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NONEQUILIBRIUM GREEN FUNCTION TECHNIQUES APPLIED TO HOT ELECTRON QUANTUM TRANSPORT

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ABSTRACT

During the last few years considerable effort has been devoted to deriving quantum transport equations for semiconductors under extreme conditions (high electric fields, spatial quantization in one or two directions). Here we review the results obtained with nonequilibrium Green function techniques as formulated by Baym and Kadanoff, or by Keldysh. In particular, the following topics will be discussed: (i) Systematic approaches to reduce the transport equation governing the correlation function to a transport equation for the Wigner function; (ii) Approximations reducing the nonmarkovian quantum transport equation to a numerically tractable form, and results for model semiconductors; (iii) Recent progress in extending the formalism to inhomogenous systems; and (iv) Nonequilibrium screening. In all sections we try to direct the reader's attention to points where the present understanding is (at best) incomplete, and indicate possible lines for future work.

KEYWORDS

Transport theory; semiconductor heterostructures; non-equilibrium Green functions; quantum transport; Monte Carlo simulations

INTRODUCTION

Transport in semiconductors has traditionally been described with the help of the Boltzmann equation (BE) (Conwell, 1967) (or by some of its simplifications, such as the drift-diffusion equation). Powerful Monte Carlo techniques have been developed to solve the BE (for a review, see e.g. Jacoboni and Reggiani, 1983), and combined with Poisson equation solvers these methods form the basis for the theoretical analysis and design of modern semiconductor microdevices. As the characteristic length scales continue to decrease, however, quantum effects begin to dominate the transport, and consequently the semiclassical BE cannot be used as a starting point. Examples of such quantum effects are space quantization (the characteristic lenght scales in one or several directions are such that plane waves are no longer appropriate wave functions for describing the charge carriers), or ballistic transport (the charge carriers experience no or only few collisions within the active region of the device; here we point out the difference between *quantum* ballistic transport, such as underlies resistance quantization in quantum point contacts (van Wees and others, 1988, and Wharam and others, 1988) and ballistic peaks observed in the distribution function in Boltzmann equation studies of microstructures (Baranger and Wilkins, 1984)). Other effects beyond the conventional Boltzmann picture include the influence of field on collision processes, the so called intra-collisional field effect (ICFE) (Barker 1973, Levinson 1970), and collisional broadening (CB), or, in other words, effects due to finite quasi-particle life-times. A common theme for all these quantum effects is that the *phase coherence* of the charge carriers is maintained longer than some characteristic length scale (e.g. the inelastic mean free path), or time scale (the collision duration time is not negligible).

A number of theoretical approaches have been developed to describe quantum transport. Methods based on the Landauer formula (Landauer 1970) have been particularly useful in analyzing transport in situations where inelastic collisions in the 'interesting' part of the system are infrequent (dissipation and loss of phase coherence takes place in the contacts). The Feynman path integral method has been applied to high field transport in semiconductors (Thornber 1978, Mason and Hess 1989), but it has not yet gained as widespread use as some other methods. A quantum Langevin equation approach has been developed by Hu and O'Connell (1987,1988,1989). Wigner distribution function methods have also been reported (Frensley 1987, and Kluksdahl and others, 1989; the latter reference contains extensive references to other papers employing the Wigner distribution function). In fact, the Wigner function bears a close connection to the nonequilibrium Green function methods (see below) which form the actual topic of this review. - The present review should be viewed as a logical continuation of the review talks on similar topics given during previous Hot Electron Conferences: Barker (1978) and (1981), and Reggiani (1985).

THEORETICAL BACKGROUND

The nonequilibrium Green functions were developed simulataneously, and independently by Baym and Kadanoff (1962), and by Keldysh (1965). These two formalisms are equivalent, and an elegant demonstration can be found in the review article by Langreth (1976). Several review articles focusing on different aspects have recently appeared (Danielewicz 1984 (an interesting application to nonequilibrium nuclear collisions); Mahan 1984, 1987 (focusing on *linear* transport); Rammer and Smith 1986 (quasi-classical Green functions; applications to degenerate Fermi systems); and Jauho 1989 (derivation of quantum kinetic equations for model systems)). We refer the interested reader to these articles for technical details, and additional references: here we try to elucidate the main physical content the various formalisms.

In equilibrium (and consequently also in linear reponse theory, which only involves equilibrium quantities) one can prove the ***fluctuation-dissipation theorem*** which connects the causal (or time-ordered) Green function to the retarded (or advanced) Green function. This relation is of great importance: Wick's theorem, and hence the diagrammatic perturbation techniques can be proved only for the causal function while physical observables are obtained from response functions which are related to retarded functions, or correlation functions. Thus equilibrium theory is in a way short-circuited: it is not necessary to develop calculational schemes for ***both*** causal ***and*** retarded functions. This fortunate situation does not hold in nonequilibrium: one has to develop a theory which contains the two types of Green functions as independent objects. This was the bad news, the good news is that nonequilibrium theory can ***formally*** be written in a form which appears entirely equivalent to the equilibrium theory for the causal Green function (and thus the diagrammatic perturbation theory exists). The price that has to be paid is that the time-labels of the Green function no longer are real: the reside on a complex path which goes form $-\infty$ to $+\infty$, and back to $-\infty$. As shown by Craig (1968), and Langreth (1976), the Baym-Kadanoff and Keldysh formalisms correspond to two slightly differenct choices of this contour in the complex time plane. Complex time integrals are not convenient to work with, and the set of rules which tell how to extract real time quantities out of objects defined on a complex contour are known as 'analytic continuation' or 'Langreth theorems' (Langreth and Wilkins, 1972; Langreth 1976). Applying these rules to the Dyson equation obeyed by the complex time Green function yields the basic equations of the non-equilibrium Green function theory:

$$[G_0^{-1} - U, G^<] - [\Sigma, G^<] - [\Sigma^<, G] = -\frac{1}{2}\{\Sigma^<, G^>\} + \frac{1}{2}\{\Sigma^>, G^<\} \tag{1}$$

$$G_{r(a)} = G_{0,r(a)} + G_{0,r(a)}\Sigma_{r(a)}G_{r(a)} \quad . \tag{2}$$

Here the we introduced the (nonequilibrium) correlation function $G^<$ and the retarded (advanced) Green function $G_{r(a)}$:

$$G^<(1,1') = i < \Psi^\dagger(1')\Psi(1) > \quad , \quad G_{r(a)}(1,1') = \mp i\theta(\pm t_1 \mp t_1') < \{\Psi(1), \Psi^\dagger(1')\} > \ , \tag{3}$$

where $1 \equiv (\mathbf{x}_1, t_1)$ etc. Further, $U(\mathbf{x}, t)$ contains single particle potentials (driving fields, heterojunction conduction band edge potentials, self-consistent Hartree potentials etc.), and the self-energy $\Sigma^< = \Sigma^<[G^<]$ contains the interactions (carrier-impurity scattering, phonon scattering, carrier-carrier scattering etc.). We also defined $\Sigma = \frac{1}{2}(\Sigma_r + \Sigma_a)$ and $G = \frac{1}{2}(G_r + G_a)$. Brackets indicate commutators and curly brackets mean anticommutators. A product of two terms implies multiplication/integration over intermediate variables.

Eqs.(1) and (2) are exact, and they form the starting point of the theory. Their physical interpretation is as follows. ¿From the definition of the correlation function it follows that the Wigner distribution function f^W can be extracted from it:

$$f^W(\mathbf{p}, \mathbf{r}, T) = -iG^<(\mathbf{p}, \tau = 0, \mathbf{R}, T) = -i\int \frac{d\omega}{2\pi} G^<(\mathbf{p}, \omega, \mathbf{R}, T) \tag{4}$$

Here we introduced the center-of-mass coordinates: $\mathbf{R} \equiv (\mathbf{x} + \mathbf{x}')/2$, $\mathbf{r} \equiv \mathbf{x} - \mathbf{x}'$; $\mathbf{p}$ is Fourier transform of $\mathbf{r}$. Analogous definitions hold for the temporal variables. The Wigner function is the quantum mechanical generalization of the semiclassical Boltzmann distribution; in particular physical observables such as density, or current density, are obtained as its moments. Thus, Eq.(1) governs the ***distribution*** of particles: it is a quantum kinetic equation which generalizes the Boltzmann equation. Its structure is also suggestive: the first term on the left hand side gives rise to a driving term, the second and third terms are renormalization terms, and the right hand side is the quantum collision term with the characteristic gain-loss structure. A demonstration of how Eq.(1) reduces to the BE can be found in Langreth (1976), or Jauho (1989). - A final point to note is that the initial value for the distribution used in the kinetic equation (1) should be chosen consistently with the Dyson equation (Barker, 1987, Kluksdahl and others, 1989).

Let us now examine the second equation, Eq.(2). We recall from equilibrium Green function theory the definition of the ***spectral density*** $A \equiv i(G_r - G_a)$. Many important objects (e.g. density of states, scattering rates, quasi-particle life-times) require the knowledge of the spectral density, and thus the solution of Eq.(2) is a prerequisite before proceeding with the kinetic equation Eg.(1). (It may occur that Σ_r involves $G^<$: in this case the two equations must be solved simultaneously. Such complications fall beyond the scope of the present discussion.) Below we analyze several model spectral densities.

Eqs.(1) and (2) are too complicated to be solved directly, and during recent years many research groups have developed simplifications and approximation schemes which allow further progress. We now turn to these applications.

QUANTUM KINETIC EQUATIONS FOR THE WIGNER FUNCTION

There are a number of reasons for trying to develop quantum kinetic equations for the Wigner function f^W rather than the correlation function $G^<$. First, f^W depends on one less variable than $G^<$, and is therefore hopefully a less complicated object. (Actual numerical studies show, however, that for spatially inhomogenous systems f^W is an exceedingly complex object (!) (Kluksdahl and others, 1989)) Second, most physical observables do not require the knowledge of the full correlation function and the knowledge of f^W suffices. Next, since f^W has many

properties of a true distribution function, its equation of motion may resemble the semi-classical BE so much that some of the vast body of experience on interpreting and solving the BE can perhaps be carried over to the quantum case. Finally, many other semiconductor quantum transport equations have been proposed (e.g. Barker and Ferry, 1979; Levinson, 1970; Seminozhenko 1982) and it would be desirable to obtain independent rederivations and/or generalizations of these results.

In order to reduce Eq.(1) to an equation governing the Wigner function some assumptions must be made. The approach chosen by many groups has been to make an Ansatz which directly expresses the $G^<$ in terms of f^W. The relation (4) between the Wigner function and the correlation function provides an important sum rule (in ω-space) or a boundary condition (in τ-space) which any guess for $G^<$ must satisfy. In *equilibrium* the exact relation

$$G^<(\mathbf{p},\omega) = iA(\mathbf{p},\omega)f_{FD}(\omega) \tag{5}$$

holds (here $f_{FD}(\omega)$ is the Fermi-Dirac distribution), and early papers (Barker, 1981; Jauho and Wilkins, 1982) employed a direct generalization of this relation to nonequilibrium:

$$G^<(\mathbf{p},\omega,\mathbf{R},T) = iA(\mathbf{p},\omega,\mathbf{R},T)f^W(\mathbf{p},\mathbf{R},T) \quad . \tag{6}$$

Here A is the nonequilibrium spectral function obtained from the solution of the Dyson equation Eq.(2). This assumption satisfies the sum rule Eq.(4), and for vanishing fields it reduces to the exact equilibrium result Eq.(5) if the spectral function A is strongly peaked at $\omega = \epsilon(\mathbf{p})$ (quasi-particle approximation). Using Eq.(6) in the kinetic equation then leads to a closed equation for f^W which one may then attempt to solve for various types of interactions. The weak point of this approach is that guessing an Ansatz does not provide any means of estimating its limits of validity. Even worse, it was soon realized (Jauho and Wilkins, 1984) that the Ansatz (6) led to a collision integral which differed slightly from the one derived with the density matrix method (Barker and Ferry, 1979; Levinson 1970). The resolution to this paradox came in two steps: it was first realized that the theory should be formulated in a gauge invariant manner (the crucial ideas occur already in the early work of Langreth, 1966, and in the linear theory of Mahan and Hänsch, 1983, but it took some time before they were adopted to semiconductor high-field transport: Sarker, 1985; Khan and others, 1987; Reggiani and others, 1987), and secondly, Vinogradov (1986), and Lipavský, Špička and Velický (1986) (from this on LSV) gave the first *systematic* derivation of an Ansatz of the type of Eq.(6). With these improvements perfect agreement was found between the Green function methods, and the earlier density matrix results. - For uniform and steady fields the new Ansatz of LSV reads

$$G^<(\mathbf{k},\tau) = iA(\mathbf{k},\tau)f^W(\mathbf{k} - \frac{1}{2}\mathbf{E}|\tau|) \quad . \tag{7}$$

Note that here one uses the kinematical momentum $\mathbf{k}$ rather than the canonical momentum $\mathbf{p}$. The point of the derivation of LSV is that the correlation function $G^<$ is written as its time-diagonal piece, and a correction term which has an integral equation structure. The integral equation does not represent a perturbative expansion in a small coupling constant but rather an expansion in the various relaxation times: the quasi-particle life-time and the charcteristic decay time for correlations. We find this approach very promising and strongly encourage further work, perhaps in the form of simple model systems which would allow a quantitative estimate of the accuracy of the various Ansätze relating the correlation function to the Wigner function.

We conclude this section by giving the quantum kinetic equation obtained with the Ansatz of LSV for non-degenerate carriers, driven by a uniform and steady driving field of arbitrary strength, and the electron-phonon interaction treated within the self-consistent Born approximation (Khan and others, 1987, from this on KDW) :

$$\begin{aligned}\mathbf{E}\cdot\nabla_{\mathbf{k}} f^W(\mathbf{k}) = \sum_{\mathbf{q}} \int_0^\infty d\tau [&P(\mathbf{k}-\mathbf{E}\tau, \mathbf{k}-\mathbf{q}-\mathbf{E}\tau;\tau) f^W(\mathbf{k}-\mathbf{q}-\mathbf{E}\tau) \\ &- P(\mathbf{k}+\mathbf{q}-\mathbf{E}\tau, \mathbf{k}-\mathbf{E}\tau;\tau) f^W(\mathbf{k}-\mathbf{E}\tau)] \ ,\end{aligned} \tag{8}$$

where

$$\begin{aligned}P(\mathbf{k}+\mathbf{q},\mathbf{k};\tau) &= 2\pi|M_{\mathbf{q}}|^2 \sum_{\eta=\pm 1} [N_{\mathbf{q}} + \frac{1}{2}(1+\eta)] \times \\ &Re\frac{1}{\pi}[A(\mathbf{k}+\mathbf{q}+\frac{1}{2}\mathbf{E}\tau,\tau)A(\mathbf{k}+\frac{1}{2}\mathbf{E}\tau,-\tau)e^{-i\eta\omega_{\mathbf{q}}\tau}] \ .\end{aligned} \tag{9}$$

Setting $\mathbf{E} = 0$ in the collision integral and using free spectral densities, $A(\mathbf{k},\tau) = exp(-i\epsilon(\mathbf{k})\tau)$, recovers the BE. Eqs.(8-9) generalize the Barker-Ferry equation (1979): the interacting nonequilibrium spectral densities allow, in principle, a rigorous and nonperturbative treatment of interference effects between driving fields and scattering.

The mathematical structure of Eq.(8) differs crucially from the BE: the additional integral appearing on the right hand side makes it unsuitable for standard Monte Carlo simulation schemes, and additional simplifications are called for. In the next section we describe some of the suggested approximation schemes. Very little is known about Eq.(8)'s formal properties and we mention only a few points where our undertanding is still incomplete: conservation laws, convergence, stability, existence of solutions, (ir)reversibility, and consistency within a given order of perturbation theory.

FURTHER APPROXIMATIONS

As mentioned above, Eq.(8) appears unsuitable for a numerical evaluation: this is because of the retardation, or memory effects in the collision integral. Rather than approximating Eq.(8) directly, it is advantageous to integrate both sides, and after some manipulations one finds (here we follow KDW)

$$f(\mathbf{k}) = \sum_{\mathbf{q}} \int_0^\infty dt[\tilde{W}(\mathbf{k}-\mathbf{E}t, \mathbf{k}-\mathbf{q}-\mathbf{E}t; t) f(\mathbf{k}-\mathbf{q}-\mathbf{E}t) - \tilde{W}f] \quad , \tag{10}$$

where

$$\tilde{W}(\mathbf{K}+\mathbf{q}, \mathbf{K}; t) = \int_0^t d\tau' P(\mathbf{K}+\mathbf{q}, \mathbf{K}, \tau') \quad . \tag{11}$$

In the second $\tilde{W}f$ term in Eq.(10) one should make the replacement $\mathbf{k} \to \mathbf{k}+\mathbf{q}$. This form bears a striking similarity to the integrated BE: the *only* formal difference is the explicit time-dependence in $\tilde{W}$. Further simplification is possible if $\tilde{W}$ approaches its asymptotic value on a time scale faster than any other relevant time scale, and hence could be replaced by its limiting value $\tilde{W}(t \to \infty)$. A condition for this is that $P(\tau)$ (Eq.(9)) be a short ranged function of τ. KDW argue that this is indeed the case (see their Appendix C), while other groups are content in passing to the asymptotic limit ('completed collisions limit') phenomenologically (Kim and others, (1987); Reggiani and others (1987), (1988a)). According to the analysis of KDW the asymptotic limit becomes the better the stronger the field is. These conclusions depend sensitively on the form of the Ansatz used to relate $G^<$ and f^W thus stressing the importance of further work on the points raised in the previous section.

Summarizing, the completed collisions limit gives rise to a Boltzmann type of transport equation with the energy conserving δ-functions replaced by a 'joint spectral density' $K(\mathbf{k}+\mathbf{q}, \mathbf{k})$:

$$\delta(\epsilon(\mathbf{k}+\mathbf{q}) - \epsilon(\mathbf{k}) - \eta\omega_{\mathbf{q}}) \to$$

$$K(\mathbf{k}+\mathbf{q}, \mathbf{k}) \equiv \int_0^\infty d\tau Re[\frac{1}{\pi} A(\mathbf{k}+\mathbf{q}+\frac{1}{2}\mathbf{E}\tau, \tau) A(\mathbf{k}+\frac{1}{2}\mathbf{E}\tau, -\tau) e^{-i\eta\omega_{\mathbf{q}}\tau}] \quad . \tag{12}$$

In recent years several groups have performed Monte Carlo simulations based on Eq.(12). A detailed description of a particular set of simulations can be found in Reggiani and others (1988a); here we summarize the main results. The simulations can be classified according to what physical effects were included in the joint spectral density.

Intracollisional field effect (ICFE). (Jauho and Reggiani, 1988; Reggiani and others, 1988b) The spectral density for free electrons in a parabolic band in the precence of a uniform static electric field can be solved analytically:

$$A(\mathbf{k}, \tau) = exp[-i(\epsilon(\mathbf{k})\tau + \frac{E^2}{24m}\tau^3)] \quad , \tag{13}$$

and the resulting joint spectral density can be expressed in terms of Fresnel integrals. Here one encounters a conceptual difficulty: the joint spectral function is not a positive semidefinite quantity, and the probabilistic interpretation required in Monte Carlo simulations breaks down. In the numerical calculations the suggestion of Barker (1978) was followed: the main peak in K was fitted with a Lorentzian thus suppressing the negative oscillations. This procedure has a physical motivation: inclusion of scattering would imply a smearing of all sharp features, and hence the rapid oscillations, which integrate to zero, should be strongly supressed. The shift and width of the Lorentzian depend on the strength on the electric field, and its orientation with respect to $\mathbf{k}$ and $\mathbf{q}$ (see Reggiani and others, 1988a for several illustrations).

Collisional broadening (CB). (Kim and others, 1987; Reggiani and others, 1987) In this case the field in Eq.(12) can be set to zero, and the integral reduces to a convolution in energy space. Kim and others (1987) solve the spectral density self-consistently for dispersionless optical phonon scattering, and use a realistic density of states, while Reggiani and others (1987) work analytically in lowest order of perturbation theory and use a free electron density of states. The resulting joint spectral densities are similar, and there are no problems with the joint spectral density going negative. It is tempting to suggest that the differing behavior between ICFE and CB is due to the singular nature of a perturbing electric field: recall that the Hamiltonian including a scalar potential $U(\mathbf{x}) = -\mathbf{x} \cdot \mathbf{E}$, giving rise to a uniform electric field, is not bounded, and that the spectral function Eq.(13) in energy space does not approach uniformly the free spectral density (but it does in the distribution sense, Jauho and Wilkins, 1984). These difficulties suggest that it maybe necessary to explicitly accout for the finiteness of the sample: a consistent nonlinear theory cannot be formulated without accounting for end effects. This would imply that even the uniform field case should be treated with a spatially inhomogenous theory.

The Monte Carlo simulation for the ICFE and CB give similar results. For low fields, $E < 10kV/cm$, the results differ very little from those obtained with the conventional semiclassical BE while for higher fields there is an increase of carriers both in the low- and the high-energy tails of the distribution function. The high-energy electron population enhancement may have some relevance to the onset of impact ionization. In simulations of this kind one should be on guard against spurious run-away effects that may occur if there is no inherent high-

energy cut-off in the problem. - To our knowledge no results have been reported where ICFE and CB have been treated with equal footing: this would require a solution to the nonequilibrium Dyson equation (2). Studies of this sort would be most opportune.

SPATIALLY INHOMOGENOUS SYSTEMS

All the applications discussed above deal with the uniform field case. However, all microdevices are (almost by definition!) extremely inhomogenous, and it is important to ask how much of the above can be generalized to spatially inhomogenous systems. This is a very difficult task and only few results have been reported. To illustrate the difficulties we write down explicitly the driving term (first commutator in Eq.(1)) for a general potential $U(x)$ (for simplicity consider a one-dimensional system):

$$[\frac{\partial}{\partial T}+\frac{p}{m}\frac{\partial}{\partial X}]f^W(p,X,T)-\int\frac{dp'}{2\pi i}M(p-p',X)f^W(p',X,T)=(\frac{\partial f}{\partial T})_{coll} \quad , \tag{14}$$

where

$$M(q,X)=\int dx e^{-iqx}[U(X+x/2)-U(X-x/2)] \quad . \tag{15}$$

The driving term is *nonlocal,* which implies considerable difficulties in numerical implementation. A few applications to the resonant tunneling diode have been reported (Frensley, 1987; Kluksdahl and others, 1989); in these calculations the collision term has been treated in the relaxation time approximation. We would like to issue a warning here: the simple relaxation time approximation violates particle conservation (Mermin, 1970), and spurious effects may result.

Very recently (Ziep and others, 1986; Jauho and Ziep, 1989; Bertoncini and others, 1989a and 1989b; however related procedures have been used earlier, e.g. Herbert and Till, 1982) an alternative idea has been proposed: rather than working directly with Eq.(14) one transforms to a new basis defined by the eigenfunctions of the potential $U(x)$. For the uniform field case, for example, this means that the kinetic equation and the Dyson equation should be 'Airy-transformed' (Bertoncini and others, 1989a and 1989b). Here we sketch the procedure for a system where the translational invariance is broken in one spatial direction. The eigenfunctions are determined by

$$[-\frac{1}{2}\frac{d^2}{dz^2}+V(z)+\frac{1}{2}k_\perp^2]\phi_n(z)=\epsilon_n(k_\perp)\phi_n(z) \quad , \tag{16}$$

where $V(z)$ is the perturbing one-dimensional potential, for example the position dependent conduction band edge found in heterostructures. The transformed Green functions are defined by (the self-energies have analogous definitions)

$$G(\mathbf{k}_\perp,z,z',\omega)=\sum_{n,n'}\phi_n(z)G_{nn'}(\mathbf{k}_\perp,\omega)\phi^*_{n'}(z') \tag{17}$$

and the transformed Dyson equation reads (the kinetic equation has a similar structure and will not be given here)

$$G_{mm'}(\mathbf{k}_\perp,\omega)=\delta_{mm'}\tilde{G}_m(\mathbf{k}_\perp,\omega)+\sum_n\tilde{G}_m(\mathbf{k}_\perp,\omega)\Sigma_{mn}(\mathbf{k}_\perp,\omega)G_{nm'}(\mathbf{k}_\perp,\omega) \tag{18}$$

with

$$\tilde{G}_n(\mathbf{k}_\perp,\omega)=\frac{1}{\omega-\epsilon_n(\mathbf{k}_\perp)+i\eta} \tag{19}$$

The transformation has achieved the field, or the non-uniform potential, has essentially been eliminated, and for many cases of interest the self-energy term may well be dominated by the diagonal term (for the uniform field case see Bertoncini and others (1989b)) in which case Eq.(18) is immediately solved. We expect to see more work along these lines in the future.

NONEQUILIBRIUM SCREENING

Screening in the presence of strong electric fields and scattering has, despite of its central importance, up to date received only little attention. Of the earlier work we mention the Monte Carlo simulations by Lugli and Ferry (1986), and the work by Lowe and Barker (1985), which is based on nonequilibrium Green functions and thus of interest for the present review. Very recently Hu and others (1989a, 1989b) considered linear nonequilibrium screening *without* assuming a specific form for the distribution function; in the first paper the Boltzmann equation was used as a starting point, while in the second paper the quantum kinetic equation Eq.(1) was used, with the collision integral approximated by a *number-conserving* generalization of the relaxation time approximation (Mermin, 1970). The procedure to calculate the density response function $\chi(\mathbf{q},\omega)$ (from which the dielectric constant can be extracted) is as follows: (i) Set up, and solve the quantum transport equation for the "unperturbed" (i.e. without the imposed density variation) Wigner function f_0^W. (ii) Linearly perturb the transport equation with a perturbation $U_1e^{i(\mathbf{q}\cdot\mathbf{R}-\omega T)}$ to produce a first order response in the Wigner function f_1^W, and solve for it. (iii) Integrate f_1^W to get the induced density n_1. The ratio n_1/U_1 then gives χ. Hu and others (1989b) find that

for $q << q_{dB}$ (q_{dB} is the deBroglie wave-length) the quantum results are practically indistinguishable from BE results (Hu and others (1989a)) while for $q > q_{dB}$ quantum effects due to spatial nonlocality begin to dominate, and the susceptibility approaches a Lindhard-like formula, with the eqilibrium distribution functions replaced by the nonequilibrium ones.

The above results are extremely interesting in their own right, but they are obtained withing a very simple model for the collisions: for example, neither intra-collisional field effect nor collisional broadening are accounted for. The obvious question is whether the scheme can be extended to a more general collision integral. The step (i) given above has been accomplished with simulations based on Eq.(12), and the crucial point is whether the step (ii) can be carried through. While no definite answer is known at the moment, we hope to see progress along this lines in near future.

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