Research

Analytical Model for the Diode Saturation Current of Point-contacted Solar Cells

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Point-contacted solar cells exhibit three-dimensional transport effects due to a spatially inhomogeneous surface recombination. Complex multi-dimensional finite element simulations are commonly applied to model such devices. This paper presents an empirical analytic equation for the diode saturation current of a point-contacted base of a solar cell that accounts for three-dimensional transport. The input parameters of the model that characterize the back surface are: recombination velocity at the contacts; recombination velocity between the contacts; fraction of surface area covered by the contacts; and the contact spacing. We test this model experimentally by conducting spatially resolved minority-carrier lifetime measurements on silicon wafers with point contacts of various sizes and spacings. The diode saturation currents derived from the lifetime measurements agree with the values predicted by the analytic model. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: point contacts; rear surface passivation; amorphous silicon; diode saturation current; analytical model

INTRODUCTION

H igh-efficiency Si solar cells have contacts that cover only a few percent of the device area, while the rest of the surface is electronically passivated by a dielectric coating. In order to reduce the series results.^{1–3} While point contacts have previously been used only in high-quality laboratory cells the application of point contacts to industrial solar cells is within reach.⁴

Figure 1 schematically sketches a solar cell with a point-contacted back surface, facing upwards. The contacted area fraction is $f = \pi r^2/p^2$, with r being the point radius and p being the period length of the square pattern. S_{pass} is the surface recombination velocity (SRV) in the passivated area between the contacts, while S_{met} is the SRV at the point contacts. The surface parameters S_{pass} , S_{met} , f, p, and the bulk minority-carrier diffusion length L control the diode saturation current j_0 and thus the device efficiency. A theoretical model to predict

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Figure 1. Sketch of a solar cell with passivated rear side and point contacts. The metallization fraction is $f = \pi r^2/p^2$ for a square pattern of circular contact points

 j_0 (S_{pass} , S_{met} , f, p, L) is thus required for an optimization of the point contact geometry and for characterizing back-contacted cells.

A transport model for point-contacted rear sides has to consider three dimensions in space due to the inhomogeneous surface recombination. Finite element device simulation programs, such as DESSIS⁵ have thus been commonly used. However, since rear point-contact patterns have a low degree of freedom, it is advantageous and feasible to derive flexible analytical solutions instead of using sophisticated modeling each time a new device design has to be optimized. Another approach has been used by Rau, who developed a three-dimensional transport model based on the Fourier decomposition of the transport equations.⁶ This Fourier approach is easier to implement and much faster than the finite element technique. This advantage comes at the price of being restricted to solar cells that obey the superposition principle.

The numerical finite element approach, as well as the Fourier approach, give access to the spatial distribution of the excess minority carrier concentration and other quantities derived there from. However, this spatially resolved information is not required if one is only interested in the device efficiency. Fischer has described an alternative fully analytic technique to calculate an effective back surface recombination velocity of a point-contacted solar cell.⁷ Fischer's approach is simple to implement and sufficient to calculate the diode saturation current j_0 , which is an important parameter for the cell efficiency.

In this paper we use Fischer's approach⁷ to calculate the diode saturation current $j_0(S_{\text{pass}}, S_{\text{met}}, f, p, L)$ with a single analytic equation. The price to pay for the simple analytic form of the model is that no spatially resolved information is being generated. We present a minor extension of Fischer's model and, more important, demonstrate agreement with experiments.

THEORY

For a full area contact with a spatially homogeneous back surface recombination velocity S_{1D} the transport is one-dimensional and the well-known result for the diode saturation current is^{8,9}

$$j_{0,1D} = \frac{Dn_0}{L} \frac{S_{1D}L + D \tanh(W/L)}{D + S_{1D}L \tanh(W/L)}$$
(1)

with *D* being the minority-carrier diffusion constant, n_0 the equilibrium minority carrier density, and *L* the bulk diffusion length in the base of thickness *W*.

For a point-contacted rear side Fischer *defines*, analogously to Equation (1), the effective recombination velocity S_{3D} to yield the correct diode saturation current.

$$j_{0,3D} = \frac{Dn_0}{L} \frac{S_{3D}L + D \tanh(W/L)}{D + S_{3D}L \tanh(W/L)}$$
(2)

of the point-contacted cell. This relation still depends on the value of the bulk diffusion length *L*. However, in high-efficiency cells we often have $L \gg W$. In the limit $L/W \rightarrow \infty$ we find

$$j_{0,3D} = Dn_0 \frac{S_{3D}}{D + S_{3D}W}$$
(3)

where the dependence of S_{3D} on the diffusion length *L* is removed. In this paper S_{3D} is, by definition, an effective recombination velocity that yields the diode saturation current $j_{0,3D}$ of a point-contacted device with negligible bulk recombination by Equation (3). A theory for $j_{0,3D}$ may thus be tested by measuring and calculating S_{3D} instead of $j_{0,3D}$ provided the bulk recombination of the samples is negligible.

We derive an expression for $j_{0,3D}$ in three steps. The first two steps follow the treatment by Fischer. We briefly outline his derivation here since the original reference, a PhD thesis,⁷ is not readily available to the reader. The third step generalizes Fischer's work and is presented here for the first time.

Step 1: $(L = \infty, S_{\text{pass}} = 0, S_{\text{met}} = \infty)$

Following Fischer,⁷ we neglect bulk recombination $(L = \infty)$, assume zero SRV ($S_{pass} = 0$) in between the contact points, and infinite SRV at the contacts ($S_{met} = \infty$). The plan for solving the minority-carrier transport problem is to exploit the formal equivalence between two physically very different entities: (i) minority-carrier concentration under forward injection through the *p*–*n* junction (injection case); and (ii) the electrostatic potential that causes the majority carriers to flow in a device of identical geometry, but with two ohmic contacts (ohmic case). This formal equivalence permits as to express the diode saturation current in terms of the series resistance of the point-contacted structure. Numerical studies and analytical formulas can be obtained from the literature.^{10–12}

Under forward injection, the excess minority-carrier concentration $n(\mathbf{r})$ in the base depends on the position $\mathbf{r} = (x, y, z)$ and is determined by the Laplace equation (injection case). The Laplace equation

$$\nabla^2 n(\mathbf{r}) = 0 \qquad \nabla^2 \Psi(\mathbf{r}) = 0 \tag{4}$$

also holds for the electrostatic potential $\Psi(\mathbf{r})$ that determines the majority-carrier flow when both sides have ohmic contacts (ohmic case). The boundary conditions at the *p*-*n* junction

$$n(x, y, 0) = n_{\text{ict}} \qquad \Psi(x, y, 0) = \Psi_{\text{ict}}$$
(5)

are also formally identical, when assuming a highly conducting emitter that causes a spatially homogeneous excess carrier concentration n_{jct} at the junction for the injection case and when assuming a highly conductive metal contact that causes a constant potential Ψ_{jct} in the ohmic case. Neither minority carriers nor majority carriers must enter the surface between the contacts since $S_{pass} = 0$. Thus

$$\frac{\partial n}{\partial z}\Big|_{z=W} = 0 \qquad \frac{\partial \Psi}{\partial z}\Big|_{z=W} = 0 \tag{6}$$

holds between the contact points. The minority-carrier concentration n_{met} vanishes at the metal contacts due to an infinite SRV $S_{\text{met}} = \infty$. Without any loss of generality we may assume the electrostatic potential at the point contacts to vanish. Thus we find

$$n(x, y, W) = 0$$
 $\Psi(x, y, W) = 0$ (7)

at the contact points. The current densities for minority and majority carriers are then

$$j_{\min} = q \int_{j_{ct}} \int D \frac{\partial n}{\partial z} \Big|_{z=0} dx dy \qquad j_{\max} = \int_{j_{ct}} \int \frac{1}{\rho} \frac{\partial \Psi}{\partial z} \Big|_{z=0} dx dy$$
(8)

respectively, where ρ is the base resistivity (in Ω cm), and q is the carrier charge. The integration extends over the junction area that is positioned at z = 0. Alternatively j_{\min} is expressed by the saturation current (Equation 3), and j_{\max} by Ohm's law

$$j_{\min} = q D n_{jct} \frac{S_{3D}}{D + S_{3D} W} \qquad j_{maj} = \frac{\Psi_{jct}}{R_S}$$
(9)

with R_S being the series resistance of the point-contacted base in the dark in Ω cm². From Equations (8) and (9) we find

$$\frac{S_{3D}}{D + S_{3D}W} = \frac{1}{n_{jct}} \int_{jct} \int \frac{\partial n}{\partial z} \Big|_{z=0} dxdy \quad \text{and} \quad \frac{\rho}{R_s} = \frac{1}{\Psi_{jct}} \int_{jct} \int \frac{\partial \Psi}{\partial z} \Big|_{z=0} dxdy \quad (10)$$

The two right-hand sides of Equations (10) are identical since n and Ψ both obey the Laplace equation with identical boundary conditions. We thus find

$$\frac{S_{\rm 3D}}{D + S_{\rm 3D}W} = \frac{\rho}{R_{\rm S}} \tag{11}$$

for a cell with $S_{\text{pass}} = 0$, $S_{\text{met}} = \infty$, and $L = \infty$. Using Equation (3), the diode saturation current is then

$$j_{0,3\mathrm{D}} = D \frac{n_0}{R_\mathrm{S}/\rho} \tag{12}$$

Solving Equation (11) for the effective surface recombination we find

$$S_{3\mathrm{D}} = D \left(\frac{R_{\mathrm{S}}}{\rho} - W\right)^{-1} \tag{13}$$

Knowing the series resistance R_S , we may thus either calculate $j_{0,3D}$ or, equivalently, an effective recombination velocity S_{3D} . The latter has no physical meaning other than to yield the correct saturation current when inserted into formulas that were originally derived for one-dimensional transport. We therefore prefer to discuss a physically sensible entity, the saturation current $j_{0,3D}$ rather than the effective surface recombination velocity S_{3D} .

Step 2: $(L = \infty, S_{\text{pass}} = 0, S_{\text{met}} < \infty)$

The approximation of $S_{\text{met}} = \infty$ that we made in Step 1 is certainly not a good one if local back surface fields are used. As a consequence of $S_{\text{met}} < \infty$, the excess minority-carrier concentration at the point contacts is not zero

but has an average value $n_{\text{met}} > 0$ under forward injection in the dark. We now consider the minority-carrier current densities for finite S_{met} as being a superposition (sum) of the Step 1 case ($S_{\text{met}} = \infty$ and thus $n_{\text{met}} = 0$, injection from the junction side thus $n_{\text{jct}} > 0$) and the case where both contacts have interchanged roles ($S_{\text{jct}} = \infty$ ∞ and thus $n_{\text{jct}} = 0$, injection trough the point contacts thus $n_{\text{met}} > 0$). Since the currents are flowing in opposite directions in both cases and since the series resistance does not depend on the direction of current flow we find a total minority current density

$$j_{\min} = D\left(\frac{n_{jct}}{R_{S}/\rho} - \frac{n_{met}}{R_{S}/\rho}\right)$$
(14)

when applying the result from Step 1 to both cases. Note that Equation (14) is only approximately valid since the excess carrier density at the point contacts is assumed to be constant, which is not the case in reality. By definition of S_{met} we also have

$$j_{\min} = f S_{\mathrm{met}} n_{\mathrm{met}} \tag{15}$$

with *f* being the metallized area fraction. The latter relation applies since recombination in the bulk and at the passivated surface in between the contact points is zero and thus the total minority-carrier current equals the recombination current at the contacts. Solving Equations (14) and (15) for $j_{0,3D} = j_{min} \times n_0/n_{jct}$ yields the diode saturation current

$$j_{0,3D} = Dn_0 \left(\frac{R_{\rm S}}{\rho} + \frac{D}{f S_{\rm met}}\right)^{-1} \tag{16}$$

Using the left-hand part of Equation of (9) the corresponding effective SRV becomes

$$S_{3\mathrm{D}} = D \left(\frac{R_{\mathrm{S}}}{\rho} + \frac{D}{f S_{\mathrm{met}}} - W \right)^{-1} \tag{17}$$

As required, Equations (12) and (13) are retrieved from Equations (16) and (17), respectively, in the limit of $S_{\text{met}} \rightarrow \infty$. Note that Equations (16) and (17) hold independently of the actual contact geometry, which enters only indirectly via *f* and R_{S} .⁷ The metallization fraction *f* and the series resistance R_{S} are the only relevant parameters.

Step 3:
$$(L = \infty, S_{\text{pass}} \ge 0, S_{\text{met}} < \infty)$$

In this step, we extend the model to a non-vanishing SRV S_{pass} in between the contact points. We apply the result from Step 2 to the two complementary structures that are shown in Figure 2(a and b).

The sample in Figure 2(a) has an emitter on the bottom surface and point contacts of area fraction *f* on the top surface. All point contacts are assumed to be interconnected (interconnection not shown in Figure 2a). As for the Step 2 case we assume $S_{\text{met}} < \infty$ (thus finite $n_{\text{met}} > 0$) at the point contacts and $S_{\text{pass}} = 0$ in between the contacts. The saturation current

$$j_{0,3D,a} = Dn_0 \left(\frac{R_S}{\rho} + \frac{D}{f S_{met}}\right)^{-1}$$
 (18)

follows directly from Step 2 and is marked here with an additional sub-index a to distinguish it from the saturation current of the structure in Figure 2(b).

Figure 2(b) shows the complementary structure with a contact having circular holes on the top and a full area emitter on the bottom. The area fraction covered by the contact is (1 - f). The contact's SRV is assumed to be $S_{\text{pass}} < \infty$ (thus $n_{\text{pass}} > 0$). At the holes we assume a recombination velocity $S_{\text{met}} = 0$. Since the results of Step 2



Figure 2. Sketch of two diodes with different rear contact geometry: (a) point-metallized diode with the SRV S_{met} under the contact points, and no recombination ($S_{pass} = 0$) between them; (b) diode with a metallization scheme complementary to (a). The SRV under the metal is S_{pass} , while we assume $S_{met} = 0$ in the holes

do not depend on the specific contact geometry we simply interchange the role of the point contacts and the passivated region to find the saturation current

$$j_{0,3D,b} = Dn_0 \left(\frac{\tilde{R}_{\rm S}}{\rho} + \frac{D}{(1-f)S_{\rm pass}}\right)^{-1}$$
(19)

of the complementary structure shown in Figure 2(b). Here \tilde{R}_{S} is the series resistance of the complementary structure of Figure 2(b).

We approximate the carrier concentration of the Step 3 case (that is $L = \infty$, $S_{\text{pass}} \ge 0$, $S_{\text{met}} < \infty$) with the sum of the carrier concentrations for the two complementary cases. Consequently, the recombination currents add and we find the saturation current

$$j_{0,3D} = Dn_0 \left\{ \left(\frac{R_{\rm S}}{\rho} + \frac{D}{f \ S_{\rm met}} \right)^{-1} + \left(\frac{\tilde{R}_{\rm S}}{\rho} + \frac{D}{(1-f)S_{\rm pass}} \right)^{-1} \right\}$$
(20)

for $L = \infty$, $S_{\text{pass}} \ge 0$, $S_{\text{met}} < \infty$ as the sum of Equations (18) and (19). Note that this 'parallel diode approximation' does not account for interaction between the passivated and the metallized areas. It is therefore only approximately valid for large diffusion lengths compared with the period length, $L \gg p$. However, our experimental data will show that Equation (20) fits the experiment for $L \ge 3p$ within the experimental error.

The corresponding surface recombination velocity follows from Equation (9):

$$S_{3\mathrm{D}} = \frac{D}{\left\{ \left(\frac{R_{\mathrm{S}}}{\rho} + \frac{D}{f \, S_{\mathrm{met}}}\right)^{-1} + \left(\frac{\tilde{R}_{\mathrm{S}}}{\rho} + \frac{D}{(1-f)S_{\mathrm{pass}}}\right)^{-1} \right\}^{-1} - W}$$
(21)

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Step 4: $(L < \infty, S_{\text{pass}} \ge 0, S_{\text{met}} < \infty)$

If bulk recombination were not negligible (*L* similar to or smaller than *W*), then inserting Equation (21) in Equation (2) should be a good estimate for the actual saturation current since both extreme limits $L \rightarrow 0$ and $L \rightarrow \infty$ give the correct result.

Modeling the series resistance $R_{\rm S}$

The theory given above is as good as is the equation for calculating the series resistance R_S of the point-contact pattern, and that of the complementary structure \tilde{R}_S . In order to calculate R_S a specific contact pattern needs to be assumed. Here we assume the pattern that is sketched in Figure 1. Following previous work,^{7,13} we use the approximate series resistance

$$R_{\rm S} = p^2 \frac{\rho}{2\pi r} \arctan\left(\frac{2W}{r}\right) + \rho W \left\{1 - \exp\left(-\frac{W}{p}\right)\right\}$$
(22)

for a square pattern of circular point contacts with radius *r* and spacing *p*. The specific resistance of the base is denoted by ρ . This approximate formula interpolates between the 'large-scale' case of $p \gg W$, and the 'small-scale' case of $p \ll W$. This approximation was shown to have an accuracy of better than 5%.^{67,14}

The large-scale case is represented by the first term on the right-hand side of Equation (22), and describes the spreading resistance of a point contact opposite to a conducting surface, normalized to the area p^2 . For a very large contact spacing *p*, the series resistance R_S approaches $R_S = \rho W/f$. The small-scale case is represented by the second term. For very small contact spacing *p*, the base series resistance R_S approaches that of a completely metallized wafer $R_S = \rho W$.

Note that $R_{\rm S}$ represents only the base resistance of a solar cell in the dark. In order to model the complete series resistance of a solar cell, the contribution of the emitter and front contact grid have to be also considered. Earlier investigations showed that the base series resistance of point-contacted solar cells is mostly independent of the actual operating point and illumination of the cell, since the current-flow pattern is governed by spreading resistance and not by injection effects.^{14–16} Thus, $R_{\rm S}$ may also be considered as an approximation for the base resistance under illumination.

The calculation of \tilde{R}_S is similar to that of Equation (22). Again, we interpolate between the small-scale case $(p \ll W)$ that leads to $\tilde{R}_S = \rho W$, and the large scale case $(p \gg W)$:

$$\frac{1}{\tilde{R}_{\rm S}} = \frac{1}{\rho W} - \frac{2\pi r}{\rho p^2 \arctan\left(\frac{2W}{r}\right) \exp\left(\frac{W}{p}\right)}$$
(23)

In the large-scale case, the series resistance is increased by the second term on the right-hand side of Equation (23) that represents the metal-free holes. A parallel connection of the holes with the perforated contact would result in a completely metallized wafer with the resistance ρW . We see that for very large contact spacing *p*, the resistance approaches the expected value $\tilde{R}_{\rm S} = \rho W/(1-f)$.

Equations (21–23) thus represent a general approximate model for the effective back-surface recombination velocity of a point-contacted base with arbitrary values for S_{met} , S_{pass} , f, and p. In combination with Equation (2) this model may be applied to calculate the diode saturation current for finite bulk diffusion lengths L.

EXPERIMENTAL

We use (100)-oriented boron-doped FZ wafers with a thickness of 300 μ m and a resistivity of 1.4 Ω cm. The doping concentration is $N_A = 1 \times 10^{16}$ cm⁻³. The sample design for the lifetime measurements is sketched in Figure 3. After removing the native oxide by a 1 min treatment in 2% HF solution in water we deposit 50-nm-thick a-Si:H films on both sides of the wafers by plasma-enhanced chemical vapor deposition

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Figure 3. Sketch of the sample designs for μ W-PCD lifetime measurements. The wafers are passivated on both sides with an a-Si:H layer. Aluminum points with radius *r* and spacing *p* are applied to one side. Dimensions are not to scale

(PECVD). The deposition parameters are described elsewhere.¹⁷ The surface recombination velocity on both wafer sides is then S_{pass} .

We form an aluminum point pattern on one side of each wafer by evaporating Al through shadow masks. Each mask is divided into six segments patterned with holes of different radius and spacing plus two sections with f = 0 and f = 1, respectively, in the center of the wafer. We use 18 different combinations of point radii r ranging from 120 to 920 µm and point spacings p ranging from 1.0 to 5.0 mm. The metallization fraction f varies from 0.013 to 0.5. The samples are then annealed at 210°C for 3 h to dissolve the amorphous Si into the Al. Thus local Al contacts form.^{17,18} The surface recombination velocity rises in the Al-covered regions during annealing from S_{pass} to S_{met} . The SRV remains unchanged in the non-Al-covered regions.¹⁷

We measure the spatially resolved effective minority carrier lifetime τ_{eff} of our samples by the microwavedetected photoconductance decay method (μ PCD). A short laser pulse illuminates the sample from the non-metallized side, and the local effective minority-carrier lifetime τ_{eff} is extracted from the transient microwave reflectance. White bias light, corresponding to 1 sun, shines on the sample. Its above-bandgap photon flux is $2 \times 10^{21} \,\mathrm{m}^{-2} \,\mathrm{s}^{-1}$ as measured with a calibrated Si solar cell.

Figure 4 shows the lifetime map of one wafer with the contact distance p = 1.5 mm. The metallization fraction *f* varies from 0.05 to 0.38 in the six outer sections. The center of the wafer is divided into a fully metallized semicircle and a non-metallized semicircle that is completely passivated. Bright areas have high effective minority-carrier lifetimes, while dark areas exhibit low lifetimes. We see the Al contact points as spots of low lifetime. Lifetime measurements are taken on a square grid with a spacing of 250 µm in *x*- and in *y*-directions.

DATA EVALUATION

This theory for the diode saturation current requires the values of the metallization fraction f and the distance p of contact points as well as S_{pass} and S_{met} . While f and p are defined by the masks used for sample preparation the values of S_{pass} and S_{met} have to be measured.

We deduce S_{pass} from the non-metallized center region of the lifetime map shown in Figure 4. To this end we solve the equation $^{19-21}$

$$\tan(\beta_0 W) = \frac{2S_{\text{pass}}D\beta_0}{D^2\beta_0^2 - S_{\text{pass}}^2}$$
(24)

with

$$\beta_0 = D^{-1} \sqrt{\left\langle \tau_{\text{eff}}^{-1} \right\rangle - \tau^{-1}} \tag{25}$$



Figure 4. Experimental map of the effective minority-carrier lifetime τ_{eff} of a wafer with p = 1.5 mm. Brighter areas correspond to higher lifetimes. The contact points are visible as dark spots

for S_{pass} . Here W is the wafer thickness, D the diffusion constant,²² and τ the bulk lifetime. For the latter we use the parameterization proposed by Kerr and Cuevas that accounts for Auger, radiative, and Shockley–Read–Hall recombination.²³ For a 1.4 Ω cm boron-doped Si wafer we calculate $\tau \simeq 3.6$ ms. The bulk diffusion length $L = (D\tau)^{0.5}$ is then 3.3 mm, which is about ten times the wafer thickness. We use the harmonic mean lifetime $\langle \tau_{\text{eff}}^{-1} \rangle$ in Equation (25) since in our case the diode saturation is proportional to the inverse effective lifetime $(j_{0,3D} \propto \tau_{\text{eff}}^{-1})$. Here $\langle x \rangle$ denotes the arithmetic mean $\langle \tau_{\text{eff}}^{-1} \rangle = N^{-1} \sum_{i,j=1}^{N} \tau_{\text{eff}}^{-1}(x_i, y_j)$ over measurement positions (x_i, y_j) . This average is determined from a sample area of about 1 cm² which we select as to exclude processinduced inhomogeneities such as the lines that appear in the f = 0.05 section of Figure 4.

The effective minority-carrier lifetime for the non-metallized sample areas is $\langle \tau_{\text{eff},f=0}^{-1} \rangle^{-1} = 620 \,\mu\text{s}$. This value corresponds to the effective surface recombination velocity $S_{\text{pass}} = 20 \,\text{cm/s}$ at the a-Si:H passivated surfaces. For metallized samples we determine S_{3D} using^{19–21}

$$\tan(\beta_0 W) = \frac{(S_{\text{pass}} + S_{3\text{D}})D\beta_0}{D^2\beta_0^2 - S_{\text{pass}}S_{3\text{D}}}$$
(26)

together with Equation (25) since one side of the metallized samples is passivated by amorphous Si. We use $S_{\text{pass}} = 20 \text{ cm/s}$ to determine S_{3D} . The evaluation of the lifetime map of the fully metallized center area in Figure 4 yields $S_{3D} = S_{\text{met}} = 5 \times 10^5 \text{ cm/s}$.

The symbols in Figure 5(a) denote the measured recombination velocities S_{3D} for metallization fractions f of 0.013, 0.03, and 0.1. The effective surface recombination velocity ranges from 400 cm/s for f=0.1 and p=1 mm to 43 cm/s for f=0.013 and p=5 mm.

The symbols in Figure 5(b) show the same experimental data after conversion into diode saturation currents with Equation (3). We use $n_0 = n_i^2/N_A$, where $n_i = 1.05 \times 10^{10} \text{ cm}^{-3}$ is the intrinsic carrier concentration of Si at 300 K in the case of $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$ as the acceptor concentration of the Si wafers.²⁴ The saturation currents range from 0.54 nA/cm² for f = 0.1 and p = 1 mm to 0.077 nA/cm² for f = 0.013 and p = 5 mm.



Figure 5. (a) Effective surface recombination velocity of point-contacted surfaces of various metallization fractions f: experimental data drived from the μ PCD-measurements (symbols) and theoretical data calculated with Equations (21, 22, and 23); (b) diode saturation current $j_{0,3D}$ for various metallization fractions f: experimental data (symbols) and theoretical data derived from Equations (20, 22, and 23)

The symbols of Figure 6 are experimental diode saturation currents $j_{0,3D}$ for a point distance p = 1.5 mm and various metallization fractions f = 0-1. The lifetime map of Figure 4 is used for calculating $j_{0,3D}$. The values of $j_{0,3D}$ rise from $j_{0,3D} = 0.11$ nA/cm² at f = 0.013 to $j_{0,3D} = 1.8$ nA/cm² at f = 1.

COMPARISON WITH THEORY

We use the experimental results for $S_{\text{pass}} = 20 \text{ cm/s}$ and $S_{\text{met}} = 5 \times 10^4 \text{ cm/s}$ in the theoretical Equations (20) and (21). The values of f and p are known from the design of the shadow masks that defined the point contacts. Our theory thus contains no free fit parameters.

The theoretical lines in Figure 5(a) show the SRV derived from the analytical model with the Equations (21–23). The theory fits the experimental data for all three metallization fractions f = 0.013, 0.03, and 0.1 within the experimental error. Typical values of f for high-efficiency solar cells are f < 0.05.²⁵

The lines in Figure 5(b) and Figure 6 are theoretical values generated with Equations (20, 22, and 23). Since the $j_{0,3D}$ data in Figure 5(b) are merely a translation of the S_{3D} data from Figure 5(a), the agreement between experiment and theory is as good as in Figure 5(a). The line in Figure 6 gives the theoretical diode saturation current $j_{0,3D}$ for a wider range of metallization fractions at p = 1.5 mm. The recombination velocity rises from $S_{3D} = S_{pass}$ for f=0 to $S_{3D} = S_{met}$ at f=1. The diode saturation current changes rapidly with metal coverage at f < 0.1. For high values $S_{3D} \gg D/W$, we see from Equation (3) that $j_{0,3D}$ saturates at Dn_0/W , which is 1.9 nA/cm² for our samples. These measurements confirm the analytic theory for the whole range of metallization fractions f=0-1.



Figure 6. The diode saturation current increases with the metallization fraction *f*. Experimental data from the μ W-PCD-measurements (symbols); theoretical data calculated with Equations (20, 22, and 23). The parameters used are $S_{\text{pass}} = 20 \text{ cm/s}$, $S_{\text{met}} = 5 \times 10^4 \text{ cm/s}$, p = 1.5 mm, and $N_A = 1.0 \times 10^{16} \text{ cm}^{-3}$

CONCLUSION

In conclusion, we report an easy to handle approximate analytical model for the effective back-surface recombination velocity and the diode saturation current in point-contacted solar cells. We test this model using life-time measurements on passivated Si samples having point contacts with distances 1 mm and a metal coverage <math>0 < f < 1. The measured data fit the theoretical calculation within the experimental error. The model has no free fit parameters.

Using the results presented here, three-dimensional transport effects in point-contacted devices can be considered without the necessity to perform finite element three-dimensional simulations. The analytical model may be helpful for device simulations, since Equations (2, 21, 22, and 23) provide the effective rear recombination velocity and diode saturation current that are necessary parameters for simulating the cell performance. However, care must be taken when calculating the actual series resistance of a solar cell, since Equation (22) represents only the base resistance in the dark. In order to model the complete series resistance of a solar cell, the contribution of the emitter and front-contact grid have to be also considered. Deviations of the base resistance under illumination from $R_{\rm S}$ have been shown to be small.^{14–16}

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