Simple model of current-voltage characteristics of a metal–insulator–semiconductor transistor

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(Submitted March 6, 1989; accepted for publication December 15, 1989)
Fiz. Tekh. Poluprovodn. 24, 777–781 (May 1990)

The equation of continuity for the current is solved and used to obtain a simple compact expression for the drain current in a metal–oxide–semiconductor transistor which provides a unified description of the gate and drain characteristics of this device operating under subthreshold, weak or strong inversion, or saturation conditions. An automatic allowance is made for the relationship between the diffusion and drift components of the current. Quantitative criteria are derived for ensuring a particular type of operation when the values of the input parameters are temperatures are varied. The distributions of the electrostatic potential and of the electric field along a channel are obtained.

The problem of analytic modeling of the current-voltage characteristics of a field-effect metal–oxide–semiconductor transistor (MOST) has remained topical for many years.[1] This has been due to, firstly, the simplicity and flexibility of the analytic approach compared with the cumbersome computer calculations and, secondly, the fact that a physically justified analytic model is always the starting point of more detailed numerical calculations. However, in spite of the large number of papers on analytic or numerical-analytic modeling of a MOST,[1,2] there is as yet no sufficiently simple approach which would provide a unified description of the operation of a MOST throughout the full range of working gate voltages, ranging from the exponential subthreshold part of the current-voltage characteristic to the linear and above-threshold operation, including the saturation effects and the transition from the diffusion to the drift current.

The current-voltage characteristic of a MOST is usually derived by direct integration of the expression for the current, which implies physically the summation of an infinite number of series resistances and this is possible if we have information on the distribution of the electric field, electrostatic potential, and carrier density throughout the channel in a transistor. This additional information requires solution of the equation of continuity in its general form, but this is usually not done. Therefore, the description is not complete and the final results can be obtained only if we use additional empirical or a priori information. We shall propose a consistent approach which makes it possible to derive a general expression for the current that describes in a unified manner the main electrical operating regimes of a MOST in a wide range of temperatures. This general expression is both compact and closed and convenient for practical applications.

When the parameters of a MOST are modeled, there are two aspects of the physics: electrostatics of the space charge layers controlling the carrier density in the channel and the transport conditions in the channel. In the case of the first aspect it is found that the electrostatics of sufficiently long channels can be described fully by the Poisson equation using the one-dimensional approximation for an electrostatic potential ϕ measured downward from the conduction band in the substrate and by an expression for the carrier density (in the case of a p-type substrate, this expression describes electrons). In practice, it is necessary to know only the first two moments of the Poisson equation along a transverse coordinate. Allowing for natural boundary conditions and for the contact potential between the semiconductor and metal ϕ_{ms}, we obtain the following two expressions:

$$N + N_{ox} = N_{ss} + N_A W + n, \tag{1}$$

$$V_g = \varphi_{ms} + \frac{eN}{C_0} + \frac{4\pi e}{\varepsilon_i}\int_0^d (d-x)\,\rho_{ox}(x)\,dx + \frac{2\pi e}{\varepsilon_s} N_A W^2. \tag{2}$$

the first expression is the equation of electrical neutrality and it relates the local surface charge densities in the gate N, in the oxide $N_{ox} = \int_0^d \rho_{ox}(x)\,dx$, in the surface states N_{SS}, in a depletion layer $N_A W$, and in an inversion layer n. Equation (2) gives the total potential drop over the whole thickness of the structure: $V_g - \phi_{ms} = \int_0^{d+W} E_x(x)\,dx$. Here, d and W are the thicknesses of the oxide and of the depletion layer, respectively; N_A is the acceptor concentration; V_g is the positive voltage applied to the gate. Equations (1) and (2) are written in the approximation of an abrupt depletion layer; moreover, Eq. (2) ignores the very small potential drop across the inversion layer, so that the change in the potential across the depletion layer is equal to the surface potential $\phi \cong \phi_d = (2\pi e/\varepsilon_s)N_A W^2$. As pointed out already, a closed description in terms of the Poisson equation requires generally an independent expression for the density of free carriers n and for the density of the surface states N_{SS}. This is necessary because these quantities are governed by the internal parameters of the problem (in particular, thermodynamic parameters) and by the conditions at the interface. We shall not consider the problem of the surface states, but simply point out that the general expression for the density of mobile electrons in a channel is as follows in the case when the statistics of the Boltzmann type, provided there is no quantization and a static confining field is applied:

$$n = \bar{n}\,\frac{kT}{eF_s}\exp\left(\frac{e\zeta}{kT}\right). \tag{3}$$

Here, $\zeta < 0$ is the chemical potential of electrons for the quasiequilibrium case, which can be written in the form $\zeta_0 = \phi - E_g/2e - \psi_B$; E_g is the energy width of the band gap; $\psi_B = (kT/e)\ln(N_A/n_i)$ is the position of the chemical potential in the substrate measured from the middle of the band gap. the average confining electric field f_S is

$$F_s = \frac{2\pi e}{\varepsilon_s}(\bar{n} + 2N_A W), \tag{4}$$

where $\bar{n}$ is the effective density of states in the conduction band near the interface. It should be pointed out that Eqs. (3) and (4) are equivalent, subject to our assumptions, to the familiar exact solution of the one-dimensional Poisson equation,[3] which is readily demonstrated by solving the quadratic equation derived from Eqs. (3) and (4) for n.

The transport description of the motion of carriers is given by the Boltzmann equation, but if we need to derive the current-voltage characteristic of a transistor it is sufficient to use the zeroth and first moments of the transport equation with respect to the velocity. Under steady-state conditions in the case of long-channel transistors these moments are given respectively by the equation of continuity for the current (per unit length)

$$dI/dy = 0 \tag{5}$$

and by the expression for the diffusion–drift current

$$I = e\mu n \frac{d\varphi}{dy} + eD\left|\frac{dn}{dy}\right|. \tag{6}$$

The ratio of the diffusion and drift parts of the total current (6) is generally speaking a function of the coordinate along the channel, whereas the total current remains constant. Using the Einstein relationship, we can readily show that the ratio of the absolute values of the diffusion and drift components of the current can be expressed in terms of a quantity $\chi \equiv |d\zeta/d\phi|$:

$$eD\left|\frac{dn}{dy}\right| = eD\left|\frac{dn}{d\zeta}\right|\left|\frac{d\zeta}{d\varphi}\right|\left|\frac{d\varphi}{dy}\right| = e\mu n\left|\frac{d\zeta}{d\varphi}\right|\left|\frac{d\varphi}{dy}\right|. \tag{7}$$

This quantity χ shows how fast does the chemical potential of electrons (and, consequently, the density of electrons) change with the electrostatic potential and it represents an internal parameter of the problem. We can find χ making the usual assumption that the main physical cause for the diffusion current in the channel is the electrostatic influence of the space charge of the depletion layer on the carrier density in the channel. Differentiation of Eqs. (1) and (2) with respect to the chemical potential ζ (when the differential is understood to be the change in a given quantity as a result of a shift along the longitudinal coordinate) and ignoring the change in the density of the surface states along the channel, we readily obtain

$$\chi = \frac{\frac{C_0}{e} + \frac{N_A W}{2\varphi}}{|dn/d\zeta|}. \tag{8a}$$

the expression for the derivative of the carrier density with respect to the chemical potential differs from the usual bulk Boltzmann equation and it can be obtained from Eqs. (3) and (4):

$$\frac{dn}{d\zeta} = \frac{en}{2kT}\left(1 + \frac{N_A W}{n + N_A W}\right). \tag{8b}$$

If we assume that the ratio of the diffusion to the drift current varies weakly in at least a small part of the channel and if we ignore the derivative $(d\chi/d_y) \sim 0$, we find that the equation of continuity (5) for the current of Eq. (6) can be reduced to

$$\frac{d^2\varphi}{dy^2} = -\frac{e}{kT}\frac{d\zeta}{d\varphi}\left(\frac{d\varphi}{dy}\right)^2.$$

It is convenient to rewrite the above equation introducing $E(y) = d\phi/dy$, which represents a longitudinal electric field in the channel, and bearing in mind that $d\zeta/d\phi = -\chi$;

$$\frac{dE}{dy} = \frac{e}{kT}\chi E^2. \tag{9}$$

Equation (9) is readily integrated:

$$E(y) = \frac{E(0)}{1 - \frac{eE(0)}{kT}\int_0^y \chi(y)\,dy}. \tag{10}$$

Subject to the above assumptions, we shall now assume that $\int_0^y \chi(y)\,dy \cong$ and $\chi(0) \equiv \chi$. The integration constant $E(0)$ in Eq. (10), which represents a longitudinal field in the source region, can be found from the natural condition

$$V_D = \int_0^L E(y)\,dy, \tag{11}$$

where L is the length of the channel and V_D is the voltage applied to the drain. Hence, we readily obtain

$$E(0) = \frac{kT}{eL\chi}\left[1 - \exp\left(-\frac{eV_D}{kT}\chi\right)\right]. \tag{12}$$

Then, Eq. (10) becomes

$$E(y) = \frac{kT}{eL\chi}\,\frac{1 - \exp\left(-\frac{eV_D}{kT}\chi\right)}{1 - \frac{y}{L}\left[1 - \exp\left(-\frac{eV_D}{kT}\chi\right)\right]}. \tag{13}$$

Integration Eq. (13) with respect to the coordinate, we find the distribution of the electrostatic surface potential along the channel satisfying the condition $\phi(L) - \phi(0) = V_D$, where

$$\varphi(y) - \varphi(0) = -\frac{kT}{e\chi}\ln\left[1 - \frac{y}{L}\left(1 - \exp\left(-\frac{eV_D}{kT}\chi\right)\right)\right]. \tag{14}$$

The constancy of the current along the channel (5) makes it possible to use simply Eq. (12) in derivation of the current-voltage characteristic. The total current of Eq. (6) in a channel of width Z is given by the following expression derived with the aid of Eq. (7):

$$I = e\frac{Z}{L}Dn\frac{1+\chi}{\chi}\left[1 - \exp\left(-\frac{eV_D}{kT}\chi\right)\right]. \tag{15}$$

Here, n and χ are taken in the region of the source and are independent of the drain voltage V_D.

Equation (15) together with Eqs. (8) and (3) provide a compact description of all the operating regimes of a MOST. Under strong inversion conditions ($n > N_A W$) a large charge in the inversion layer is controlled mainly by the gate, it depends weakly on V_D, and the electron density along the channel (and, consequently, the chemical potential) is independent of the change in the electrostatic potential $\chi \approx (C_0 kT/en) \to 0$. However, if $\chi eV_D/kT < 1$, this potential varies linearly and the field is constant in a part of the channel where $(\chi eV_D/kT)(y/L) \ll 1$

Expanding the exponential function in Eq. (15) up to the quadratic term, allowing for the explicit form of χ given by Eq. (8), and bearing in mind that above the threshold we have $n = C_0(V_g - V_t)$, we obtain the familiar result[1,4]

$$I=\frac{Z}{L}\mu C_0\left[(V_g-V_T)V_D-\left(1+\frac{eN_AW}{2C_0\varphi}\right)\frac{V_D^2}{2}\right]. \tag{16}$$

However, it should be pointed out that in the usual approach Eq. (16) is obtained subject to a stringent restriction requiring that $V_D \ll 2\psi_B \lesssim 1$ V and then, in spite of this restriction, it is used to describe the saturation regime. In fact, the condition for saturation of the current above the threshold is the inequality $\chi eV_D/kT > 1$, which is opposite to the condition of validity of Eq. (16). In the saturation case Eq. (15) yields the following value for the saturation current[1]

$$I_{sat}=e\frac{Z}{L}Dn\frac{1+\chi}{\chi}=\frac{Z}{L}\mu\frac{1}{2}\frac{C_0(V_g-V_T)^2}{1+\dfrac{eN_AW}{2C_0\varphi}}. \tag{17}$$

Finally, in the subthreshold region when $n < N_AW$, and $\chi \gg 1$, the general expression (15) yields the observed exponential dependence[1,4]

$$I\cong eZD\frac{n}{L}=e\frac{Z}{L}D\bar{n}\frac{kT}{eF_s}\exp\left(\frac{e\varphi-E_g/2-e\psi_B}{kT}\right). \tag{18}$$

A longitudinal field of Eq. (13) tends to zero over a large part of the channel and the electrostatic potential of Eq. (14) is practically constant, i.e., we now have the diffusion regime when the current is independent of the drain voltage. The critical point at which the drift current begins to predominate over the diffusion mechanism corresponds, in accordance with Eq. (15), to saturation of the parameters that $\chi eV_D/kT \sim 1$. For given values of the drain voltage and temperature the change in χ in accordance with Eq. (8) from a very small quantity to a very large one results in a gradual transition from the drift to the diffusion current, and it describes the saturation effects.

It therefore follows that Eq. (15) – together with Eqs. (8), (3), and (4) – represents a closed set of calculation formulas suitable for the description of the current-voltage characteristic of a MOST throughout the full range of the electrical regimes. The above treatment deals only with the electrical model of the current-voltage characteristic of a transistor omitting details of little importance such as mobility, surface states, etc. Nevertheless, our experience in modeling of real devices based on the above approach demonstrates its flexibility from the theoretical point of view and, which is equally important in the applications, its simplicity in practice.

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Translated by A. Tybulewicz