Thin Solid Films, *146* (1987) 15–20
ELECTRONICS AND OPTICS

INVESTIGATION OF RING STRUCTURES FOR METAL–SEMICONDUCTOR CONTACT RESISTANCE DETERMINATION

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(Received April 14, 1986; accepted June 2, 1986)

Solutions to voltage profiles obtained from radial current flow under metal ring structures on thin semiconductor layers are examined in the light of results obtained for flow under rectangular structures. By using series expansions it is shown that expressions obtained in the case of circular structures reduce to those of the rectangular case for values of ring radius $r \gtrsim 30L_t$, at all points on the ring, and where L_t is the transfer length—a property of the materials concerned and the specific contact resistance. In addition, expressions are developed to cover the range of values $3L_t \lesssim r \lesssim 30L_t$ and $r \lesssim 3L_t$. These are then used to obtain new expressions from which values of specific contact resistance and semiconductor sheet resistance (under contacts) can be found.

1. INTRODUCTION

In recent years much interest has been centred on the fabrication of low resistance ohmic contacts to semiconductors, which have become important in the manufacture of integrated circuits[1], laser diodes[2] and many other semiconductor devices ranging from metal Schottky field effect transistors to Gunn diodes. One of the most important properties of ohmic contacts is the specific contact resistance[3] r_c, defined in terms of the voltage drop V across a metal–semiconductor interface, passing a current density $\boldsymbol{J}$:

$$r_c = \left.\frac{\mathrm{d}V}{\mathrm{d}J}\right|_{V\to 0} \tag{1}$$

If the direction of $\boldsymbol{J}$ is chosen normal to the interface, r_c should be independent of the contact area and represents a metal–semiconductor "resistivity" characteristic of contact quality. Much research has been devoted to production of contacts with values of $r_c \lesssim 10^{-6}\ \Omega\,\mathrm{cm}^2$ and, since theoretical knowledge is insufficient to predict *a priori* specific contact resistance values, techniques have been developed which allow such measurements to be made.

These techniques usually consist of the construction of models resulting from consideration of current flow in the interface region[4–6]. By mathematical manipu-

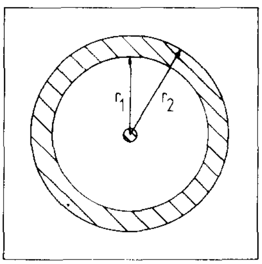
0040-6090/87/$3.50

lation, it is then generally possible to obtain parameters relevent to the contact in terms of measureable quantities. One of the most versatile models in this regard is the transmission line model[6] (TLM), which involves construction of an equivalent transmission line to facilitate analysis of current flow under and through the interface. Up until recently most attention was focused on models involving rectangular geometry, and consequently the mathematics of this structure has been extensively developed[6,7].

However, the rectangular TLM pattern must be fabricated by a process involving a mesa etch step which can sometimes be a complicated procedure[8]. In addition, edge effects—not accounted for in the TLM analysis—can reduce the accuracy of the analysis. To avoid this problem, work has been done on the development of a mathematical model describing a pattern in the form of concentric circular contacts[8,9], suitable for evaluation of specific contact resistance. However, the resultant equations can sometimes be considerably more complex than those obtained from the rectangular model. The purpose of this paper is to investigate whether the circular TLM equations reduce to those of the rectangular TLM and, if so, under what conditions.

2. THEORETICAL ANALYSIS

As is common in TLM problems the flow under the current-collecting contact of Fig. 1 is described by a lumped resistance network. Consideration of flow under an annulus of inner radius r and width dr gives rise to the lumped values shown in Fig. 2.



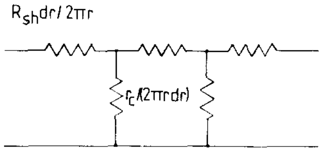


Fig. 1. Plan view of a metal ring contact of thickness $r_2 - r_1$ on a semiconductor substrate. The shaded region represents metallization.

Fig. 2. Lumped resistance equivalent circuit for a metal–semiconductor contact. The sheet resistance is given by R_{sh}, and the specific contact resistance by r_c.

The TLM equations are then formed by considering the flow of current through the contact and the voltage drop parallel to the interface:

$$\mathrm{d}i = \frac{V}{r_c/2\pi r\,\mathrm{d}r} \qquad \frac{\mathrm{d}i}{\mathrm{d}r} = \frac{2\pi V r}{r_c} \tag{2}$$

$$\mathrm{d}V = \frac{R_{sh}\,\mathrm{d}r\,i}{2\pi r} \qquad \frac{\mathrm{d}V}{\mathrm{d}r} = \frac{R_{sh}\,i}{2\pi r} \tag{3}$$

The sheet resistance R_{sh} under the contact is defined as the ratio of the semiconductor resistivity to thickness of the current-carrying layer. Eliminating i from eqns. (2) and (3) gives

$$\frac{d^2V}{dr^2}+\frac{1}{r}\frac{dV}{dr}-\frac{V}{L_t^2}=0 \tag{4}$$

where the transfer length L_t is defined as

$$L_t=\left(\frac{r_c}{R_{sh}}\right)^{1/2}$$

Imposing the boundary conditions for current flow that $i(r_1)=i_0$ and $i(r_2)=0$ and solving using the recurrence relations[9]

$$\frac{dI_\nu(x)}{dx}=I_{\nu+1}(x)+\frac{\nu}{x}I_\nu(x) \tag{5}$$

$$\frac{dK_\nu(x)}{dx}=\frac{\nu}{x}K_\nu(x)-K_{\nu+1}(x) \tag{6}$$

gives

$$V(r)=-\frac{i_0R_{sh}L_t}{2\pi r_1}\frac{I_0(r/L_t)K_1(r_2/L_t)+K_0(r/L_t)I_1(r_2/L_t)}{I_1(r_1/L_t)K_1(r_2/L_t)-K_1(r_1/L_t)I_1(r_2/L_t)} \tag{7}$$

where I_ν and K_ν are Bessel functions of the first and second kind respectively, of order ν, but with imaginary arguments.

In the limiting case of rings with $r_2-r_1 \ll r_1$ it might be expected that the voltage profile would exhibit the behaviour associated with a rectangular contact, since the geometry of a given small section of the ring then approximates to a section of a rectangular contact. To see how this situation may arise, let us consider once again eqn. (4):

$$L_t^2\frac{d^2V}{dr^2}+\frac{L_t^2}{r}\frac{dV}{dr}-V=0 \tag{8}$$

For a ring of constant thickness, it follows from eqns. (2) and (3) that the second term of eqn. (8) will become small in comparison with the other terms for large enough values of r_1. On application of the above boundary conditions in this regime, eqn. (8) becomes

$$L_t^2\frac{dV}{dr^2}-V=0 \tag{9}$$

or

$$V=\frac{i_0R_{sh}L_t}{2\pi r_1}\frac{\cosh\{(r_2-r)/L_t\}}{\sinh\{(r_2-r_1)/L_t\}} \tag{10}$$

This is the equation for the voltage profile under a rectangular contact[7].

On substitution of eqn. (10) into eqn. (8) the following equation results:

$$L_t^2\frac{d^2V}{dr^2}-\left\{1+\frac{L_t}{r}\tanh\left(\frac{r_2-r}{L_t}\right)\right\}V=0 \tag{11}$$

for $r_1 \leqslant r \leqslant r_2$.

It now would seem plausible to suppose that in the limit of r_1 and $r_2 \gg L_t$, eqn. (7) could be approximated by eqn. (10) and the ring would behave as a rectangular contact.

Obviously, the above argument is not an exact analysis. For this it is necessary to examine the full solution of eqn. (4) to facilitate a more detailed investigation of transition between the two regimes.

Let us consider the following asymptotic forms[10] for the Bessel functions $K_\nu(t)$ and $I_\nu(t)$:

$$K_\nu(t) = \left(\frac{\pi}{2t}\right)^{1/2} e^{-t}\left\{1+\frac{(4\nu^2-1^2)}{1!8t}+\frac{(4\nu^2-1^2)(4\nu^2-3^2)}{2!(8t)^2}+\ldots\right\} \tag{12}$$

$$I_\nu(t) = \frac{1}{(2\pi t)^{1/2}} e^{t}\left\{1-\frac{(4\nu^2-1^2)}{1!(8t)}+\frac{(4\nu^2-1^2)(4\nu^2-3^2)}{2!(8t)^2}-\ldots\right\} \tag{13}$$

Provided that for all r ($r_1 \leqslant r \leqslant r_2$), $8r/L_t \gg 3$ or $r/L_t \gg 3/8$, only the first terms of the series in eqns. (12) and (13) are retained. Then it follows that eqn. (7) reduces to eqn. (10) and the voltage profile under the contact behaves as if the ring were a rectangular structure of length r_2-r_1. Generally, for the region $r \geqslant 100(3/8)L_t$ or $r \gtrsim 30L_t$, eqn. (10) can be used with a considerable degree of confidence, since the second terms of eqns. (12) and (13) are then 1% or less of the first term in each case.

However, if structures are being used for which $3L_t \lesssim r \lesssim 30L_t$ then the second term of each series becomes important, so that after some algebra:

$$V(r) = -\frac{i_0 R_{sh}}{2\pi r_1} L_t\left(\frac{r_1}{r}\right)^{1/2} \frac{1+(3/rr_2)(L_t/8)^2}{1-(3L_t/8)^2/r_1 r_2}\left\{\frac{\cosh\{(r-r_2)/L_t\}}{\sinh\{(r_1-r_2)/L_t\}} + \frac{L_t(1/r+3/r_2)/8}{1+(3/rr_2)(L_t/8)^2}\right\} \tag{14}$$

or, to a good approximation,

$$V(r) \approx -\frac{i_0 R_{sh} L_t}{2\pi(rr_1)^{1/2}}\left\{\frac{\cosh\{(r-r_2)/L_t\}}{\sinh\{(r_1-r_2)/L_t\}}+\frac{L_t}{8}\left(\frac{1}{r}+\frac{3}{r_2}\right)\right\} \tag{15}$$

Obviously, for the smallest structures, for which $r \lesssim 3L_t$, eqn. (15) is inadequate, and in these cases it will be necessary to use the full expression given by eqn. (7). Thus, broadly three regimes of behaviour can be identified: small rings with $r \lesssim 3L_t$; intermediate cases $3L_t \lesssim r \lesssim 30L_t$; and large structures having $r \gtrsim 30L_t$.

Two quantities readily measured in contact resistance experiments are the input and contact end resistance[7], defined in terms of nomenclature here as

$$R_{in} = \left.\frac{V(r)}{i_0}\right|_{r=r_1} \tag{16}$$

$$R_e = \left.\frac{V(r)}{i_0}\right|_{r=r_2} \tag{17}$$

(i) *Input resistance.* It follows from eqns. (15) and (16) that

$$R_{in} = \frac{R_{sh} L_t}{2\pi r_1}\left\{\coth\left(\frac{r_2-r_1}{L_t}\right)-\frac{L_t}{8}\left(\frac{1}{r_1}+\frac{3}{r_2}\right)\right\} \tag{18}$$

provided that $r \geqslant 3L_t$ as described above. For $r \geqslant 30L_t$ it can be seen that this reduces to R_{in} as obtained from eqn. (10), giving the familiar result[7]

$$R_{in} = \frac{R_{sh}L_t}{2\pi r_1}\coth\left(\frac{r_2-r_1}{L_t}\right)$$

(ii) *End resistance.* To obtain an exact expression for R_e, eqn. (7) can be simplified by application of the following wronksian formula for modified Bessel functions[11]:

$$I_\nu(x)K_{\nu+1}(x)+I_{\nu+1}(x)K_\nu(x) = \frac{1}{x} \tag{19}$$

Combining eqns. (7), (16) and (19) the contact end resistance becomes

$$R_e = \frac{R_{sh}}{2\pi r_1 r_2}L_t\left\{\frac{1}{I_1(r_1/L_t)K_1(r_2/L_t)-K_1(r_1/L_t)I_1(r_2/L_t)}\right\} \tag{20}$$

Applying the series expansions of eqns. (12) and (13) and retaining the first two terms of each gives

$$R_e = \frac{R_{sh}L_t}{2\pi(r_1r_2)^{1/2}}\,\frac{1}{\{1-(3L_t/8)^2/r_1r_2\}\sinh\{(r_2-r_1)/L_t\}} \tag{21}$$

which for all but the smallest ring contacts ($r \lesssim 3L_t$) can be written as

$$R_e = \frac{R_{sh}L_t}{2\pi(r_1r_2)^{1/2}}\,\frac{1}{\sinh\{(r_2-r_1)/L_t\}} \tag{22}$$

3. APPLICATION OF RESULTS

Dividing eqn. (22) into eqn. (18) results in an equation for input and contact end resistance in terms of L_t:

$$\frac{R_{in}}{R_e} = \left(\frac{r_2}{r_1}\right)^{1/2}\left\{\cosh\left(\frac{r_2-r_1}{L_t}\right)-\frac{L_t}{8}\sinh\left(\frac{r_2-r_1}{L_t}\right)\left(\frac{1}{r_1}+\frac{3}{r_2}\right)\right\} \tag{23}$$

for $r \gtrsim 3L_t$. This can be solved numerically or graphically to yield L_t, from which R_{sh} can be deduced from eqn. (18). Then, since the transfer length L_t is defined as $L_t^2 = r_c/R_{sh}$ and R_{sh} is known, it becomes possible to obtain the specific contact resistance r_c. For most ring structures the condition on eqn. (23) should easily be satisfied but as a check on applicability it might be advisable to make a first-order estimate using typical values available from the literature. Alternatively, L_t can be obtained by using a standard technique[5,6] in the first instance. This information should then aid in fabrication of a suitable test pattern.

As mentioned previously, in the case of structures for which $r \gtrsim 30L_t$ eqn. (18) reduces to the formula obtainable from the voltage profile of eqn. (10):

$$R_{in} = \frac{R_{sh}L_t}{2\pi r_1}\coth\left(\frac{r_2-r_1}{L_t}\right) \tag{24}$$

It is possible to obtain R_{sh} explicitly by combining eqn. (24) with eqn. (22):

$$R_{sh} = \frac{2\pi R_e^2}{R_{in}(r_2 - r_1)} \left[r_1 \left\{ r_2 - \left(\frac{R_{in}}{R_e} \right)^2 r_1 \right\} \right]^{1/2} \cosh^{-1} \left\{ \left(\frac{r_1}{r_2} \right)^{1/2} \frac{R_{in}}{R_e} \right\} \quad (25)$$

for $r \gtrsim 30L_t$.

Calculation of r_c then proceeds as before.

4. CONCLUSION

In summary, the case of current flow under a ring contact was examined. The voltage profile was shown to be identical with that obtained for a rectangular contact provided that $r \gtrsim 30L_t$. For smaller radial values approximate expressions were developed. These expressions were used to deduce formulae for the input and contact end resistance, two experimentally measurable parameters, from which it is possible to calculate sheet resistance values under a contact as well as values for r_c. New expressions were developed for this purpose.

REFERENCES

1 R. Warner and J. Fordemwalt, *Integrated Circuits*, McGraw-Hill, New York, 1965.
2 I. Ladany and D. P. Marinelli, *RCA Rev.*, *44* (1983) 101.
3 B. L. Sharma, *Semicond. Semimet.*, *15* (1981) 18.
4 S. B. Schuldt, *Solid-State Electron.*, *21* (1978) 715.
5 R. H. Cox and H. Strack, *Solid-State Electron.*, *10* (1967) 1213.
6 H. H. Berger, *Solid State Electron.*, *15* (1972) 145.
7 H. B. Harrison, *Proc. IREE Aust.*, (1980) 95.
8 G. K. Reeves, *Solid-State Electron.*, *23* (1979) 487.
9 V. Niskov and G. Kubetskii, *Sov. Phys. Semicond.*, *4* (1971) 1553.
10 M. Abramowitz and I. A. Segun (eds.), *Handbook of Mathematical Functions*, Dover Publications, New York, 1965.
11 I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, Academic Press, New York, 1980.