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NONLINEAR DYNAMIC STUDY OF A BISTABLE MEMS BEAM TOGGLED BY OPTICAL ACTUATION

Marc Sulfridge
Dept. of Mechanical and Ind. Eng.
Univ. of Ill. at Urbana-Champaign
Urbana, Illinois 61801
Email: sulfridg@uiuc.edu

Taher Saif
Dept. of Mechanical and Ind. Eng.
Univ. of Ill. at Urbana-Champaign
Urbana, Illinois 61801
Email: saif@uiuc.edu

Norman Miller
Dept. of Mechanical and Ind. Eng.
Univ. of Ill. at Urbana-Champaign
Urbana, Illinois 61801
Email: nr-millr@uiuc.edu

ABSTRACT

In (Sulfridge, et al., 2001) we presented an actuation scheme for toggling a bistable MEMS from one stable state to the other using the radiation pressure of light. The experiments on radiation pressure revealed some unexpected behavior of the beam when toggled by short duration laser pulses. While long duration laser pulses reliably toggled the beam every time, short duration pulses only seemed to toggle the beam about half of the time. This paper presents a new, nonlinear dynamic analysis of the bistable beam device used in those experiments that explains this anomalous behavior.

INTRODUCTION

Figure 1 shows the bistable MEMS device used to experimentally verify that radiation pressure could be used to actuate a MEMS device (see (Sulfridge, et al., 2001) and (Sulfridge, et al., 2002)). This device was first studied in (Saif & Miller, 1999) and (Saif, 2000). It consists of two long slender structural beams, made of single crystal silicon, and a comb drive actuator that applies an axial compressive force, F on the beams to buckle them to a transverse displacement D. There are two possible buckled states and hence the system is bi-stable. Each beam is 1000 μm long, 1 μm wide and 15 μm deep. They are attached in the middle so that they both buckle along the same direction with stability against rotation. The two beams can thus be considered as one beam AB. There is a small 3-comb actuator at the middle which may be employed to generate a threshold transverse force, F_{th}, on AB to switch its state. F_{th} can also be generated by the radiation pressure of a laser focused on the lateral surface of the silicon beam.

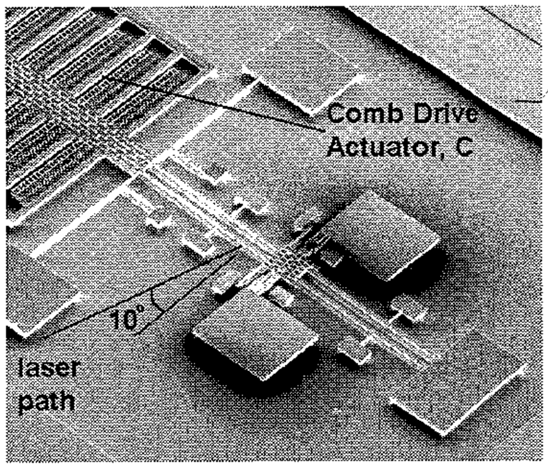

Figure 1. SEM IMAGE OF THE BISTABLE MEMS DEVICE.

The experimental setup designed to demonstrate the feasibility of toggling the silicon beam using light is illustrated in Figure 2. In this setup, a beam of 1064 nm infrared laser light generated by a Nd:YAG continuous wave laser operating at about 3 W of output power is fed into a beam attenuator and a set of filters. The output of the laser is most stable when operated at relatively high power. This laser is used because it is powerful

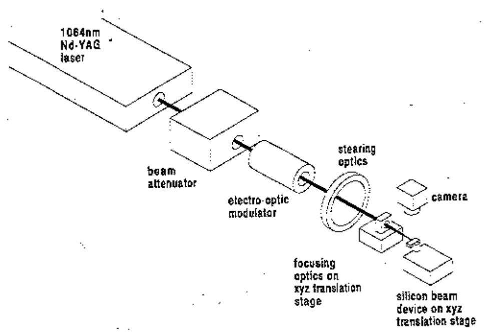

Figure 2. EXPERIMENTAL SETUP.

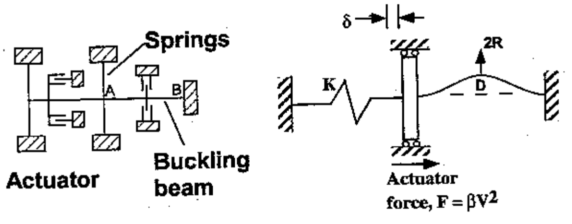

Figure 3. SCHEMATIC AND MODEL OF THE BISTABLE MEMS DEVICE USED TO TEST ACTUATION BY LIGHT.

enough to switch the state of the MEMS beam from a range of buckled states, and is reasonably reflective (72%) with very little absorption (2%) to silicon that is 1 micron thick (Sulfridge, et al., 2002). The filters are used for discrete, coarse adjustment on the laser power, and the attenuator is used to fine tune the power level. The beam then enters an electro-optic modulator, which is used as a shutter for the laser beam. It is controlled digitally, blocking most of the laser power while receiving a logic low input, and passing most of the laser power while receiving a logic high input. The input signal is supplied by a square wave function generator for tuning purposes, and by a single shot bounceless switch with adjustable pulse duration during actual operation of the experiment.

From the modulator, the laser light passes through a system of lenses used to stear the beam, and then it enters an objective lens, used to focus the light onto the side of the MEMS beam. Both the objective lens and the MEMS device are placed on micrometer controlled xyz translation stages for maximum flexibility in focusing the laser light on the side of the beam. A CCD camera with a microscope objective is mounted above the MEMS device in order to view the device during operation.

While this experiment successfully demonstrated the feasibility of radiation pressure as an actuation scheme, it did produce some unexpected results. Using the formula in (Saif, 2000), the time of flight for this beam was expected to be about 2 msec. However, we found experimentally that in order to reliably toggle the beam every time, it was necessary to use a laser pulse of about 20 msec. For pulses between 2 msec and 20 msec, we found that the beam only toggled on average about half of the time. These results prompted us to develop a dynamic model of the system in order to better understand the beam's behavior.

DYNAMIC MODEL

In (Saif, 2000), it was shown that the bistable MEMS device statically obeys the equation, $-K_1D + K_3D^3 = 2R$, where K_1 and K_3 are respectively the linear and cubic spring constants, and $2R$ is the force applied transverse to the buckled beam in order to get it to toggle. This system is shown schematically in Figure 3. A very simple dynamic model of the system could thus be generated by adding an inertial term, $m\ddot{D}$, where m is the effective mass of the beam (1/3 of the total mass for this geometry), and a dissipation term $c_1\dot{D}$, where c_1 is the dissipation coefficient. This dissipation term would take into account both the low Reynolds number viscous dissipation (Hoerner, 1965), as well as the internal structural damping (Lazan, 1968). The low Reynolds number assumption is well justified considering the extremely small scale of the device ($Re \approx 10^{-3}$). Thus a simple dynamic model of the system would be:

$$m\ddot{D} + c_1\dot{D} - K_1D + K_3D^3 = 2R \tag{1}$$

The full system also includes the massive array of comb drives at the end of the beam. We define $\delta = \delta_b + \delta_e$ as in (Saif, 2000), where δ is the displacement of the end of the beam, δ_b is that part of the displacement due to buckling, and δ_e is that part of the displacement due to elastic compression of the beam. Saif (2000) showed that $\delta_e = (F - K\delta_b)/(K + AE/L)$ where K is the cumulative spring constant of the beam's support springs, A is the cross sectional area of the beam, E is its elastic modulus, and L is its length. If we define $\gamma = KL/AE$, then the end displacement becomes $\delta = (\delta_b + FL/AE)/(1+\gamma)$. Saif (2000) further showed that $\delta_b = \pi^2D^2/4L$, which gives us δ in terms of only D and F. If we call the mass of the comb actuators M, then its inertial term is simply $M\ddot{\delta}\mathrm{d}\delta/\mathrm{d}D$, and its dissipation term is $c_2\dot{\delta}\mathrm{d}\delta/\mathrm{d}D$. We may thus add these terms to Equation 1 to obtain the full dynamics of the system. After resolving all variables in terms of D and F and extensive simplification, we arrive at:

$$
\begin{aligned}
m\ddot{D} &+ c_1\dot{D} - K_1 D + K_3 D^3 \\
&+ \frac{\pi^2 D}{2AE(1+\gamma)^2}\left[M\ddot{F} + c_2\dot{F}\right] \\
&+ M\left(\frac{\pi^2}{2L(1+\gamma)}\right)^2 D\left[\dot{D}^2 + D\ddot{D}\right] \\
&+ c_2\left(\frac{\pi^2}{2L(1+\gamma)}\right)^2 D^2\dot{D} = 2R
\end{aligned} \tag{2}
$$

Now, for any given buckled configuration, F is fixed. So, if we define $\varepsilon = \left(\frac{\pi^2}{2L(1+\gamma)}\right)^2$, our dynamic equation simplifies to:

$$
\begin{aligned}
m\ddot{D} &+ c_1\dot{D} - K_1 D + K_3 D^3 \\
&+ \varepsilon\left[M\left(D\dot{D}^2 + D^2\ddot{D}\right) + c_2 D^2\dot{D}\right] = 2R
\end{aligned} \tag{3}
$$

In order to get a better feel for this equation, it is helpful to nondimensionalize it. We define $y = D/L$, $\tau = t\omega_0$ where ω_0 is a yet to be determined constant, and $u = 2R/2P_{cr}$ where P_{cr} is the first critical buckling load of the beam. With these substitutions, the equation becomes:

$$
\begin{aligned}
\ddot{y} &- \frac{K_1}{m\omega_0^2}y + \frac{K_3L^2}{m\omega_0^2}y^3 + \frac{c_1}{m\omega_0}\dot{y} \\
&+ \varepsilon\left[\frac{ML^2}{m}\left(y\dot{y}^2 + y^2\ddot{y}\right) + \frac{c_2L^2}{m\omega_0}y^2\dot{y}\right] = \frac{2P_{cr}}{mL\omega_0^2}u
\end{aligned} \tag{4}
$$

Clearly, it is helpful to define $\omega_0 = \sqrt{K_1/m}$ and $2\xi\omega_0 = c_1/m$. Doing so, we obtain:

$$
\begin{aligned}
\ddot{y} &- y + \frac{K_3L^2}{K_1}y^3 + 2\xi\dot{y} \\
&+ L^2\varepsilon\left[\frac{M}{m}\left(y\dot{y}^2 + y^2\ddot{y}\right) + 2\xi\frac{c_2}{c_1}y^2\dot{y}\right] = \frac{2P_{cr}}{K_1L}u
\end{aligned} \tag{5}
$$

Finally, we define $K_r = K_3L^2/K_1$, $m_r = M/m$, $c_r = c_2/c_1$, $\varepsilon_0 = L^2\varepsilon$, and $K_{1b} = 2LK_1/P_{cr}$. Substituting these into the equation and rearranging the terms we arrive at:

$$
\begin{aligned}
(1+\varepsilon_0 m_r y^2)\ddot{y} &+ 2\xi(1+\varepsilon_0 c_r y^2)\dot{y} \\
&- (1-\varepsilon_0 m_r \dot{y}^2)y + K_r y^3 = \frac{4}{K_{1b}}u
\end{aligned} \tag{6}
$$

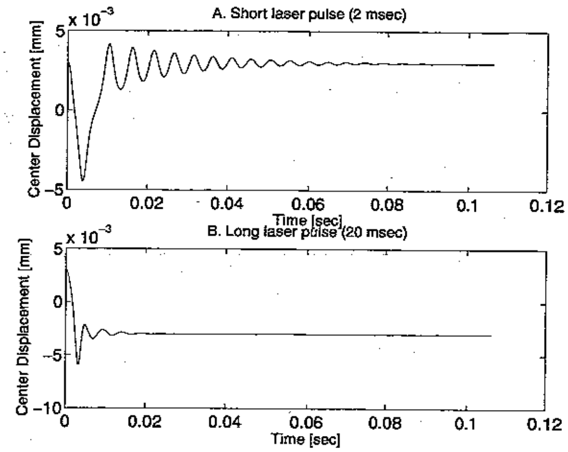

Figure 4. A. MODEL RESULTS FOR THE POSITION OF A BEAM BUCKLED BY 3 MICRONS AND SUBJECTED TO A 2 MSEC LASER PULSE. B. A LONGER DURATION 20 MSEC PULSE ALLOWS THE BEAM TO SETTLE IN THE TOGGLED POSITION.

Thus, in order for the dynamics to simplify to those of the beam alone, we must have $\varepsilon_0 m_r y^2 \ll 1$, $\varepsilon_0 c_r y^2 \ll 1$, and $\varepsilon_0 m_r \dot{y}^2 \ll 1$. For the system under consideration, $\varepsilon_0 \approx 25$, $c_r \approx 5$, and $m_r \approx 1000$. Clearly, if the first condition is met, then the second one will be as well. In addition, as long as the beam is moving at or below the resonant frequency of the beam (ω_0), if the first condition is met then the third one will be as well. So for the approximation to be accurate we must have $y \ll 6\times10^{-3}$, which corresponds to a displacement much less than 6 microns. Thus, under most circumstances, the full dynamics must be considered.

Model Results

Figure 4 shows typical simulated results for the full dynamic model given in Equation 3. In this simulation, the beam is initially buckled to a displacement of 3 microns. In Figure 4A, the buckled beam is then subjected to a 2 msec laser pulse powerful enough to toggle the beam. However, rather than remaining in its toggled state, the beam then bounces back to its original state before settling. In Figure 4B, the beam is subjected to a much longer, 20 msec laser pulse. In this case, the laser toggles the beam and holds it in its toggled state long enough for the beam to settle there, so that when the laser is turned off, the beam remains toggled. This behavior nicely explains the unexpected results that were observed during the actual experiments on the beam. Short duration pulses are strong enough to toggle the beam, as predicted. However, because they are short duration pulses, the beam is free to bounce back and forth between buckled states until it settles. Hence, the beam's final state is essentially a random variable. On the other hand, for long duration pulses, the laser always holds the beam in its toggled state until it settles, and hence the long duration pulse experiments are very repeatable. None of the beam motion could be seen on the videos of these experiments since the video refresh rate of 60 Hz was far too slow to capture the beam dynamics.

CONCLUSIONS

We have presented a new, nonlinear dynamic model of an optically actuated bistable MEMS device. This model takes into account the full dynamics of the system, including both the beam and the actuators attached to the beam. Simulated results from this model nicely explain the unexpected results obtained in prior experiments conducted using the bistable MEMS device. We are currently developing an experiment to capture the full dynamics of the beam by measuring how the capacitance of the central electrostatic combs changes as a function of time, in order to further verify the model we have developed.

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