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ESTIMATION OF THE DIFFUSE RADIATION FRACTION FOR HOURLY, DAILY AND MONTHLY-AVERAGE GLOBAL RADIATION

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Abstract—Hourly pyrheliometer and pyranometer data from four U.S. locations are used to establish a relationship between the hourly diffuse fraction and the hourly clearness index k_T. This relationship is compared to the relationship established by Orgill and Hollands and to a set of data from Highett, Australia, and agreement is within a few percent in both cases. The transient simulation program TRNSYS is used to calculate the annual performance of solar energy systems using several correlations. For the systems investigated, the effect of simulating the random distribution of the hourly diffuse fraction is negligible. A seasonally dependent daily diffuse correlation is developed from the data, and this daily relationship is used to derive a correlation for the monthly-average diffuse fraction.

1. INTRODUCTION

Solar radiation incident on buildings or collection surfaces must be known in order to perform thermal analyses. In general, only measurements of the total horizontal (global) radiation are available. As most surfaces of interest are inclined, it is necessary to estimate the radiation on a tilted surface from measurements of global radiation. Estimation procedures usually require the beam and diffuse components of global radiation.

The beam and diffuse components of global radiation can be estimated from empirical relationships. Existing relationships correlate the fraction of the global radiation which is beam or diffuse to an index of atmospheric clarity. Correlations of this type have been developed for use with hourly, daily, and monthly-average values of global radiation.

The hourly correlations of Boes[1], Orgill and Hollands[2], and Bruno[3] can be expressed as relationships of I_d/I, the ratio of the hourly diffuse radiation to the hourly global radiation, to k_T, the ratio of the hourly global radiation to the hourly extraterrestrial radiation, I/I_0. Correlations of the hourly diffuse fraction, I_d/I, to k_c, the ratio of the hourly global radiation to an estimate of hourly "clear sky" radiation, I/I_c, have been developed by Bugler[4] and by Stauter and Klein[5], each with different definitions of "clear sky" radiation. The statistical algorithm developed by Randall and Whitson[6] cannot be expressed analytically; this algorithm was used[7] to estimate beam radiation for the SOLMET data base.

Relationships for estimating the beam and diffuse components of daily global radiation have also been developed by numerous authors. The correlations of Liu and Jordan[8], Choudhury[9], Stanhill[10], Tuller[11], Ruth and Chant[12], and Collares-Pereira and Rabl[13] all relate H_d/H, the daily diffuse fraction, to K_T, the ratio of daily global to daily extra-terrestrial radiation, H/H_0.

On a monthly-average basis, relationships between $\bar{H}_d/\bar{H}$ and $\bar{K}_T$ have been developed by Liu and Jordan[8], Page[14], Tuller[11], Collares-Pereira and Rabl[13], and Iqbal[15]. Hay[16] developed a correlation which includes the effect of multiple reflections between the ground and sky.

Within each group of existing correlations (i.e. the hourly, daily, and monthly-average relationships) there is considerable disagreement. The significance of this disagreement depends upon what the diffuse correlation is used for. If the annual total radiation on a tilted surface is estimated using each of the existing relationships, the results will generally be within a few percent. If a computer simulation is used to estimate the annual performance of a system with concentrating collectors, the results obtained using different correlations may vary by more than 10 per cent. In addition, there are inconsistencies between daily and monthly correlations. These discrepancies may be the result of variations in instrumentation and measurement techniques, different methods of correlating the data, locational dependence of the data, or insufficient data.

The objectives of this study are to develop, from a new data base, relationships for estimating the diffuse fraction of hourly, daily, and monthly-average global radiation, to determine the degree to which the relationships developed are dependent on season and location, and to compare these relationships to the existing relationships.

2. THE DATA BASE

The data used to develop the correlations presented here are thought to be among the best data available at this time. These data were recorded at the four U.S. cities listed in Table 1. The data for Livermore were recorded by Sandia; those for Raleigh by the Environmental Protection Agency; and the data from Fort Hood and Maynard were recorded by the Army Atmospheric Sciences Laboratory. The Aerospace Corporation edited the raw data, reduced them to the International Pyrheliometric Scale, and placed them on magnetic tape. Information included for each hour are direct normal radiation, total radiation, mean solar altitude, declination, date, and extraterrestrial radiation. The direct normal radiation was measured with a pyrheliometer and the

Table 1. Cities and duration of records for aerospace data base

Station Name	Fort Hood, TX	Livermore, CA	Raleigh, NC	Maynard, MA	Albuquerque, NM
Station Number	03902	32899	32900	-0042	23050
Latitude (Deg. N)	31.08	37.70	35.87	42.42	35.05
Longitude (Deg. W)	97.85	121.70	78.78	71.48	106.62
Altitude (ft)	1080	486	441	203	5314
Data Period					
Begin (Day/Mo/Yr)	1/9/74	1/8/74	20/3/75	1/1/75	1/1/61
End (Day/Mo/Yr)	30/6/76	30/10/75	1/4/76	31/12/76	31/12/64

total radiation was measured with a pyranometer. It should be noted that while the use of a pyrheliometer eliminates the needs for shade ring corrections, there can be problems with tracking systems and calibration.

To test the applicability of the correlation at locations other than the four from which it was derived, a set of data recorded in Highett, Victoria, Australia (latitude 38°S) during the years 1966–69 was obtained[17]. These data, measured using an unshaded pyranometer and a pyranometer with a shade ring, are also thought to be of high quality.

In addition, hourly data for a month (Feb. 1980) recorded in Albany, NY were obtained from the Atmospheric Sciences Research Center of the State University of New York[18]. These data are numerical integrations of minute values. Included in this data set are total insolation on a horizontal surface, direct normal insolation, diffuse insolation using a shadow band, diffuse insolation using an occulting disk, and the number of minutes of beam radiation during each hour. The presence of three independent measurements of the diffuse radiation made it possible to monitor the internal consistency of the diffuse data.

All of the data were checked for the following inconsistencies: zero global radiation after sunrise and before sunset, beam radiation exceeding global, global radiation exceeding extraterrestrial, and no beam radiation when k_T is large. Data exhibiting any of these problems (less than 1 per cent of the total number of hours) were deleted from the data set.

3. ESTIMATING THE DIFFUSE FRACTION OF HOURLY GLOBAL RADIATION

(a) *Correlating the diffuse fraction with* k_T

The diffuse fraction of the hourly total radiation is strongly correlated with k_T[8,2,3]. The parameter k_T is an indicator of the relative clearness of the atmosphere. In general, when the atmosphere is clearer, a smaller fraction of the radiation is scattered. A relationship was developed between I_d/I and k_T using the combined data of the four U.S. locations. The individual k_T and I_d/I values were weighted with the total radiation for the hour, and average values calculated for each interval in k_T of 0.025. The resulting correlation is shown with the average data in Fig. 1. It can be represented by:

$$
\begin{aligned}
I_d/I &= 1.0 - 0.09\,k_T \text{ for } k_T \le 0.22 \\
I_d/I &= 0.9511 - 0.1604\,k_T + 4.388\,k_T^2 - 16.638\,k_T^3 \\
&\quad + 12.336\,k_T^4 \text{ for } 0.22 < k_T \le 0.80 \\
I_d/I &= 0.165 \text{ for } k_T > 0.80.
\end{aligned} \tag{1}
$$

For values of k_T greater than 0.8, eqn (1) was not fit to the data. Following the procedure of Orgill and Hollands[2], a constant value of I_d/I was chosen for k_T in this range. Orgill and Hollands attribute the observed increase in the diffuse fraction as k_T increases from 0.8 to beam radiation being reflected from clouds and recorded as diffuse radiation during periods when the sun is unobscured by the surrounding clouds. The data in this region of k_T represent only 0.2 per cent of the points in the combined data set, and they are not understood well enough to justify fitting a curve to them.

The mean bias error ($\bar{d}$) and the standard deviation (σ) are used to indicate how closely the hourly correlation agrees with the data. The mean bias error is the weighted difference between the diffuse fractions estimated from a correlation and the measured diffuse fractions.

$$
\bar{d} = \frac{\sum^{N} ((I_{d,c}/I) - (I_{d,m}/I))I}{\sum^{N} I}. \tag{2}
$$

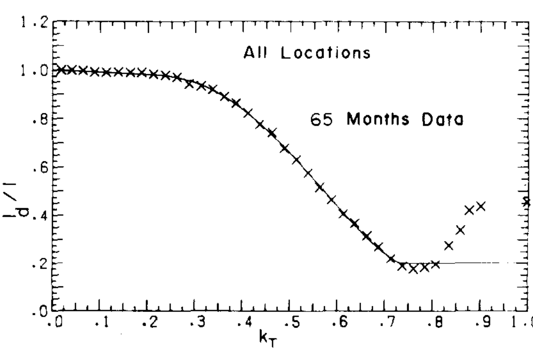

Fig. 1. Hourly correlation between I_d/I and k_T compared to average hourly U.S. data.

In eqn (2), $I_{d,m}$ is the measured diffuse radiation, I is the measured total radiation, and $I_{d,c}$ is the diffuse radiation estimated from the correlation. The standard deviation is an indication of how much the measured hourly diffuse fractions vary from the correlation. The standard deviation is defined by:

$$\sigma = \frac{\left(\sum^{N} (I_{d,m} - I_{d,c})^2\right)^{1/2}}{\left(\sum^{N} I\right)^{1/2}}. \tag{3}$$

In general, 95 per cent of the data lie within plus and minus 2σ of the correlation.

The mean bias error and standard deviation were calculated for the data from each of the four U.S. locations. Figure 2 shows the hourly correlation (solid line), average values of the data for each interval in k_T of 0.0125 (x's), and plus and minus one standard deviation of the hourly diffuse fractions from the correlation (dashed lines). The mean bias error exhibits only a slight locational dependence, and except for Livermore, the average data lie very close to the correlation for all values of k_T less than 0.8. The standard deviation is also relatively independent of location. However, the large size of the standard deviation indicates that there may be considerable error (roughly plus or minus two standard deviations) in estimating the diffuse fraction for any particular hour.

A comparison between the Highett data and eqn (1) is shown in Fig. 3. For values of k_T between 0.2 and 0.8 (which include 90 per cent of the data points), the average data for k_T intervals of 0.0125 (x's) are within 3 per cent of the correlation. The diffuse fraction of the Highett data approaches 0.85 as k_T approaches 0, which does not appear reasonable. A similar trend in the data from Raleigh, NC was traced to erroneous pyrheliometer readings. The nature of the Highett diffuse radiation measurements (shaded pyranometer) makes it impossible to determine whether erroneous data are present. If the diffuse fraction approached 1.0 as expected, the agreement between the Highett data and this correlation would be excellent.

Agreement between eqn (1) and the Highett data supports the contention that the correlation is location-independent. Further support can be drawn from a comparison between eqn (1) and the relationship developed by Orgill and Hollands. The two correlations, compared in Fig. 4, are within 4 per cent of each other for all values of k_T and within 2 per cent for values of k_T greater than 0.5. The nominal accuracy of the instruments is 5 per cent; thus, the agreement between the data recorded at Highett, Australia and Toronto, Canada and eqn (1) is within the uncertainty of the measurements.

The large standard deviation of the hourly data from eqn (1) is due, in part, to a seasonal variation in the average diffuse fraction. The U.S. data were grouped into four seasons, and the mean bias error and the standard deviation were found for each of the four locations; these results appear in Table 2. The correlation tends to overpredict the diffuse radiation in the fall and winter and underpredict in the spring and summer. Due to the large variation in the mean bias error for each of the seasons, only the overprediction in fall and winter is statistically significant.

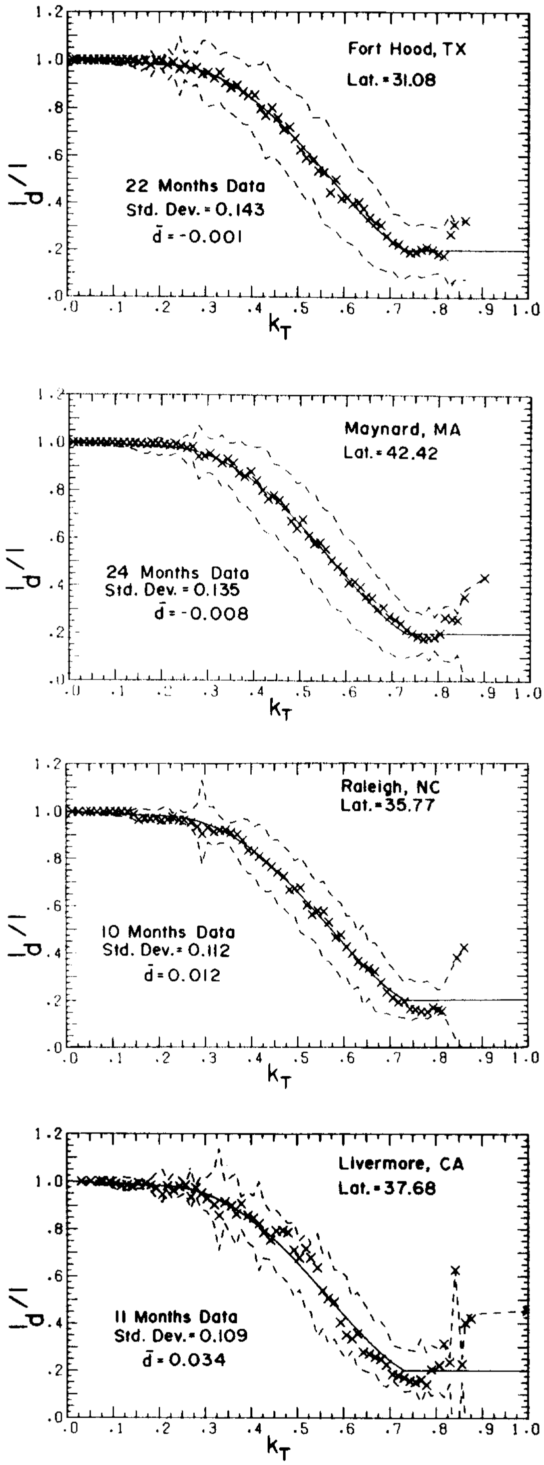

Fig. 2. Comparisons of the U.S. data with the modified k_T correlations.

(b) *Correlating the diffuse fraction with* k_c

The large standard deviation of the hourly diffuse data from the correlation between I_d/I and k_T led to an

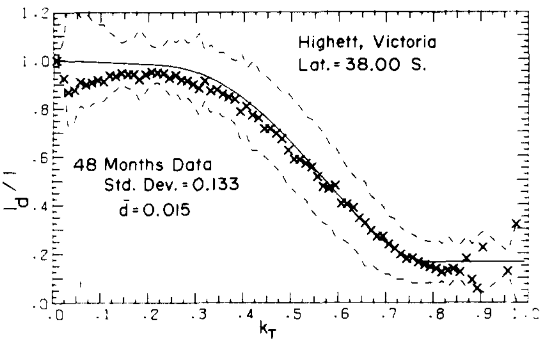

Fig. 3. A comparison of the hourly Highett data with the present correlation between I_d/I and k_T.

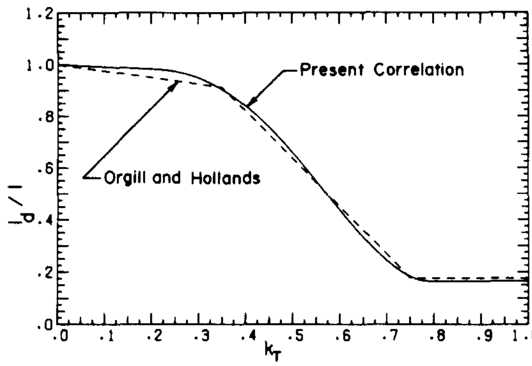

Fig. 4. A comparison of the present k_T correlation and the Orgill and Hollands correlation.

investigation of correlating I_d/I to k_c, the ratio of hourly global to hourly "clear sky" radiation, to determine if including air mass, season, and receiver altitude as parameters could reduce this standard deviation. The beam component of the "clear sky" radiation was obtained from a model developed by Hottel[19], while the diffuse "clear sky" radiation was calculated using a correlation of Liu and Jordan[8].

The standard deviations of the data from the correlation were only slightly smaller when k_c was used in place of k_T. There was no change in the mean bias errors on an annual basis, although the seasonal dependence of the diffuse fraction was reduced somewhat when the data were correlated with k_c instead of k_T. The use of k_c as the independent variable did not reduce the uncertainty of the estimated hourly diffuse fraction sufficiently to warrant the extra calculations required, when compared to the use of k_T.

(c) *The use of per cent possible sunshine data in a correlation*

The diffuse radiation fraction for an hour is strongly dependent on the type and distribution of clouds in the sky during the hour. Neither k_T nor k_c is a function of the per cent possible sunshine. Intermediate values of k_T (or k_c) can be the result of a thin, continuous cloud cover or a heavy, intermittent cloud cover. For a constant value of k_T, thin continuous clouds will result in a higher diffuse fraction than will heavy intermittent clouds.

One month of data from Albany, NY were used to develop a correlation which demonstrates the dependence of the hourly diffuse fraction on the nature of the cloud cover. The data were separated into three bins using the hourly per cent possible sunshine. The first bin, 0–20 per cent, corresponds to mostly cloudy conditions. The second, 21–80 per cent, corresponds to partly cloudy skies, while the third, 81–100 per cent, is the clear sky region. A curve relating the diffuse fraction to k_T was then constructed for each per cent possible sunshine bin.

The three curves are shown in Fig. 5 along with eqn (1). Despite the large uncertainty associated with the curves as a result of the small data base used, the dependence of I_d/I on per cent possible sunshine is very evident. At a value of k_T of 0.5, the difference between the average diffuse fractions of the first and third bins is 35 per cent. A correlation which includes the hourly per cent possible sunshine as a parameter could significantly reduce the standard deviation of the hourly diffuse fractions from the correlation.

(d) *The dependence of simulation results on hourly correlations*

The disagreements that exist among the correlations available for estimating the I_d/I lead to different estimates of the radiation incident on an inclined surface. It is of interest to know how large of an effect the choice of diffuse fraction correlation will have on calculated solar system performance. The correlations chosen for a comparison of simulated system performance are: (1) The Liu and Jordan daily K_T correlation (often used on an hourly basis); (2) eqn (1); (3) A statistical correlation based on eqn (1); and (4) The Aerospace model of Randall and Whitson.

The statistical correlation adds a diffuse fraction devi-

Table 2. Seasonal bias errors and standard deviations

Season		Winter	Spring	Summer	Fall
Location					
Fort Hood, TX	$\bar{d}$	.092	-.035	-.040	.042
	σ	.172	.140	.154	.140
Livermore, CA	$\bar{d}$	.051	-.019	.048	.045
	σ	.113	.116	.113	.105
Raleigh, NC	$\bar{d}$	.048	-.013	.000	.032
	σ	.130	.102	.099	.130
Maynard, MA	$\bar{d}$	.058	-.031	-.027	.055
	σ	.141	.131	.142	.142

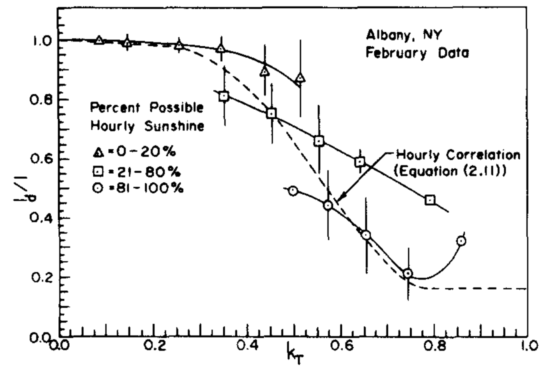

Fig. 5. The dependence of the hourly diffuse fraction on hourly fraction of possible sunshine.

ation to the diffuse fraction estimated from eqn (1). For the first hour in each day, this deviation is random and normally distributed, with the variance a function of k_T. For all subsequent hours in the day, the diffuse fraction deviation is found using the AR(1) model[20]:

$$d_j = \rho_1 d_{j-1} + a_j \tag{4}$$

where d_{j-1} is the deviation for the preceding hour. The residual a_j is random and normally distributed with a mean of 0 and variance σ_a^2. The values of ρ_1 and σ_a used (0.604 and 0.135) are radiation weighted averages of the values for the four U.S. locations.

SOLMET Typical Meteorological Year (TMY) data for Seattle–Tacoma, Washington were used in the simulations. Seattle is notable for low values of k_T in the winter and high values of k_T in the summer. (Earlier simulations using data from Madison, WI, Albuquerque, NM and Seattle, WA, indicated that the Seattle data produced the largest disagreement among correlations.)

TRNSYS[21] was used to model the performance of two different types of systems for a wide range of system parameter values. The difference between the annual solar fractions obtained using eqn (1) and those obtained using the other three correlations were calculated from the simulation results and used as the responses in a factorial design[22] for each system.

The solar fraction, $\mathcal{F}$, is defined here as one minus the fraction of the load met with auxiliary energy. A factorial design was used to determine the effect of changing certain system parameter values on the differences between the annual solar fractions. Several system parameters were designated design variables, and a low and high level chosen for each. All possible combinations of the design variables were simulated. The difference in a response resulting from changing a design variable from the low to the high level, referred to as a main effect, was found for each design variable and response. The effects of varying more than one design variable at a time, referred to as interactions, were also calculated, as were the average values of the responses for each of the systems.

The first series of simulations investigated five variables of a house heating system with a flat-plate collector. The five design variables, their low and high levels, and the annual solar fractions are given in Table 3. The main effects (i.e. the changes in the differences between the annual solar fractions resulting from changing the levels of the design variables) are given in Table 4; the interaction effects are not given, as all were less than 0.4 per cent solar fraction.

The results in Table 4 indicate that the only significant difference in annual solar fractions is between the Liu and Jordan correlation and eqn (1). For the range of system parameters investigated, the effect of adding a statistical variance to eqn (1) on the simulation results is

Table 3. Design variables and annual solar fractions for flat plate system

Level of Design Variable					Annual Solar Fraction			
Collector Azimuth	Collector Slope	Tank Volume / Collector Area	Collector Quality	Collector Area	Liu and Jordan	Equation [1]	Aerospace	Statistical Eq. [1]
0°	45°	35 l/m²	"Poor"	45 m²	.39	.37	.37	.37
45°	45°	35 l/m²	"Good"	45 m²	.52	.48	.49	.49
0°	90°	35 l/m²	"Good"	45 m²	.45	.41	.42	.43
45°	90°	35 l/m²	"Poor"	45 m²	.28	.26	.26	.26
0°	45°	150 l/m²	"Good"	45 m²	.60	.55	.56	.57
45°	45°	150 l/m²	"Poor"	45 m²	.40	.38	.38	.38
0°	90°	150 l/m²	"Poor"	45 m²	.36	.33	.33	.33
45°	90°	150 l/m²	"Good"	45 m²	.45	.41	.41	.42
0°	45°	35 l/m²	"Good"	90 m²	.72	.66	.67	.69
45°	45°	35 l/m²	"Poor"	90 m²	.49	.45	.46	.47
0°	90°	35 l/m²	"Poor"	90 m²	.44	.40	.41	.41
45°	90°	35 l/m²	"Good"	90 m²	.57	.51	.52	.54
0°	45°	150 l/m²	"Poor"	90 m²	.58	.54	.55	.55
45°	45°	150 l/m²	"Good"	90 m²	.69	.64	.66	.66
0°	90°	150 l/m²	"Good"	90 m²	.70	.63	.65	.65
45°	90°	150 l/m²	"Poor"	90 m²	.45	.41	.41	.42
Average for All Runs					.51	.46	.47	.48

Collector Type	F'	ε	α	KL	U_{be} (w/m² - °C)	Covers
"Poor"	.75	.95	.95	.06	1.4	single
"Good"	.95	.10	.95	.012	.28	single

UA_{HOUSE} = 333 w/m² - °C	U_{TANK} = .28 w/m² - °C	$\dot{M}_{COLLECTOR}$ = 60 kg/m² - hr

Table 4. Effect of design variables on differences in estimated solar fractions for flat plate system

Difference in Annual Solar Fraction			
	Main Effect of Design Variable		
Design Variable	(Liu and Jordan - Equation [1])	(Statistical Correlation -Equation [1])	(Aerospace -Equation [1])
Average	.043	.014	.007
Azimuth	-.005	-.003	-.004
Slope	.006	-.001	.000
Tank Volume Ratio	.006	.000	.001
Collector Quality	.017	.010	-.004
Collector Area	.017	.008	.006

negligible. The difference between the statistical Aerospace model of Randall and Whitson and eqn (1) (both developed from the same data base) is also negligible.

The second series of simulations investigated three variables in an industrial process heating system. The collectors are two-axis tracking linear concentrators with a concentration ratio of 15. The load consists of generating 100°C steam from 8:00 am to 12:00 pm, 5 days a week. Table 5 lists the design variables, their low and high levels (along with other system parameters), and the annual solar fractions. The main effects are presented in Table 6. Once again, all interactions were less than 0.4 per cent fraction by solar.

Based on the results in Table 6, the only significant difference in annual solar fractions is again between the Liu and Jordan correlation and eqn (1). Although larger than they were for the flat-plate system, the differences between the statistical version of eqn (1) and eqn (1) and between the Aerospace method and eqn (1) are still negligible. The concentrating collectors only utilize beam radiation, and as a result, they are much more sensitive to the split of global radiation into the beam and diffuse components. These comparisons show that a consideration of the statistical variation of the diffuse fraction, as in the Randall and Whitson algorithm, is unnecessary for estimating the long-term performance of solar energy systems.

4. ESTIMATION OF THE DIFFUSE FRACTION OF DAILY GLOBAL RADIATION

A correlation was developed between the daily diffuse fraction, H_d/H, and K_T using the hourly data from the four U.S. locations. It was shown earlier that the correlation between I_d/I and k_T is seasonally dependent. To determine whether the correlation between H_d/H and K_T was also dependent upon season, the daily data were grouped into seasonal bins. Following the procedure of Collares-Periera and Rabl[13], 3 bins were chosen:

$$\begin{aligned} &\text{Winter} && \omega_s < 1.4208 \\ &\text{Spring and Fall} && 1.4208 \le \omega_s \le 1.7208 \\ &\text{Summer} && \omega_s > 1.7208. \end{aligned} \tag{5}$$

Separating the data in this manner resulted in stronger seasonal trends than were obtained when the data were separated into seasons by month.

Only the correlation for winter was noticeably different from the other two seasonal correlations. For values of K_T greater than 0.45, the winter data have a smaller average diffuse fraction than the data for the remainder of the year. The air in winter is generally drier and less dusty than during other seasons, which tends to lower the diffuse fraction during the winter for large values of K_T. In addition, the lower solar altitude in the winter allows less of the scattered radiation to reach the

Table 5. Design variables and annual solar fractions for concentrating system

Level of Design Variable			Annual Solar Fraction			
Tank Volume / Collector Area	Tank Loss Coefficient	Collector Area	Liu and Jordan	Equation [1]	Aerospace	Statistical [1]
50 l/m^2	0.70 w/m^2-°C	40 m^2	.38	.31	.29	.31
200 l/m^2	0.70 w/m^2-°C	40 m^2	.31	.24	.23	.24
50 l/m^2	0.14 w/m^2-°C	40 m^2	.44	.35	.33	.35
200 l/m^2	0.14 w/m^2-°C	40 m^2	.43	.36	.35	.36
50 l/m^2	0.70 w/m^2-°C	80 m^2	.65	.56	.54	.58
200 l/m^2	0.70 w/m^2-°C	80 m^2	.58	.49	.46	.49
50 l/m^2	0.14 w/m^2-°C	80 m^2	.72	.64	.62	.65
200 l/m^2	0.14 w/m^2-°C	80 m^2	.75	.65	.63	.66
Average for All Runs			.53	.45	.43	.46

F_R = .85 $(\tau\alpha)_n$ = .75 Evacuated tubular receivers
Collector area mass flow rate = 50 kg/m² - hr *
Collector loss coefficient = .13 w/m² - °C *
Load = 16,670 W *per nominal aperature area

Table 6. Effect of design variables on differences in estimated solar fractions for concentrating system

	Difference in Annual Solar Fraction		
	Main Effect of Design Variable		
Design Variable	(Liu and Jordan -Equation [1])	(Statistical Correlation -Equation [1])	(Aerospace -Equation [1])
Average	.083	.008	-.016
Tank Volume Ratio	.006	-.004	-.004
Tank Loss Coefficient	.004	.003	.004
Collector Area	.018	.012	-.003

ground, and if clouds are present, the fraction of the hour during which there is beam radiation must be larger due to the lower transmittance of the clouds and the atmosphere.

Each of the seasonal correlations was fit with an equation. The equations obtained for the summer data and the combined fall and spring data were virtually the same. The seasonal correlations are represented by the following equation:

For $\omega_s < 1.4208$

$$H_d/H = 1.0 - 0.2727\, K_T + 2.4495\, K_T^2 - 11.9514\, K_T^3 + 9.3879\, K_T^4 \textit{ for } K_T < 0.715$$

$$H_d/H = 0.143 \textit{ for } K_T \geq 0.715.$$

For $\omega_s \geq 1.4208$

$$H_d/H = 1.0 + 0.2832\, K_T - 2.5557\, K_T^2 + 0.8448\, K_T^3 \text{ for } K_T < 0.722$$

$$H_d/H = 0.175 \text{ for } K_T \geq 0.722. \tag{6}$$

Equation (6) is shown with the average seasonal bin values in Fig. 6.

5. ESTIMATION OF THE DIFFUSE FRACTION OF MONTHLY-AVERAGE RADIATION

One method of developing a relationship between $\bar{H}_d/\bar{H}$ and $\bar{K}_T$ is to sum the daily values of H_d, H and H_0 for all days in each month and correlate the resulting values of $\bar{H}_d/\bar{H}$ and $\bar{K}_T$. However, it was shown by Liu and Jordan[8] that a relationship between $\bar{H}_d/\bar{H}$ and $\bar{K}_T$ can be obtained from the relationship between H_d/H and K_T if the long-term average distribution of K_T is known. There are several advantages to deriving the monthly-average correlation from the daily correlation. The

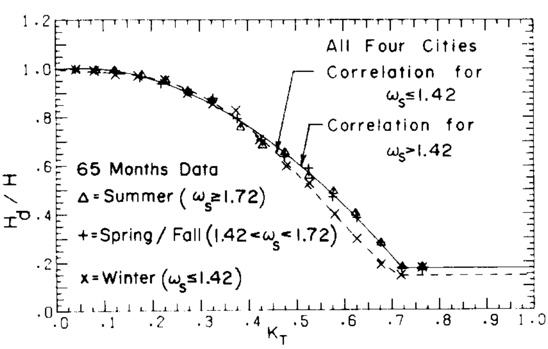

Fig. 6. Comparison of the seasomal U.S. data with the seasonal correlation between the daily diffuse fraction and K_T.

amount of data available for developing a daily correlation will be 30 times larger than that for the monthly-average data. The daily correlation will be more smoothly defined and there will be greater precision in fitting the daily correlation when compared to the monthly-average correlation developed from the data. In addition, the monthly-average correlation derived from the daily correlation should be more accurate over the long-term than a correlation of the monthly-average data because it reflects the long-term average radiation distribution. A final advantage is that a monthly-average correlation derived using eqn (7) covers the range of $\bar{K}_T$ from 0.3 to 0.7, even though measured values of $\bar{K}_T$ may not extend over this large a range.

The relationship between $\bar{H}_d/\bar{H}$ and H_d/H suggested by Liu and Jordan can be written as:

$$\frac{\bar{H}_d}{\bar{H}} = \frac{1}{N\bar{K}_T} \sum^{N} \frac{H_d}{H} K_T. \tag{7}$$

To use eqn (7), N values of H_d/H and K_T must be known for each value of $\bar{K}_T$, where N is a large number, sufficient to represent long-term average conditions. The values of H_d/H are obtained from a daily correlation between H_d/H and K_T. All that remains is to determine the distribution of K_T characteristic of the long-term average for each value of $\bar{K}_T$.

The definition of $\bar{K}_T$ is given by the equation:

$$\bar{K}_T = \int_0^1 K_T \, \mathrm{d}f \tag{8}$$

where f is the fraction of the time K_T was less than a particular value. Liu and Jordan found that for widely spread locations, cumulative distributions of K_T having the same value of $\bar{K}_T$ were remarkably similar. They used data from 27 locations to develop generalized cumulative frequency distributions of K_T for the 5 values of $\bar{K}_T$ between 0.3 and 0.7. Recently, Bendt *et al.*[23] used data from 90 locations, with 20 yr at each location, to develop generalized K_T distributions. Theilacker[24] has also recently developed generalized K_T distributions using 23 yr records for several locations. The agreement with the distributions of Liu and Jordan is generally very good.

The generalized K_T distributions of Liu and Jordan can be used to generate values of K_T for any value of $\bar{K}_T$. The procedure is to choose N values of f which are equally spaced between 0 and 1. The corresponding values of K_T are then found from the appropriate $\bar{K}_T$ curve. This allows eqn (7) to be evaluated for values of $\bar{K}_T$ between 0.3 and 0.7. The seasonal daily diffuse correlation, eqn (6), was used along with eqn (7) and the K_T distributions of Liu and Jordan as curve fit by Cole[25] to derive a seasonal monthly-average daily diffuse correlation. The following equation was fit to the correlation:

For $\omega_s \le 1.4208$ *and* $0.3 \le \bar{K}_T \le 0.8$

$$\bar{H}_d/\bar{H} = 1.391 - 3.560\,\bar{K}_T + 4.189\,\bar{K}_T^2 - 2.137\,\bar{K}_T^3$$

For $\omega_s > 1.4208$ and $0.3 \le \bar{K}_T \le 0.8$

$$\bar{H}_d/\bar{H} = 1.311 - 3.022\,\bar{K}_T + 3.427\,\bar{K}_T^2 - 1.821\,\bar{K}_T^3. \qquad (9)$$

The monthly-average data were grouped into 3 seasonal bins according to the monthly-average value of the sunset hour angle. The 3 bins are defined by eqn (5). Figure 7 is a comparison of the seasonal monthly-average correlation (eqn 9) with the combined United States and Highett data. There are no distinguishable seasonal trends in the Highett data, while the seasonal variation in the diffuse fraction of the U.S. data is larger than the seasonal variation of the correlation. The monthly-average diffuse fraction is more strongly dependent upon season than the daily diffuse fraction. One explanation for this, which is supported by Theilacker and by Bendt *et al.*, is that the generalized K_T distributions have more variation in K_T (They are "steeper") in winter and less variation in K_T ("flatter") in the summer, which would increase the dependence of the monthly-average data from that of the daily data.

For some locations, such as Highett, Australia, the monthly-average diffuse fraction does not exhibit seasonal dependence. The proximity of Highett to the ocean may tend to damp seasonal variations in the moisture and dust content of the air and in the distribution of cloud cover. The hourly data for the four U.S. locations were also used to develop a nonseasonal daily diffuse correlation from which the following nonseasonal monthly-average daily diffuse correlation was derived:

For $0.3 \le \bar{K}_T \le 0.8$

$$\bar{H}_d/\bar{H} = 1.317 - 3.023\,\bar{K}_T + 3.372\,\bar{K}_T - 1.769\,\bar{K}_T^3. \qquad (10)$$

Equation (10) compares with a mean bias error of 0.002 to the monthly-average data for Highett and the four U.S. locations.

The monthly-average correlations of Liu and Jordan, Page, Collares–Periera and Rabl, Hay, and eqn (9) were compared to the data from the 4 U.S. locations and from Highett. The relationship developed by Hay requires monthly-average values of ground reflectance; a value of 0.2 was used for all months and locations. The standard deviations and mean bias errors from each correlation were calculated for each location. The results are

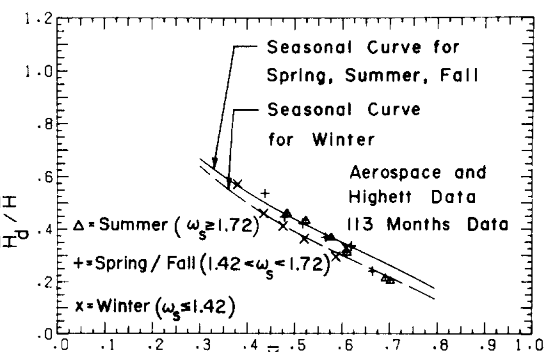

Fig. 7. Comparisons between the derived seasonally dependent correlation and the seasonal monthly-average data.

Table 7. Comparison of existing monthly-average correlations with present U.S. and Highett data

Location		L + J	Page	C - P + Rabl	Hay (1)	Eq. [8]
Fort Hood, TX	$\bar{d}$	-0.041	0.007	0.017	-0.011	0.011
	σ	0.079	0.078	0.048	0.068	0.058
Livermore, CA	$\bar{d}$	-0.018	-0.008	0.076	0.032	0.032
	σ	0.058	0.037	0.109	0.072	0.061
Raleigh, NC	$\bar{d}$	-0.070	-0.026	-0.014	-0.043	-0.026
	σ	0.085	0.048	0.042	0.065	0.044
Maynard, MA	$\bar{d}$	-0.069	-0.003	-0.000	-0.041	-0.011
	σ	0.086	0.058	0.033	0.068	0.046
Combined U.S.	$\bar{d}$	-0.051	-0.003	0.016	-0.019	0.002
	σ	0.080	0.063	0.058	0.068	0.053
Highett Australia	$\bar{d}$	-0.049	0.002	0.024	-0.021	0.007
	σ	0.067	0.040	0.071	0.050	0.048
All Data Combined	$\bar{d}$	-0.050	-0.001	0.019	-0.036	0.004
	σ	0.074	0.054	0.064	0.061	0.051

presented in Table 7. The Liu and Jordan correlation and eqn (9) were derived from daily correlations, while the other relationships were developed using monthly-average data.

6. CONCLUSIONS

The correlation developed between the hourly diffuse fraction and k_T was found to be essentially the same as the relationship previously developed by Orgill and Hollands [2], although different data were used in each case. Data recorded in Highett, Australia were also found to agree to within a few per cent with the hourly relationship presented. While the uncertainty in the estimated diffuse fraction for an hour is significant, the correlation predicts the long-term average hourly diffuse fraction accurately. For the systems investigated, the importance of simulating the distribution of hourly diffuse fractions about the long-term average in computer simulations is minor. Only the long-term relationship between I_d/I and k_T, and not the random nature of I_d/I, appears to be important. For simulations involving a flat-plate collector system, the results obtained with different correlations were generally within 5 per cent of each other, but for a concentrating collector system, the Liu and Jordan daily diffuse correlation resulted in significantly higher estimates of system performance. The seasonal monthly-average daily diffuse correlation given by eqn (9), which was derived from a seasonal daily diffuse correlation, agrees closely with the monthly-average U.S. and Highett data. The monthly-average correlations of Collares-Pereira and Rabl and of Page agree with the U.S. and Highett data nearly as well as the monthly-average correlations presented.

NOMENCLATURE

- a diffuse fraction residual
- d bias error
- $\bar{d}$ mean bias error
- f cumulative fraction of occurrence
- $\mathcal{F}$ annual solar fraction
- F' collector efficiency factor
- F_R collector heat removal factor
- H daily total radiation incident on a horizontal surface
- H_d daily diffuse radiation incident on a horizontal surface
- H_0 daily extraterrestrial radiation incident on a horizontal surface
- $\bar{H}$ monthly-average daily total radiation incident on a horizontal surface
- $\bar{H}_d$ monthly-average daily diffuse radiation incident on a horizontal surface
- $\bar{H}_0$ monthly-average daily extraterrestrial radiation incident on a horizontal surface
- I hourly total radiation incident on a horizontal surface
- I_c hourly "clear sky" total radiation incident on a horizontal surface
- I_d hourly diffuse radiation incident on a horizontal surface
- I_0 hourly extraterrestrial radiation incident on a horizontal surface
- K extinction coefficient
- k_c hourly clearness index (ratio of I to I_c)
- k_T hourly clearness index (ratio of I to I_0)
- K_T daily clearness index (ratio of H to H_0)
- $\bar{K}_T$ monthly-average daily clearness index (ratio of $\bar{H}$ to $\bar{H}_0$)
- L length, thickness
- N number of days
- U loss coefficient
- α absorptance
- ϵ emittance
- ρ coefficient of autocorrelation
- σ standard deviation
- τ transmittance
- ω_s sunset hour angle

Subscripts

- b back
- c calculated
- e edge
- m measured
- n normal

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