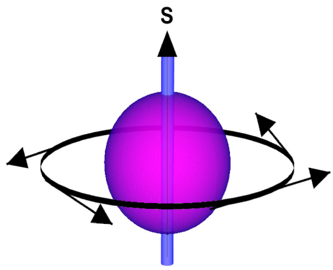


Figure 3a. Spin angular momentum is related to the rotation of the polarization of the field and there is no unique centre of the rotation. A classical analogy is the rotation of all parts of the spinning earth.

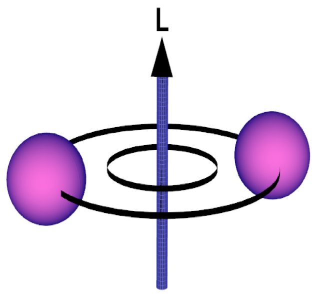


Figure 3b. The orbital angular momentum is from the rotation of the phase between different parts of the field with respect to a centre. In this case the phase rotates around the central axis. The classical analogy is that of the orbiting planets around the sun at the centre.

circular polarization. The equator then represents linear polarization of any direction and intermediate points correspond to light beams with elliptical polarization. Since light has two orthogonal components, the circular-handed decomposition is a more natural representation (figure 3a).

To extend the Poincaré sphere to complex light, the complex beam is described by opposite-handed basis sets in terms of the phase structure, and a higher-order Poincaré sphere is constructed of combinations of these complex-light basis states. Two spheres are needed in order to fully describe a complex light beam using this higher-order Poincaré sphere (figure 3b). This is because there are two possible symmetries of the system: one case in which the spin angular momentum (from the polarization of light) is along the same direction as the orbital angular momentum (from the spatial phase of the light field); the other case is for when the spin angular momentum is oppositely directed to the orbital angular momentum vector. In this way, a complex light beam can be fully described on a geometrical representation as has been done for classical optics on the Poincaré sphere.

Generation of complex light

In the early days of the laser, the occurrence of beam profiles in the form of donuts and side lobes was considered a nuisance. Generally, the TEM_{00} Gaussian mode, which has a circularly symmetric intensity pattern with a maximum at the centre, is in many ways easier to work with mathematically and experimentally. However, growing interest in the field of complex light has brought with it a rapid diversification in the methods used to generate such complex light beams. Some of the more important approaches are elaborated in the following discussion.

One of the earliest means of generating higher-order modes from a laser cavity is to introduce wire grids and amplitude masks into the laser cavity, breaking the symmetry of the cavity and forcing the oscillations into higher-order modes. This approach, however, is not very user-friendly since it requires modifications to the internal workings of the laser. The cylindrical mode converters described in the previous section are an elegant method to make Laguerre–Gaussian modes but require Hermite–Gaussian modes and then a series of focusing optical elements (for mode conversion).

Other types of complex light fields can be generated with refractive elements. The Bessel beam, which in some ways is considered to be 'non-diffracting' in the Rayleigh length[3], can be generated with an axicon lens (which is a conical-lens-type optical element). Bessel beams can also be made with circular apertures, but this greatly reduces the total intensity of the beam.

Technological advances over the past decade have led to relatively simple benchtop methods to generate different types of complex light beams, and not only Laguerre–Gaussian modes. Diffractive methods, for example, can produce complex light fields as illustrated by the interference pattern in figure 2. If the interference pattern of the complex beam with a Gaussian beam produces this bifurcated fringe pattern, then holographic principles tell us that shining Gaussian light at a phase mask with the bifurcated pattern will produce the complex light beam in the transmitted or diffracted light. Thus, one needs only make a ruled diffraction grating with a bifurcation in the pattern and the resulting first-order diffracted spot will carry the complex phase profile in the beam.

The bifurcated diffraction pattern can be constructed in the form a projection slide, making it very simple to incorporate into an optical system. However, low diffraction efficiencies and the generation of other unwanted diffraction orders represent the downsides to this method. What's more, these static elements generate only one type of complex light, which corresponds to the pattern.

An alternative method uses computer-generated holograms to overcome some of these issues. Such holograms enable the generation of any diffraction pattern (limited mainly by the size of the pixels) and in turn any corresponding complex light field. The user-defined holographic patterns modulate the light beam's spatial property, and thus are appropriately named spatial light modulators (SLMs). These SLMs act as tiny, precisely controlled birefringent pixel elements that can write almost any phase profile onto the beam and, as a consequence, they have become commonplace in research labs working with complex light.

[3] The Rayleigh length is a characteristic length scale of a light focused light beam. It is defined as the distance over which the beam waist expands to $\sqrt{2}$ of its smallest size.

A different approach to generating complex light is to use specially designed optical wave-plates. Conventional wave retarders make use of birefringence of a large crystal so that orthogonal polarization components of the beam acquire different phase delays. The result is still a homogenous phase profile in the beam cross-section, but the net polarization direction rotates as the beam propagates, giving the light a bit of 'spin' in the electric-field vector.

In the case of complex light, the aim is to achieve a phase profile that is non-homogeneous in the beam profile, so conventional wave retarders will not generate complex beams. A method developed by Lorenzo Marucci and colleagues, however, makes use of a transmissive optical wave-plate, called a Q-plate, with a non-homogeneous optical axis to drive a spin-to-orbit conversion. This results in output light with a helical phase profile and orbital angular momentum [4].

The output of the Q-plate is determined by the input polarization. Circularly polarized input leads to oppositely circular-polarized output with orbital angular momentum, while linear input will lead to a superposition of positive and negative orbital angular momentum (thus net zero of the output beam). However, linearly polarized input with no spin angular momentum generates a complex polarization profile (termed vector beams). For a Q-plate that can generate one unit of orbital angular momentum, horizontal/vertical input leads to radial/azimuthal polarized output. In this way, by using a SLM in tandem with a Q-plate, it is possible to generate beams of any value of orbital angular momentum with a complex polarization structure.

Current directions

Having reviewed the fundamental science and various approaches for generating complex light, we now take a look at some of the more topical applications of structured light beams. What's hopefully apparent is the utility, diversity and possibility of those applications—including biological imaging, particle trapping, next-generation optical communication schemes and quantum information processing.

Super-resolution microscopy and image enhancement

One of the simplest applications of light is to form an image of an object, with the properties of the light determining the properties of the image. For example, different illumination geometries will change the quality of the image, while polarizers can be used to reduce glare and colored filters can highlight one feature over another. Given its unique characteristics, it's perhaps no surprise to see that complex light has triggered fundamental breakthroughs in imaging science. The 2014 Nobel Prize in Chemistry is a case in point—awarded to Stefan Hell, Eric Betzig and Eric Moerner for their development of super-resolution microscopy to surpass the classical diffraction limit due to the point-spread function (as described by Ernst Abbé more than a hundred years previously) [5].

The super-resolution method relies on stimulated emission depletion (STED) of parts of the sample under study, with small regions that have not been depleted emitting normal fluorescence after the depletion pulse. In practice, an ultrashort

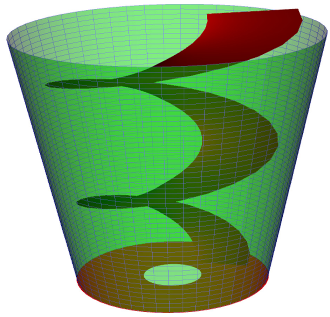


Figure 4. In STED illumination, the intense homogenous laser pulse (green) excites emitting species within the beam profile. A second structured beam (in red) causes stimulated emission of the species except for the region in the intensity null of the structured beam. Thus, only the species within the small central region will remain bright at later times.

light pulse of tens to hundreds of femtoseconds (10^{-15} s) duration excites fluorophores in a material. Under normal conditions the fluorophores would lose the absorbed energy by non-radiative (vibrations) and radiative (fluorescence) transitions. The duration of the fluorescence is determined by the properties of the fluorophore[4] and its surrounding environment. Generally, the emission can persist for femtoseconds or for as long as a few nanoseconds (and even longer than seconds in the case of phosphorescence).

The principle that leads to enhanced resolution is that a second pulse, interacting with the system after the first excitation pulse, will modify the radiative properties. The mechanism in which the second pulse affects the system is analogous to the way that light can induce a transition into the excited state. The symmetry of the Einstein A and B coefficients that determine absorption and emission says that the second pulse can cause stimulated emission, which returns the system to the ground state via the interaction with light.

In essence, the second pulse can be used to turn off the fluorescence faster than would have occurred without the second pulse. The trick to break the classical diffraction limit is to impart a complex profile in the second depletion pulse. In Hell's original experimental demonstration, the donut profile was not created by a helical phase profile. Instead the researchers used an alternative method by retarding only the central portion of a beam by half a wavelength, so that at the focal plane the central region will be dark (owing to destructive interference of the beam with itself). The first pulse excites a large region. Next, the following depletion pulse will stimulate prompt fluorescence from the emitters in the donut-shaped region of the intensity, while the centre region is unaffected because the intensity decreases at the centre of the complex light beam. The excited fluorophores in the intensity null of the complex beam will go on to emit their characteristic fluorescence at later times (figure 4).

[4] The emission from the fluorophore has characteristic properties, such as emission wavelength and how long the emission lasts. The duration of the emission is known as the fluorescence lifetime. In the case of STED spectroscopy, it is the fluorescence lifetime that is exploited to achieve enhanced resolution.

The second component that leads to super-resolution, as shown by Mats G L Gustafsson [6], is the nonlinear dependence of the stimulated emission process, which further enhances radial contrast of the emission locations. The circular region and nonlinear effects taken together determine the fluorophore position to length scales less than 100 nm, well below the classical diffraction limit. With a vortex beam, the size of the intensity null in light with an azimuthal phase can be controlled with the value of the topological charge of the beam. This means that simply changing the phase profile of the complex light beam generates successive increases in resolution and spatial selectivity. It's not overstating things to note that a burgeoning field of cellular and subcellular imaging has emerged from a simple donut-shaped complex light beam.

Further improvements to STED incorporate the supercontinuum laser, a source of high-brightness and broadband coherent laser radiation. The combination of supercontinuum and complex light Laguerre–Gaussian beams shown by Henry Sztul and colleagues brought forward the possibility of multicolored STED experiments with precise phase control offered by the phase properties of the supercontinuum [7]. The different wavelength components of the supercontinuum are in phase (i.e. a white-light laser) and this further improves wavelength-selective STED microscopy with the intense, coherent field of a laser needed to efficiently drive the stimulated emission.

Species-selective STED is also possible using the supercontinuum, since different fluorophores can be addressed with different wavelengths in the spectral bandwidth of the laser. Further, the realization of compact supercontinuum laser sources with pulse durations in the picosecond range has made STED microscopes a commercially available tool that is now very much a cornerstone of biological imaging.

While STED makes use of the intensity profile of complex light, there are other imaging methods that make use of the more subtle phase properties of complex beams. The light from an object can be filtered by colour or by its polarization to produce different images. The object light can also be filtered by amplitude or phase, the latter providing the basis of so-called phase-contrast microscopy. The key question here: what happens when the phase filter is one that is used to generate complex light?

A helical phase mask results in the donut beam profile with diminishing intensity near the centre. If this phase mask is placed in the Fourier plane of a 4F optical system[5], then the object light rays near the optical axis between the two lenses will be reduced in intensity at the image plane. These rays near the optical axis at the Fourier plane are those with lower spatial frequencies, and so reducing their intensity in the image plane results in an image with edge-contrast enhancement. The process of edge enhancement is not new, similar to dark-field microscopy where

[5] A 4F system is a common imaging arrangement in optics. Two identical lenses are used each with focal length F, and they are separated by twice the focal length. If the object is placed at the focal length of one lens, the image will appear at the focus of the second lens. Then the total distance between object and image is 4F. The Fourier plane, which is the plane in between the two lenses, is a special region where the light field of the object is transformed into its Fourier spectrum and can be easily modified in phase space.

the background of the image is dark and the edges of objects are more apparent than other regions.

Dark-field images are traditionally obtained by a circular intensity mask and require bulky condenser lenses and modest light intensities (since much of the light is cut out by the amplitude mask). Edge contrast comes about because the annular region of light that falls on the object is comprised of higher spatial frequencies. Instead, using a complex light Fourier filter removes those components with lower spatial frequencies to achieve edge contrast. The fast digital control of the SLM allows a series of images to be obtained in rapid succession, each filtered with different complex phase profiles. Looking ahead, it will be intriguing to see what other powerful imaging capabilities emerge using complex-light phase filters.

Optical spanners and transfer of angular momentum

The interaction of light with matter is an intensively studied field of physical optics and is fundamental for many advanced photonic applications. Usually, classical mechanical effects are disregarded since the momentum of the photon is much less than the object. The momentum for any quantum particle is $p = h/\lambda$ (where h is the Planck constant = 6.626×10^{-34} J · s and λ the wavelength of the quantum particle). Since the wavelengths for visible light are much larger than molecular or quantum-mechanical length scales, the momentum of the photons can be orders of magnitude smaller than the material system under study. Under specific conditions, however, an intense light beam can mechanically trap small particles—a so-called 'optical tweezers' configuration.

A physical theory that describes how radiation pressure can be used to trap atoms was developed by Arthur Ashkin in the 1970s [8]; experimental demonstrations followed on a range of particle sizes in 1986 [9]. The researchers found that the trapping force is proportional to the spatial gradient in the electric field. For ordinary light, this gradient is negligible. However, the gradient around the tight focus of a laser can become large enough to stabilize a particle against external forces such as gravity and turbulence. Perhaps we could soon realize optical tractor beams with high enough powers and sufficient control of the waveforms. This could enable micro-, even possibly molecular-scale control for construction of complex electronic structures not achievable with current fabrication methods. Somewhere off in the distant future, one could even imagine manipulating large objects from a distance with tailored light fields.

Using structured light, which allows precise control of the field gradients and more complex arrangements, can undoubtedly enhance the optical trapping and manipulation process. For example, N B Simpson and colleagues calculated a Laguerre–Gaussian mode to have a greater trapping force than the fundamental mode beam (because of the annular intensity and focusing properties of the complex light beam) [10].

Perhaps more fundamental to the interaction of light with matter is the observation that complex light fields have the ability to rotate small particles depending on the value of the orbital angular momentum (or equivalently the azimuthal phase gradient of the Laguerre–Gaussian beams). Questions surrounding the angular momentum of light go back at least 100 years. Chandrasekhara Venkata Raman (best known for the discovery named after him in which scattered light is frequency-shifted because of vibrational absorptions in a material) pondered the physics of light's angular momentum in the 1930s. In 1936, Richard Beth demonstrated a mechanical torque exerted by polarized light on a doubly refracting plate, unequivocally establishing transfer of angular momentum of a light field to matter [1]. Furthermore, Beth found that each photon carried the precise unit of angular momentum of positive or negative h depending on the direction of polarization spin. This wonderful discovery of the quantum of photon spin connected the theory of quantum-mechanical angular momentum of particles and the angular momentum of light.

Much of the early work focused on the spin angular momentum of light, since complex light was not at that point an established field. Classical pictures require a centrosymmetric force in order to have orbital angular momentum, and since there is no such obvious force for light there was no clear distinction of the orbital versus spin angular momentum of light in the early experiments and theories.

Fast forward 50 years and the theory of orbital angular momentum of light emerged. An excellent review concerning the fundamental physics by Stephan Barnett and colleagues considers the distinctions between the spin angular momentum and the orbital angular momentum of light [11]. It seems that the spin angular momentum and orbital angular momentum are separate quantities with their own descriptions; both can cause rotations of particles and their rotational effects can be additive or subtractive.

Regardless of the final description of angular momentum of the photons, it has been demonstrated from a series of experiments that complex light fields with helical phase profiles can control the angular momentum of a particle. In 1992, Les Allen and co-workers showed that Laguerre–Gaussian modes have well defined orbital angular momentum and proposed experiments similar to Beth's work to observe a mechanical torque associated with the transfer of that momentum [12]. Shortly after, Hao He and colleagues experimentally demonstrated that a helically phased complex light beam causes rotation of small particles, and that the rotation can be controlled by the orientation of the helicity [13]. A review by Miles Padgett and Richard Bowman covers some of the more recent advances in optical tweezers with complex light and angular momentum [14].

Complex light propagation

We now move on to consider aspects of complex light propagation, acknowledging that the physics of electromagnetic waves has transformed every aspect of our daily lives over the past 100 years, from satellite communications to the humble wireless garage-door opener. Looking ahead, it's evident that the next phase of technology development in communications and computing will exploit more advanced properties of light transmission—i.e., speed, polarization and, ultimately, the exotic possibilities afforded by quantum entanglement.

Let's start with speed, though, and the opportunities that follow from scientists' ability to precisely manipulate and control the detection of complex light beams. Because a fundamental aspect of wireless communication is the propagation of the signal, we will review some of the new possibilities of how signal propagation and detection can be altered with complex light.

The invention of the laser in 1960 gave scientists the ability to study light's properties to extreme precision and also in a variety of conditions that were not previously possible. The coherence of laser light, for example, allows one to look at very precise phase relationships in a light field that were not easily accessible with incoherent sources such as lamps. Furthermore, the laser cavity is the natural environment for the paraxial approximation (as described previously), so well defined cavity spatial modes of light could be generated easily with high power and coherence.

As laser science advanced to produce shorter and shorter pulses of light, the finer details of light's propagation came under closer scrutiny. A more detailed description of light propagation shows that there are at least two speeds to consider in regards to the propagation of the wave. The first is the phase velocity, representing the speed at which the wave fronts move in the medium, equal to c divided by the refractive index of the medium in question (which could be wavelength-dependent). This is the ratio of the spatial and temporal terms in the oscillating phase. The second speed is the group velocity, equal to the derivative of the frequency with respect to the wave vector. The group velocity is the speed at which the energy (and thus the signal) propagate.

For a pulse, one can consider the group velocity as the speed at which the overall pulse shape propagates, while the phase velocity represents the motion of the wave fronts within the pulse shape. Inside a material, dispersion means that the index of refraction is dependent on wavelength, such that the group velocities and phase velocities can be different from each other and can deviate from c. The regions of normal dispersion lead to group velocities reduced relative to c and one can observe a retardation of the light pulse due to an index of refraction, n, greater than 1. For large enough dispersion, the light can be slowed considerably and in some special cases scientists have almost achieved 'stopped' light.

Within frequency ranges where there is anomalous dispersion, usually near an absorption band, the effective index of refraction is less than one, which can lead to 'fast' light. Because these effects of fast and slow light depend on the material dispersion, there is not expected to be any difference in the speed of propagation through free space—i.e. the group and phase velocity of the light are both c.

This idea, however, is now being reconsidered for complex light fields. Since the wave front is defined as the tangent surface to the propagation vector, complex wave fronts may modify the propagation vector, essentially spreading the light's momentum over a range of directions instead of a strictly linear ray-like trajectory. In 2015, Daniel Giovanni and colleagues reported a signal arrival time of structured light that implied a speed of signal propagation less than c in free space [15]. The 'subluminal' speed, which is attributed to the transverse spatial structure of the beam, was observed both for Gaussian and for Bessel structured beams. Indeed, solving Maxwell's equations for the unphysical situation of an infinite plane wave gives the expected value for the speed of light.

For a physical beam which must have some finite transverse size, the complete description of the light field must include transverse wave vectors. It is this transverse momentum, which is not along the propagation direction, that reduces the component of the momentum along the propagation and thus reduces the speed of the light pulse. This theory was supported by further experiments that modified the transverse profile of the light, showing that the signal arrival time could be changed controllably by the beam's transverse profile. Additional theoretical studies, published earlier this year by Robert Alfano and Dan Nolan, found that any amount of transverse wave vector of the beam reduces the longitudinal group velocity of the light [16].

Also this year, Frédéric Bouchard and colleagues published a study of the propagation speed of Laguerre–Gaussian modes and Gaussian modes [17]. The calculations for the entire family of Laguerre–Gaussian modes related the speed of propagation to the mode indices l and p. The model predicts speeds different from c are found mainly within the Rayleigh length of the beam, and higher-order p modes would have a group velocity above c within the Rayleigh length, but less than c beyond the Rayleigh length (to give a final speed less than c over longer distance). Experiments in a non-collinear geometry showed a slowing comparable to the predicted values. A similar study by Nestor Bareza and Nathaniel Hermosa regarding Laguerre–Gaussian beams also found a delay in the arrival of the complex light beam. This was explained by the complex wave vector adding extra path length to the propagation distance [18].

It is important to recall that an infinite plane wave is a mathematical idealization but not a physical reality and so any light source will have a finite transverse spatial profile. Applying classical Fourier analysis leads to a non-zero distribution of wave vectors in directions non-parallel to the initial propagation direction, which essentially is the self-diffraction of a light beam.

Consequently, according to these recent models of light propagation, any finite-sized light beam will have a group velocity different from c. Yet these experiments on the light's group velocity reference the light beam propagation to itself and thus are not absolute measures of the speed of light. Nevertheless, it seems to be the case that the signal detection time can be controllably modified by structuring the light field. As data rates become faster, and channels become denser with information, it is therefore critical and advantageous to understand the smallest details of signal propagation and detection.

Data transfer with complex light

The seemingly strange propagation properties of complex light beams notwithstanding, there are straightforward advances shaping up in regards to data transfer rates of optical signals. Specifically, the orthogonal spatial modes of complex light fields are expected to offer much higher transmission rates if orthogonal modes can be used as independent channels of communication along the same optical transmission

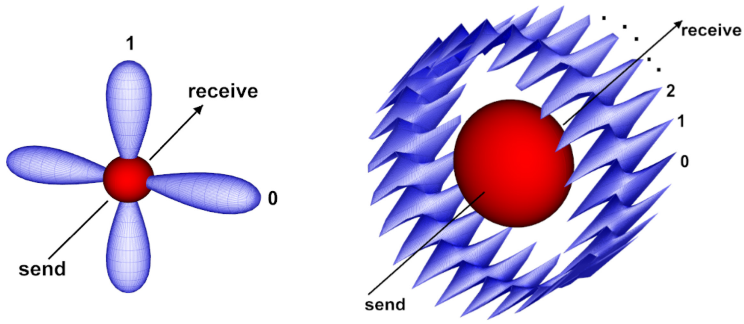


Figure 5. Classical optical communication depends on orthogonal polarizations and gives a simple binary encoding of the photon. With complex light, there are potentially an infinite number of orthogonal states, which means a much higher data content can be encoded in the photon.

line (figure 5). For example, classical polarization-dependent information transfer is limited by the binary orthogonal states of polarization of a light field, which are physically separated by a polarizer to decode the light pulse into its information. Complex light fields, on the other hand, have an infinite number of orthogonal states, pointing the way to data transfer speeds in the petabit/s (10^{15} bit/s) bandwidth regime and beyond.

A recent review by Alan Willner and colleagues [19] highlights the major engineering issues that need to be addressed in order for optical data transmission with orbital angular momentum to achieve these ultrafast data-transfer rates. One of the main hurdles is the efficiency of multiplexing and demultiplexing the complex light states at the send and receive end of the optical transmission line. Classical optical elements for polarization and intensity are well established in optical communication schemes. One method of combining several modes into a single channel uses a cascade of beam splitters. The drawback to this approach, however, is the reduction in intensity every time a new channel is added (because of an additional beam splitter), so this method is not practical for combining large numbers of different signals. What this means is that complex light fields will require alternative technologies to sort the signal based on the new complex mode structures.

One option to demultiplex the signal is to use a hologram that generates the complex light field, but in reverse. The snag is that this approach has similar drawbacks to beam splitters, in that it can give poor efficiency and a loss of power into other unintended modes generated by the hologram (though wavelength correction factors and specially designed holographic patterns can increase the efficiency in some cases).

Another option to discriminate the modes is the use of Q-plates. This is particularly appealing since the Q-plate can be dropped into existing systems without major modifications to alignment. Yet another new technology, developed by Miles Padgett's group, employs a log-polar optical transformation of the light to sort angular momentum modes in a spatial direction, similar to the way a grating separates wavelengths [20]. This sorter can be effective both for combining and sorting the angular momentum modes up to very high order, which allows data transfer over many simultaneous channels.

The other major challenge facing complex light data transfer is the efficiency of conventional optical fibres in faithfully transmitting the complex modes. Fundamentally, the data signal will be compromised if the mode structure is not preserved on the receiving end of the fibre. Conventional high-bandwidth optical fibres are normally designed for singlemode use (such as the TEM_{00} mode, though not necessarily this mode), while multimode fibres with a larger core can support many modes (though the cross-talk between the modes is an issue for long fibre paths). New fibre geometries and technologies are currently being developed to faithfully transmit orbital angular momentum modes.

Quantum entanglement of complex modes

Quantum entanglement of photons is a physical realization of the Schrödinger cat paradox (or more formally quantum non-locality). The basis of an optical entanglement experiment is that two photons, generated from the exact same source, contain information between the two photons even after they are separated in space. This is the 'spooky action at a distance' that Einstein and many other scientists have pondered over the past 100 years.

Experimentally, entangled states are often measured by a Hong–Ou–Mandel (HOM) interferometric technique (figure 6). When the conditions are set for maximum indistinguishability of the entangled particles, there is a marked dip in the coincidence counts in the interferometer's separate detection arms, the so-called the HOM dip.

Entanglement measurements generally require extreme precision and single-photon detectors. And while still in the early days of experimentation, this

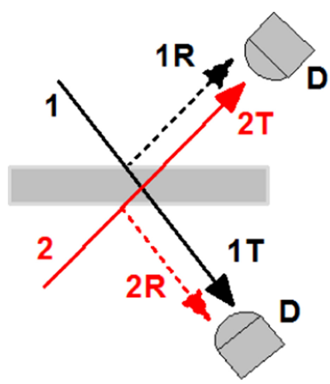


Figure 6. Schematic description of the Hong–Ou–Mandel (HOM) interferometer. When a photon wave-function is sent into a beamsplitter from two different possible directions (1 and 2), there are four possible outcomes. Either both are reflected, both are transmitted, or one is transmitted and the other reflected. When the beamsplitter is perfectly symmetric, the outcomes are indistinguishable and the two detectors will not record a coincidence detection. If the beamsplitter is not symmetric, the two detectors can produce a coincidence signal.

quantum-mechanical phenomenon of information entanglement has brought with it a new method of computation using quantum entangled bits (dubbed qubits), where the computational power increases dramatically since each logical element can take on simultaneous states.

There is a vast body of research in progress on entanglement and quantum computing, but here we will restrict ourselves to discuss aspects of the research effort that are specific to complex light. Complex light is appealing in quantum entanglement experiments for the same reason that it increases data transmission rates: using complex light can multiply the number of orthogonal states available for entanglement phenomena.

Some of the first demonstrations of quantum entanglement used the orthogonal states of polarization of the photon. One of the pioneering groups in entanglement research, led by Anton Zeilinger, demonstrated entangled photons with orbital angular momentum in 2001 [21]. By generating orbital angular momentum in one of the photons in an entangled pair, they found that the second photon carried with it the angular momentum information of the other. This is an essential first step towards using higher-dimensional entangled states for quantum information processing because it makes clear that orthogonal complex spatial profiles can be maintained through quantum entanglement processes.

But not only can the entanglement be spatially separated, it can be mapped out in space, and also has finite-sized entanglement volumes. The phase singularity in a Laguerre–Gaussian mode is found to be a conserved entity throughout the 3D light field, and these topological features have consequences for quantum entanglement.

In 2011, using their theory on optical vortex knots, Jacquiline Romero and colleagues experimentally mapped out the entanglement of vortex knots in a volume of 3D space over (relatively) large length scales [22], showing that the entanglement has particular topographic features that can extend over finite sizes. How this finding can be applied to finite-sized material systems is an intriguing possibility as it could lead to the control of macroscopic quantum-entangled states in matter, a possible route to quantum computation and instantaneous communications.

Outlook

The topological properties of the electromagnetic field are opening up a wealth of new possibilities in fundamental and applied optical science. For the most part light has, up until the development of complex light theories, been treated as a two-dimensional field and a third propagation direction. Now, though, the spatial phase and polarization structures require a broader set of parameters to completely describe complex light in three (space) + one (time) dimensions.

One of the most striking features of some forms of complex light is the orbital angular momentum associated with a helical phase distribution of the beam profile, manifesting as a helicoidal phase-front surface. These types of light beams have already found use in micromechanical particle traps, advanced biological imaging schemes and high-data-rate optical communication experiments. At the same time, the more abstract features of complex light are being studied in the context of quantum communication and computing, since the higher-dimensional description of the complex beams versus classical linear polarized light affords many more options for quantum entanglement and computing.

Going forward, it's inevitable that further ground-breaking advances will emerge from the fundamental interactions of matter with complex light. While not covered in this book, there are a number of complex light effects specific to so-called metamaterials (a class of engineered material with properties that don't occur naturally). Those effects show promise for controlling nanoscale light propagation and in turn point toward a new type of complex light spectroscopy [22].

Elsewhere, researchers are pursuing promising lines of enquiry by applying complex light to fields as diverse as atomtronics (a branch of ultracold atom physics that seeks to implement analogues of electronic circuits in superfluid systems), structured electron-matter waves (targeting new capabilities for electron microscopy and spectroscopy), and optical metrology (including studies of chiral molecules and fluid vortices) [23, 24]. Additional research on beautiful and exotic flower-like localizations [25] and OAM lasers [26] offer more versatility in complex light generation and applications.

For complex light, it seems, the future's bright.

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