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Department of Applied Mathematics and Mathematical Physics, University College, Cardiff

Recombination in Semiconductors by a Light Hole Auger Transition

By

A. R. BEATTIE and G. SMITH

The transition rates for an Auger collision process in semiconductors which involves the light hole band is calculated using a quantum mechanical perturbation method. Spherical energy surfaces are assumed although non parabolic energy bands are allowed for. The temperature dependence of the lifetime of excess carriers due to this process is investigated and the results applied to InSb and InAs in the temperature range 200 to 500 °K. The shape of the temperature dependence of this theoretical lifetime for InSb agrees well with experiment at room temperature and above, and when estimates of overlap parameters which occur in the theory are made the absolute magnitude of the lifetime also agrees with experiment. The probability per unit time that a light hole created by a photon of energy $h\nu$ will take part in an impact ionizing transition is also given as a function of $h\nu$. It is concluded that the transition rate for this process is at least comparable to that of the more usual Auger transitions involving the heavy hole and conduction bands only.

Die Übergangsraten für einen Augerkollisionsprozeß in Halbleitern werden unter Einbeziehung der Energiebänder leichter Löcher mit einer quantenmechanischen Störungsrechnung berechnet. Es werden sphärische Energieflächen angenommen, obwohl auch nichtparabolische Energiebänder erlaubt sind. Die Temperaturabhängigkeit der Lebensdauer der durch diesen Prozeß erzeugten überschüssigen Ladungsträger wird untersucht und die Ergebnisse werden auf InSb und InAs im Temperaturbereich 200 bis 500 °K angewendet. Die Temperaturabhängigkeit dieser theoretischen Lebensdauer stimmt für InSb bei Zimmertemperatur und darüber gut mit dem Experiment überein. Wenn die Überlappungsparameter der benutzten Theorie abgeschätzt werden, stimmt die absolute Größe der Lebensdauer ebenfalls mit dem Experiment überein. Die Wahrscheinlichkeit pro Zeiteinheit, daß ein leichtes Loch, durch ein Photon der Energie $h\nu$ erzeugt, an einem Stoßionisationsübergang teilnimmt, wird als Funktion von $h\nu$ gegeben. Es wird geschlossen, daß die Übergangsrate für diesen Prozeß mindestens vergleichbar ist mit dem gebräuchlicheren Augerübergang, an dem nur das Band der schweren Löcher und das Leitungsband beteiligt sind.

1. Introduction

If the conduction, heavy hole, and light hole bands in semiconductors are considered, there are ten Auger transitions which lead to recombination, or, if the reverse processes are considered, to impact ionization [1]. A theory for the two transitions involving the conduction and heavy hole bands only has been developed by Beattie and Landsberg [2, 3] who applied it to the problem of recombination of excess carriers. The theory for the transitions shown in Fig. 1 is now developed. In Fig. 1a two electrons in the heavy hole band interact via their Coulomb repulsion so that one electron drops into a hole in the light hole band while the other goes into the conduction band, conserving energy and

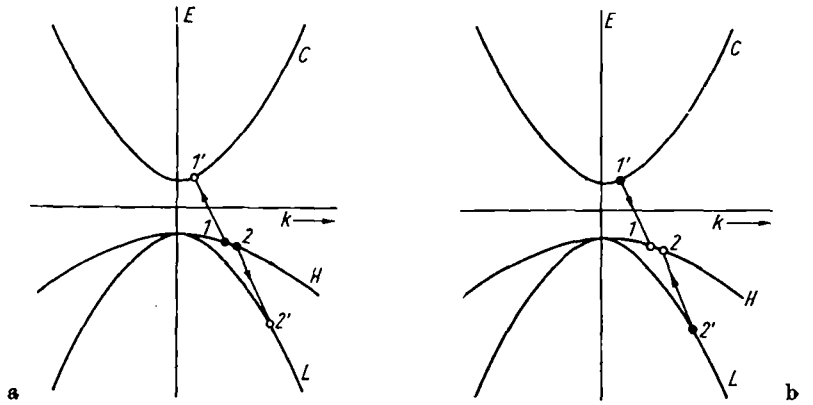

Fig. 1. Auger transitions involving the light hole band for a) recombination and b) impact ionization are shown. Curves C, H, L represent the conduction, heavy hole, and light hole bands, respectively. The initial and final states are 1, 2 and 1', 2', respectively

momentum in the process. Fig. 1b shows the reverse of Fig. 1a and leads to recombination.

The processes of Fig. 1 have been considered because their possible importance has been suggested by measurements of the quantum efficiency of incident photons in InSb [4, 5]. When Beattie [1] examined the threshold energies of all ten possible impact ionization transitions, he found that only two would be likely to cause a quantum efficiency greater than unity in the energy range of interest. These were the electron collision process of Beattie and Landsberg and the transition of Fig. 1a. In Tauc's [4] measurement of the quantum efficiency in InSb as a function of photon energy the two threshold energies which appeared agreed well with the theoretical values and, although the experimental error was such that there was some doubt about the existence of the second threshold (Antoncik [6]), its presence has since been confirmed by Nasledov [5]. Although the threshold photon energies are different for the two processes the threshold kinetic energies of the ionizing carriers are almost the same ($1.069\times$energy gap for the Beattie-Landsberg transition and $1.073\times$energy gap for the present transition). Thus, if the transition of Fig. 1a is important in determining the quantum efficiency, then, the transitions of Fig. 1 should also be important in the recombination of excess carriers. The calculation of recombination lifetimes is preferred to a direct calculation of quantum efficiency which entails not only considering the impact ionization transitions but also all other energy loss mechanisms.

In calculating the transition probability in Section 2 a quantum mechanical perturbation treatment [2] is followed. The net transition rate, the difference between the recombination and impact ionization rates, is found for small departures from equilibrium by integrating over all allowed states the transition probability multiplied by the appropriate probabilities of occupancy and vacancy of the initial and final states. The overlap integrals which occur in the theory are discussed in Section 3 where their functional dependence on $\boldsymbol{k}$-vectors is given for small wave vector differences, and estimates of an overlap parameter are obtained by three methods which, in the case of InSb, agree within an order

of magnitude. In the numerical work of Section 4, where the theory is used to evaluate intrinsic carrier lifetimes in InSb and InAs and to give the probability per unit time that a light hole of energy E will impact ionize, it is assumed that transitions between light and heavy holes are sufficiently rapid to permit the use of the same quasi-Fermi level for these two bands.

2. Transition Rates and Lifetimes

In calculating the net transition rate for the processes of Fig. 1 Bloch wave functions are used in the quantum mechanical perturbation treatment of [2]. Umklapp processes are neglected and the assumptions of spherical energy surfaces, quasi-Fermi levels and non-degenerate semiconductors are made. However, the possibility of non-parabolic energy bands is taken into account. The net transition rate per unit volume follows directly from equation (3.6) of [2] and is given by

$$R = \frac{e^4 t}{8 \hbar^2 \varepsilon^2 \pi^7} (\delta F_C + \delta F_L - 2\,\delta F_H) \iiint \theta(1, 1', 2, 2') \times$$
$$\times \left\{ \left| \frac{F(1,1')\,F(2,2')}{\lambda^2 + g^2} \right|^2 + \left| \frac{F(1, 2')\,F(2, 1')}{\lambda^2 + l^2} \right|^2 + \left| \frac{F(1, 1')\,F(2, 2')}{\lambda^2 + g^2} - \frac{F(1, 2')\,F(2, 1')}{\lambda^2 + l^2} \right|^2 \right\} \times$$
$$\times \frac{1 - \cos x}{x^2}\, d\boldsymbol{k}_1\, d\boldsymbol{k}_2\, d\boldsymbol{k}_2' , \tag{1}$$

where e is the electronic charge, t is time, $\hbar$ is Dirac's constant, ε is the dielectric constant, δF_i is the departure of the quasi-Fermi level from its equilibrium value for the i-th band, subscripts C, H, L refer to the conduction, heavy hole, and light hole bands, θ (1, 1′, 2, 2′) is the appropriate probability of occupancy or vacancy of states 1, 1′, 2, 2′, $\boldsymbol{g} = \boldsymbol{k}_1 - \boldsymbol{k}_1'$, $\boldsymbol{l} = \boldsymbol{k}_2 - \boldsymbol{k}_1'$, where

$$x = \frac{t}{\hbar} |E_1 + E_2 - E_1' - E_2'| \tag{2a}$$

and where

$$F(i, 1') = \int u_H^*(\boldsymbol{k}_i, \boldsymbol{r})\, u_C(\boldsymbol{k}_{1'}, \boldsymbol{r})\, d\boldsymbol{r}$$

and

$$F(i, 2') = \int u_H^*(\boldsymbol{k}_i, \boldsymbol{r})\, u_L(\boldsymbol{k}_{2'}, \boldsymbol{r})\, d\boldsymbol{r} \tag{2b}$$

are overlap integrals of the modulating parts of Bloch functions. The neglect of the screening parameter λ in (1) results in a slight overestimate of the transition rate. This effect, however, is small since the screening length in non-degenerate semiconductors is large and the correction to the Auger lifetime due to screening was found in [2] to be no more than 5%.

The overlap functions which are discussed in Section 3 are taken to be of the form

$$|F(i, j)|^2 = \alpha_{ij}\, |\boldsymbol{k}_i - \boldsymbol{k}_j|^2 / (E_i - E_j) , \tag{3}$$

where E_i is the energy of the i-th state and α_{ij} is a parameter. Since the α_{ij} factors are slowly varying in comparison with the rest of the integrand they may be removed from the integral of (1) and evaluated for the most probable transition. For the most probable transition $\boldsymbol{k}_1 = \boldsymbol{k}_2$ (see [1]), and hence

$\alpha_{11'} = \alpha_{21'}$ and $\alpha_{22'} = \alpha_{12'}$ so that with (3) the net transition rate of (1) becomes

$$R = \frac{\alpha_{11'}\,\alpha_{22'}\,e^4\,t}{4\,\pi^7\,\varepsilon^2\,E_g^2\,\hbar^2}(\delta F_C + \delta F_L - 2\,\delta F_H)\exp(\eta_v - F_0) \times$$
$$\times \iiint \exp\{-(E_v - E_{2'})/kT\}\frac{1-\cos x}{x^2}\,d\boldsymbol{k}_1\,d\boldsymbol{k}_2\,d\boldsymbol{k}_{2'}\,. \qquad (4)$$

It is convenient to write the band structure of the three bands in the form

$$E_{1'} - E_C = \frac{\hbar^2 k_{1'}^2}{2\,m_C}\,, \qquad E_v - E_2 = \frac{\hbar^2 k_2^2}{2\,m_H}\,, \qquad E_v - E_{2'} = \frac{\hbar^2 k_{2'}^2}{2\,m_L}\,, \qquad (5)$$

where for non-parabolic bands the effective masses m_C, m_H, and m_L are, in general, energy dependent. However, since states, 1, 1′ and 2 of Fig. 1 are near their respective band edges their effective masses are taken to be constant with values m_{C0} and m_{H0}.

It is also convenient to define new variables of integration, $\boldsymbol{h}$,$\boldsymbol{j}$, and $\boldsymbol{k}_{2'}$, where

$$\boldsymbol{h} = \boldsymbol{k}_2 - \boldsymbol{k}_1\,, \qquad \boldsymbol{j} = (1 + \mu_H/2)^{1/2}(\boldsymbol{k}_1 + \boldsymbol{k}_2) - (1 + \mu_H/2)^{-1/2}\,\boldsymbol{k}_{2'} \qquad (6)$$

so that (2a) becomes

$$x = \frac{t\,\hbar}{m_{C0}}\left(a\,k_{2'}^2 - j^2 - \frac{1}{2}\,\mu_H\,h^2 - k_g^2\right), \qquad (7)$$

where

$$a = \mu_L - 1 + (1 + \mu_H/2)^{-1}\,; \qquad \mu_L = m_{C0}/m_L \quad \text{and} \quad \mu_H = m_{C0}/m_{H0}\,. \qquad (8)$$

Thus with the use of (4), (5), (6), and (7) and when the changes in Fermi level are related to their excess carrier densities δn, δp_H, δp_L and corresponding thermal equilibrium values n_0, p_{H0}, p_{L0}, the recombination rate becomes

$$R = \frac{8\,e^4\,m_{C0}^{3/2}\,\alpha_{11'}\,\alpha_{22'}}{m_L^2\,\pi^{1/2}\,\varepsilon^2\,\hbar^7\,a^2(E_T)\,(2 + \mu_{H0})^{3/2}}\,\eta_T^{5/2}\,\frac{\delta n\,p_{H0}}{n_0}\left[1 + \frac{2\,n_0\,\delta p_H}{p_{H0}\,\delta n} - \frac{n_0\,\delta p_L}{p_{L0}\,\delta n}\right] I(\eta_T)\,, \qquad (9)$$

where

$$I(\eta_T) = \int_1^\infty e^{-\eta_T y}\,a^2\,m_L^{5/2}\left(m_L + y\,\frac{dm_L}{dy}\right) y^{1/2}\,(y-1)^2\,dy \qquad (10)$$

and where $E_T\,(= \eta_T\,k\,T)$ is the threshold energy which is related to the energy gap E_g by solving the equation

$$E_T = \left[\frac{2\,m_{H0} + m_{C0}}{2\,m_{H0} + m_{C0} - m_L(E_T)}\right] E_g\,. \qquad (11)$$

The integral of (10) can be computed numerically. However, for $\eta_T \gg 1$ the exponential term in (10) ensures that there is a sharp maximum in the integrand so that slowly varying functions may be evaluated at this maximum y_m and removed from the integral. Thus (10) may be approximated to

$$I(\eta_T) = \left\{a^2\,m_L^{5/2}\left(m_L + y\,\frac{dm_L}{dy}\right)\right\}_{y=y_m}[I_2(\eta_T) - 2\,I_1(\eta_T) + I_0(\eta_T)]\,, \qquad (12)$$

where the $I_n(\eta_T)$ are related to incomplete gamma functions and are defined by

$$I_n(\eta_T) = \eta_T^{-(n+3/2)}\int_{\eta_T}^\infty e^{-z}\,z^{n+1/2}\,dz\,.$$

The lifetime of excess carriers which is given by $\tau = \delta n/R$ is then

$$\tau = D(E_T)\left\{1 + \frac{2\,n_0\,\delta p_H}{p_{H0}\,\delta n} - \frac{\delta p_L\,n_0}{\delta n\,p_{L0}}\right\}^{-1} \frac{\eta_T^{-3/2}}{I(\eta_T)}, \tag{13}$$

where

$$D(E_T) = \frac{\pi^{1/2}\,e^2\,\hbar^7\,m_L^2(E_T)\,a^2(E_T)\,(2+\mu_{H0})^{3/2}\,n_0}{8\,m_{C0}^{3/2}\,\alpha_{11'}\,\alpha_{22'}\,e^4\,p_{H0}}. \tag{14}$$

In the case of an intrinsic semiconductor where the transition rate between the light and heavy hole bands is sufficiently great that they may be considered to have the same quasi-Fermi level, then (13) reduces to

$$\tau = \frac{D(E_T)\,\eta_T^{3/2}}{2\,I(\eta_T)} \tag{15}$$

and when asymptotic expressions for the integrals in (12) are used (15) reduces to

$$\tau = \frac{D(E_T)\,e^{\eta_T}\,\eta_T^{3/2}}{4\left\{a^2\,m_L^{5/2}\left(m_L + y\frac{dm_L}{dy}\right)\right\}_{y=y_m}}\left[1 + \frac{3}{2\,\eta_T} + \frac{3}{2\,\eta_T^2} + \cdots\right]^{-1}. \tag{16}$$

3. Overlap Functions

The overlap functions of the modulating parts of Bloch functions F_{ij} which occur in the matrix element of (1) and for which the states are in different bands, may be related to the matrix elements of the momentum operator [9, 10] by the equation

$$|F(i\,j)|^2 = \alpha_{ij}\,g^2/(E_i - E_j), \tag{17}$$

where

$$\alpha_{ij} = \left(\frac{\hbar}{m}\right)^2 \frac{|\langle i|\,\boldsymbol{e}\cdot\boldsymbol{p}|\,j\rangle|^2}{(E_i'' - E_j'')^2}\,(E_i - E_j), \tag{18}$$

where $E_i'' = E_i - \hbar^2\,k_i^2/2\,m$, $\boldsymbol{e}$ is the unit vector in the direction of $\boldsymbol{g}$ $(= \boldsymbol{k}_i - \boldsymbol{k}_j)$, m is the free electron mass, and $\boldsymbol{p}$ is the momentum operator. Thus to the first order in g^2 the overlap functions are obtained by evaluating α_{ij} for $\boldsymbol{k}_i = \boldsymbol{k}_j$, so that

$$\alpha_{ij} = \left(\frac{\hbar}{m}\right)^2 \frac{|\langle i|\,\boldsymbol{e}\cdot\boldsymbol{p}|j\rangle|^2}{E_i - E_j}; \qquad (\boldsymbol{k}_i = \boldsymbol{k}_j). \tag{19}$$

Three ways of estimating values for α_{ij} are now considered.

3.1 Effective mass sum rule[1])

For spherical energy surfaces the effective mass sum rule [12] may be written in terms of α_{ij} as

$$\sum_{j\neq i} \alpha_{ij} = \frac{\hbar^2}{2\,m}\left(\frac{m}{m_i^*} - 1\right), \tag{20}$$

where

$$\frac{1}{m_i^*} = \hbar^{-2}\frac{\partial^2 E_i}{\partial k^2}.$$

[1]) See Antončik and Landsberg [9].

Although the summation is over all bands at the same wave-vector it is usually possible to include in the sum (20) only those bands which are closest in energy to the ones of interest. However, it should be noted that, if (20) is summed over all i, then, since $\alpha_{ij} = -\alpha_{ji}$,

$$\sum \left(\frac{m}{m_i^*} - 1\right) = 0 \tag{21}$$

and sufficient numbers of bands should be included so that (21) is at least approximately satisfied. Even when only the conduction, heavy hole, light hole, and split-off bands are taken into account, further approximation must be made. If, in general, the states which are closest in energy have the greatest value of overlap, then reasonable approximations[2])
are a) near $k = 0$:

$$\alpha_{CH} = \alpha_{CL} \gg \alpha_{CS}$$

so that

$$\alpha_{CH} = \frac{\hbar^2}{4m}\left(\frac{m}{m_C^*} - 1\right) \tag{22}$$

and b) for large k:

$$\alpha_{LS} \gg \alpha_{CS}\,, \quad \alpha_{HS} \text{ and } \alpha_{HL} \gg \alpha_{CL}$$

so that

$$\alpha_{HL} = -\frac{\hbar^2}{2m}\left(\frac{m}{m_L^*} - 1\right) - \frac{\hbar^2}{2m}\left(\frac{m}{m_S^*} - 1\right). \tag{23}$$

Equation (22) is used for α_{CH} and (23) for α_{HL}.

3.2 Optical absorption

The optical absorption constant for transitions between bands i and j at a wave vector $\boldsymbol{k}$ is given by [13, 14] as

$$a_{ij} = \frac{4\,e^2\,\hbar^2\,M_{ij}^2\,k^2}{m^2\,c\,n\,(E_i - E_j)}\left|\frac{dE_i}{dk} - \frac{dE_j}{dk}\right|^{-1}\theta(i)\,\theta(j)\,, \tag{24}$$

where α_{ij} is the optical absorption coefficient, M_{ij}^2 is the square of the optical matrix element averaged over direction, $\theta(i)$ is the probability of vacancy of state i and $\theta(j)$ is the probability of occupation of state j, c is the speed of light and n is the refractive index. Thus from (19) and (24)

$$\alpha_{ij} = \frac{\hbar\,c\,n\left|\frac{dE_i}{dk} - \frac{dE_j}{dk}\right| a_{ij}}{4\,e^2\,k^2\,\theta(i)\,\theta(j)}\,. \tag{25}$$

3.3 Band structure calculations

If the wave functions for III—V compounds (equation (14) of [11]) are used the α_{ij} may be evaluated directly. When one of the states is in the heavy hole band the result is

$$|\alpha_{iH}| = \left|\frac{2\,Q^2\,\Phi(E_i')}{9\,[E_i' - \hbar^2\,k^2/2\,m]}\right|, \tag{26}$$

where

$$\Phi(E_i') = \frac{E_i'\,(E_i' + \Delta)\,(E_i' + 2\,\Delta/3)}{[2\,E_i'^3 + (3\,\overline{\Delta - E_g})\,E_i'^2 + (4\,\Delta/3)\,(\Delta - E_g)\,E_i' - 2\,\overline{\Delta^2 E_g/3}]}\,. \tag{27}$$

[2]) Different approximation to that of Antončik and Landsberg [9] are used.

E_i' is a solution of the equation

$$E_i' (E_i' - E_g) (E_i' + \Delta) - k^2 Q^2 (E_i' + 2\Delta/3) = 0 \quad (E_C' > E_L' > E_S') . \quad (27a)$$

Δ is the spin–orbit splitting
and

$$Q^2 = \frac{3 E_g \hbar^2}{4 m} \left[1 + \frac{m}{m_{L0}}\right] . \quad (27b)$$

Strictly speaking m_{L0} in (27b) should be the effective mass corresponding to E_L' at $k = 0$. Thus, $\alpha_{11'}$ is evaluated from (26) at $\boldsymbol{k}_1 = 0$ and $\alpha_{22'}$ at $E_2' = E_T$.

There are difficulties in connection with all three methods. In Section 3.1 qualitative assumptions about relative magnitudes of the α_{ij} have to be made, in Section 3.2 the experimental optical absorption represents the total absorption and experiments have to be found where the transitions of interest are the dominant ones, and in Section 3.3 since a variational approach is used, the wave functions, although accurate at $k = 0$, may not be accurate for $k \neq 0$. However, all three methods should give an order of magnitude estimate for the overlap parameters.

4. Results

The general theory of Section 2 is applied to InSb and InAs. For III—V compounds the energies of the conduction, light hole, and split-off bands are given by the roots of the cubic equation (27a) so that

$$E_i - E_v = \frac{\hbar^2 k_i^2}{2 m} + E_i' + \delta(E_i') , \quad (28)$$

where $\delta(E_i')$ is a correction term due to interaction with bands other than the conduction, heavy hole, light hole, and split-off bands.

For the conduction, light hole, and split-off bands $\delta(E_i')$ is unimportant in InSb, but it is important in InAs. The results given below are appropriate for InSb where $\delta(E_i')$ is neglected but for InAs the correction term is taken into account. From (27a) the effective mass m_L defined in (5) is given by

$$\left(\frac{m}{m_L} - 1\right) = \left(\frac{m}{m_{L0}} - 1\right)\left(1 - \frac{E_L'}{E_g}\right)^{-1} (\Delta + E_L')^{-1} (\Delta + 3 E_L'/2) , \quad (29)$$

and when

$$m_L \ll m , \quad \Delta \gg |E_L'| ,$$

then

$$m_L = m_{L0} \{1 + (E_V - E_L)/E_g\} . \quad (30)$$

With the use of (29) or (30) the threshold energy is calculated from (11) and the lifetime is found from (15). The effective mass m_L^* is obtained from (27a) by finding the second derivative of the energy with respect to k. Since the conduction, heavy hole, light hole, and split-off bands only are included in the sum rules determination of α_{ij}, the correction term $\delta(E_i')$ is also omitted in finding m_L^* for InAs.

The material parameters which were used are summarized in Table 1, and the values of the overlap parameters corresponding to the threshold transition are given in Table 2. The total optical absorption near the band edge includes not

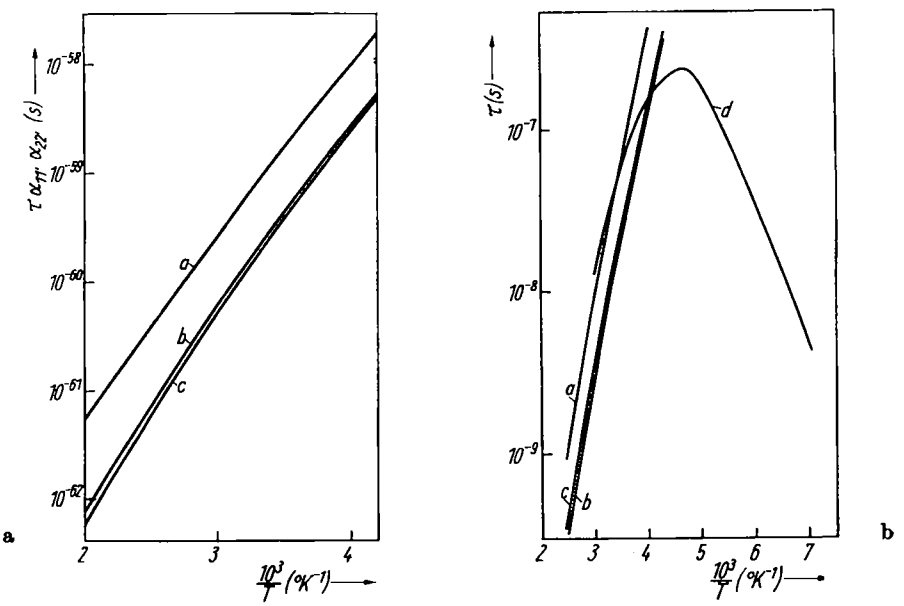

Fig. 2. Lifetimes for intrinsic InSb based on the theory of Section 2 are plotted as functions of inverse temperature. In a) where the lifetimes are multiplied by the overlap parameters, curve (a) is based on the assumption of parabolic bands, curve (b) represents the approximation to the non-parabolic case, and curve (c) gives the result using the computed integral. In b) the overlap parameters have been estimated by (a) sum rules, (b) optical absorption, and (c) band structure calculations. Curve (d) shows the experimental lifetimes from Zitter et al. [8]

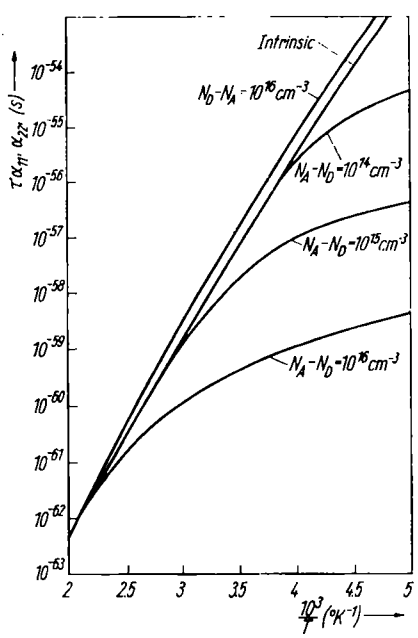

Fig. 3. Theoretical lifetimes multiplied by the overlap parameters for InAs are plotted as functions of inverse temperature. The donors and acceptors are assumed to be completely ionized

Fig. 4. The probability per unit time that a light hole of wave vector k will impact ionize is plotted as a function of the photon energy required to produce this light hole

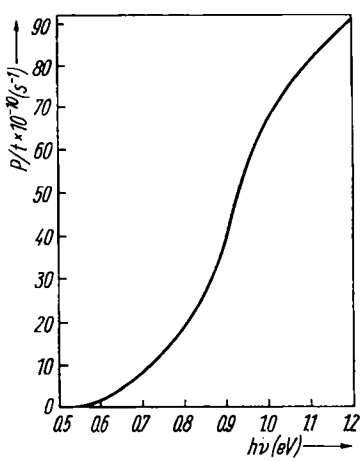

Table 1
Material parameters for InSb and InAs

Parameter	InSb	InAs
E_g	$(0.24-0.26\times10^{-3}\ T)$ eV	$(0.426-0.22\times10^{-3}\ T)$ eV
E_T	$1.073\ E_g$	$1.303\ E_g$
m_{C0}	$0.0114\ m$	$0.026\ m$
m_{H0}	$0.18\ m$	$0.4\ m$
m_{L0}	$0.0117\ m$	$0.025\ m$
ε	16.8	10.9
n	4.1	

Table 2
Overlap parameters for the threshold transition

	Sum rules	Optical absorption	Direct evaluation from wave function
InSb α_{CH}	2.57×10^{-26}	2.83×10^{-26}	2.42×10^{-26}
α_{HL}	0.24×10^{-26}	0.78×10^{-26}	0.71×10^{-26}
$\alpha_{CH}\alpha_{HL}$	0.62×10^{-52}	2.21×10^{-52}	1.71×10^{-52}
InAs α_{CH}	1.42×10^{-26}		1.12×10^{-26}
α_{HL}	1.48×10^{-28}		2.21×10^{-28}
$\alpha_{CH}\alpha_{HL}$	2.11×10^{-54}		2.48×10^{-54}

only the heavy hole–conduction band absorption, but also the light hole–conduction band absorption and this was allowed for.[3]) For the light hole–heavy hole transition, data on degenerate p-type material were used so that only this transition was being observed (Gobeli and Fan [7]).

The results of the lifetime calculations are shown in Fig. 2 and Fig. 3. It can be seen from graph a) of Fig. 2 that the assumption of parabolic bands leads to a lifetime which is about an order of magnitude greater than the computed lifetime, while with the approximation to the non-parabolic case there is a discrepancy of at most 14% which occurs at the higher temperatures ($\approx$500 °K).

On the basis of the theory of Section 2 the probability per unit time that a hole in the light hole band with a wave-vector $\boldsymbol{k}$ and energy E will take part in an impact ionization process (Fig. 1 b) is

$$\frac{P}{t} = \frac{4\,\alpha_{11'}\,\alpha_{22'}\,e^4\,m_{H0}^{3/2}}{\varepsilon^2\,\hbar^7\,m_{C0}^{1/2}\,(2+\mu_{H0})^{3/2}}\,a^2(E)\,m_L^2\,\frac{(E_V-E-E_T)^2}{E_g^2} \tag{31}$$

and is shown in Fig. 4 as a function of the photon energy required to produce the light hole.

[3]) If the only difference between them were the density of states the ratio of the light hole optical absorption to the heavy hole optical absorption would be 1:2.

5. Conclusions

The recombination mechanism investigated here yields a temperature dependence which agrees with experimentally measured lifetimes in InSb at room temperature and above. When estimates of the overlap function are made the magnitude of the lifetime appears to give values which are somewhat smaller than the experimental lifetime (Fig. 2). The closest fit is obtained when the overlap parameters are estimated from the effective mass sum rules (a factor 1.4 too small), while the worst fit is obtained using optical absorption data (a factor 4 too small). However, all these methods of estimating the overlap parameters can only be hoped to give an order of magnitude estimate so that to this order it can be concluded that the magnitudes of the experimental and theoretical lifetimes are in agreement. In particular for the case of the optical absorption calculation an underestimate of the lifetime would be expected since included in the experimental optical absorption coefficient will be all allowed transitions and not just the direct transitions of interest.

The Auger theory of Beattie and Landsberg has an almost identical temperature dependence. Their calculation could be extended to non-parabolic energy bands, but direct comparison with the present process is difficult due to the overlap functions. For the dominant transitions in the present case the differences in $\boldsymbol{k}$-vectors are small, but for the other case these differences are large precluding the use of the results of Section 3. However, the process considered here seems comparable with the more usual Auger process for intrinsic semiconductors becoming less important for heavy n-type material and more important for heavy p-type material.

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