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Size-Dependent Electric Conductivity in Semiconducting Thin Wires

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A quantum theory for electric conductivity in a thin rectangular-shaped wire is developed taking into account the wave character of an electron. It is shown that the quasi-one-dimensional (QOD) gas behaves somewhat like the strictly-one-dimensional (SOD) gas when the wire is ultra-thin, so that the spacing between the quantized levels is quite large, and all the electrons are assumed to be in the lowest quantized level. In the ultra-thin limit (UTL), the ratio of longitudinal resistivity to bulk resistivity is shown to be proportional to λ_D^2/A, where $\lambda_D = \hbar/(2m^* k_B T)^{1/2}$ is the de-Broglie wavelength of an electron of effective mass m^* at temperature T, and A is the area of the cross-section of the wire. The transverse resistivity ratio depends strongly on the scattering parameters and is inversely proportional to the area for wires of square cross-section.

Es wird eine Quantentheorie für die elektrische Leitfähigkeit in einem dünnen rechteckförmigen Draht entwickelt, wobei der Wellencharakter des Elektrons berücksichtigt wird. Es wird gezeigt, daß sich das quasi-eindimensionale (QOD) Gas etwa ähnlich wie das strikt-eindimensionale (SOD) Gas verhält, wenn der Draht extrem dünn ist, so daß der Abstand zwischen den Quantenniveaus sehr groß ist und sich alle Elektronen im niedrigsten Quantenniveau befinden. In dem ultradünnen Grenzfall (UTL) ist das Verhältnis des longitudinalen Widerstands zum Volumenwiderstand proportional zu λ_D^2/A, wobei $\lambda_D = \hbar/(2m^* k_B T)^{1/2}$ die de-Broglie-Wellenlänge eines Elektrons der effektiven Masse m^* bei der Temperatur T und A die Querschnittsfläche des Drahtes ist. Das transversale Widerstandsverhältnis hängt stark von den Streuparametern ab und ist proportional zur Fläche der Drähte mit rechteckigem Querschnitt.

1. Introduction

In the last few years, there has been a considerable interest in electronic transport limited by the size of the sample. The existence of quantized energy levels in thin films [1] is now well known. When the de-Broglie wavelength λ_D of an electron is comparable to dimension of the sample, quantum effects are important. Under degenerate conditions, these effects are of oscillatory type [2]. But, for non-degenerate semiconductors, monotonically increasing or decreasing behavior of the transport properties with sample size is expected. The quantum size effect (QSE) and perspectives of its practical applications have been discussed by Elinson et al. [2].

Recently, it has been predicted [3, 4] that thin metal films never exhibit true metallic conductivity. Non-metallic behavior at low temperatures for thin wires whose impurity resistance exceeded 10 kΩ is predicted by Thouless [4]. The Fermi energy ζ of a metal lies well above in the conduction band. But, when quantization is taken into account, the lowest quantized energy $\varepsilon_{y,z}$ (see Section 3) of an electron may become larger than the Fermi energy. And, for a sufficiently thin sample, when $\varepsilon_{y,z} \gg \zeta$, metals may behave like semiconductors. In the ultra-thin limit (UTL), when only the lowest quantized level is assumed to contain most of the electrons, the system

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behaves somewhat like a strictly low-dimensional system [1, 5]. Electric transport parameters are then strongly size-dependent [1, 5]. An analogous situation occurs for transport properties of electrons confined to a strong magnetic field [6], when $\lambda_D \gtrsim \lambda$, the radius of the cyclotron orbit. Transport parameters are then strongly magnetic-field-dependent. When magnetic field in a degenerate sample is sufficiently strong, so that $\lambda_D \gg \lambda$, the metal behaves like a semiconductor [7]. This kind of behavior has not yet been seen in metals in a magnetic field as the magnetic field required for this to happen is expected to be quite large. This may not be too difficult to test in future when ultrahigh magnetic fields are available.

The QSE in thin films has been well investigated, but the study of QSE in thin wires is just beginning to emerge. So far, the theoretical work is largely based on the model of localization, according to which the electronic states become localized when variation in potential wells in juxtaposition with their spatial distribution is great enough to prevent transitions between them [8]. Several theories had predicted that the electric conductivity in strictly one-dimensional (SOD) systems would go exponentially to zero with temperature [8], but such conclusions could not be experimentally tested until Thouless [4] generalized the arguments to apply to thin wires for which the length was sufficiently long compared to cross-sectional area. Experimental results tend to be in qualitative agreement with the theory, but there is a lack of quantitative agreement [9]. Dolan and Osheroff [10] have also done similar experiments to test Thouless's theory and have indicated one-dimensional localization.

In a more recent study [1], we have tried to explain the behavior of a strictly two-dimensional (STD) and a quasi-two-dimensional (QTD) gas in terms of a quantum transport theory. In the UTL, QTD gas is found to behave somewhat like an STD gas. Relative longitudinal (parallel to the film) resistivity is found to be proportional to λ_D/d, where d is the film thickness. This study showed that the quantitative relation between resistance, temperature, and size of the sample is determined mainly by the scattering mechanisms affected by the size quantization of the electron [1]. Motivated by the success of the above calculations, we present here a study, in terms of quantum theory, of transport parameters in long and thin wires, taken to be rectangular in cross-section. In this work, we will only consider the electron scattering via acoustic phonons and point defects represented by a δ-function potential. The study of the electric conductivity in the domain when ionized-impurity and other scattering mechanisms are important is deferred to a future work.

In Section 4, we discuss the transport properties of an SOD gas when electron waves are constrained to move in one direction only. In Section 3, we indicate the quantum properties of QOD gas and show its relationship with SOD and bulk properties. The results so obtained are analyzed in the UTL. For completeness sake, in Section 2, we present a more elaborate quantum transport theory, that involving the density matrix to find the expressions for conductivity in transverse (perpendicular to the length of the wire) as well as longitudinal (parallel to the length of the wire) configuration.

2. SOD Model

In an SOD model, the electronic motion parallel to the length a of the wire is characterized by a plane wave,

$$\psi_k = (1/a)^{1/2} \exp(ikx) \tag{2.1}$$

with associated energy given by

$$\varepsilon_k = \hbar^2 k^2/2m^* . \tag{2.2}$$

The matrix elements of the velocity operator v in the representation of (2.1) are

$$\langle k'|\, v_x\, |k\rangle = (\hbar k/m^*)\, \delta_{k'k}\,. \tag{2.3}$$

The density of states $N_0(\varepsilon) \equiv \sum_{ks} \delta(\varepsilon - \varepsilon_k)$, where s stands for spin, is readily calculated as

$$N_0(\varepsilon) = a(2m^*)^{1/2}\, \varepsilon^{-1/2}/\pi\hbar\,. \tag{2.4}$$

$N_0(\varepsilon)$, unlike in STD system, varies inversely as the square root of energy. The Fermi energy ζ, as obtained from the normalization $\left(N = \sum_{ks} f_0(\varepsilon_k)\right)$ of the distribution function $f_0(\varepsilon_k) \approx \exp\left[(\zeta - \varepsilon_k)/k_BT\right]$, is given by

$$\zeta = -\, k_BT \ln\left[(2\pi m^*\, k_BT)^{1/2}/n_a\pi\hbar\right], \tag{2.5}$$

where $n_a = N/a$ is the number of electrons per unit length, m^* the effective mass of the electron and T the temperature. The inverse relaxation time $\tau^{-1}(\varepsilon_k)$ for electron scattering via acoustic phonons and point defects is proportional to the density of states [1]. For a thin wire, following the procedure used earlier [1], $\tau^{-1}(\varepsilon_k)$ is obtained as follows:

$$\tau^{-1}(\varepsilon_k) = (E_1^2 k_BT\, (2m^*)^{1/2}/\varrho u^2 bd\hbar^2)\, \varepsilon_k^{-1/2} \qquad \text{(acoustic phonons)}\,, \tag{2.6}$$

$$= (n_i V_0^2 (2m^*)^{1/2}/bd\hbar^2)\, \varepsilon_k^{-1/2} \qquad \text{(point defects)}\,, \tag{2.7}$$

where E_1 is the deformation potential constant, ϱ the volume density of wire, u the sound velocity, b and d are transverse lengths of the rectangular wire, n_i is the volume density of point defects, and V_0 the potential parameter for point defect potential $V = V_0 \sum_{i=1}^{n_i} \delta(x - x_i)$.

The electric conductivity parallel to the wire as obtained from the solution of the Boltzmann transport equation (BTE) is

$$\sigma_s = -\, \frac{e^2}{\Omega} \sum_{ks} \left(\frac{\hbar k}{m^*}\right)^2 \frac{\partial f_0}{\partial \varepsilon_k}\, \tau(\varepsilon_k)\,, \tag{2.8}$$

where $\Omega = abd$ is the volume of the wire. It is worth noting that the use of BTE is still valid as the matrix elements of velocity, as given by (3), are diagonal. Now, converting summation to integration by $\sum_k \to (a/2\pi) \int \mathrm{d}k$ and using (5) after integration, we have for σ_s a simple expression,

$$\sigma_s = 2n_a e^2 \lambda_D \hbar \varrho u^2/\sqrt{\pi} m^* E_1^2 \qquad \text{(acoustic phonons)}\,, \tag{2.9}$$

$$= 2n_a e^2 \lambda_D \hbar k_BT/\sqrt{\pi} m^* n_i V_0^2\,, \qquad \text{(point defects)}\,, \tag{2.10}$$

where

$$\lambda_D = \hbar/(2m^* k_BT)^{1/2}\,. \tag{2.11}$$

Comparing the above expressions with the bulk conductivity σ_b,

$$\sigma_b = 4ne^2 \varrho u^2 \pi \hbar^4/3(2\pi m^* k_BT)^{1/2}\, m^{*2} E_1^2 k_BT \qquad \text{(acoustic phonons)}\,, \tag{2.12}$$

$$= 4ne^2 \pi \hbar^4/3(2\pi m^* k_BT)^{1/2}\, m^{*2} n_i V_0^2 \qquad \text{(point defects)}\,, \tag{2.13}$$

we get for the relative resistivity ratio $\varrho_s/\varrho_b = \sigma_b/\sigma_s$ (both for acoustic phonon and point defect scattering) the simplified result

$$\varrho_s/\varrho_b = 4\pi\lambda_D^2/3A\,. \tag{2.14}$$

The relative resistivity is, therefore, inversely proportional to the area of cross-section $A = bd$ of the wire, is inversely proportional to T^{-1}, and is independent of the scattering parameters. No transverse conductivity is expected in SOD model as the transverse electronic motion is zero.

3. QOD Model

In a QOD model, the electronic motion in the transverse direction is quantized, with eigenfunctions ψ_{plk} and eigenvalues ε_{plk} given by

$$\psi_{plk} = (2/\Omega)^{1/2} \exp(ikx) \sin(p\pi y/b) \sin(l\pi z/d)\ ; \qquad p, l = 1, 2, 3\ , \tag{3.1}$$

$$\varepsilon_{plk} = \hbar^2 k^2/2m^* + p^2 \varepsilon_y + l^2 \varepsilon_z\ , \tag{3.2}$$

respectively, with

$$\varepsilon_y = \pi^2 \hbar^2/2m^* b^2\ , \qquad \varepsilon_z = \pi^2 \hbar^2/2m^* d^2\ . \tag{3.3}$$

The matrix elements of the components v_x, v_y, and v_z of the velocity vector in the representation of (3.1) are given by

$$\langle p'l'k'|\, v_x\, |plk\rangle = (\hbar k_x/m^*)\, \delta_{l'l} \delta_{p'p} \delta_{k'k}\ , \tag{3.4}$$

$$\left.\begin{aligned}\langle p'l'k'|\, v_y\, |plk\rangle &= 2\hbar pp' \{1 - \exp[i\pi(p' - p)]\}\ \delta_{l'l} \delta_{k'k}/im^* b(p'^2 - p^2) \\ &= 0 \quad \text{when} \quad p = p'\ ,\end{aligned}\right\} \tag{3.5}$$

$$\left.\begin{aligned}\langle p'l'k'|\, v_z\, |plk\rangle &= 2\hbar ll' \{1 - \exp[i\pi(l' - l)]\}\ \delta_{p'p} \delta_{k'k}/im^* d(l'^2 - l^2) \\ &= 0 \quad \text{when} \quad l = l'\ .\end{aligned}\right\} \tag{3.6}$$

As we see from (3.4) to (3.6), the matrix elements of v_x are diagonal, while those of v_y and v_z are non-diagonal. The BTE, therefore, can be successfully used for calculating the expectation value of the electronic current parallel to the wire. But, for v_y and v_z a more elaborate technique, that of using the density matrix [11], is required.

The density of states, $N(\varepsilon) \equiv \sum_{plks} \delta(\varepsilon - \varepsilon_{plk})$, are evaluated as follows:

$$N(\varepsilon) = [a(2m^*)^{1/2}/\pi\hbar] \sum_{pl}{}' (\varepsilon - p^2 \varepsilon_y - l^2 \varepsilon_z)^{-1/2}\ , \tag{3.7}$$

where the prime on the summation indicates $x^{-1/2}$ is zero when $x < 0$. The Fermi energy ζ, evaluated from the normalization condition, is given by

$$\zeta = -k_B T \ln\left((2\pi m^* k_B T)^{1/2} \gamma_y \gamma_z/n_a \pi\hbar\right) \tag{3.8}$$

with

$$\gamma_y(\varepsilon_y, T) = \sum_p \exp(-p^2 \varepsilon_y/k_B T)\ , \tag{3.9}$$

$$\gamma_z(\varepsilon_z, T) = \sum_l \exp(-l^2 \varepsilon_z/k_B T)\ . \tag{3.10}$$

(3.8) agrees with (2.5) if γ_y and γ_z are taken unity. The infinite series $\gamma_y(\varepsilon_y, T)$ and $\gamma_z(\varepsilon_z, T)$ cannot be evaluated analytically. In the limiting cases ($\varepsilon_{y,z} \gg k_B T$) or $\varepsilon_{y,z} \ll k_B T$), these can be evaluated to give

$$\gamma_y(\varepsilon_y, T) = \exp(-\varepsilon_y/k_B T)\ ; \qquad \varepsilon_y \gg k_B T\ , \tag{3.11}$$

$$= (\pi k_B T/4\varepsilon_y)^{1/2}; \qquad \varepsilon_y \ll k_B T\ , \tag{3.12}$$

$$\gamma_z(\varepsilon_z, T) = \exp(-\varepsilon_z/k_B T); \qquad \varepsilon_z \gg k_B T\ , \tag{3.13}$$

$$= (\pi k_B T/4\varepsilon_z)^{1/2}; \qquad \varepsilon_z \ll k_B T\ . \tag{3.14}$$

The BTE, for the conductivity in the longitudinal direction, gives for conductivity σ_{xx}^Q an expression

$$\sigma_{xx}^Q = -\frac{e^2}{\Omega} \sum_{plks} \left(\frac{\hbar k}{m^*}\right)^2 \frac{\partial f_0}{\partial \varepsilon_{plk}}\, \tau(\varepsilon_{plk})\ , \tag{3.15}$$

where $\tau^{-1}(\varepsilon_{plk})$, by following the procedure outlined earlier [1], is given by

$$\tau^{-1}(\varepsilon_{plk}) = [E_1^2 k_B T (2m^*)^{1/2}/bd\varrho u^2 \hbar^2] [\sum_{p'l'} (\varepsilon_{plk} - p'^2\varepsilon_y - l'^2\varepsilon_z)^{-1/2} + \\ + \tfrac{1}{2} \sum_{p'} (\varepsilon_{plk} - p'^2\varepsilon_y - l^2\varepsilon_z)^{-1/2} + \tfrac{1}{2} \sum_{l'} (\varepsilon_{plk} - p^2\varepsilon_y - l'^2\varepsilon_z)^{-1/2} + \\ + \tfrac{1}{4} (\varepsilon_{plk} - p^2\varepsilon_y - l^2\varepsilon_z)^{-1/2}] \quad \text{(acoustic phonons)}\,. \tag{3.16}$$

In the UTL ($p' = 1$, $l' = 1$), the above expression for the relaxation time yields

$$\tau_{\text{UTL}}^{-1}(\varepsilon_k) = [qE_1^2 k_B T (2m^*)^{1/2}/4\varrho u^2 bd\hbar^2] \varepsilon_k^{-1/2} \qquad \text{(acoustic phonons)} \tag{3.17}$$

which differs from $\tau^{-1}(\varepsilon_k)$ in SOD model, as given in (2.6), by a factor $\frac{9}{4}$. For point defect scattering, the above expressions are valid if $E_1^2 k_B T/\varrho u^2$ is replaced by $n_i V_0^2$.

The conductivity expression given by (3.15) in the UTL ($\varepsilon_{y,z} \gg k_B T$) simplifies to

$$\sigma_{xx}^Q = 4\sigma_s/9\,. \tag{3.18}$$

Thus, the longitudinal conductivity in QOD model differs from that in SOD model by a factor $\frac{4}{9}$. The relative resistivity ratio $\varrho_{xx}^Q/\varrho_b = \sigma_b/\sigma_{xx}^Q$ is given by

$$\varrho_{xx}^Q/\varrho_b = 3\pi\lambda_D^2/A\,. \tag{3.19}$$

We thus see that a QOD gas behaves somewhat like an SOD gas in the UTL only. In the bulk limit ($\varepsilon_{y,z} \ll k_B T$), it can be shown that ϱ_{xx}^Q/ϱ_b approaches unity. Thus, two limiting cases (bulk and SOD) bracket the QOD case very well.

4. Quantum Transport Theory

As discussed in the previous section, BTE has its own limitations. It cannot be used, for example, to find the expectation value of current flowing perpendicular to the wire as the average value of the velocity is zero in a transverse configuration [11]. We then resort to a more complete quantum-mechanical theory, that of using the density matrix ϱ. The expectation value of the current is then obtained from

$$\boldsymbol{J} = -\frac{e}{\Omega} \sum_{\alpha\alpha'} \langle\alpha|\,\varrho\,|\alpha'\rangle \langle\alpha'|\,\boldsymbol{v}\,|\alpha\rangle\,, \tag{4.1}$$

where $\langle\alpha|\,\varrho\,|\alpha'\rangle$ are evaluated from the solution of Liouville's equation

$$i\hbar\, \partial\varrho/\partial t = [H, \varrho]\,. \tag{4.2}$$

Here $H = H_0 + V + F$ is the Hamiltonian of the system, which consists of unperturbed part H_0, electron–lattice interaction V, and electron–electric-field interaction $F = e\mathscr{E} \cdot \boldsymbol{r}$. The eigenfunctions and eigenvalues of the electronic part of H_0 are those given by (3.1) and (3.2), respectively. A steady state linearized solution, in the representation $|\alpha\rangle = |plk\rangle$ of H_0, is given by

$$\langle\alpha'|\,\varrho\,|\alpha\rangle = f_0(\varepsilon_\alpha)\,\delta_{\alpha'\alpha} + \frac{\langle\alpha'|\,[\varrho_0, F]\,|\alpha\rangle}{\varepsilon_{\alpha'} - \varepsilon_\alpha - i\hbar\tau_{\alpha'\alpha}^{-1}} \tag{4.3}$$

with

$$\tau_{\alpha'\alpha}^{-1} = \tfrac{1}{2}\tau_{\alpha'}^{-1} + \tfrac{1}{2}\tau_\alpha^{-1}\,, \tag{4.4}$$

where $\tau_{\alpha'}^{-1} = \tau^{-1}(\varepsilon_{plk})$ is that given by (3.16). The matrix elements of commutator

$[\varrho_0, F]$ for an electric field $\vec{\mathscr{E}} = (\mathscr{E}_x, \mathscr{E}_y, \mathscr{E}_z)$ are given by

$$\langle\alpha'|\,[\varrho_0, F]\,|\alpha\rangle = \frac{\mathrm{d}f}{\mathrm{d}\varepsilon_{plk}}\,\frac{e\hbar^2}{im^*}\,k\mathscr{E}_x\,\delta_{l'l}\delta_{p'p}\delta_{k'k} - $$
$$- \frac{2e\hbar^2 pp'\,\{1-\exp\,[i\pi(p'-p)]\}}{m^*b(p'^2-p^2)^2\,\varepsilon_y}\,[f_0(\varepsilon_{p'lk})-f_0(\varepsilon_{plk})]\,\mathscr{E}_y\delta_{l'l}\delta_{k'k} -$$
$$- \frac{2l\hbar^2 ll'\,\{1-\exp\,[i\pi(l'-l)]\}}{m^*d(l'^2-l^2)^2\,\varepsilon_z}\,[f_0(\varepsilon_{pl'k})-f_0(\varepsilon_{plk})]\,\mathscr{E}_z\delta_{l'l}\delta_{k'k}\,. \tag{4.5}$$

Using (4.3) to (4.5) along with (4.1) and using $\langle \boldsymbol{J}\rangle = \boldsymbol{\sigma}\cdot\vec{\mathscr{E}}$, we get for the components of $\boldsymbol{\sigma}$ the expressions

$$\sigma_{xx} = \sigma_{xx}^{\mathrm{Q}}\,, \tag{4.6}$$

$$\sigma_{yy} = -\frac{16\hbar^3e^2}{m^{*2}b^2\varepsilon_y\Omega}\,i\sum_{pp'lk}\frac{(pp')^2\sin^2(\pi(p'-p)/2)\,[f_0(\varepsilon_{p'lk})-f_0(\varepsilon_{plk})]\,i\hbar\tau^{-1}_{p'lk,plk}}{(p'^2-p^2)^3\,[(p'^2-p^2)\,\varepsilon_z^2+\hbar^2\tau^{-2}_{l'pk,plk}]}\,, \tag{4.7}$$

$$\sigma_{zz} = -\frac{16\hbar^2e^2}{m^{*2}d^2\varepsilon_z\Omega}\,i\sum_{ll'pk}\frac{(ll')^2\sin^2(\pi(l'-l)/2)\,[f_0(\varepsilon_{pl'k})-f_0(\varepsilon_{plk})]\,i\hbar\tau^{-1}_{pl'k,plk}}{(l'^2-l^2)^3\,[(l'^2-l^2)^2\,\varepsilon_z^2+\hbar^2\tau^{-2}_{p'lk,plk}]}\,, \tag{4.8}$$

$$\sigma_{ij} = 0\,; \qquad i \neq j\,. \tag{4.9}$$

We thus see that quantum transport theory gives results equivalent to those obtained from BTE for σ_{xx}. But expressions for σ_{yy} and σ_{zz} could not be obtained from BTE. In the UTL, these components are simplified to give

$$\sigma_{xx} = 4\sigma_{\mathrm{s}}/9\,, \tag{4.10}$$

$$\sigma_{yy} = \frac{64ne^2\tau_0^{-1}\hbar^6}{81m^{*3}b^3d\varepsilon_y^3\,k_{\mathrm{B}}T}\,\exp\left(\frac{b\hbar/\tau_0}{2^{1/2}\pi^{3/2}dk_{\mathrm{B}}T}\right)^2 \times$$
$$\times\,\mathrm{Ei}\left(-\left(\frac{b\hbar/\tau_0}{2^{1/2}\pi^{3/2}\,dk_{\mathrm{B}}T}\right)^2\right), \tag{4.11}$$

$$\sigma_{zz} = \frac{64ne^2\tau_0^{-1}\hbar^6}{81m^{*3}d^3b\varepsilon_z^3\,k_{\mathrm{B}}T}\,\exp\left(\frac{d\hbar/\tau_0}{2^{1/2}\pi^{3/2}bk_{\mathrm{B}}T}\right)^2 \times$$
$$\times\,\mathrm{Ei}\left(-\left(\frac{d\hbar/\tau_0}{2^{1/2}\pi^{3/2}bk_{\mathrm{B}}T}\right)^2\right) \tag{4.12}$$

with

$$\tau_0^{-1} = 3(2m^*k_{\mathrm{B}}T)^{3/2}\,\mathrm{E}_1^2/8\pi^{1/2}\varrho u^2\hbar^4\,, \tag{4.13}$$

$$\mathrm{Ei}(-\alpha) = \int_\alpha^\infty \frac{\mathrm{e}^{-t}}{t}\,\mathrm{d}t\,. \tag{4.14}$$

Under the conditions when brodening due to collisions is small ($\hbar/\tau_0 \ll k_{\mathrm{B}}T$), $\mathrm{Ei}(-\alpha)$ can be approximated as

$$\mathrm{Ei}(-\alpha) \approx -\ln\alpha - \gamma\,; \qquad \alpha \ll 1\,, \tag{4.15}$$

where $\gamma = 0.577$ is Euler's constant. The relative resistivity ratios (as compared to bulk resistivity) are then given by

$$\frac{\varrho_{xx}}{\varrho_b} = \frac{\varrho_s}{\varrho_b}, \tag{4.16}$$

$$\frac{\varrho_{yy}}{\varrho_b} = \frac{81 d\pi^6 k_B T}{512 \tau_0^{-2} b^3 m^*} \left[\ln\left(\frac{2^{1/2}\pi^{3/2} d k_B T}{b h/\tau_0}\right) - \gamma\right]^{-1}, \tag{4.17}$$

$$\frac{\varrho_{zz}}{\varrho_b} = \frac{81\, b\pi^6 k_B T}{512 \tau_0^{-2} d^3 m^*} \left[\ln\left(\frac{2^{1/2}\pi^{3/2} b k_B T}{d h/\tau_0}\right)^2 - \gamma\right]^{-1}. \tag{4.18}$$

As is expected, the quantum transport theory gives results equivalent to those obtained for BTE for longitudinal conductivity. The longitudinal resistivity ratio is independent of scattering parameters. But, transverse resistivity ratios are strongly dependent on scattering parameters. For a wire with square cross-section, the transverse resistivity varies inversely with the cross-sectional area of the wire. The longitudinal conductivity is inversely proportional to the area for any rectangular shape. The theoretical results for transverse resistivity, at present, may be difficult to test experimentally as measurements of transverse resistivity may be quite difficult. But, in future, when technology develops to that extent, it may be possible to verify the above expressions for the conductivity by including other possible scattering mechanisms.

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