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Analysis of a GaAs Laser

J. R. Biard, W. N. Carr, and B. S. Reed

An analysis of the semiconductor injection laser is presented which is based on a phenomenological model using device and material parameters. The intent of the laser threshold analysis is not to predict from theory the actual threshold current density but rather to provide a logical means of interpreting experimental results. The device and material parameters have been selected such that they describe both the threshold conditions and quantum efficiency of the laser. A technique is proposed whereby the current required to obtain a population inversion is predicted from device I-V *characteristics. Experimental data are presented which support the analysis.*

THIS investigation is concerned with the characteristics of GaAs lasers fabricated in this laboratory. However, the assumptions used in these calculations should not limit their application to GaAs alone. It is felt that the analysis is sufficiently general to be useful in describing the operation of other semiconductor-laser materials.

ANALYSIS

Mechanism of Laser Gain. The strength of band-to-band optical transitions in direct-gap semiconductors is described by the intrinsic optical absorption coefficient $\alpha_i(E)$. Ideally $\alpha_i(E)$ is the band-to-band absorption coefficient of the undoped semiconductor. However, in this analysis $\alpha_i(E)$ must be corrected for the reduction in band gap and change in the density of states function associated with the heavily doped semiconductors to be considered. The absorption and/or generation of optical radiation may be completely specified in terms of $\alpha_i(E)$ and the probability functions $f_v(E)$ and $f_c(E)$ which define the occupancy of the valence-band and conduction-band levels, respectively. The symbols used in this analysis are listed at the end.

Following the approach of Hall[1] it can be shown that the effective gain coefficient, g, in the plane of a forward-biased P-N junction is

$$g(E) = -\alpha_i(E)\,[1 - f_v(E) - f_c(E)] \qquad [1]$$

In particular Eq. [1] holds for the photon energy at which lasing occurs, E_p. Throughout the following discussion Eq. [1] is considered only for the lasing transition $E = E_p$. For simplicity the functional energy dependence of the various factors is omitted in subsequent equations. Unless otherwise stated the symbols used represent the values at E_p.

If the analysis is limited to the P-type side of the laser junction, the function $\alpha_i(1-f_v)$ is recognized as the band-to-band absorption in the P-type material at thermal equilibrium.

$$g_0 = \alpha_i(1 - f_v) \qquad [2]$$

From Eqs. [1] and [2],

$$g = -g_0 + \alpha_i f_c \qquad [3]$$

Under forward-bias conditions the function f_c is determined by the electron quasi Fermi level in P-type material. For sufficiently large forward bias the second term in Eq. [3] will become dominant and net gain will result; *i.e.*, g will become positive.

In order to simplify the subsequent analysis several assumptions are in order at this point.

a) No conductivity modulation occurs. f_v is independent of f_c and therefore g_0 is a constant.

b) The P-type material is heavily doped such that the degeneracy of the valence band permits popu-

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lation inversion and lasing to be achieved without the minority-carrier (electron) level reaching a degenerate concentration. This allows the use of Boltzman statistics in describing f_c.

c) The assumption of momentum conservation in the allowed optical transitions is implicit in the entire analysis.

Using assumptions *a*) and *b*), Eq. [3] may be written

$$g = -g_0 + \frac{\alpha_i H}{N_c} n \qquad [4]$$

where H is a probability function which describes the distribution of the minority carriers, n, among the conduction-band levels.

Population inversion occurs when $g = 0$, and at this particular value of forward bias $n = n_p$. From Eq. [4],

$$n_p = \frac{g_0 N_c}{\alpha_i H} \qquad [5]$$

and from Eqs. [4] and [5],

$$g = g_0\left[\frac{n}{n_p} - 1\right] \qquad [6]$$

A similar conclusion may also be obtained by considering the N-type side of the junction.

Effect of Diode Geometry. In the preceeding analysis it has been shown that for a given sample of material characterized by α_i, H, and N_c the gain coefficient, g, is a function only of the minority-carrier concentration, n. Since the minority-carrier concentration at the junction is determined uniquely by the voltage, V, applied to the junction, the gain coefficient, g, may also be thought of as being primarily a function of applied voltage. Furthermore, it can be shown from Eq. [1] that population inversion between a pair of levels occurs when the difference between the quasi Fermi levels is equal to the energy-level difference of the pair.[2] Applied voltage, V, is therefore the most fundamental parameter associated with specifying population inversion and stimulated emission. The diode current is a function of both applied voltage and bulk lifetime. Even though diode current is a secondary or dependent parameter, it has the greatest practical value in specifying laser performance since it can be measured directly. More difficulty is encountered in specifying V for a given device because the series IR drop is not known.

The current-voltage characteristic of a diode made from a direct-recombination semiconductor is somewhat difficult to specify because the lifetime is affected significantly by the shape and size of the diode wafer, free-carrier absorption, and the rate at which photons leave the device. The reason for this dependence can be visualized by considering a semiconductor in which recombination is dominated by radiative processes. In such a case, a photon could be absorbed and re-emitted several

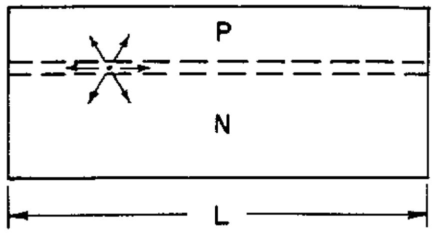

Fig. 1—Model of injection-laser structure.

times before finally leaving the sample or being converted to heat by free-carrier absorption.

The model of the injection-laser structure used to determine its diode characteristics is shown in Fig. 1. For this model it is assumed that radiative recombination occurs only near the plane of the junction where the injection level is high. Hence, absorption-reradiation effects and stimulated emission are confined to a narrow region in and around the plane of the P-N junction. The expression for diode current density, J, resulting from this model is given in Eq. [7].

$$J = J' + b\eta_i J' \frac{g}{g-\alpha}\left[\frac{\beta - 1}{(g-\alpha)L}\left(1 + \frac{R\beta - R}{1 - R\beta}\right) - 1\right] \qquad [7]$$

where

$$\beta = \exp[(g-\alpha)L] \qquad [8]$$

The absorption coefficient α includes not only free-carrier absorption but also the diffraction loss described by Lasher.[3]

The current density J' in Eq. [7] represents the idealized diode current which would flow if all photons generated by recombination immediately left the wafer without being reabsorbed or stimulating additional recombination. This idealized diode current is given by

$$J' = \frac{q\sqrt{D}}{\sqrt{\tau}} n \qquad [9]$$

From assumption *a*), it follows that the idealized lifetime, τ, is a constant. The junction voltage, V, is implicit in Eq. [7] in that it specifies both J' by Eq. [9] and g through Eqs. [1] and [6]. The somewhat formidable term on the right side of Eq. [7] represents the effect of the small fraction, b, of spontaneously emitted photons which produce a significant interaction in the active region of the laser structure. This term carries the sign of g and represents absorption and reradiation when g is negative and stimulation when g is positive. At population inversion when g is zero, this term vanishes and $J = J'$.

The diode current density may be expressed by the form

$$J = \frac{q\sqrt{D}}{\sqrt{\tau_L}} n \qquad [10]$$

where τ_L, the effective lifetime, reflects the total recombination—both spontaneous and stimulated. Substituting Eq. [10] into Eq. [7] results in Eq. [11]:

$$\frac{\tau}{\tau_L} = \left\{1 + b\eta_i \frac{g}{g-\alpha}\left[\frac{\beta - 1}{(g-\alpha)L}\left(1 + \frac{R\beta - R}{1 - R\beta}\right) - 1\right]\right\}^2 \quad [11]$$

At the point of population inverstion, $g = 0$, and Eq. [11] simplifies to the relationship $\tau = \tau_L$. Taking advantage of this relationship, Eq. [10] may be written

$$J_p = \frac{q\sqrt{D}}{\sqrt{\tau}} n_p \quad [12]$$

for $g = 0$. Substituting Eqs. [9] and [12] into Eq. [6] to eliminate n/n_p gives

$$g = g_0\left[\frac{J\sqrt{\tau_L}}{J_p\sqrt{\tau}} - 1\right] \quad [13]$$

Laser Threshold. By substituting Eq. [11] into Eq. [13] to eliminate $\sqrt{\tau_L}/\sqrt{\tau}$ an explicit function of g in terms of diode current density results. Fig. 2 is a typical plot of this explicit relationship. When g is negative the small fraction of spontaneously emitted photons which are reabsorbed in the active region produces a negligible change in the g vs J characteristic. When g is positive, but still less than the value required to overcome the total cavity losses, each spontaneously emitted photon which traverses the active region of the device will stimulate other hole-election pairs to recombine. However, the number of hole-election pairs which are stimulated to recombine is still small compared to the total number which recombine spontaneously. This is not to imply that the external quantum efficiency of the diode will not increase in this bias range. In fact a degree of superlinearity in the light output, line narrowing, and a small shift of the peak energy is usually observed in the super-radiance region indicated in Fig. 2.

An inspection of Eq. [7] will show that as $(1 - R\beta)$ approaches zero the current density increases without limit. The current density at which the onset of this change in the g vs J characteristic occurs is taken to be the laser threshold. Experimental data verifies the abrupt nature of the threshold transition. Both b and η_i have finite values and the current density can take on any value without the factor $(1 - R\beta)$ vanishing. Because of this the junction laser saturates in the process of amplifying a portion of the internal spontaneous emission and is therefore incapable of true coherent oscillation.

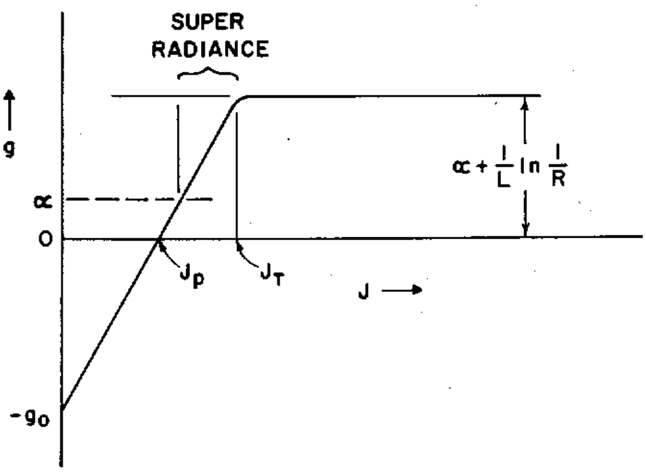

Fig. 2—Typical plot of gain coefficient, g, vs current density, J.

For all practical purposes the plot of Fig. 2 is made up of two straight-line segments and may be represented by

$$g = g_0\left[\frac{J}{J_p} - 1\right] \quad [14]$$

for all values of current density up to and including J_T. Detailed calculations were made from Eqs. [11] and [13] using $b = 0.01$ and representative values of the laser parameters. The departive from Eq. [14] was less than 0.6 pct up to $0.7\,J_T$ and reached 3.0 pct at $0.9\,J_T$.

From Eqs. [7] and [8] the laser-threshold condition is specified by

$$1 - R\exp[(g-\alpha)L] = 0 \quad [15]$$

Clearing the exponential in this expression results in Eq. [16]:

$$g_T = \alpha + \frac{1}{L}\ln\frac{1}{R} \quad [16]$$

This may be combined with Eq. [14] to give the desired expression for laser threshold in terms of device and material parameters,

$$J_T = J_p\left(1 + \frac{\alpha}{g_0} + \frac{1}{g_0 L}\ln\frac{1}{R}\right) \quad [17]$$

Quantum Efficiency for Stimulated Emission. One of the consequences of this analysis is the fact that the injection laser does not treat the band-to-band absorption coefficient, g_0, as an optical loss. This concept is particularly important in specifying the quantum efficiency for stimulated emission η_s:

$$\eta_s = q\frac{d\phi}{dI}\bigg|_{I > I_p} \quad [18]$$

From Eq. [18], η_s does not refer to the total external quantum efficiency but rather to the change in light output for a change in current above the threshold for lasing. From Fig. 3 the gain coefficient, g, has a constant value g_T for all current densities in excess of J_T. It follows from simple considerations that the quantum efficiency for stimulated emission may be expressed as

$$\eta_s = \frac{g_T - \alpha}{g_T} \quad [19]$$

The numerator of this expression is the difference between the stimulation rate and the absorption rate and therefore represents the external photon-emission rate from the laser. Combining Eq. [19] with Eq. [16] gives an expression for η_s in terms of device parameters:

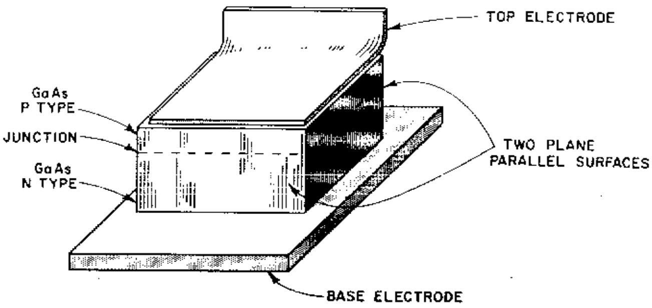

Fig. 3—GaAs laser structure.

$$\eta_s = \frac{1}{\dfrac{\alpha L}{\ln 1/R} + 1} \qquad [20]$$

No quantitative experimental verification of this equation has been undertaken at this laboratory. However, the expression does give the expected result of $\eta_s = 0$ for the total internal-reflection laser and predicts that a short cavity length, L, is required for high efficiency. Using Eqs. [17] and [20] it is possible to describe the total external quantum efficiency of the laser.

EXPERIMENTAL

Lasers were fabricated by diffusion from a zinc-doped oxide source[4] into the (111) face of N-type GaAs; $N_d = 10^{17}$ cm^{-3}. Contacts were alloyed to both the P-type and N-type regions. After cleaving on $\{110\}$ planes to form the Fabry-Perot interferometer, the lasers were mounted on T0-18 headers with solder. The laser structure is illustrated in Fig. 3.

Fig. 4 shows a plot of J_T vs L for lasers fabricated from two different diffusion runs on the same crystal. Note that all the experimental points lie above the minimum-threshold hyperbola which has been fitted to the data. This result is in general agreement with the work of Pilkuhn and Rupprecht.[5]

The value of J_p may be determined experimentally by measuring current, voltage, and relative light output at low current levels where the drop across the series resistance is negligible and, in the same physical setup, measuring current and relative light output at a higher current level where the external quantum efficiency is constant. These data may be used to extrapolate to the value of current which will flow when the voltage across the junction corresponds to E_p. The results of such an experiment performed on three units are indicated in Fig. 5. The values of J_p are also plotted on the graph of Fig. 4 for comparison. The close agreement between the measured J_p values and the horizontal asymptote of the minimum-threshold hyperbola indicates that α has a negligible effect in determining the threshold for units which plot close to the minimum-threshold hyperbola. The excess threshold current exhibited by most of the lasers may be attributed to laser cavity defects such as:

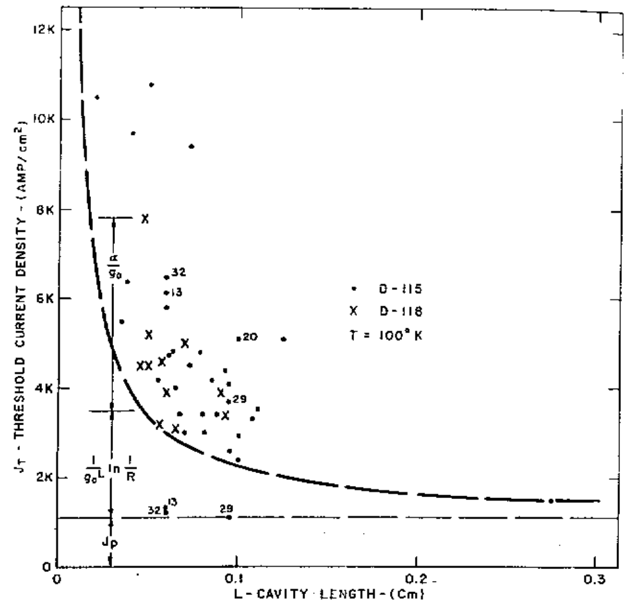

Fig. 4—Threshold current density, J_T, vs laser cavity length, L.

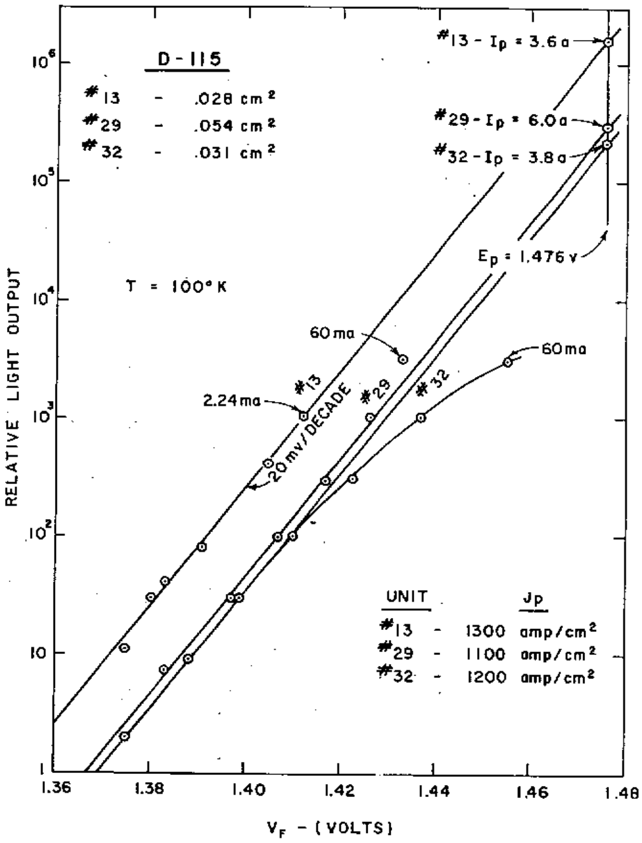

Fig. 5—Plot of experimental data illustrating the extrapolation to determine the population-inversion current density, J_p.

1) lack of planarity or roughness in the P-N junction;
2) lensing of the slice prior to diffusion resulting in the junction not being perpendicular to the end faces;

3) poor cleaving resulting in scattering of the reflected light at the interferometer faces.

In Eq. [17] R, L, g_0, and J_p are all defined in such a way that they are independent of these laser cavity defects. For this reason the excess threshold current must be described in terms of α as indicated in Fig. 4. It should be noted that the diodes which plot close to the minimum-threshold hyperbola exhibit exceptionally low thresholds for a temperature as high as 100°K.

If the factor R in Eq. [17] is assumed to have the value for a normal GaAs-air interface, the parameters of the minimum-threshold hyperbola may be used to calculate g_0. Results of this calculation for the data of Fig. 4 give $g_0 = 11\ cm^{-1}$ at $E_p = 1.476$ ev.

CONCLUSIONS

An analysis has been presented for the GaAs injection laser which emphasizes measurable device and material parameters. Particular significance has been given to the population-inversion bias point and an experimental technique has been described for determining the population-inversion current density, J_p. The effective gain coefficient, g, has been shown to vary linearly with $(J - J_p)$ and expressions have been developed which indicate that the injection laser does not treat the band-to-band absorption coefficient, g_0, as an optical loss.

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LIST OF SYMBOLS

$\alpha_i(E)$ - intrinsic optical absorption coefficient

$\alpha(E)$ - absorption coefficient for free-carrier absorption and other nonrecoverable absorption

b - fraction of spontaneously emitted photons which produce a significant interaction in the active region

D - minority-carrier diffusion coefficient

E_p - transition or photon energy at which lasing occurs

$f_c(E)$ - the occupation probability in the conduction-band levels for electrons

$f_v(E)$ - the occupation probability in the valence-band levels for holes

$g(E)$ - effective gain coefficient at the diode junction. A negative value of $g(E)$ represents net absorption. Population inversion exists for $g(E) = 0$.

$g_0(E)$ - band-to-band absorption coefficient for extrinsic material at thermal equilibrium

H - Boltzman occupation probability for conduction-band levels

I - diode current

I_p - diode current at population inversion

J - diode durrent density

J' - idealized diode current density

J_p - diode current density at which population inversion occurs

L - length of the laser cavity

η_i - internal quantum efficiency (photons per electron) for spontaneous emission

η_s - quantum efficiency for stimulated emission defined in Eq. [18]

n - minority-carrier concentration at the diode junction

n_p - minority-carrier concentration at the diode junction for population inversion

N_c - equivalent density of states in the conduction band

R - reflectivity at the GaAs-air interface

τ - idealized bulk lifetime

τ_L - effective lifetime of total recombination (spontaneous and stimulated)

ϕ - external radiated photon flux (photons per sec)

V - forward voltage applied to the device junction region

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