Contributions to Wavelength Shifts of DFB Fiber Lasers used as Acoustic Sensors in Air.

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1 Introduction

Our aim is to investigate the use of distributed feedback (DFB) fiber lasers [1] as acoustic sensors. In this paper we will discuss the various contributions to the frequency shifts that make such sensing possible, and compare the theoretical results with preliminary experimental results. In model used in this paper the fiber is spanned perpendicularly to the wave vector of an acoustic plane wave. The fiber laser used so far has no coating and a center wavelength of $\lambda = 1550$nm, but the model is easily extendable to coated fiber lasers.

2 Temperature and pressure variations in the fiber at the acoustic frequency

When the pressure of the air varies due to the acoustic wave, the work applied to compress and decompress the air leads to temporal temperature variations, following the first principle of thermodynamics, which may be expressed by [2]:

$$\begin{aligned} dq &= \rho c_V \, dT + (c_p - c_V)\, \rho \left(\frac{\partial T}{\partial V}\right)_p dV \\ &= \rho c_p \, dT - (c_p - c_V)\, \rho \frac{T}{p}\, dp \end{aligned} \tag{1}$$

Here q is heat per unit volume added to a differential amount of air, T is the temperature, p is the pressure, ρ is the density and c_p and c_V are the specific heat capacities at constant pressure and temperature, respectively. In the last line of equation (1) the ideal gas approximation is used. dq must equal heat conducted from the surroundings [3]:

$$dq = \kappa \nabla^2 T \, dt \tag{2}$$

Here κ is the thermal conductivity. Due to the relative smallness of acoustic pressure compared with the static pressure, we can usually set $T/p \approx T_{\text{static}}/p_{\text{static}}$. By combining equations (1)-(2) and assuming a harmonic acoustic field with frequency $\omega = 2\pi f$ we thus get:

$$\begin{gathered} j\omega\,\Delta T = \frac{\kappa}{\rho c_p}\nabla^2 T + j\omega\frac{c_p - c_V}{c_p}\cdot\frac{T_{\text{static}}}{p_{\text{static}}}\Delta p = D_{T_{\text{air}}}\nabla^2 T + j\omega\,\Delta T_0 \\ T(\vec{r},t) = T_{\text{static}} + \Delta T(\vec{r})\cdot e^{j\omega t}, \qquad p(\vec{r},t) = p_{\text{static}} + \Delta p(\vec{r})\cdot e^{j\omega t} \end{gathered} \tag{3}$$

Here $D_{T_{\text{air}}}$ is the thermal diffusivity of air. We may neglect the Laplacian term in equation (3) far away from the fiber, and assume that the process is adiabatic, if:

$$\frac{D_{T_{\text{air}}}\left(\frac{\omega}{c}\right)^2}{\omega} = \frac{D_{T_{\text{air}}}2\pi f}{c^2} = \frac{22.5\cdot 10^{-6}\frac{\text{m}^2}{\text{s}}\cdot 2\pi f}{\left(350\frac{\text{m}}{\text{s}}\right)^2} \approx 1.15\cdot 10^{-9}\text{s}\cdot f \ll 1 \tag{4}$$

Here c is the velocity of sound, and typical values [3] for air at 300K are inserted. Thus, for frequencies up to several MHz, the only significant contributions to the Laplacian term in equation (3) come from the inhomogeneities caused by the presence of the fiber. In this paper we are working with acoustic frequencies in the range 100-20kHz, and we may therefore also safely ignore the spatial dependence of the acoustic pressure in the vicinity of the fiber. Thus, the temperature field is spatially dependent only on the radius r in cylinder coordinates, and equation (3) simplifies to an inhomogeneous Bessel equation of the zeroth order with general solution:

$$\Delta T_{\text{air}}(r) = \Delta T_0 + C_1\cdot J_0\left(\sqrt{\frac{\omega}{jD_{T_{\text{air}}}}}\cdot r\right) + C_2\cdot Y_0\left(\sqrt{\frac{\omega}{jD_{T_{\text{air}}}}}\cdot r\right) \tag{5}$$

In silica there is no significant acoustic generation of heat, and thus the temperature in the fiber is determined from a homogenous Bessel equation with general solution:

$$\Delta T_{\text{fiber}}(r) = C_3\cdot J_0\left(\sqrt{\frac{\omega}{jD_{T_{\text{silica}}}}}\cdot r\right) + C_4\cdot Y_0\left(\sqrt{\frac{\omega}{jD_{T_{\text{silica}}}}}\cdot r\right) \tag{6}$$

The constants $C_{1,2,3,4}$ in equations (5)-(6) are found by using the boundary condition $\lim_{r\to\infty}\Delta T(r) = \Delta T_0$ and requiring that both $T(r)$ and its derivative are finite and continuous everywhere.

The total frequency shift of the laser at the acoustic frequency can now be found by adding the frequency shifts due to the temperature and pressure variations in the fiber. The temperature sensitivity of the Bragg wavelength is approximately $\Delta\lambda/\lambda = 8.85\cdot 10^{-6}\text{K}^{-1}\Delta T$ [4, 5] , and $\Delta T_0 \approx 0.85\text{mKPa}^{-1}\,\Delta p$ at 300 K and atmospheric pressure. The acoustic pressure sensitivity has a predicted value of $\Delta\lambda/\lambda = 4.5\cdot 10^{-12}\text{Pa}^{-1}\Delta p$ [6, 7]. Because of the large ratio between the wavelength of the acoustic wave and the diameter of the fiber, we assume that the pressure is uniform across the transverse area of the fiber. We assume a Gaussian optical mode [8], and by integrating over the fiber cross section the product between the intensity of the mode and the temperature transfer function, the spatial variation of the latter may be accounted for. This correction was however found to have only negligible effect. The total frequency shift per rms Pa sound pressure at a temperature of 300K is shown in figure 1. At low frequencies the total shift is many orders of magnitude larger than the contribution from the pressure variations alone, and this is confirmed by our preliminary experiments. The thermal effect is negligible for frequencies above ~8kHz.

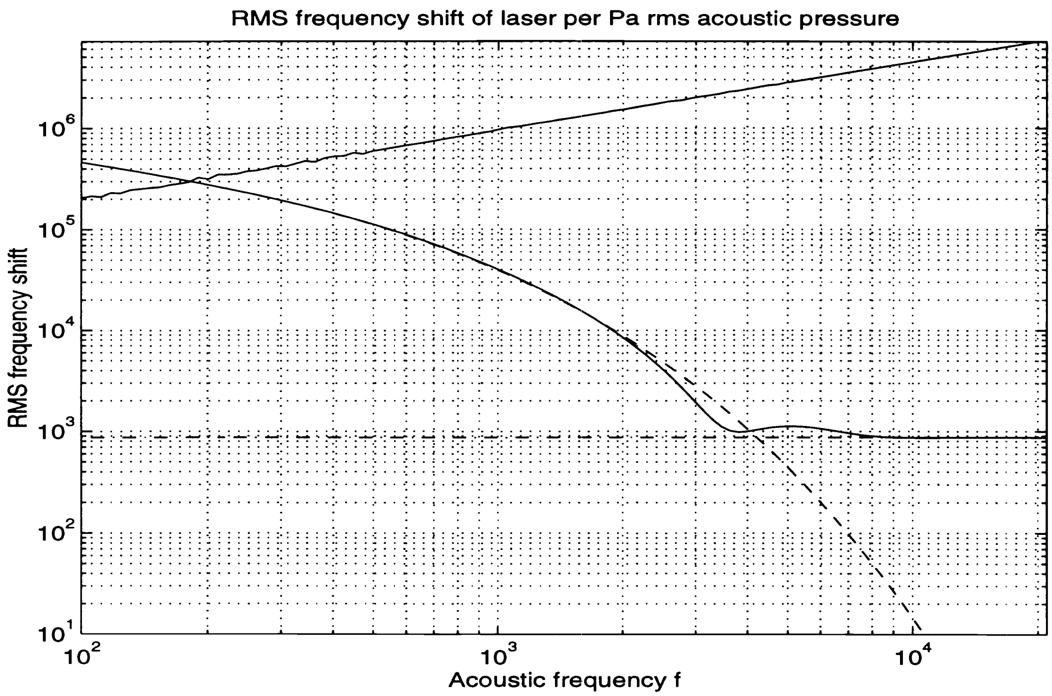

Figure 1: *The various contributions to frequency shifts per Pa rms acoustic pressure of a stripped fiber laser. The straight dash-dotted line is frequency shift of the laser due to pressure variations in the fiber. The dashed line is the contribution from the temperature variation, while the solid declining line is the sum of the two. The other solid line is the frequency shift due to longitudinal strain in a tightly spanned fiber laser as discussed in section 3.*

3 Longitudinal strain in fiber due to pressure gradients in the acoustic wave.

Although the pressure gradients of the acoustical field are small for the modest frequencies we have been working with here, the gradients will lead to a transverse force on the fiber. If the fiber is spanned tightly, this will lead to a longitudinal stress and strain in the fiber, and thus to a frequency shift of the laser. If the acoustic field propagates perpendicularly to the longitudinal axis of the fiber, it is straight forward to show that the rms transverse force F_{rms} per unit length of a fiber with radius R and rms acoustic pressure Δp_{rms} is:

$$F_{\text{rms}} = 4\sqrt{2}\,\Delta p_{\text{rms}} R \int_{0}^{\pi/2} \cos\theta \sin\left(\frac{\omega}{c} R \cos\theta\right) d\theta \tag{7}$$

It may also be shown that the longitudinal stress ϵ in the fiber can be found by numerical solution of the following set of equations:

$$\begin{aligned} \Delta l &= \frac{S_z^2}{KF}\left[\sqrt{1+\left(\frac{Fl_0}{2S_z}\right)^2}\cdot\frac{Fl_0}{2S_z} + \operatorname{arcsinh}\left(\frac{Fl_0}{2S_z}\right)\right] \\ \epsilon &= \frac{S_z}{K} = \frac{\Delta l}{l_0 + \Delta l} \end{aligned} \tag{8}$$

Here l_0 is the distance between the points of suspension of the fiber, Δl is the elongation of the fiber due to the transverse force, S_z is the longitudinal force along the fiber axis and $E = K/(\pi R^2)$ is Young's modulus. Note that due to the nonlinearity of equations (8), the wavelength shift will vary with an even multiplum of the acoustic frequency, and will therefore not interfere directly with the shifts discussed in section 2. Note also that this shift would be considerably reduced if the fiber is loosely spanned or is close to a maximum in a standing wave pattern. Using the same logic, we would suspect larger noise due to drifts in the air etc. when the fiber is tightly spanned, something that is confirmed in our preliminary experiments. Equation (8) also requires that F is constant along the span of the fiber, which is unlikely for very high frequencies. In order to give an idea of the magnitude of the effect, we solved equations (8) by inserting the rms value of F for 1Pa rms acoustic pressure, and using $l_0 = 10$cm, $E = 7.2 \cdot 10^{10}$Pa, and a strain sensitivity of $\Delta\lambda/\lambda = 0.78\epsilon$ [5] we get the result shown in figure 1.

4 Conclusions

We have found that adiabatic processes in the surrounding air are important when using fiber lasers as acoustic sensors for low frequencies. Likewise we have found that the way the fiber laser is spanned is important for its sensing capabilities. More experimental work will be done in order to confirm these theoretical results.

References

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