

where $F : R^n \rightarrow R^n$, $F_D : R^{n-k} \rightarrow R^{n-k}$, $F_R : R^n \rightarrow R^k$, $F_d : R^{n-k} \rightarrow R^m$, $F_{nd} : R^{n-k} \rightarrow R^{n-k-m}$, and $F_r : R^{n+m-k} \rightarrow R^k$.

If the response subsystem is a stable one, its trajectory $\mathbf{x}_r(t)$ should be immune to small displacement of its initial conditions. That is, for a fixed set of drive signal from the drive subsystem, no matter where $\mathbf{x}_r(t_0)$ is, $\mathbf{x}_r(t)$ "will always converge to the same trajectory and at each time always be at the same predictable place on that trajectory" as $t \rightarrow \infty$. Mathematically speaking, if starting from $\mathbf{x}_r(t_0)$, the response subsystem $\dot{\mathbf{x}}_r = F_r(\mathbf{x}_d, \mathbf{x}_r)$ has a trajectory $\mathbf{x}_r(t)$, then starting from a nearby initial position $\mathbf{x}'_r(t_0)$, the same response subsystem will follow the trajectory $\mathbf{x}'_r(t)$, which obeys the equation $\dot{\mathbf{x}}'_r = F_r(\mathbf{x}_d, \mathbf{x}'_r)$. And the stability of the response subsystem means $\Delta \mathbf{x}_r(t) = \mathbf{x}'_r(t) - \mathbf{x}_r(t) \rightarrow 0$ as $t \rightarrow \infty$.

To probe the stability property of the response subsystem, Pecora and Carroll started by first looking at its variational equation

$$\Delta \dot{\mathbf{x}}_r = \dot{\mathbf{x}}'_r - \dot{\mathbf{x}}_r = F_r(\mathbf{x}_d, \mathbf{x}'_r) - F_r(\mathbf{x}_d, \mathbf{x}_r) \approx J_{F_r} \Delta \mathbf{x}_r, \quad (72)$$

where $J_{F_r} = (\partial F_r / \partial \mathbf{x}_r)|_{\mathbf{x}_r}$ is the Jacobian of the \mathbf{x}_r -subsystem, assuming a very small $\Delta \mathbf{x}_r$.

The behavior of the \mathbf{x}_r -response subsystem depends on the Lyapunov exponents of the above equation, which depend on the chaotic driving signals \mathbf{x}_d and are termed "conditional Lyapunov exponents" in Pecora & Carroll [1991]. The negativity of all these exponents will guarantee the asymptotic stability of the response subsystem, which means that the small displacement of adjacent trajectories of this subsystem will eventually decay to zero as time elapses. It is also assured by stability theory that there exists a nonempty set of initial positions $\mathbf{x}'_r(t_0)$ for the trajectory $\mathbf{x}'_r(t)$ to converge to $\mathbf{x}_r(t)$ as $t \rightarrow \infty$.

5.4.2. Synchronizing dynamical systems

Pecora and Carroll defined $\dot{\mathbf{x}} = F(\mathbf{x})$ (of Eq. 71) as a heterogeneous driving system for the case when the response subsystem differs from that part of the drive subsystem, which does not offer any driving variable, i.e., $F_r \neq F_{nd}$. The system $\dot{\mathbf{x}} = F(\mathbf{x})$ is an homogeneous driving system if, after decomposition, $F_r = F_{nd}$. Homogeneous driving is closely related to the concept of synchronizing chaotic subsystems. One way to construct a synchronizing

chaotic system involves dividing a nonlinear system into two parts: the \mathbf{x}_d -subsystem, which will be used for driving a response subsystem, and a stable \mathbf{x}_{nd} -subsystem, which will not be used for driving. Duplicating the \mathbf{x}_{nd} -subsystem results in a third subsystem, the \mathbf{x}'_{nd} -subsystem, which will be used as the response subsystem. This construction process is

$$\begin{aligned} \dot{\mathbf{x}} = F(\mathbf{x}) &\xrightarrow{\text{divide}} \begin{cases} \dot{\mathbf{x}}_d = F_d(\mathbf{x}_d, \mathbf{x}_{nd}) \\ \dot{\mathbf{x}}_{nd} = F_{nd}(\mathbf{x}_d, \mathbf{x}_{nd}) \end{cases} \\ &\xrightarrow{\text{duplicate}} \begin{cases} \begin{cases} \dot{\mathbf{x}}_d = F_d(\mathbf{x}_d, \mathbf{x}_{nd}), \\ \dot{\mathbf{x}}_{nd} = F_{nd}(\mathbf{x}_d, \mathbf{x}_{nd}), \end{cases} \\ \dot{\mathbf{x}}'_{nd} = F_{nd}(\mathbf{x}_d, \mathbf{x}'_{nd}). \end{cases} \end{aligned} \quad (73)$$

Then there is an open set of initial conditions containing $\mathbf{x}_{nd}(t_0)$ and $\mathbf{x}'_{nd}(t_0)$ for which the trajectory $\mathbf{x}'_{nd}(t)$ will converge to $\mathbf{x}_{nd}(t)$. That is, the two subsystems \mathbf{x}_{nd} and \mathbf{x}'_{nd} are synchronized. Note that the synchronization between the drive subsystem ($\mathbf{x}_d, \mathbf{x}_{nd}$) and the response subsystem (\mathbf{x}'_{nd}) is only possible when they are both driven by a proper (chaotic) signal \mathbf{x}_d , which actually determines the stability of \mathbf{x}_{nd} -subsystem. The above construction process will ensure this stability.

As an example, one way of decomposition and duplication of the well-known Lorenz system

$$\begin{cases} \dot{x} = \sigma(y - x), \\ \dot{y} = -xz + \gamma x - y, \\ \dot{z} = xy - \beta z, \end{cases} \quad (74)$$

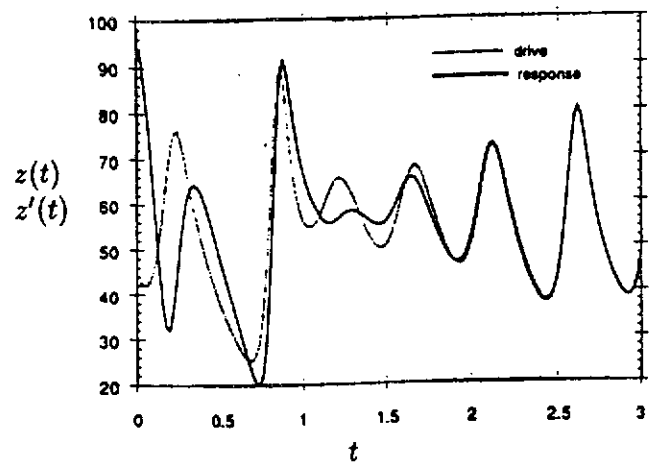


Fig. 45. Convergence of the z' component of the response subsystem (y', z') to the z component of the (y, z) -subsystem in time series. (Figure from Pecora & Carroll [1991], courtesy of The American Physical Society.)

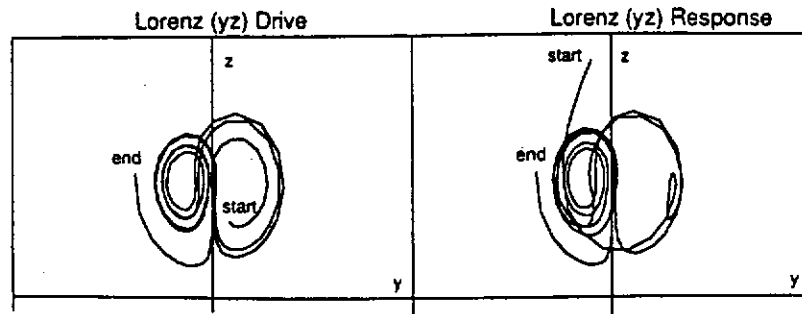


Fig. 46. Convergence of the trajectory of the (y', z') -response subsystem to the trajectory of the (y, z) -subsystem in phase plane. (Figure from Pecora & Carroll [1991], courtesy of The American Physical Society.)

where σ , γ , and β are system parameters, leads to the following compound system,

$$\left\{ \begin{array}{l} \dot{x} = \sigma(y - x) \\ \dot{y} = -xz + \gamma x - y \\ \dot{z} = xy - \beta z \end{array} \right\} \text{ drive subsystem,}$$

$$\left\{ \begin{array}{l} \dot{y}' = -xz' + \gamma x - y' \\ \dot{z}' = xy' - \beta z' \end{array} \right\} \text{ response subsystem,}$$
(75)

where the subsystem (y, z) is indeed stable if one checks its conditional Lyapunov exponents. Figures 45 and 46 clearly illustrate the gradual synchronization of the (y', z') -response subsystem with the (y, z) -subsystem in the drive subsystem, under the driving signal x from the drive subsystem.

Pecora and Carroll also demonstrated a successful synchronization on a set of electronic circuits — the hysteretic circuit [Pecora & Carroll, 1991; Carrol & Pecora, 1991].

6. Summary and Discussions

6.1. Summary

Controlling chaos is a scientific as well as engineering challenge. It is anticipated that with its promising initiation described in this survey, future development of the subject will not only provide satisfactory solutions to many old problems but also bring novel ideas and techniques into the field of nonlinear systems dynamics and control.

We have described some existing efficient techniques for ordering or controlling chaos, to offer an overview of this challenging research direction. In addition to describing the interesting parametric

variation method and entrainment-migration technique, we have also discussed how to design a conventional linear or nonlinear feedback controller to drive a chaotic trajectory of some well-known continuous-time and discrete-time nonlinear dynamic systems to their (unstable) equilibria or (multi-)periodic orbits, where the latter was recently developed by the present authors. The parametric variation approach is especially suitable for signal processing and time series analysis since it does not require a dynamic equation model for the chaotic system, while the entrainment-migration and feedback control strategies are efficient for physical dynamic systems described by conventional nonlinear ordinary differential or difference equations. We have also reviewed, albeit briefly, some other related literature, including a stochastic control approach for chaotic systems, a design of two-degree-of-freedom robust controller for reduction of chaos, ordering chaos of distributed artificial intelligence systems with a reward policy, a neural network strategy for intelligent control of chaos, a signal encoding method under a chaotic environment, and some success in using chaotic signals to synchronize nonlinear dynamic systems. Other related references which we have known but not been able to cover are collected in the bibliographic section of this survey article for the reader's information.

Although only second- or third-order systems have been included in the above presentation, many proposed approaches can be extended to large-scale (higher-dimensional) systems. On the other hand, many of the analyses developed for continuous-time systems can be converted to the discrete-time setting, and vice versa. All the techniques discussed are valuable in their own rights, and many of them have been verified theoretically and/or experimentally. It should be pointed out, however, that the achievements reached so far in developing the

methodologies of controlling chaos allow no single approach to claim that it is all-embracing, or that it is the only valid one. As a matter of fact, many of the proposed approaches that we have discussed are still in their early stage of development, and they probably are only some pieces of the "puzzle." In light of this, when mentioning the virtues or drawbacks of a particular approach, we have reserved opinions and judgment regarding its overall comparative merit, and chosen instead to concentrate on the idea, procedure, and possible application of that approach.

6.2. Discussions

It is deemed that the general problem of controlling chaos in nonlinear dynamic systems deserves a great deal of further research efforts. While pursuing deeper understanding and further amelioration of the existing control methods, we should be aware of that new problems are still evolving, and we must be willing to venture into any possible new avenue that might lead us to better techniques for controlling chaos and its potential applications.

One can easily find many challenging open questions that need to be addressed. For example, the reader may have observed that some of the proposed control methods are based more or less on intuition or experience rather than rigorous mathematical arguments. The major shortcoming of this is the lack of a universally operational theory for a systematic analysis and synthesis of the control methodologies. As may have been observed also, many successful techniques are applicable only to case studies and have to be further improved and reorganized to establish a unified framework for general settings. These challenges are therefore calling for new endeavors in the theoretical and experimental investigation of the subjects and further development of the control techniques.

There have been enough indications that engineers and scientists have started tackling the problem of using the very features of chaos. This includes synchronizing systems with chaotic signals (Sec. 5.4), performing chaotic signal processing [Corcoran, 1991], database management based on the science of chaos and fractal geometry [Cortese, 1992], using chaotic actions to control various kinds of processes, and the attempt by the Prediction Company in Santa Fe, New Mexico to understand and predict financial market chaos [Berreby, 1993]. It will be exciting to see just how far we can go in

this direction. Nevertheless, the realm, where the developed, developing, or to-be-developed theories on controlling (or ordering) chaos are applicable or commercializable, seems to be beyond our wildest imagination. For example, would it be possible for the research on controlling chaos to benefit some aspects of current brain research? Can this research help scientists discover methods for improving our memory or creativity, and to develop better medicine for mental disorders like depression, panic attacks, and schizophrenia? Considering natural disasters, would it be possible for us someday to weaken or control a hurricane, once it has been detected, and to direct it to some designated place, or even more significantly, to generate electricity from its otherwise disastrous power? These are some of the pressing issues confronting us today. The impact of successful answers to them would be enormous and far-reaching.

If we have raised the awareness of the significance of controlling chaos, and if we have generated some excitement about what have been and can be accomplished in this new direction of research, we have then achieved the rest of our objectives which motivated us to start the writing of this article.

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