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Magnetoresistance in Two-Dimensional Disordered Systems: Effects of Zeeman Splitting and Spin-Orbit Scattering

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Effects of Zeeman splitting and spin-orbit scattering on the resistance in two-dimensional disordered systems are theoretically studied. The field dependence of the magnetoresistance is shown to have the characteristic anisotropy. The present theory explains the qualitative features of the experimental observations by Komori *et al.* in Cu granular films.

§1. Introduction

Recent scaling theory by Abrahams *et al.*[1] and Anderson *et al.*[2] have demonstrated that two-dimensional metals are not truly metallic but that the conductivity of a macroscopic sample vanishes at absolute zero if electrons are scattered only by normal impurities. The precursor of this complete localization at $T=0$ is seen at higher temperatures as a small correction to the metallic conductivity, which depends on temperature logarithmically.[3-7] This logarithmic region can be called as weakly localized regime (WLR). Since a perturbational treatment from the metallic limit is applicable,[8] we can perform rather detailed comparison between theory and experiments in WLR,[9] and it is now known that the logarithmic correction is sensitive to such scattering mechanisms as inelastic,[2] spin-orbit,[2,11] paramagnetic impurity[10,11] and mutual interactions.[12-14]

One of characteristic features of WLR is the existence of magnetoresistance (MR). In the case of Si-MOS[6] and cesiated Si surface[5] MR is negative and depends only on the component of the magnetic field perpendicular to the interface. This experimental fact has been explained by the theory which takes account of scattering due to normal impurities and some unspecified inelastic scattering.[6,11]

Besides interfaces of silicon, recent experiments by Komori *et al.*[15,16] demonstrated that the resistivity of Cu granular films of average thickness around 30 Å also exhibits the characteristic temperature dependence of WLR, *e.g.* samples with sheet resistance of several hundreds Ω show $\ln T$ correction to the resistivity. However, MR in this system has turned out to be not so simple as in MOS: It is finite even if the magnetic field is applied parallel to the surface and, moreover, MR in a perpendicular field is positive at weak fields and becomes negative at higher fields.

The purpose of this paper is to explain such features of MR in WLR. Basic assumption here is that electrons in the metallic granular films are represented as two-dimensional electrons with some effective mass and potential scattering. Since spin-orbit scattering is known to play a role in an isolated fine particle of Cu,[17] we also assume scattering due to spin-orbit interaction in the two-dimensional electrons. The existence of finite MR in a parallel field indicates the importance of Zeeman splitting, and we will investigate the interplay between Zeeman splitting and spin-orbit scattering. Such interplay has been investigated in superconductors in a magnetic field.[18] In the former discussions of MR[11] this effect of Zeeman splitting was neglected.

In §2 we define our model and calculate MR in a magnetic field applied parallel to the surface. MR in a perpendicular field is evaluated in §3. In §4 we show some examples of numerical results and compare these with experimental data by Komori *et al.* In Appendix A, the theoretical expression of MR in the presence of both spin-orbit scattering potentials and paramagnetic impurities is given. In this paper we take units $\hbar = k_{\mathrm{B}} = 1$.

§2. Model and Magnetoresistance in a Parallel Field

Our model is two-dimensional independent electrons scattered both by normal impurities and spin-orbit interactions. The actual interfaces between metallic fine particles of a diameter around 30 Å,[15,16] where normal and spin-orbit scatterings take place, are represented as impurities for convenience, since the logarithmic correction to the conductivity in WLR is not sensitive to the details in an atomic scale. Then, our Hamiltonian is written as

$$\begin{aligned}\mathscr{H} = &\frac{1}{2m}\left(\boldsymbol{p}+\frac{e}{c}\boldsymbol{A}\right)^2 \\ &+\sum_l\left[1+\frac{1}{4m^2c^2}\left(\boldsymbol{p}+\frac{e}{c}\boldsymbol{A}\right)\cdot\boldsymbol{\sigma}\times\boldsymbol{V}\right] \\ &\times u(\boldsymbol{r}-\boldsymbol{R}_l)+\frac{g}{2}\mu_{\mathrm{B}}\boldsymbol{H}\cdot\boldsymbol{\sigma},\end{aligned} \tag{2.1}$$

where m and g are the effective mass and the g-factor of electrons, respectively, $\boldsymbol{A}$ is the vector potential, $\boldsymbol{\sigma}$ is the Pauli spin matrix, and $u(\boldsymbol{r}-\boldsymbol{R}_l)$ is the potential by an impurity located at $\boldsymbol{R}_l$. We note that m may be different from the free electron mass m_0 because the conduction is partly due to the tunneling between grains. Because of the same reason the Fermi energy, ε_{F}, may be considered to be a parameter.

The Fourier transform of the impurity scattering potential given in eq. (2.1) and the relaxation time are written as

$$u(\boldsymbol{k}-\boldsymbol{k}') = u_0 + \mathrm{i}u_{so}(\hat{k}\times\hat{k}')\cdot\boldsymbol{\sigma}, \tag{2.2}$$

$$\tau^{-1} = \tau_0^{-1}+\tau_{so,x}^{-1}+\tau_{so,y}^{-1}+\tau_{so,z}^{-1}, \tag{2.3}$$

$$\tau_0^{-1} = 2n_i u_0^2 \pi N(0), \tag{2.4}$$

$$\tau_{so,i}^{-1} = 2n_i u_{so}^2 \pi N(0)\overline{(\hat{k}\times\hat{k}')_i^2}, \quad (i=x, y, z), \tag{2.5}$$

where $\overline{(\hat{k}\times\hat{k}')_i^2}$ denotes the i-th component of the angular average, $N(0)$ is the density of states of electrons at the Fermi level, and n_i is the number density of impurities. Both $N(0)$ and n_i are defined per unit area in the film. It should be noted that one-electron states here are essentially three-dimensional, since the Fermi energy is much larger than the energy spacing of the quantized perpendicular motion. We take coordinate axes x and y in a film and the z axis normal to the film, and then $\tau_{so,x} = \tau_{so,y}$ However, $\tau_{so,z}$ may be different from others. Instead of the three-dimensionality of one-electron states, the conduction process can be treated as two-dimensional because the distance L_τ an electron diffuses between inelastic collisions is estimated to be $L_\tau \approx 210$ Å for our choice of parameters for Cu granular films (Appendix B) and then the relationship, ($L_\tau >$ film thickness, grain size), is satisfied.

Let us first examine the case where a magnetic field is in the film plane, *e.g.* x-direction. In this case, the effect of the field on orbital motions can be ignored as far as $L_\tau \gg$ film thickness and $\omega_c\tau \ll 1$, where $\omega_c = eH/mc$ is the cyclotron frequency. However, the effect of Zeeman splitting given by the last term in eq. (2.1) should be taken into account. The electron Green's function with spin $v = \pm 1$ is written as

$$G_v(k, \varepsilon_n) = \left(\mathrm{i}\varepsilon_n - \xi_{k,v} + \frac{\mathrm{i}}{2\tau}\,\mathrm{sgn}\,\varepsilon_n\right)^{-1}, \tag{2.6}$$

$$\xi_{k,v} = \frac{k^2}{2m}+\frac{g}{2}v\mu_{\mathrm{B}}H - \varepsilon_{\mathrm{F}}, \tag{2.7}$$

where $\varepsilon_n = 2\pi T(n+\frac{1}{2})$ with n being integer. In order to evaluate the correction to the conductivity in WLR, we have to determine the diffusion processes defined in Fig. 1. In the figure, the Greek letters denote the spin indices of electrons, and the crosses denote impurity potentials, eq. (2.2). The upper propagators of electrons have the frequency ε_n and the lower ones have $\varepsilon_n - \omega_l$. The equation in Fig. 1 is written as

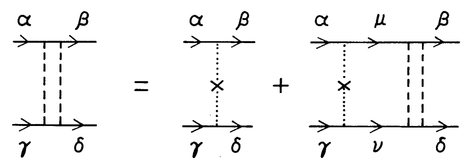

Fig. 1. Equations for the diffusion propagators. The upper propagators of electrons have the frequency ε_n and the lower ones have $\varepsilon_n - \omega_l$, where $\varepsilon_n(\varepsilon_n - \omega_l) < 0$. The Greak letters denote the spin states. The crosses denote impurity potentials, eq. (2.2).

$$\Gamma_{\alpha\beta,\gamma\delta}=\Gamma^0_{\alpha\beta,\gamma\delta}+\sum_{\mu,\nu}\Gamma^0_{\alpha\mu,\gamma\nu}\Pi_{\mu\nu}\Gamma_{\mu\beta,\nu\delta}, \tag{2.8}$$

$$\Gamma^0_{\alpha\beta,\gamma\delta}=(\tau_0^{-1}\delta_{\alpha\beta}\delta_{\gamma\delta}-\sum_i\tau_{so,i}^{-1}\sigma^i_{\alpha\beta}\sigma^i_{\gamma\delta})/2\pi N(0), \tag{2.9}$$

$$\Pi_{\mu\nu}=\sum_k G_\nu(-\boldsymbol{k},\,\varepsilon_n-\omega_l)G_\mu(\boldsymbol{k}+\boldsymbol{q},\,\varepsilon_n). \tag{2.10}$$

If $\varepsilon_n(\varepsilon_n-\omega_l)<0$ and $|\omega_l|\tau\ll 1$, $Dq^2\tau\ll 1$ and $h\tau\ll 1$ where $D=\varepsilon_F\tau/m$ is the diffusion constant and $h=(g/2)\mu_B H$, we obtain

$$\Pi_{\nu,\nu}=2\pi N(0)\tau(1-|\omega_l|\tau-Dq^2\tau)\equiv\Pi(q,\,\omega_l), \tag{2.11}$$

$$\Pi_{\nu,-\nu}=2\pi N(0)\tau(1-|\omega_l|\tau-Dq^2\tau+2i\nu h\tau). \tag{2.12}$$

In terms of $\Gamma_{\alpha\beta,\gamma\delta}$, the quantum correction to the conductivity in the order of $(\varepsilon_F\tau)^{-1}$ is given by

$$\sigma'(H)=-\frac{e^2}{\pi m}\varepsilon_F\tau^3N(0)\int_0^{q_c}q\mathrm{d}q(\Gamma_{++,++}+\Gamma_{--,--}+\Gamma_{+-,-+}+\Gamma_{-+,+-}), \tag{2.13}$$

where $q_c=(D\tau)^{-1/2}$ is the cut-off momentum. By solving eqs. (2.8) and (2.9), we obtain

$$\Gamma_{++,++}=\Gamma_{--,--}=\frac{(x-z)+(x+y-2z)(y-x)\Pi(q,\,\omega_l)}{(1-(x-z)\Pi(q,\,\omega_l))^2-(y-z)^2\Pi^2(q,\,\omega_l)}, \tag{2.14}$$

$$\Gamma_{+-,-+}=\Gamma_{-+,+-}=-\frac{(y+z)}{(1-(x+z)\Pi_{+-})(1-(x+z)\Pi_{-+})-(y+z)^2\Pi_{-+}\Pi_{+-}}, \tag{2.15}$$

where

$$x=\frac{1}{2\pi N(0)\tau_0}, \tag{2.16a}$$

$$y=\frac{1}{2\pi N(0)\tau_{so,z}}, \tag{2.16b}$$

$$z=\frac{1}{2\pi N(0)\tau_{so,x}}=\frac{1}{2\pi N(0)\tau_{so,y}}. \tag{2.16c}$$

Using eqs. (2.11) and (2.12) in eqs. (2.14) and (2.15), and replacing $|\omega_l|$ by the inverse of the energy relaxation time τ_ε, we find

$$\frac{\sigma'(H)}{\sigma_0}=\frac{1}{2\pi\varepsilon_F\tau}\left\{\frac{1}{2}\ln\left(\frac{\tau}{\tau_\varepsilon}+\frac{4z}{x+y-2z}\right)+\frac{1}{2}\ln\left(\frac{\tau}{\tau_\varepsilon}+\frac{2(y+z)}{x-y}\right)\right.$$
$$\left.+\frac{1}{2\sqrt{1-\gamma_{//}}}\ln\left[\frac{\dfrac{\tau}{\tau_\varepsilon}+\left(\dfrac{y+z}{x-y}\right)(1+\sqrt{1-\gamma_{//}})}{\dfrac{\tau}{\tau_\varepsilon}+\left(\dfrac{y+z}{x-y}\right)(1-\sqrt{1-\gamma_{//}})}\right]\right\}, \tag{2.17}$$

where $\sigma_0=\varepsilon_F\tau e^2/\pi$ and $\gamma_{//}$ is defined as follows,

$$\gamma_{//}=\left(2h\tau\frac{x-y}{y+z}\right)^2. \tag{2.18}$$

The last term of r.h.s. of eq. (2.17) is due to the spin-flip processes, which depends on magnetic fields. In the absence of the field, $\sigma'(0)$ is given by

$$\frac{\sigma'(0)}{\sigma_0}=\frac{1}{2\pi\varepsilon_F\tau}\left\{\ln\left(\frac{\tau}{\tau_\varepsilon}+\frac{2(y+z)}{x-y}\right)+\frac{1}{2}\ln\left(1+\frac{4\tau_\varepsilon z}{\tau(x+y-2z)}\right)\right\}. \tag{2.19}$$

In ordinary cases of weak spin-orbit scattering, i.e. y, $z\ll x$, eq. (2.19) takes the following limiting values,

$$\frac{\sigma'(0)}{\sigma_0}=\begin{cases}-\dfrac{1}{2\pi\varepsilon_F\tau}\ln\left(\dfrac{\tau_\varepsilon}{\tau}\right), & \text{for } \dfrac{\tau}{\tau_\varepsilon}\gg\dfrac{y}{x}\text{ and }\dfrac{z}{x}, \quad (2.20\text{a})\\[2ex] -\dfrac{1}{2\pi\varepsilon_F\tau}\left\{\ln\left(\dfrac{x-y}{2(y+z)}\right)-\dfrac{1}{2}\ln\left(\dfrac{4\tau_\varepsilon z}{\tau(x+y-2z)}\right)\right\}, & \text{for } \dfrac{\tau}{\tau_\varepsilon}\ll\dfrac{y}{x}\text{ and }\dfrac{z}{x}. \quad (2.20\text{b})\end{cases}$$

On the other hand, $\sigma'(H)$ in weak fields is given by

$$\begin{aligned}\frac{\Delta\sigma'(H)}{\sigma_0}&\equiv\frac{\sigma'(H)-\sigma'(0)}{\sigma_0}\\ &=-\frac{\gamma_{//}}{8\pi\varepsilon_F\tau}\left\{\left(\frac{y+z}{x-y}\right)\left(\frac{\tau_\varepsilon}{\tau}\right)\left(1+\frac{1}{1+2\left(\frac{\tau_\varepsilon}{\tau}\right)\left(\frac{y+z}{x-y}\right)}\right)-\ln\left(1+2\left(\frac{\tau_\varepsilon}{\tau}\right)\left(\frac{y+z}{x-y}\right)\right)\right\}+O(h^4).\end{aligned} \quad (2.21)$$

If $\tau/\tau_\varepsilon\gg(y+z)/(x-y)$ is satisfied, eq. (2.21) leads to

$$\frac{\Delta\sigma'(H)}{\sigma_0}=-\frac{1}{2\pi\varepsilon_F\tau}\cdot\frac{\tau_\varepsilon(y+z)}{3\tau(x-y)}(2h\tau_\varepsilon)^2. \quad (2.22)$$

As seen in eqs. (2.21) and (2.22), MR is positive; the resistivity increases quadratically in weak fields and it saturates once $h\gtrsim\tau_\varepsilon^{-1}$. The total amount of variation due to the field is

$$\frac{\Delta\sigma'(\infty)}{\sigma_0}=-\frac{1}{2\pi\varepsilon_F\tau}\cdot\frac{\tau_\varepsilon(y+z)}{\tau(x-y)},\quad\text{for }\frac{\tau}{\tau_\varepsilon}\gg\frac{y+z}{x-y}. \quad (2.23)$$

§3. Magnetoresistance in a Perpendicular Field

When the magnetic field is applied perpendicularly to the film, the equations for the diffusion processes (Fig. 1) are solved as follows:

$$\begin{aligned}\Gamma_{++,++}=\Gamma_{--,--}&=\frac{x-y}{1-(x-y)\Pi(q,\omega_l)}\\ &=\frac{1}{2\pi N(0)\tau}\left[Dq^2\tau+|\omega_l|\tau+\frac{2(y+z)}{x-y}\right]^{-1},\end{aligned} \quad (3.1)$$

$$\begin{aligned}\Gamma_{+-,-+}=\Gamma_{-+,+-}&=\frac{-2z}{(1-(x+y)\Pi_{+-})(1-(x+y)\Pi_{-+})-(2z)^2\Pi_{+-}\Pi_{-+}}\\ &=\frac{1}{4\pi N(0)\tau\sqrt{1-\gamma_\perp}}\left\{\left[Dq^2\tau+|\omega_l|\tau+\frac{2z}{x+y-2z}(1+\sqrt{1-\gamma_\perp})\right]^{-1}\right.\\ &\quad\left.-\left[Dq^2\tau+|\omega_l|\tau+\frac{2z}{x+y-2z}(1-\sqrt{1-\gamma_\perp})\right]^{-1}\right\},\end{aligned} \quad (3.2)$$

where

$$\gamma_\perp=\left(\frac{x+y-2z}{z}h\tau\right)^2. \quad (3.3)$$

For this geometry the orbital motion is also affected by the field and Dq^2 in eqs. (3.1) and (3.2) is to be replaced by $4Dm(N+\frac{1}{2})\omega_c$ as far as $\omega_c\tau\ll1$, where N is the Landau quantum number. The correction to the conductivity, eq. (2.13), is now given by the summation over N instead of integration over q and the result is

$$\frac{\sigma'(H)}{\sigma_0}=-\frac{1}{2\pi\varepsilon_F\tau}\left[\psi\left(\frac{1}{2}+\frac{1}{a\tau}\right)-\psi\left(\frac{1}{2}+X\right)-\frac{1}{2\sqrt{1-\gamma_\perp}}\left(\psi\left(\frac{1}{2}+Y_+\right)-\psi\left(\frac{1}{2}+Y_-\right)\right)\right], \quad (3.4)$$

where $\psi(z)$ is the di-gamma function, $a=4Dm\omega_c$, and

$$X=\frac{1}{a}\left(\frac{1}{\tau_\varepsilon}+\frac{2}{\tau}\left(\frac{y+z}{x-y}\right)\right), \quad (3.5)$$

$$Y_\pm=\frac{1}{a}\left[\frac{1}{\tau_\varepsilon}+\frac{1}{\tau}\left(\frac{2z}{x+y-2z}\right)(1\pm\sqrt{1-\gamma_\perp})\right]. \quad (3.6)$$

In the absence of the Zeeman effect and in the case of weak spin-orbit scattering ($y, z \ll x$), eq. (3.4) is reduced to the results in ref. 11.* If we assume $\tau_{so,x}=\tau_{so,y}=\tau_{so,z}\equiv\tau_{so}$ and $\tau_{so}\gg\tau$, eq. (3.4) yields the following in weak magnetic fields,

$$\frac{\Delta\sigma'(H)}{\sigma_0}=\frac{1}{2\pi\varepsilon_F\tau}\left\{\frac{(a\tau_\varepsilon)^2}{48}\left[\frac{3}{\left(1+\dfrac{4\tau_\varepsilon}{\tau_{so}}\right)^2}-1\right]-\frac{2\tau_\varepsilon}{3\tau_{so}}(2h\tau_\varepsilon)^2\right\}. \tag{3.7}$$

The first term of r.h.s. in eq. (3.7) is due to the orbital motion of electrons, whereas the second term is due to the Zeeman effect. In a weak perpendicular field MR is mainly determined by $(a\tau_\varepsilon)^2$ and is stronger than that for a parallel field, where the $(h\tau_\varepsilon)^2$ dependence is expected (eq. (2.22)). As seen in eq. (3.7), the relative magnitude of τ_ε and τ_{so}, i.e. $\tau_\varepsilon/\tau_{so}\lessgtr(\sqrt{3}-1)/4=0.183$, determines the sign of MR in weak perpendicular fields. If spin-orbit scattering is weak, $\tau_\varepsilon/\tau_{so}<(\sqrt{3}-1)/4$, MR remains to be negative for any strength of the field, whereas in the case of stronger spin-orbit scattering, $(\sqrt{3}-1)/4<\tau_\varepsilon/\tau_{so}$, MR is positive in weak fields and changes sign at higher fields.

§4. Numerical Results and Discussions

Komori *et al.*[16] have measured MR in Cu granular films with various sheet resistances $R_\square$. As a typical example we consider the film with $R_\square=809.21\ \Omega$ at 2.02 K. The temperature dependence of the sheet resistance is shown in Fig. 2. The field dependence of MR is shown in Fig. 3 for both geometries of parallel and perpendicular fields at $T=1.67$ K and 2.97 K. The characteristic features of MR are the following: (i) In a parallel field, MR increases with increasing the field, attains a maximum at several tesla and then decreases. (ii) In a perpendicular field, MR is positive at low fields and then changes sign at $H\sim1\ T$.

We have numerically calculated MR by using eqs. (2.21) and (3.4). The relative MR (which is given in terms of the magnetoconductivity $\Delta\sigma'(H)/\sigma'(0)$) is shown for various values of τ_ε in a parallel field in Fig. 4 and in a perpendicular field in Fig. 5. Since direct measurements of the parameters τ_0, τ_{so}, and m have not been done, we tentatively took the values $\tau_0^{-1}=100$ K, $\tau_{so}^{-1}=3$ K, and $m/m_0=3.3$ for our qualitative comparison with experiments. We

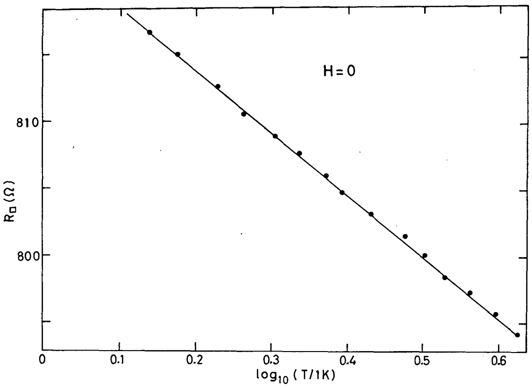

Fig. 2. Experimental data by F. Komori, S. Kobayashi, Y. Ootuka, and W. Sasaki (private communication) of the temperature dependence of the sheet resistance in a Cu granular film.

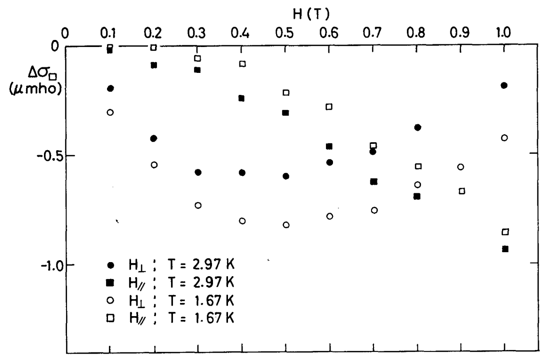

Fig. 3. Experimental data by F. Komori, S. Kobayashi, Y. Ootuka, and W. Sasaki (private communication) of the magnetoresistance ($-\Delta\sigma'_\square(H)$) in a parallel field (□ and ■) and in a perpendicular field (○ and ●) in the film shown in Fig. 2.

may conclude that our theoretical results of Figs. 4 and 5 are in rough agreement with experimental results of Fig. 3. Especially the existence of positive MR in weak fields for both geometries of the magnetic field is understood as due to spin-orbit scattering weaker than inelastic scattering. Such positive MR is not clearly seen in Si-MOS. The Figs. 4 and 5 also predict the temperature dependence of positive

* In ref. 11, τ_1^{-1} should be revised as

$$\frac{1}{\tau_1}=\frac{2}{\tau_{so}^z}+\frac{2}{\tau_{so}^x}+\frac{2}{\tau_s^x}+\frac{1}{\tau_\varepsilon}-i\omega.$$

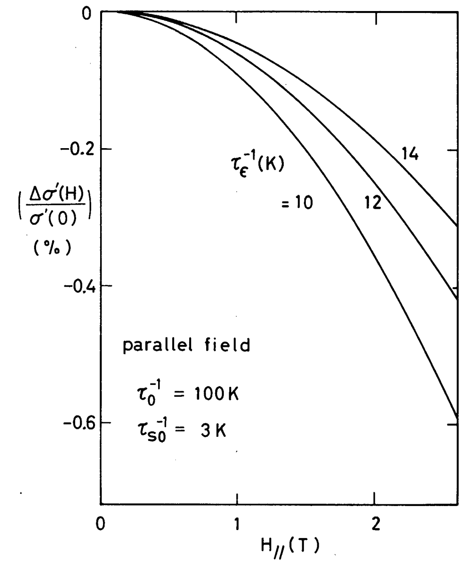

Fig. 4. Theoretical results of the magnetoresistance $(-\Delta\sigma'(H))$ in a parallel field $H_{//}$ for various values of τ_ε for the choice of $\tau_0^{-1}=100$ K and $\tau_{so}^{-1}=3$ K.

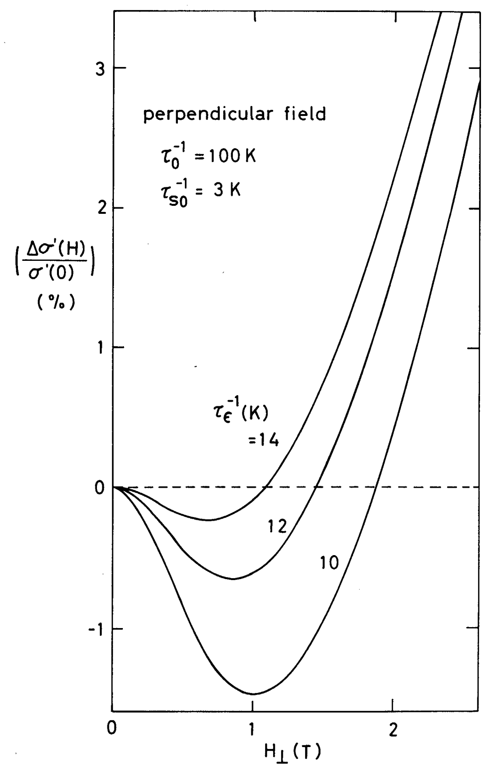

Fig. 5. Theoretical results of the magnetoresistance $(-\Delta\sigma'(H))$ in a perpendicular field $H_\perp$ for various values of τ_ε for the choice of $\tau_0^{-1}=100$ K, $\tau_{so}^{-1}=3$ K, and $m/m_0=3.3$.

MR. The temperature dependence in the perpendicular field was experimentally confirmed, but the definite experimental conclusion in the case of the parallel field has not been drawn.

The resistivity is proportional to $\ln \tau_\varepsilon$ in the range $7\,\mathrm{K} \lesssim \tau_\varepsilon^{-1} \lesssim 20\,\mathrm{K}$ for the above choices of τ and τ_{so}. This $\ln \tau_\varepsilon$ dependence will be consistent with the experimentally observed $\ln T$ dependence in the temperature range $1.3\,\mathrm{K} \lesssim T \lesssim 4\,\mathrm{K}$ if $\tau_\varepsilon \lesssim T^{-p}$ with $p \sim 1$. In some samples[15] the $\ln T$ dependence is observed in the wide temperature range $0.1\,\mathrm{K} \lesssim T \lesssim 10\,\mathrm{K}$. Within our present model we need very weak spin-orbit scattering in such cases, and we expect positive MR in the weak perpendicular field on the low temperature side in the range and negative MR on the high temperature side. Positive MR may be seen in the parallel field in the whole temperature range.

Our theory fails to explain the negative MR observed at higher fields applied parallel to the film. One possible mechanism for this negative MR is the reduction of the inelastic scattering rate, τ_ε, due to a small amount of paramagnetic impurities,[20,21] which are known[22] to exist in the oxide layers of Cu grains. This τ_ε affects the classical conductivity, σ_0, as $\sigma_0=(e^2\varepsilon_F/\pi)(\tau^{-1}+\tau_\varepsilon^{-1})^{-1}$. Existence of such paramagnetic impurities, however, does not change the qualitative conclusions of MR in WLR drawn in the text as is shown in Appendix A.

Quite recently, Giordano[23] has observed the resistance rise in AuPd films in a magnetic field of 75 kOe applied in the plane of the film at 4 K. These experimental results may also be explained by the present theory.

Acknowledgements

We are thankful to Professor S. Kobayashi for informing the experimental results prior to publication and for valuable discussions.

Appendix A

In this Appendix, we examine the effect of magnetic impurities on the magnetoresistance (MR). The impurity potential is written as

$$u(\boldsymbol{k}-\boldsymbol{k}')=u_0+\mathrm{i}u_{so}(\hat{k}\times\hat{k}')\cdot\boldsymbol{\sigma}+I\boldsymbol{S}\cdot\boldsymbol{\sigma}, \tag{A·1}$$

where S is the operator of an impurity spin. The relaxation time due to eq. (A·1) is given by

$$\tau^{-1}=\tau_0^{-1}+\sum_{i=x,y,z}(\tau_{so,i}^{-1}+\tau_{s,i}^{-1}), \tag{A·2}$$

$$\tau_{s,i}^{-1}=2\pi N(0)n_i I^2\overline{(S^i)^2}, \quad (i=x,\ y,\ z), \tag{A·3}$$

where $\tau_{so,i}$ is given in eq. (2.5) and $\overline{(S^i)^2}$ denotes the thermal average of $(S^i)^2$. In order to evaluate the diffusion processes given in Fig. 1, we first write the function $\Gamma^0_{\alpha\beta,\gamma\delta}$ as

$$\Gamma^0_{\alpha\beta,\gamma\delta}=[\tau_0^{-1}\delta_{\alpha\beta}\delta_{\gamma\delta}+\sum_i(\tau_{s,i}^{-1}-\tau_{so,i}^{-1})\sigma^i_{\alpha\beta}\sigma^i_{\gamma\delta}]/2\pi N(0). \tag{A·4}$$

In a parallel magnetic field ($H//x$), we find

$$\Gamma_{++,++}=\Gamma_{--,--}=\frac{(x-z+z_s)+[(y-z)^2-(x-z+z_s)^2]\Pi(q,\ \omega_l)}{[1-(x-z+z_s)\Pi(q,\ \omega_l)]^2-(y-z)^2\Pi^2(q,\ \omega_l)}, \tag{A·5}$$

$$\Gamma_{+-,-+}=\Gamma_{-+,+-}=\frac{2y_s-y-z}{[1-(x+z-z_s)\Pi_{+-}][1-(x+z-z_s)\Pi_{-+}]-(2y_s-y-z)^2\Pi_{+-}\Pi_{-+}}, \tag{A·6}$$

where

$$y_s=(2\pi N(0)\tau_{s,y})^{-1}=(2\pi N(0)\tau_{s,z})^{-1}, \tag{A·7}$$

$$z_s=(2\pi N(0)\tau_{s,x})^{-1}. \tag{A·8}$$

Then, the quantum correction to the conductivity given in eq. (2.13) is calculated to be

$$\frac{\sigma'(H)}{\sigma_0}=\frac{1}{2\pi\varepsilon_F\tau}\left\{\ln\left(\frac{\tau}{\tau_\varepsilon}+\frac{2(2y+y_s)}{x-y+y_s}\right)+\frac{(y_s-y)}{(x+y-y_s)\gamma_s}\ln\left(\frac{\frac{\tau}{\tau_\varepsilon}-1+\eta-\eta\gamma_s}{\frac{\tau}{\tau_\varepsilon}-1+\eta+\eta\gamma_s}\right)\right\}, \tag{A·9}$$

$$\eta=\frac{(x+y-y_s)(x+3y+3y_s)}{(x-y+y_s)(x+3y-3y_s)}, \tag{A·10}$$

$$\zeta=\frac{2(y_s-y)}{x+y-y_s}, \tag{A·11}$$

$$\gamma_s=\sqrt{\zeta^2-(2hz/\eta)^2}. \tag{A·12}$$

The last term of r.h.s. in eq. (A·9) is due to the spin-flip processes. Here, we took the value of an impurity spin S to be 1/2, so that $\overline{(S^i)^2}=1/4$ and $\tau_{s,i}=\tau_s$ for $i=x,\ y,\ z$. We also took $\tau_{so,i}=\tau_{so}$. Assuming $\tau_s\gg\tau_\varepsilon$ and $\tau_{so}\gg\tau_\varepsilon$, we find MR in weak fields as

$$\frac{\Delta\sigma'(H)}{\sigma_0}=\frac{1}{2\pi\varepsilon_F\tau}\cdot\frac{2}{3}(2h\tau_\varepsilon)^2\left(\frac{\tau_\varepsilon}{\tau_s}-\frac{\tau_\varepsilon}{\tau_{so}}\right). \tag{A·13}$$

MR saturates once $h\gtrsim\tau_\varepsilon^{-1}$. The total amount of the variation due to the field is

$$\frac{\Delta\sigma'(\infty)}{\sigma_0}=\frac{1}{\pi\varepsilon_F\tau}\left(\frac{\tau_\varepsilon}{\tau_s}-\frac{\tau_\varepsilon}{\tau_{so}}\right). \tag{A·14}$$

The sign of MR in a parallel field depends on the ratio τ_{so}/τ_s. When the effect of paramagnetic impurities overcomes spin-orbit interaction ($\tau_s<\tau_{so}$), MR is negative at any value of the parallel field. On the other hand, for $\tau_s>\tau_{so}$, MR is positive at any value of the field.

In a perpendicular field ($H//z$), we obtain

$$\begin{aligned}\Gamma_{++,++}=\Gamma_{--,--}&=\frac{x-y+z_s}{1-(x-y+z_s)\Pi(q,\ \omega_l)}\\&=\frac{1}{2\pi N(0)\tau}\cdot\frac{1}{Dq^2\tau+|\omega_l|\tau+2\left(\frac{y+z+y_s}{x-y+z_s}\right)},\end{aligned} \tag{A·15}$$

$$\Gamma_{+-,-+}=\Gamma_{-+,+-}$$
$$=\frac{2(y_s-z)}{[1-(x+y-z_s)\Pi_{+-}][1-(x+y-z_s)\Pi_{-+}]-4(y_s-z)^2\Pi_{+-}\Pi_{-+}}$$
$$=\frac{(y_s-z)}{2\pi N(0)\tau(x+y-z_s)\gamma_s}\left[\frac{1}{Dq^2\tau+|\omega_l|\tau-1+\eta-\eta\gamma_s}-\frac{1}{Dq^2\tau+|\omega_l|\tau-1+\eta+\eta\gamma_s}\right], \quad (\mathrm{A}\cdot 16)$$

$$\eta=\frac{(x+y+2z+2y_s+z_s)(x+y-z_s)}{(x+y-z_s)^2-4(y_s-z)^2}, \quad (\mathrm{A}\cdot 17)$$

$$\gamma_s=\sqrt{\zeta^2-(2h\tau/\eta)^2}, \quad (\mathrm{A}\cdot 18)$$

$$\zeta=\frac{2(y_s-z)}{x+y-z_s}. \quad (\mathrm{A}\cdot 19)$$

Then, taking into account the orbital motion of electron, we find the correction to the conductivity as

$$\frac{\sigma'(H)}{\sigma_0}=-\frac{1}{2\pi\varepsilon_F\tau}\left[\psi\left(\frac{1}{2}+\frac{1}{a\tau}\right)-\psi\left(\frac{1}{2}+X_s\right)\right.$$
$$\left.-\frac{(y_s-z)}{(x+y-z_s)\gamma_s}\left(\psi\left(\frac{1}{2}+Y_{s+}\right)-\psi\left(\frac{1}{2}+Y_{s-}\right)\right)\right], \quad (\mathrm{A}\cdot 20)$$

$$X_s=\frac{1}{a}\left[\frac{1}{\tau_\varepsilon}+\frac{2}{\tau}\left(\frac{y+z+y_s}{x-y+z_s}\right)\right], \quad (\mathrm{A}\cdot 21)$$

$$Y_{s\pm}=\frac{1}{a}\left[\frac{1}{\tau_\varepsilon}-\frac{1}{\tau}(1-\eta\pm\eta\gamma_s)\right], \quad (\mathrm{A}\cdot 22)$$

where $a=4Dm\omega_c$. If we assume $\tau_{so,i}=\tau_{so}\gg\tau$ and $\tau_{s,i}=\tau_s\gg\tau$ for $i=x, y, z$, MR in weak magnetic fields is written as

$$\frac{\Delta\sigma'(H)}{\sigma_0}=\frac{1}{2\pi\varepsilon_F\tau}\left\{\frac{2}{3}(2h\tau_\varepsilon)^2\left(\frac{\tau_\varepsilon}{\tau_s}-\frac{\tau_\varepsilon}{\tau_{so}}\right)+\frac{(a\tau_\varepsilon)^2}{48}\left[\frac{3}{\left(1+2\tau_\varepsilon\left(\frac{2}{\tau_{so}}+\frac{1}{\tau_s}\right)\right)^2}-\frac{1}{\left(1+\frac{6\tau_\varepsilon}{\tau_s}\right)^2}\right]\right\}. \quad (\mathrm{A}\cdot 23)$$

As seen in eq. (A·23), the sign of MR in weak fields depends on the relative magnitude of τ_{so} and τ_s. For $\tau_{so}\gg\tau_s$, MR is negative irrespective of the ratio of τ_ε and τ_s.

Appendix B

In this Appendix, we evaluate the characteristic diatance of the localization, L_τ, for a typical Cu granular film. According to Thouless,[19] L_τ is given by

$$L_\tau=(v_F^2\tau\tau_\varepsilon/d)^{1/2}, \quad (\mathrm{B}\cdot 1)$$

where v_F and d are the Fermi velocity and the dimension of the system, respectively. The sheet resistance $R_\square=809.21\ \Omega$ corresponds to the value $\varepsilon_F\tau=16.3$ if the relation $R_\square=\sigma_0^{-1}$ with $e^2/\pi=7.54\times10^{-5}\ \Omega^{-1}$ is used. This value gives $\varepsilon_F=1.6\times10^3$ K for $\tau^{-1}=100$ K. Using the value of τ, ε_F, and $m/m_0=3.3$, we have $v_F=1.2\times10^7$ cm/sec. and the mean free path due to elastic collisions $l=v_F\tau=100$ Å. Taking $\tau_\varepsilon^{-1}\approx10$ K, we obtain $L_\tau\approx210$ Å.

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