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Hydrophone for Measuring Particle Velocity

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Mathematical expressions are derived for the motion of a rigid, uniform sphere in an underwater sound field. The results show that at low frequencies a neutrally buoyant sphere of this type has a velocity equal to the particle velocity of the water at the same location when the sphere is removed. A velocity pickup may be mounted inside such a sphere to form a hydrophone sensitive to particle velocity. Two practical design applications are presented. A low-frequency hydrophone with a diameter of five inches was constructed having an essentially flat response between 15 and 700 cps with a sensitivity of 0.18 volt/cm/sec working into a 500-ohm load. A higher frequency design with a 2.5-inch diameter operated from 70 to 7000 cps with somewhat lower sensitivity and impedance level. Velocity hydrophones of this type are easy to build, easily calibrated, stable, and possess good directionality.

INTRODUCTION

If a uniform, rigid sphere of neutral buoyancy is placed in a low-frequency underwater sound field, it can be shown that the velocity of the sphere is the same as the particle velocity of the water at the same location when the sphere is removed. This suggests constructing a velocity hydrophone by mounting a velocity sensitive device inside of a rigid, spherical shell. The hydrophone should be designed so that the assembly composed of the shell and the parts attached rigidly thereto is neutrally buoyant with its center of mass coincident with the geometrical center of the shell. Such an instrument can be used over a limited frequency range to measure the component of particle velocity parallel to the axis of sensitivity of the pickup.

Two such velocity hydrophones have been constructed at the Naval Ordnance Laboratory. The first was developed by one of us[1] in 1941 before reciprocity calibration techniques had been generally applied to underwater acoustics. It was used as an absolute standard for the calibration of transducers in the audio-frequency range and was designated as the SV-1. Another model, the SV-2, was designed and assembled recently. It is being used in conjunction with a pressure hydrophone for detailed measurements near a lake bottom in order to determine the acoustic characteristics of the bottom.

THEORY

The theory is very similar to the usual analysis of scattering by a rigid sphere,[2] although a slight extension results from the fact that the sphere is free to move rather than fixed. Referring to Fig. 1 the pressure of the incident plane wave can be expressed as

$$p_I = P_+ e^{ik(x-ct)} = P_+ e^{ik(r\cos\theta - ct)}$$

$$= P_+ e^{-i\omega t} \sum_{q=0}^{\infty} (2q+1) i^q P_q(\cos\theta) j_q(kr), \qquad (1)$$

where $k=\omega/c$ is the wave number, $P_q(\cos\theta)$ denotes a Legendre polynomial of order q, and $j_q(kr)$ denotes a spherical Bessel function of order q. The radial particle velocity of this wave is

$$u_{I,R} = \frac{-i}{\omega\rho}\frac{\partial p_I}{\partial r}$$

$$= \frac{P_+ e^{-i\omega t}}{i\rho c} \sum_{q=0}^{\infty} i^q P_q(\cos\theta) \times [q j_{q-1}(kr) - (q+1) j_{q+1}(kr)], \qquad (2)$$

where ρ is the density of water. The presence of the sphere causes a scattered (outgoing) wave of the form

$$p_S = e^{-i\omega t} \sum_{q=0}^{\infty} A_q P_q(\cos\theta)[j_q(kr) + i n_q(kr)], \qquad (3)$$

where each A_q is a coefficient to be determined and $n_q(kr)$ denotes a spherical Neuman function of order q.

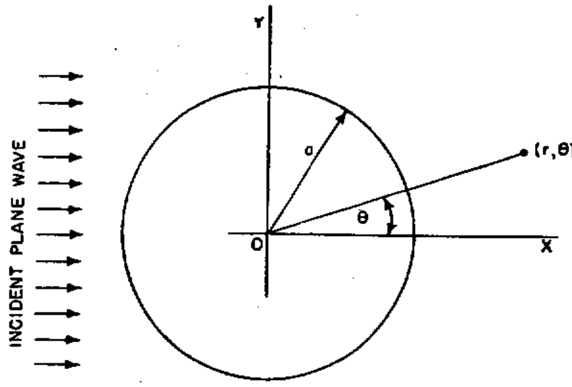

Fig. 1. Sphere and incident plane wave.

[1] James M. Kendall, J. Acoust. Soc. Am. 23, 625(A) (1951). See also James M. Kendall, U. S. Patents 2,582,994 and 2,597,005 (1952).

[2] P. M. Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., New York, 1948), second edition, p. 354. See also Alfred Wolf, Geophysics 10, 91 (1945).

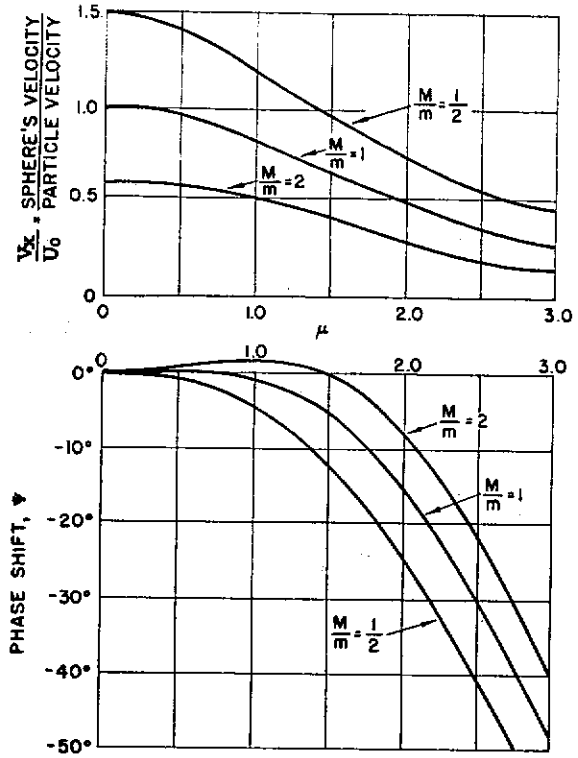

FIG. 2. Velocity ratio and phase shift of a sphere in a sound field for three density ratios.

The corresponding radial particle velocity is

$$u_{S,R}=\frac{-i}{\omega\rho}\frac{\partial p_S}{\partial r}$$

$$=\frac{e^{-i\omega t}}{i\rho c}\sum_{q=0}^{\infty}\frac{A_q P_q(\cos\theta)}{2q+1}\{q[j_{q-1}(kr)+in_{q-1}(kr)]$$

$$-[q+1][j_{q+1}(kr)+in_{q+1}(kr)]\}. \quad (4)$$

From symmetry it is clear that the impinging plane wave causes the sphere to move only in the X direction. Since the displacement of the sphere is very small, its surface is for all practical purposes always at $r=a$. Consequently, the boundary condition to be satisfied is

$$u_{I,R}|_{r=a}+u_{S,R}|_{r=a}=v_X\cos\theta, \quad (5)$$

where v_X denotes the sphere's velocity. The coefficients, A_q, then turn out to be

$$A_q=-(2q+1)P_+ i^{q+1}e^{-i\delta_q}\sin\delta_q, \quad q=0, 2, 3, 4, \cdots,$$

$$A_1=\frac{\rho c V_X e^{-i\delta_1}+3P_+D_1e^{-i\delta_1}\sin\delta_1}{D_1}, \quad (6)$$

where we have written $v_X(t)=V_Xe^{-i\omega t}$, and D and δ_q are defined by the equation[3]

$$iD_qe^{i\delta_q}=\frac{d}{d\mu}[j_q(\mu)+in_q(\mu)] \quad (7)$$

with $\mu=ka$. The total pressure on the surface of the sphere is

$$p_T=p_I|_{r=a}+p_S|_{r=a}=\frac{\rho c V_X e^{-i(\omega t+\delta_1)}}{D_1}P_1(\cos\theta)$$

$$\times[j_1(\mu)+in_1(\mu)]+\frac{P_+e^{-i\omega t}}{\mu^2}$$

$$\times\sum_{q=0}^{\infty}\frac{2q+1}{D_q}P_q(\cos\theta)e^{-i(\delta_q-\frac{1}{2}\pi q)}, \quad (8)$$

and the resultant force on the sphere is

$$f_X=-2\pi a^2\int_0^{\pi}p_T\sin\theta\cos\theta d\theta$$

$$=-\frac{4\pi e^{-i(\omega t+\delta_1)}}{D_1}\left\{\frac{a^2\rho c[j_1(\mu)+in_1(\mu)]}{3}V_X+\frac{iP_+}{k^2}\right\}, \quad (9)$$

since the Legendre polynomials are orthogonal. Applying Newton's second law and solving for V_X

$$V_X=\frac{12\pi P_+}{3\omega MD_1k^2e^{i\delta_1}-4\pi\rho c\mu^2[n_1(\mu)-ij_1(\mu)]}, \quad (10)$$

where M is the sphere's mass. Denote the particle velocity of the incident plane wave at the point 0 (with the sphere removed) by $u_0(t)=U_0e^{-i\omega t}$ so that $U_0=P_+/\rho c$ and

$$\frac{V_X}{U_0}=\frac{3/\mu^2}{\frac{M}{m}\mu D_1e^{i\delta_1}-n_1(\mu)+ij_1(\mu)}$$

$$=\frac{3}{\left[\left(\frac{2M}{m}+1\right)^2+\left(\frac{2M}{m}+1\right)\mu^2+\left(\frac{M}{m}\right)^2\mu^4\right]^{\frac{1}{2}}}e^{-i\psi}, \quad (11)$$

where $m=(4/3)\pi a^3\rho$, the mass of the water displaced by the sphere, and

$$\psi=-\mu+\arctan\frac{\left(1+\frac{2M}{m}\right)\mu}{1+\frac{2M}{m}-\frac{M}{m}\mu^2}. \quad (12)$$

Although the previous discussion has assumed an incident plane wave, Eqs. (11) and (12) are clearly valid for much more complicated sound fields which

[3] Tabulations of D_q and δ_q are available in reference 2, p. 450.

can be described by a superposition of plane waves traveling in many directions. If $\mu \ll 1$, the phase angle, ψ, goes to zero and $V_X/U_0 = 3m/(2M+m)$. This shows that at low frequencies the motion of a sphere whose mass is equal to that of the water it displaces is identical to the motion of the water particles at this location when the sphere is removed. Equations (11) and (12) are plotted in Fig. 2. The curves show that the response of the sphere is flat to within about 1 db up to frequencies where the circumference of the sphere is 0.8 the wavelength, and that the phase shift is negligible over this frequency range.

The above discussion has considered a rigid sphere in a sound field. When a velocity pickup is mounted inside of the sphere the situation is somewhat more complicated. The type of velocity pickup under consideration consists of a coil operating in the air gap of a permanent magnet. The coil and magnetic structure are loosely coupled by means of a spring so that the assembly has a low natural frequency. The operating range of the pickup is well above its natural frequency so that the seismically suspended element remains essentially fixed in space while the other part is driven with the velocity to be measured. In commercially available pickups the light coil is normally the seismically suspended element and the heavy magnetic structure is the driven element. However, it is advantageous for some applications to reverse this design. When a velocity pickup of the latter design is used in a velocity hydrophone, the heavy, seismically suspended structure has a considerable effect on the sphere's motion at frequencies in the neighborhood of resonance. This matter will now be investigated quantitatively.

Figure 3 shows the mechanical system under discussion, where from now on it is understood that M includes all parts of the assembly which are rigidly attached to the sphere, and the term neutral buoyancy refers to the condition where $M=m$. The mass of the resiliently supported structure is denoted by M' and its velocity by V_X'. At frequencies well above resonance, it is clear that neither the mass of the resiliently supported structure nor the location of its center of mass affects the performance of the velocity hydrophone. However, at lower frequencies this is no longer the case. It is assumed that the center of mass of M' (as well as that of M) coincides with the center of the sphere so that only translational motion need be con-

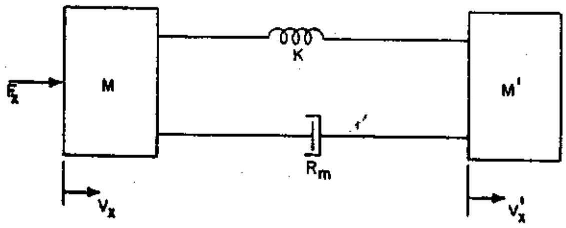

FIG. 3. Mechanical system of a velocity hydrophone.

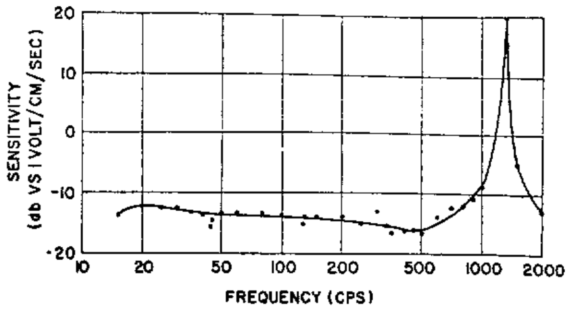

FIG. 4. Acoustic calibration of an SV-2 velocity hydrophone operated into a 500-ohm load.

sidered. The equations of motion are

$$\begin{aligned}&[R_m - i(\omega M - K/\omega)]V_X - [R_m + iK/\omega]V_X' = F_X,\\ &-[R_m + iK/\omega]V_X + [R_m - i(\omega M' - K/\omega)]V_X' = 0,\end{aligned} \qquad (13)$$

where we have written $f_X(t) = F_X e^{-i\omega t}$. Over the frequency range of interest $\mu \ll 1$, so that Eq. (9) simplifies to

$$F_X = i(\tfrac{1}{2}\omega m V_X - 2\pi P_{+}\mu a^2). \qquad (14)$$

Substituting this result in Eq. (13) and solving for $(V_X' - V_X)/U_0$, which is proportional to the output voltage of the velocity pickup,[4] we obtain

$$\frac{V_X' - V_X}{U_0} = \frac{3m}{2M+m} \times \frac{(\omega/\omega_0)^2}{1-(\omega/\omega_0)^2 - i(2R_m/R_{mc}')(\omega/\omega_0)}, \qquad (15)$$

where

$$\omega_0 = \omega_n \left[\frac{2M+2M'+m}{2M+m}\right]^{\frac{1}{2}}, \qquad (16)$$

$$\frac{R_m}{R_{mc}'} = \frac{R_m}{R_{mc}}\left[\frac{2M+2M'+m}{2M+m}\right]^{\frac{1}{2}}, \qquad (17)$$

and $\omega_n = (K/M')^{\frac{1}{2}}$ and R_m/R_{mc} are, respectively, the undamped natural circular frequency and the damping ratio of the velocity pickup with the sphere held fixed. This shows that over the frequency range under consideration, the response of the velocity hydrophone is identical to that of a velocity pickup having an undamped natural circular frequency ω_0 and a damping ratio R_m/R_{mc}'. In other words the effect of the coupled-mass system is to increase both the effective natural frequency and the damping over the values which are appropriate when the mass M is held fixed. If M' is sufficiently small this increase is negligible.

[4] It is assumed that the velocity pickup is damped mechanically by a viscous fluid or by some other means so that a large load resistor can be used. If electrical damping is employed, this proportionality no longer holds at the higher frequencies where the reactance due to the self-inductance of the coil becomes appreciable compared to the relatively small load resistance required to realize a significant amount of damping.

DESIGN AND PERFORMANCE

It is important that the center of mass of the assembly of mass M coincide with the geometrical center of the sphere to eliminate turning moments. Also neither the pickup nor the sphere should have spurious resonances in the operating frequency range. If the effective damping of the hydrophone is about 50 or 60% of critical, its response will be flat almost down to the natural frequency. However, if the lowest frequency of interest is several times the hydrophone's natural frequency, the amount of damping is not important.

A velocity hydrophone of this type does not have to be neutrally buoyant. Equation (11) shows that positive buoyancy increases the sensitivity, but the maximum increase is only a factor of two. Furthermore, any substantial increase in sensitivity would require a much larger sphere, which would limit the high-frequency response. Negative buoyancy on the other hand decreases the sensitivity quite drastically. Another consideration which may be of importance if the velocity hydrophone is to be used in close proximity to other measuring equipment is minimizing the distortion of the sound field caused by the presence of the sphere. As might be expected, neutral buoyancy is optimum in this regard and of course the sphere should be made as small as possible consistent with this requirement and the size and weight of the pickup. From the above considerations, it is clear that for normal use, neutral buoyancy represents about the best compromise, and both types of hydrophones constructed at NOL were so designed.

The SV-2 velocity hydrophone design was not concerned with operation above 1 or 2 kc, so a fairly large sphere diameter could be tolerated. As a result the magnetic structure of the pickup is rigidly attached to the case and a light coil is suspended in the air gap. The advantage of this design is that such pickups are readily available commercially. A brass sphere with a diameter of five inches is required to accommodate the pickup and maintain neutral buoyancy. From Fig. 2 it is seen that this should give an essentially flat response up to 3.5 kc. However, the useful high-frequency range is limited by the sensitive element. The case of the pickup unit has a resonance at about 1.5 kc. In addition, its design is such that the load resistance controls the damping. If the resistor is chosen small enough to give optimum low-frequency response the coil inductance causes the response to fall off 1 db at about 600 cps. The pickup has an undamped natural frequency of 14 cps, and approximately 50% of critical damping is obtained by the use of a 500-ohm load resistor.

The SV-2 was calibrated at the Naval Ordnance Laboratory's Acoustic Facility at Brighton, Maryland in 1955. Figure 4 shows the output voltage as a function of frequency when operated into a 500-ohm load. Below 500 cps, these results are within about 1 db of the sensitivity determined by calibrating the pickup on a mechanical vibrator. At low frequencies, the directivity pattern of a velocity hydrophone of this type follows almost exactly the cosine law which theory predicts. At higher frequencies the structures do not have sufficient mechanical rigidity, and hence there is an appreciable transverse response. Figure 5 shows two typical directivity patterns for the SV-2.

Some means must be provided for positioning the sphere in the water. Figure 6 shows how the hydrophone is attached to a metal frame by means of four springs which are sufficiently soft so that the natural frequency of the assembly is well below that of the pickup.

The design of the SV-1 was aimed at a somewhat higher frequency range which required a smaller diameter sphere. To achieve neutral buoyancy it was necessary to use a specially constructed pickup which had the coil as the driven element. A 2.5-inch-diameter sphere was made of 0.125-inch-thick aluminum alloy, to which the coil was firmly attached. The heavy magnetic structure is resiliently supported by rubber blocks and by a rubber spider which maintains concentric alignment of the air gap and coil. The natural fre-

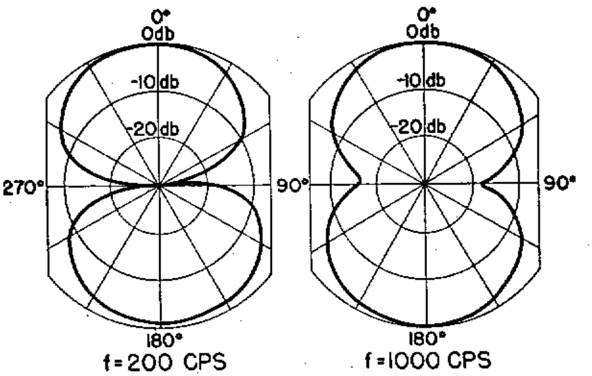

FIG. 5. Directivity patterns of an SV-2 velocity hydrophone at 200 and 1000 cps.

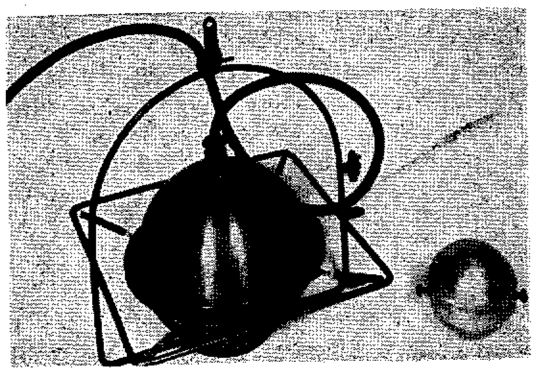

FIG. 6. Left: view of assembled SV-2 hydrophone showing method of suspending the sphere from the frame by four springs. Right: view of the SV-1 hydrophone without suspension.

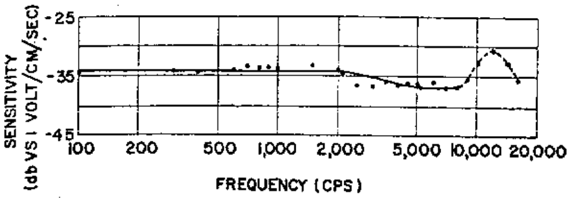

FIG. 7. Acoustic calibration of an SV-1 velocity hydrophone operating into an open circuit.

quency of the assembly (with the sphere held fixed) is about 10 cps. With this design, $M' \simeq 3M \simeq 3m$, so that from Eq. (16) the effective undamped natural frequency of the hydrophone is calculated to be approximately 17 cps. The damping results primarily from losses in the rubber and consequently is a small fraction of critical. The higher resonant frequency plus the small damping limits the hydrophone to frequencies not much lower than 70 cps. The high-frequency response, being limited only by the sphere size, is essentially flat past 7 kc and with a simple reliable correction is quite usable up to 10 kc when operated into a fairly high load resistance.

The sensitivity of the SV-1 was calculated from fluxmeter measurements of the flux cut during a small, known displacement of the coil. This hydrophone was also calibrated by the substitution method in 1942 at the OSRD facility in Orlando, Florida. Figure 7 shows the measured open-circuit sensitivity as a function of frequency. At low frequencies, these results and the calculated sensitivity agree within 1 db. Figure 8 presents directivity patterns measured at two different frequencies.

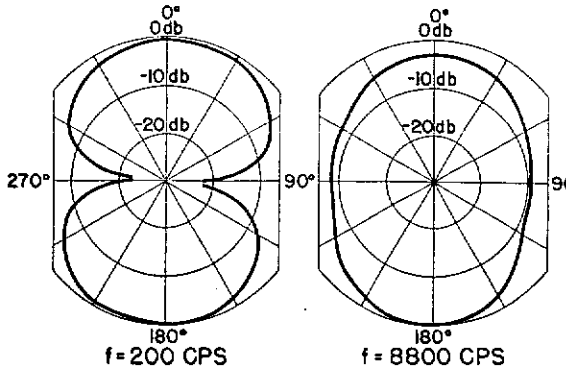

FIG. 8. Directivity patterns of an SV-1 velocity hydrophone at 200 and 8800 cps.

CONCLUSIONS

It has been shown both theoretically and experimentally that a simple, practical hydrophone which measures water particle velocity can be constructed by mounting a velocity pickup in a rigid, spherical housing. In addition to applications where a direct measurement of particle velocity is required, such a hydrophone has certain advantages for some acoustic measurements where pressure hydrophones are normally used. Its sensitivity can be accurately determined by calibrating the pickup, it has no significant change in sensitivity with temperature or static pressure, and it is quite directional at low frequencies without the large size of the usual directional pressure-hydrophone array. While it has a low sensitivity, it also has a low impedance and low self-noise level, so that measurements can be made down into the background water noise in most locations.