Performance analysis for indoor location determination

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Abstract: This work analyses theoretical localisation limits and proposes a precision bound to evaluate the performance of indoor systems under various configuration settings and environmental dynamics. By deploying a set of localisation systems in two different buildings, this study showed that database-based indoor systems perform slightly better than distance-based systems, and while it appears difficult to interpret the relationship between localisation performance and various environmental or configuration factors, the achievable precision of a localisation system for a given building is bounded and predictable. Thus this research provides a single standard criterion for the evaluation of various location determination systems.

Keywords: signal strength; location determination; precision bound; benchmark.

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1 Introduction

Location management and mobility management are critical issues for providing seamless interactions and ubiquitous computing for mobile users. As concluded independently in several market surveys (ABI Research, 2004; Insight Research Corp., 2007), the worldwide market for Location Based Services (LBS) is projected to be \$7~\$8 billion revenue over the next few years. For more than a decade, researchers have proposed and studied many different mechanisms for both indoor and outdoor localisations, and the focus is to find an efficient localisation technique that is accurate, cheap, and is able to provide reliable services to common users.

The underlying principle of most localisation research relies on either range or angle measurements using a wide range of technologies including:

- infrared (Want et al., 1992)
- ultrasound (Priyantha et al., 2000; Savvides et al., 2001)
- vision (Krumm et al., 2000; Kais et al., 2004; Agrawal and Konolige, 2006)
- Radio Frequency (RF) (Bahl and Padmanabhan, 2000; Ji et al., 2006; LaMarca et al., 2005; Cheng et al., 2005)
- landmarks (Miller et al., 2006; Borkowski and Lempiäinen, 2006; Yeluri, 2003; Russell, 1995).

Consequently, existing localisation systems can be categorised according to many different criteria, including communication techniques (centralised vs. decentralised), technology parameters (time, angle, signal signature, Cell-ID, and landmarks), environments (outdoor, indoor, and underwater), security (secure vs. open), localisation entity (mobile-based vs. network-based), and many others. Among these localisation systems, *lateration* (Caffery and Stuber, 1998; Lim et al., 2006; Fontana et al., 2003; Cheng et al., 2004; Youssef et al., 2006; Hightower and Boriello, 2001; Sayed et al., 2005; Thrun, 2002; Elnahrawy et al., 2004), *triangulation* (Niculescu and Nath, 2003; Sakagami et al., 1992; Deng and Fan, 2000; Venkatraman and Caffery, 2004), *database mapping* (Bahl and Padmanabhan, 2000; Ji et al., 2006; Chen et al., 2006a; LaMarca et al., 2005; Ladd et al., 2004; Krishnan et al., 2004; http://www.skyhookwireless.com/; Haeberlen et al., 2004; Cheng et al., 2005; Hatami and Pahlavan, 2004; Starner et al., 1998; Jenkin et al., 1993; Abdel Aziz and Karara, 1971; Mesaki and Masuda, 1992; Borenstein et al., 1996), and *dead reckoning* (DR) (Roston and Krotkov, 1991; Craig, 1986; Satthamnuwong, 2002; Gebre-Egziabher et al., 2001) are the four principal localisation techniques.

For indoor location determination, latest research has shown great interests in Wi-Fi networks, where Received Signal Strength (RSS) values (instead of time or angles from proprietary hardware sensors) are exploited for the location determination process. However, Wi-Fi signals are noisy due to building structures, multipath transmission delays, antenna directions, people movement, and other environmental factors such as temperature and humidity. Therefore, reported accuracies from existing systems are not directly comparable because the conditions under which the tests were carried out could be very different. Thus very limited testing cases and the lack of benchmark standards have greatly restricted the evaluation of existing systems. Consequently, despite advances in data processing techniques and micro-sensor technologies, most indoor localisation technologies are not well understood.

These challenges have been raised and researchers have begun to develop benchmark theories (Wallbaum and Diepolder, 2005) as well as common data sets for all indoor systems (IEEE ICDM, 2007; http://crawdad.cs.dartmouth.edu/index.php). It appears that two different approaches would contribute to this direction: First, analyse individual (environmental and system) factor and develop a dependence formula between each factor and the indoor system (Ji et al., 2007; Chen et al., 2006b); and Second, integrate all factors in a given environment and derive a theoretical limit, i.e., precision bound, that an indoor system might achieve in that environment. The first method is valuable in that it would produce standard and reproducible test beds through which indoor location determination systems would be evaluated. On the other hand, the second method considers an indoor system and its deployed building as an integral unit such that the performance of the system would be evaluated by both the theoretical precision bound and its actual achieved results in that test bed.

This paper will analyse the performance impact of representative indoor localisation systems from a broad range of environmental factors, but the main focus is on the second approach, and its contributions include:

- a theoretical precision bound for location determination, through which the best performance of an indoor system in a given building would be determined
- a through evaluation of four indoor localisation mechanisms according to commonly concerned environmental factors (such as complex partitions, sniffers deployment, and reference measurement) using two very different buildings, including a typical office building and a basement building
- a comparison study of various location searching algorithms including MultiDimensional Scaling (MDS) and lateration.

This paper focuses mainly on dynamic localisation methods where no RSS values will be manually collected across the building and no static data-training process will be required before localisation. It will show that although individual indoor mechanism and deployed environments are two most important determination factors for the performance of location determination, indoor systems also depend on many other factors including the system (sniffers) deployment method and the selection of reference RSS measurements. Therefore, the approach to identify and further to understand those individual factors is a very complex task. Instead, by using the precision bound to uniquely integrate all impacting factors, this research presents a unique method to evaluate all location determination systems.

The rest of the paper is organised as follows: Section 2 will briefly introduce unique properties of indoor radio propagation; Section 3 will model the location determination errors and derive the precision bound. Section 4 will describe various dynamic localisation mechanisms based only on RSS values; Section 5 introduces two test buildings, compares the system performance from various perspectives, and evaluates the theory of precision bound. Section 6 introduces related research, and finally, Section 7 concludes the paper and outlines future research.

2 Indoor radio propagation property

Indoor radio propagation poses a serious challenge to location determination due to the harsh multipath environment. The propagation behaviour changes at different buildings or even within a single floor when objects are added into the environment or when people enter or move in the vicinity. As indicated in Rappaport (2001), there are two methods to study the signal strength for an arbitrary T-R separation distance:

- applying radio propagation models to simulate local average received power
- designing high resolution devices and algorithms to minimise power measurement errors resulted from multipath waves at different amplitudes, phases, or multipath delays.

Time-Of-Arrival (TOA)-based indoor location determination belongs to the second technique that measures or identifies the arrival time of the wave with the strongest power, and it has been used in many location determination systems. However, in case of the existence of strong obstructions (such as multiple walls and furniture, or metallic objects) between the transmitter and the receiver, radio waves from other none-line-of-sight paths could be much stronger than the wave from the direct path, thus this 'undetected-direct-path' situation will cause significant error in indoor location determination. Unfortunately, none-line-of-sight radio propagation is a common phenomenon in indoor environment, and according to Alsindi's research (2004, p.82), the 'undetected-direct-path' problem typically results an average of 8.5 metres of range estimation errors, and it is *NOT* able to be solved or even mitigated by simply increasing the bandwidth of the signal transmission.

Therefore, this study will exploit the RSS from radio propagation models or from the local measurement average for indoor location determination, and the next section will first analyse the precision bound for typical location estimation.

3 Bounding localisation errors

3.1 The modelling of location errors

For a position P covered by three or more sniffers O_i, $(i = 1, 2, \dots, n)$, if each sniffer O_i is able to accurately estimate its distance $r_{\text{true},i}$ to the position P, then P would be uniquely determined by the intersection of three (or more) circles in a two dimensional space. However, the distance estimation is not accurate in general, and if the T-R range r_i is given by:

$$r_i \in [r_{\text{true},i} \cdot (1 - \delta_i), r_{\text{true},i} \cdot (1 + \delta_i)] \tag{1}$$

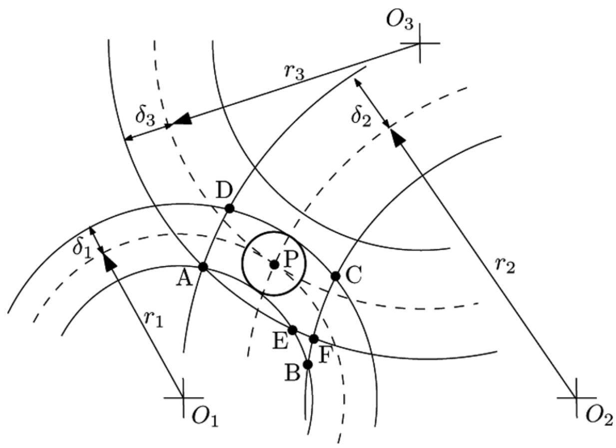
where δ_i is the uncertainty of the radio range estimation at sniffer i. Then the position P is determined by an overlapping area formed by a set of ring surfaces with radii of r_i, $(i = 1, 2, \dots, n)$ centred at those sniffers. Suppose a ring surface from each sniffer composes a point set $\{q_{r,i}\}$, then the common area of those rings determines an *Uncertainty Area* (UA) for the position P:

$$S_{ua,P} = \{q_{r,1}\} \cap \{q_{r,2}\} \cap \ldots \cap \{q_{r,n}\}. \tag{2}$$

Figure 1 illustrates the scenario with three sniffers of $O1, O2, O3$ and an interested position P. If there is no measurement error, the true radio range of the three sniffers would be r_i, $(i = 1, 2, 3)$, which uniquely determine the position P. Considering measurement errors, and let δ_i, $(i = 1, 2, 3)$ denote range perturbations of all three sniffers, the possible sensing area at each sniffer would be a circular belt (or a ring surface between radii $r_i \cdot (1 \pm \delta_i)$, $(i = 1, 2, 3)$, consequently, the candidate location for the position P would be an *area*, instead of a *point*, that is formed by the overlapping of the three rings and is represented by the surface of $AEFCDA$.

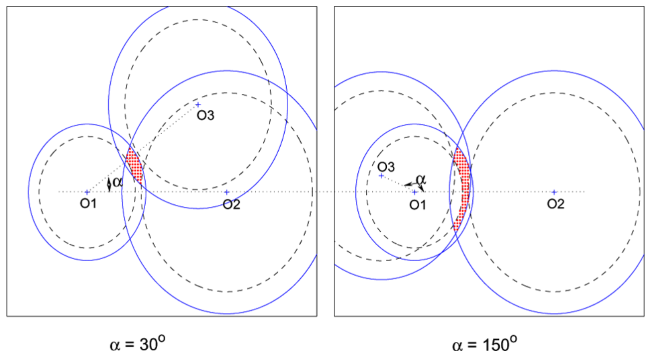
Clearly, different sniffers deployment methods will generate very different *UA*s, and smaller and regular *UA* will give higher confidence in the location determination.

Figure 1 Location error analysis



For example, Figure 2 shows two *UA*s for a same position P using two different sniffers deployment strategies. It shows that location estimation from the left configuration (i.e., $\alpha = 30°$) is more precise.

Figure 2 Uncertainty area of two sniffers deployment (see online version for colours)



Therefore, in order to improve the performance of location determination, research may concentrate on two different approaches: First) minimising the T-R range uncertainty δ; and Second) optimally deploying sniffers or even increasing the number of deployed sniffers on the site. Obviously, the first approach is straightforward, but it requires specialised hardware devices to measure the time or angle of radio transmission; moreover, as addressed in Section 2, even TOA at a high bandwidth may not achieve accurate range measurement because of multipath effects and obstructed line-of-sight transmission (Alsindi, 2004). Therefore, indoor localisation research is inherently imprecise in nature, and the question is that whether it is possible to achieve reasonable accurate location determination even with the range uncertainty δ.

Pursing the second approach (i.e., sniffers deployment strategy), Ji et al. (2007), addressed the potential to improve the performance of indoor location determination, and research results revealed that if a position is surrounded by three nearby sniffers (ideally, inside an *equilateral* triangle of three sniffers), then the location estimation of that position is significantly better than other positions outside of the triangle. Consequently, Ji et al. (2007) proposed a *mesh-grid* deployment method such that every position inside the building will be covered by at least three nearby sniffers (i.e., full coverage). However, the study in Ji et al. (2007) was only evaluated by the ARIADNE system (which is a *database mapping method*), more importantly, the achievable precision bound for a given uncertainty δ is still not clear.

3.2 Precision bound

In Ji et al. (2007), the authors defined an *average uncertainty distance* d_ε to measure the average distance of all positions in the *UA*, and it is given by:

$$d_\varepsilon = \frac{\sum_{i=1}^{n} \sum_{j=i+1}^{n} d_{ij}}{n(n-1)/2} \tag{3}$$

where n is the total points in an uncertainty area UA, and d_{ij} is the distance between two points i and j, $(i, j \in [1, n], i \neq j)$. For a given deployment for a floor plan, the d_ε presents the average estimation error (or *precision bound*) for that building. However, for a given range measurement uncertainty δ from surrounding sniffers, the question is how the precision bound should be uniquely determined.

In order to analyse the precision bound, we take a close look at the *UA* for the position P in Figure 1. To simplify the analysis, let's assume that the distance measurement uncertainty δ from all three sniffers is uniformly distributed, i.e., $\delta_1 = \delta_2 = \delta_3$. Therefore, if sniffer O_1 is the closest sniffer to the position P, then from the figure, it is clear that the *UA* for position P is mainly determined by sniffer O_1, which is a ring surface with radius between $r_1\ (1 \pm \delta)$. The second sniffer O_2 bounds the *UA* to a smaller space of $ABCDA$, and the third sniffer O_3 further fine-tunes the *UA* to an even smaller space of $AEFCDA$. Therefore, if more sniffers would be deployed while O_1 is the closest sniffer to position P and δ values are same for all sniffers, then the *UA* will ultimately form a circular disk centred at P. Thus this circle with radius of $r_1\delta$ would define the bound of *UA* for the position P.

Consequently, we define the *precision bound* as the average distance of all candidate positions inside the circle of radius $r_1\delta$. Detailed mathematical formulation or calculation is beyond the scope of this paper, but according to a geometric probability study by Dunbar (1997) and Santalo (1977), the d_ε for any two points in a s-dimensional ball is determined by:

$$d_\varepsilon = \frac{s}{2s+1} \beta_s R(x) \tag{4}$$

where $R(x)$ is the diameter, and β_s is given by:

$$\beta_s = \begin{cases} \dfrac{2^{(3s+1)}((s/2)!)^2 S!}{(s+1)(2s)!\pi} & \text{for even } s \\ \dfrac{2^{(s+1)}(s!)^3}{(s+1)(((s-1)/2)!)^2(2s)!} & \text{for odd } s \end{cases}. \tag{5}$$

For a 2-dimensional disk or 3-dimensional sphere with diameter of $R(x)$ (which is $2r_1\delta$ in this example), the *precision bound* d_ε is thus given by:

$$d_\varepsilon = \begin{cases} \dfrac{64}{45\pi} R(x) = \dfrac{64}{45\pi} \cdot (2r_1\delta) & \text{for 2D space} \\ \dfrac{18}{35} R(x) = \dfrac{18}{35} \cdot (2r_1\delta) & \text{for 3D space} \end{cases}. \tag{6}$$

Therefore, if the closest distance to a sniffer from a location P is r_1, and the distance measurement uncertainty δ is same at all sniffers,[1] then the average location estimation error at P is bounded by equation (6).

This paper will evaluate the localisation performance of various mechanisms, and special interest will be in the analysis of performance dependence on different environmental and system factors, and the assessment of the precision bound described in equation (6). We first introduce different localisation methods in the next section.

4 Dynamic location mechanisms

This section introduces basic algorithms of four dynamic systems selected for this study. The Signal-Location Map (SLM) is based on the ARIADNE system (Ji et al., 2006), and the other three (indoor Radio Propagation Modelling (RM), Signal Distance Mapping (SD) and Distance Fitting (DF)) are mainly derived, respectively, from existing systems including Lim et al.'s zero-configuration system in Lim et al. (2006), Sánchez et al.'s triangulation in Sánchez et al. (2006), and Smailagic et al.'s CMU-TMI in Smailagic et al. (2001).

4.1 Signal-location map (SLM)

SLM is a *database-based* indoor system that uses a two-phase localisation mechanism (Bahl and Padmanabhan, 2000; Ji et al., 2006): *Phase I* is called *Map Generation*, where RSS values at a grid of locations on a plane (or 3-D space) are either manually measured or theoretically estimated; then a signal-location map that connects location coordinates and RSS values is generated; and *Phase II* is the *Location Search*, where current RSS measurement from a mobile is used to search the signal-location map for the 'closest' hit.

The ARIADNE (Ji et al., 2006) is a representative SLM system that dynamically generates signal-location maps without manual measurements. ARIADNE used the following radio propagation model:

$$P = \sum_{i=1}^{N_{r,j}} (P_0 - 20\log_{10}(d_i) - \gamma \cdot N_{i,\mathrm{ref}} - \alpha \cdot N_{i,\mathrm{trans}}) \tag{7}$$

where P is the power (in dB) at receiver, $N_{r,j}$ is the total number of rays received at the receiver j; P_0 is the power (in dB) at a distance of 1 meter; d_i, $N_{i,\mathrm{ref}}$, and $N_{i,\mathrm{trans}}$ represent transmission distance, total number of reflections and total number of walls passed by the ith ray, respectively. γ is the reflection coefficient, and α is the transmission coefficient. In the equation, site-specific parameters ($N_{r,j}$, d_i, $N_{i,\text{ref}}$, $N_{i,\text{trans}}$) would be derived directly by the ray tracing processing. For an indoor system with at least three sniffers, the other three parameters (P_0, γ, α) would be determined, theoretically, with only one SS measurement at a given reference location.

Based on the model, a SLM could be built over a grid of locations inside the building, therefore the location search becomes a 'trivial' task. In this paper, a simple least mean square method will be used to search the location.

4.2 Indoor Radio Modelling (RM)

RM is one of the most important methods that builds a relation between RSS values and the distance. For more than a decade, many wonderful models have been proposed and evaluated (Rappaport, 2001). When considering large-scale attenuations, most researchers model the radio propagation path loss as a function of the attenuation exponent n, which is two for free space but often statistically determined to provide a best fit with measurement readings.

$$P(d)[\text{dB}] = P(d_0)[\text{dB}] - 10 \times n \times \log_{10}\left(\frac{d}{d_0}\right) \tag{8}$$

where $P(d)$ is the power at distance d to the transmitter in metres; $P(d_0)$ is the power at a reference distance d_0, usually set to 1.0 metre.

It should be noted that many other advanced models exist as well. For example, *partition model* (Phaiboon, 2002; Bahl and Padmanabhan, 2000), including ARIADNE in equation (7), reduces the path loss effect from attenuation exponent by additional consideration of attenuation effects from indoor partitions, like walls and floors; and *Site-specific model* (Hassan-Ali and Pahlavan, 2002; Lott and Forkel, 2001) exploits path loss from site-specific parameters such as geometric structure, materials, and partition thickness. Compared with the model in equation (8), these models are more sophisticated, but they generally adapt well to most building environments. However, distance-SS relationship from these models are not straightforward and computation is usually very expensive. Consequently, many researchers (Mahtab Hossain et al., 2007; Hills et al., 2004) still consider equation (8) in their systems. This paper will also use this model to derive the Transmitter-Receiver (T-R) distance.

4.3 Signal Distance Mapping (SD)

SD method is based on the concept that there exists an immediate (or linear) relationship between a RSS value and the geographic T-R distance, which can be expressed as follows:

$$S \cdot T = D \tag{9}$$

where S is a $m \times n$ matrix of RSS values between m sniffers and n reference locations. D is also a $m \times n$ matrix of geographic distance values corresponding to RSS values in matrix S; and T is a $n \times n$ linear transformation matrix that maps the RSS value to a T-R distance by a scaling factor unique to a transmitter-receiver pair.

With reference RSS measurements and known T-R distance values among sniffers and reference locations, matrix T would be easily obtained from equation (10). Thus with the transformation matrix T, any instant RSS measurement (S_{now}) would be translated into a T-R distance (D_{now}) transparently:

$$T = (S \cdot S^T)^{-1} \cdot S^T \cdot D \tag{10a}$$

$$D_{\text{now}} = S_{\text{now}} \cdot T. \tag{10b}$$

This mechanism was reported by Gwon and Jain (2004) and Lim et al. (2006). Originally, the SD method considers only the RSS and T-R distance values among a set of m reference APs, and therefore both matrixes of S and D are symmetric square $m \times m$ matrixes with zero diagonal entries. Clearly, for a complex indoor environment, the modification in equation (9) would provide a comprehensive map for the interested building.

4.4 Distance Fitting (DF)

The DF method is similar to the SD described in the previous section. Different from the simple linear relationship between SS and the distance, many researchers believe that the SS-distance relationship could be very complex. For example, Yin et al. (2005), proposed a two-phase approach that is very similar to SD, where a sophisticated function was used to map SS values with the distance.

In order to better model the SS-distance relationship, other researchers adopted a polynomial function. For example, Smailagic et al. (2001) used the following formula:

$$d = A \cdot S_i^2 + B \cdot S_i + C \tag{11}$$

where d is the transmitter-receiver distance corresponding to the SS measurement S_i; A, B, and C are coefficients that are unique to the building environment.

The DF method in this paper will also use equation (11). Theoretically, for a floor plan with three deployed sniffers, a single SS measurement from a given reference position will generate three equations (with three unknowns of A, B, C). If more reference positions are available, average results would be used for the model.

4.5 Distance based location search

With distance values from a mobile client to a set of reference positions using RM, SD, DF, or other techniques, various methods can be exploited to find the location $\mathbb{X}$ of the mobile. The most straight method is the lateration, where a linear equation (equation (12)) will be easily derived and mobile's location will be uniquely determined.

$$\begin{aligned} &\mathbb{AX} = \mathbb{B} \\ \text{where:}\quad &\mathbb{X} = (\mathbb{A}^T\mathbb{A})^{-1}\mathbb{A}^T\mathbb{B}. \end{aligned} \tag{12}$$

Alternatively, MDS could be used to determine the location. Different from the lateration method, MDS takes pair-wise distance values d_{ij} between the mobile and reference positions (such as sniffers) and those among all reference positions, then it generates a low dimensional representation of position relationships such that the distance values between objects (mobiles and sniffers) fit as well as possible with the given measures and estimates δ_{ij} (from RM, SD, and DF). Basically, MDS iteratively exploits the mobile's position such that the goodness-of-fit stress ϕ is minimal:

$$\phi = \sum [d_{ij} - \delta_{ij}]^2. \tag{13}$$

Many software packages are now available for MDS, but the application of this method in indoor research is not still common. In this research, we will adapt this method in indoor environment and will compare its performance with the lateration.

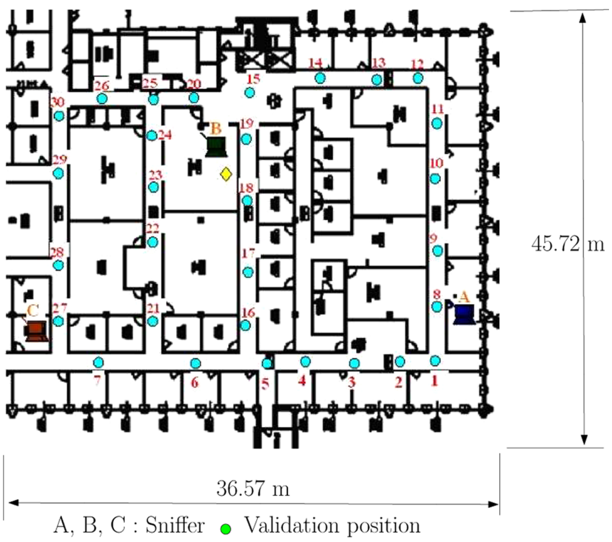
5 Simulations and system comparison

5.1 Testing environments

Two very different buildings will be used in this study.

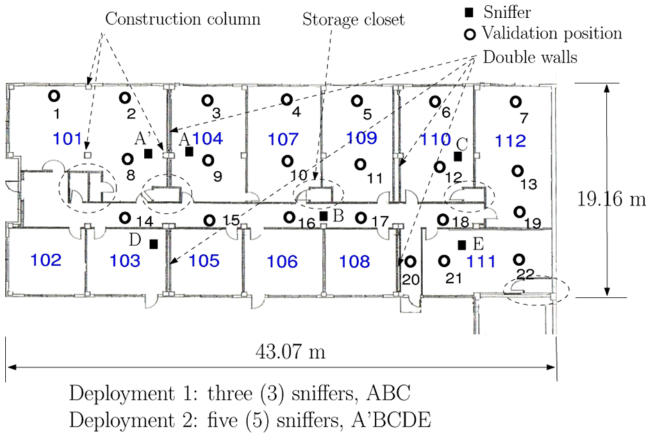
- The first building (Building I, see Figure 3) is from Telcordia Technologies, and the data was initially collected and reported by Pandey et al. (2005). In this building, three sniffers, using IBM T30 ThinkPad with RedHat 9, were deployed.
- The second building (Building II, see Figure 4) is the shop building from Auburn University. The ground level is an underground floor and will be used in this experiment.

Figure 3 Building I (see online version for colours)



As indicated in Figure 4, this ground floor includes four (4) double walls (brick and concrete), five (5) storage closets (for utility and emergency supplies), and many construction columns. Room 101 is a computer classroom, room 110 is a computer lab with metal cabinets (1.0∼1.7 metres) around the room. Room 107, 109, 111 and 112 are classrooms and the rest are offices or labs shared by graduate students. Typical office includes computers, servers, as well as bookshelves and cabinets of various sizes and materials. In this second building, three (Deployment 1 in Figure 4) or five sniffers (Deployment 2), using HP Pavilion V2000 laptops with Linux Fedora II, were deployed.

Figure 4 Building II (see online version for colours)



As illustrated in Figures 3 and 4, Building I includes thirty (30) data validation positions, and Building II has twenty-two (22) data validation positions. To measure signal strength, in Building I, 100 sample packets were collected at each data validation position and the average RSS values were used for location determination process. The experiment was carried for six days in order to evaluate the consistency of considered localisation algorithms. Similarly, in Building II, we measured all data packets in 10 seconds and we continued the measurement for a period of four months.

5.2 Experimental strategy

In order to evaluate proposed localisation mechanisms, data sets from both buildings (Figures 3 and 4) will be similarly applied to each system. As shown in Table 1, reference SS measurements of different configurations will be used to determined average values of all parameters for models in equations (7)–(9), and (11). Later in Section 5.4, dependence on reference positions will also be analysed.

For the location search, existing SS values will be reapplied against the constructed maps or models for the localisation process. For distance based methods, both lateration and classical MDS methods will be used in the

Table 1 Simulation strategies

Simulation scenarios	*Distance estimation or map construction*		*Location search*	
	Sniffers	*Ref. RSS positions*	*Sniffers*	*Building*
(A)	3	Varies	3	I and II
(B)	5	Varies	3	II
(C)	5	Varies	5	II

location search process. If five sniffers are deployed, the location search will also optimally select three sniffers (according to the SS values, scenarios (B) in Table 1) in order to determine the mobile's location, and the results will be compared with those from other scenarios (i.e., (A) and (C)).

5.3 Simulations results

The results in this section are based on the simulation scenario (A) (Table 1), where only three sniffers were deployed in both buildings (Figures 3 and 4). We will first introduce the map construction and distance estimation, and then we will compare the performance of location determination of all mechanisms.

5.3.1 Signal-location map construction using SLM

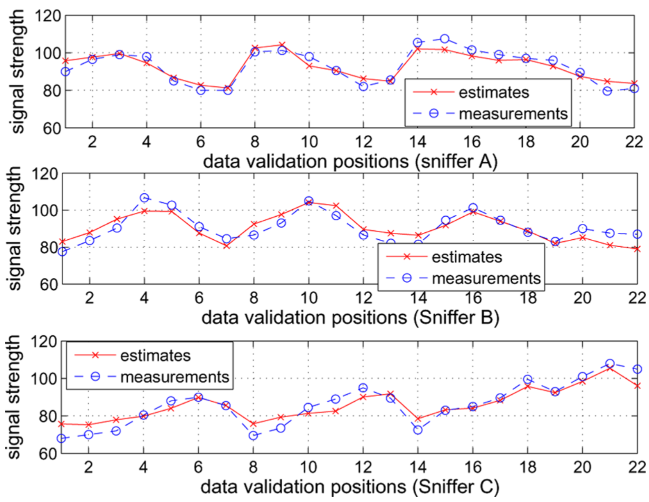
Using SLM model in equation (7), extensive simulations were carried out for both data sets from the two buildings in Figures 3 and 4. Similar to the ARIADNE system in Ji et al. (2006), for each test run, only one reference SS measurements was randomly selected among all positions (30 and 22, respectively) to derive the unknowns, then the results were plugged back to the model to estimate RSS at all data validation locations.

Typical comparison results are shown in Figure 5. The figure consists of three plots respectively for sniffers A, B, and C (see Figure 3). For each plot, the x-axis represents data validation positions and the y-axis denotes the signal strength measured as Received Signal Strength Indicator (RSSI). In the figure, the points with symbol '$\ominus$' are the signal strength measurements and the points with symbol '$\times$' denote the estimates.
This result is very similar to the ARIADNE system in Ji et al. (2006), where Building I was used for the study. It is clear that the indoor radio propagation model in equation (7) provides reliable RSS estimates.

5.3.2 Distance estimation

Different from the simulation in previous section where only one reference RSS was randomly selected in the process, in order to derive unknowns for mechanisms of RM, SD and DF, RSS values from all reference positions would be used in the simulation in this section (Section 5.4 will study other settings where less reference positions were selected in this process), then the *average* values

Figure 5 SLM RSS estimation for Building II (see online version for colours)



of all parameters (i.e., $P(d_0)$, n in equation (8), T in equation (10), and A, B, and C in equation (11)) will be used in equations (i.e., equation (8), (10), and (11)) to regenerate distance values (between sniffers and all data validation positions). We summarise distance estimation errors in Table 2.

Table 2 Distance estimation errors (Scenario A, metres)

	Mean error	*Error in percentile* 50%	70%	85%	90%
Building I					
RM	3.1	2.7	4.2	5.1	5.5
SD	2.4	1.6	3.0	3.9	4.5
DF	2.5	1.4	2.5	5.4	5.5
Building II					
RM	4.7	2.7	4.8	7.6	10.2
SD	2.9	2.3	3.2	4.8	5.6
DF	3.2	2.7	2.8	5.2	5.8

In the table, the mean error is the average distance estimation error in metres, and the error in percentile gives the probability of each estimation when comparing to the true distance result. It can be seen that all three mechanisms provide relatively reliable distance estimates for both buildings. Comparing the estimation for the two buildings, it seems that Building I provides better results for all three indoor systems.

5.3.3 Localisation results

Based on the signal-location map and the distance estimates, the mobile's location would be easily determined. Table 3 gives the localisation results for all mechanisms. In the table, the SLM method is based on a map with grid resolutions of 1.5×1.5 in metres for Building I and 2.0×2.0 in metres for Building II, in addition, the results for both buildings are based only on three (3) sniffers as indicated in Figures 3 and 4.

Table 3 Location errors (Scenario A; meters)

		Mean error	*Error in percentile*			
			50%	*70%*	*85%*	*90%*
Building I						
RM	Lateration	5.5	4.0	8.2	9.8	10.8
	MDS	4.4	4.2	5.2	5.7	6.0
SD	Lateration	3.9	3.5	4.6	5.5	6.5
	MDS	3.8	3.0	4.4	5.0	7.9
DF	Lateration	6.7	6.0	8.3	13.8	14.5
	MDS	5.8	4.0	6.8	10.2	12.0
	SLM	3.7	3.5	4.6	6.2	6.7
Building II						
RM	Lateration	34.2	–	–	–	–
	MDS	11.2	8.0	12.8	20.3	21.3
SD	Lateration	5.8	4.5	7.7	8.9	9.4
	MDS	6.4	4.5	7.7	9.4	10.8
DF	Lateration	9.3	7.5	11.3	12.4	14.8
	MDS	7.6	6.5	8.2	12.3	13.2
	SLM	4.1	3.7	6.3	7.5	7.8

From the table, it appears that for all localisation mechanisms, location determination results for the basement building (Building II) are not comparable with those from Building I because of the severe multipath radio propagation environment. This indicates that a system that works well in one building may not equally perform well in other buildings. Comparing the results from two location searching algorithms, i.e., the MDS and the lateration, it seems that the MDS method would generally provide much better estimation than the lateration method. And of all four indoor mechanisms, SLM method and SD method perform better than others.

5.4 Dependence on the number of deployed sniffers and reference measurements

This section will study the systems' dependence on the number of deployed sniffers and the number of reference measurements for all indoor mechanisms. The simulation will be based on scenarios (b) and (c) as described in Table 1, where only the basement building (Building II) will be considered.

5.4.1 Number of deployed sniffers

With all five sniffers in Figure 4, simulation using SLM gives 3.2 metres localisation errors. Compared with 4.1 metres with 3 sniffers, the performance improvement with five sniffers is impressive. This shows that more deployed sniffers provide better performance for the database based method. This agrees with other research results including Ladd et al. (2004), and Haeberlen et al. (2004).

Similarly, for the other three methods (i.e., RM, SD, and DF), when using five sniffers in Building II (i.e., 'Deployment 2' with sniffers A'BCDE in Figure 4), average distance estimation errors are similarly determined. The results are compared with those from three sniffers (from the previous section), and they are given in Figure 6. The figure shows the distance distribution probability (x-axis) with distance estimation errors (y-axis) for all three mechanisms, the red lines with '+' denote results with five sniffers, and the blue lines with 'o' represent results from three sniffers. The legends in figures also give average errors for each method. It is interesting to see that more deployed sniffers do not 'significantly' improve the accuracy of the distance estimation when compared with the SLM method. For all mechanisms, more deployed sniffers do provide slightly better (or similar) results when distribution probability is within 90%; at a larger probability, the results become more complex, and estimation errors from all five deployed sniffers seem to increase faster for both DF and RM, resulting larger average errors eventually (see the legends from the figure).

Based on the estimated distance values, the location of the client could be determined using two different techniques: using all five sniffers positions as reference (scenario (C) in Table 1) or selecting only three best reference positions from all five sniffers (scenario (B)). In this paper, three closer sniffers' positions were selected (according to a stronger received signal strength from the client), and the simulation results are given in Figure 7. Table 4 further shows the average localisation errors for both techniques (column 4 and column 5). Column 3 references the results from Table 3 (see Section 5.3.3).

It can be seen that while the deployment of more sniffers would greatly improve the localisation performance for the SLM mechanism, for distance-based indoor systems (i.e., RM, SD and DF), the performance impact from the number of deployed sniffers is not straightforward. It appears that only the SD method welcomes the extra deployed sniffers. For other two methods (RM and DF), the averaging

Figure 6 Distance errors using different sniffers (see online version for colours)

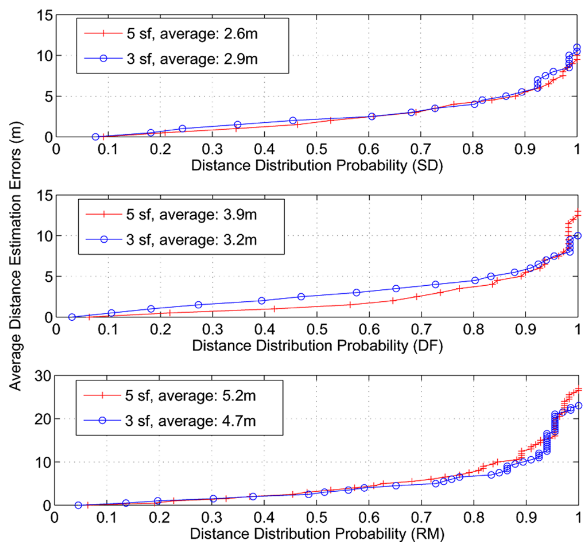


Table 4 Dependence on the number of sniffers

		Building II		
		3 Deployed sniffers	*5 Deployed sniffers*	
			3 positions	*All 5 sinffers*
RM	Lateration	34.2	11.1	40.0
	MDS	11.2	7.4	13.6
SD	Lateration	5.8	4.4	4.6
	MDS	6.4	4.5	4.6
DF	Lateration	9.3	10.5	13.6
	MDS	7.6	8.4	10.8
	SLM	4.1	–	3.2

over all five sniffers may actually overshadow certain critical location parameters (because of the imprecise information) and therefore brings considerable errors to the distance estimation and the location determination. On the other hand, if five sniffers were used to determine the parameter of the models, the selection of three closer sniffers as reference positions in the location determination process will help improve the performance of all methods. This is verified in Table 4 between column 4 (3 selected positions) and column 5 (all 5 deployed sniffers).

5.4.2 Dependence on the number of reference measurements

This section addresses the question whether multiple reference measurements would yield better (both RSS and distance) estimates that are closer to actual measurements at data validation positions. We will first consider the SLM mechanism and then discuss the distance-based methods (i.e., RM, SD and DF).

- For the SLM system, the result is a bit of interesting: one reference measurement triplet (from three sniffers) will yield estimates as good as estimates from 2, 3, or 10 reference measurement triplets. The results from both building agree well with the ARIADNE in Ji et al. (2006).
- Different from the SLM approach, distance-based mechanisms would require more validation positions to be referenced in order to achieve reasonable distance estimation performance (and thus acceptable localisation results). Table 5 gives the simulation results for the Building II, in the table, the 2nd column indicates the distance estimation errors (in metres) with all 22 validation positions; the 3rd column shows the results with only five selected reference positions which include: point 1, 7, 10, 14 and 22 (see Figure 4); and the 4th column gives the results with 10 reference positions (point 1, 3, 5, 7, 8, 10, 12, 14 16, and 22). These positions were selected in order to provide better coverage for most representative locations for the floor plan (Section 4.3). From the table, it can be seen that the RM mechanism does not suggest stronger dependence on the available reference positions (which is similar to the SLM mechanism); but the other two methods (DF and DF) obviously requires more than reference positions, and the more available reference positions (and more deployed sniffers), the better the distance estimation results. From the table, it seems that 10 reference positions would generate decent results for SD method in Building II.

Figure 7 Localisation errors using various ref. positions (see online version for colours)

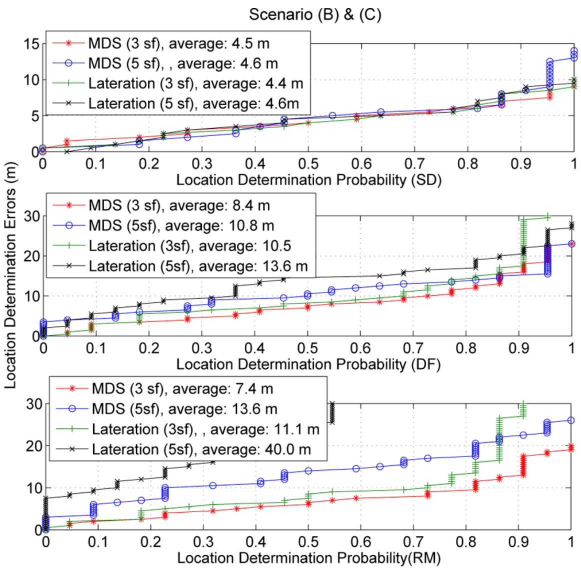


Table 5 Distance errors with different ref. positions

	Number of validation positions (for building II with 5 sniffers)		
	All 22 positions	*5 selected*	*10 selected*
RM	5.2	5.4	5.3
SD	2.6	19.7	2.8
DF	2.4	11.7	11.1

5.5 Precision bounds for testing buildings

To determine the achievable precision bound for a location determination mechanism in both buildings, the shortest distance r_1 (see equation (6)) from each data validation position to deployed sniffers were first identified, then the average results for all data validation positions are given in the second column in Table 6. Accordingly, for a given range perturbation δ, the precision bound d_ε of the location determination in these buildings would be easily determined from equation (6) and selected results with range perturbation δ of 20%, 30% and 50% of distance r_1 are given in column 3–5 in Table 6.

Considering the three distance-based location determination mechanisms (i.e., RM, SD, and DF) studied in this paper, achievable precision bounds of each

Table 6 Precision bounds for both buildings

d_ε	*Average distance* r_1	*Range perturbation* δ *20%*	*30%*	*50%*
Building I (3 sniffers)	8.50	1.54	2.31	3.85
Building II (3 sniffers)	5.92	1.07	1.61	2.67
Building II (5 sniffers)	4.78	0.87	1.31	2.17

system could be determined by using distance estimation errors (i.e., δ) from Tables 2 and 5, and the results (in metres) are shown in bold in Table 7. Comparing the localisation results from Tables 3 and 4 (which are also given in column 4 in Table 7), it shows that all three distance-based localisation mechanisms only achieve 64% at the best of the theoretical error limits (or bounds) by the system, consequently, there are still a lot of space for the future improvement.

Table 7 Precision bounds and localisation results

	Building I – 3 sniffers		
	δ	d_ε	*Results from Table V*
RM	3.1	**2.81**	4.4
SD	2.4	**2.17**	3.8
DF	2.5	**2.26**	5.8
SLM	2.7	**2.44**	3.7
	Building II – 3 sniffers		
	δ	d_ε	*Results from Table V*
RM	4.7	**4.25**	11.2
SD	2.9	**2.62**	5.8
DF	3.2	**2.90**	7.6
SLM	3.3	**2.99**	4.1
	Building II – 5 sniffers		
	δ	d_ε	*Results from Table VII*
RM	5.2	**4.71**	7.4
SD	2.6	**2.35**	4.4
DF	2.4	**2.17**	8.4
SLM	3.3*	**2.99***	3.2

The SLM method does not give direct distance information for the location determination (and thus the uncertainty value δ), consequently, in order to obtain a rough precision bound for the SLM method, this study first determined the difference of both measurement RSS values and the distance values among all data validation positions, then according to the actual RSS estimation from the radio propagation model (for example, Figure 5), range perturbation values δ would be roughly estimated for both buildings. Note that this processing technique did not consider the difference between different propagation paths, and thus the range perturbation δ is same for both configurations (i.e., no dependence on the number of deployed sniffers in the Building). In reality, more deployed sniffers may result smaller range perturbation, and thus values of both the δ and the d_ε with '*' in the last row in Table 7 could be much smaller.

Comparing the location determination results with their corresponding precision bounds, it can be seen that both SD and DF methods would potentially present smaller bounds while the SLM and the RM methods generate slightly larger bounds. Of all four methods studied in this paper, it appears that only the SLM is able to offer smaller location determination error which is also much closer to the theoretical limit or the precision bound.

Consequently, due to multipath radio propagation and complex indoor environment, the performance of an indoor system really depends on many factors including building environments, number of available sniffers and their deployment methods, the number of reference SS measurements and their positions, and many others. Moreover, for different localisation mechanisms, each of these factors would pose unique effects to those indoor systems, therefore it is very difficult, if not impossible, to interpret these effects individually. From the analysis in this section, it can be seen that the precision bound would be able to connect or integrate these factors uniquely and to offer a standard criterion that could be used to evaluate each system. For a given indoor system in a given floor plan, there exists a theoretical precision bound that would provide an achievable limit for the location determination, and both the precision bound and its actual achieved localisation performance would determine the potential of the system.

The SLM method appears to work better when compared with other distance-based mechanisms, but it should be noted that, as indicated in Table 7, this method does not provide direct estimation for the range perturbation (i.e., δ) and the estimation from multiple reference positions does not give an optimal result. When imprecise information (such as distance or RSS) must be used in the location determination process, it seems that the fingerprinting or the database mapping is capable to identify a better match than that from distance based mechanisms.

6 Related research

Many location determination systems have been developed for the indoor environment, however, the comparison study has been very challenging because of the complexity of the indoor radio transmission. Overtime, researchers have begun to analyse those critical environmental factors that impact the indoor system. For example, Ji et al. (2007) and Chen et al. (2006b) studied the optimal mechanism of sniffers deployment. Later Ji (2009), also reported the impact of other factors (including the number of sniffers, humidity, furniture, and other indoor partitions such as supporting columns and grocery storage closet) to indoor systems. However, impacts from those factors were not linked with each other and thus ultimate effects to an indoor system in a given building are still not clear.

In order to better understand an indoor system, researchers also tried to develop benchmark standards with the hope to contribute standard and reproducible test beds. For example, Wallbaum and Diepolder (2005) enumerated a list of factors that could impact indoor localisation. The list covers virtually every aspect of an indoor system, which includes building environment, wireless equipment, data sampling method, and evaluation techniques. However, the authors did not prioritise or provide a way to standardise those factors, and an effective benchmark standard is still not available.

Different from the benchmark approach, more researchers took a more *practical* method which is to apply common data set for the systems' comparison study. For example, the IEEE ICDM (2007) offered a concrete data set for an academic building of 145.5 m × 37.5 m, where a mobile's location would be estimated using current RSS values against those from nearby reference locations. The released data set was collected at a very fine resolution (1.5 m × 1.5 m) over a grid of 247 units, however, it does not include either the building structure or APs locations, and therefore this data set is useful mainly for the evaluation of data processing techniques such as classification and machine learning. In addition, the CRAWDAD, a community resource for archiving wireless data at Dartmouth (http://crawdad.cs.dartmouth.edu/index.php), also offers wireless trace data (indoor and outdoor using Wi-Fi, Bluetooth, and cellular) from many contributing locations for collaborative research in location determination, routing algorithms, and communication protocols. However, there are still not many valuable comparison studies using one or more common data sets in recent literature.

7 Conclusion

Using data measurements from two very different buildings, this paper examined four dynamic indoor localisation systems from several different perspectives including signal-location map construction, distance estimation, environments impacts, system deployment mechanisms, reference measurements, and detailed location searching methods. Research indicated that these factors pose unique impacts to different localisation mechanisms, and thus the effort to understand the performance of various indoor systems by setting a set of benchmark standards based on each individual factor could be very difficult.

Consequently, instead of analysing individual dependence between the location determination performance and the environmental or system factors, this research studied a theoretical location determination limit and proposed a precision bound as a standard criterion for the evaluation of various localisation systems. Experimental results validated the precision bound for all localisation mechanisms at both buildings, thus this work provides critical insights to the research in dynamic location determination.

From the study, this paper also indicated that the signal-location map mechanism, a database-based method, delivers better results and relies less on reference RSS measurements. On the other hand, distance-based systems present location estimates with less dependence on the number of deployed sniffers, but they require large number of reference signal measurements that should be carefully selected across the building. This paper also evaluated two distance-based searching algorithms and results indicated that the multidimensional scaling performs better than the lateration method in all simulation evaluations. This suggests that better location search algorithms are critical for robust location determination.

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Note

[1] If the distance measurement uncertainty δ is different at all sniffers, the ultimate limit of *UA* for the mobile will still be a smallest circle or sphere with some diameter of $2r\delta_i$.

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