will be discussed, while the former one will be treated in the next chapter.

In this chapter, we will extend the coupled mode theory, especially the phase matching concept to the quasiperiodic optic superlattice. We will use them to study the transmission spectrum generated in the quasiperiodic optic superlattice by the electrooptic effect.

§5—1. Coupled mode theory for the FOS

A. General case

The conventional Fibonacci superlattice is constructed according to the concatenation rule $S_j=S_{j-1}|S_{j-2}$ for $j>3$, with $S_1=A$ and $S_2=AB$. A and B are the so-called building blocks, each composed of two layers of different constituent materials. According to Merlin et al.[2], the thicknesses of the first layers of these two blocks are the same(let it be l)and the thicknesses of the second layers are differently chosen such that $l+l_{A2}=\tau(l+l_{B2})$, $\tau=(\sqrt{5}+1)/2$. Here l_{A2} and l_{B2} are the thicknesses of the second layers of blocks A and B, respectively. Hereafter we will assume that the blocks A and B are arranged along the x axis.

The coupled mode theory for the periodic structure has already been developed[1]. Its main idea is that the periodic variation of the dielectric tensor is considered as a perturbation that couples the unperturbed normal modes of the structure. In the theory, the dielectric tensor as a function of space is written as

$$\varepsilon(x,y,z)=\varepsilon_0(y,z)+\Delta\varepsilon(x,y,z), \qquad (5—1)$$

where $\varepsilon_0(y,z)$ is the unperturbed part of the dielectric tensor, and $\Delta\varepsilon(x,y,z)$ is periodic in the x direction and is the only periodically varying part of the dielectric tensor.

In the Fibonacci superlattice, Eq.(5—1) still holds except that $\Delta\varepsilon(x,y,z)$ is no longer periodic but quasiperiodic in the x direction. In the following we will assume

$$\Delta\varepsilon(x,y,z)=\Delta\varepsilon(y,z)f(x), \tag{5—2}$$

where
$$f(x)=\begin{cases}+1, \text{if } x \text{ is in the first layer of blocks,}\\ -1, \text{if } x \text{ is in the second layer of blocks.}\end{cases}$$

The following derivation is much the same as the derivation used for the periodic superlattice[1]. The only difference is that in the periodic superlattice f(x) is expanded as a Fourier series, whereas in the Fibonacci superlattice, it must be expanded as a Fourier integral. By the use of the direct or the projection method,[3,4] f(x) can be written in the form

$$f(x)=\int f(k)e^{-ikx}dk=\sum_{m,n} A_{mn}e^{-iG_{m,n}x}, \tag{5—3}$$

with

$$A_{mn} \propto e^{i(\frac{1}{2}G_{m,n}l-X_{m,n})}\frac{\sin\frac{1}{2}G_{m,n}l}{\frac{1}{2}G_{m,n}}\frac{\sin X_{m,n}}{X_{m,n}}. \tag{5—4}$$

Starting from Maxwell's equations and using the parabolic approximation[1] gives

$$\frac{dA_r}{dx}=-i\frac{\beta_r}{|\beta_r|}\sum_t \sum_{m,n} C_{rt}^{(m,n)} A_t e^{i(\beta_r-\beta_t-G_{m,n})x}, \qquad (5\text{—}5)$$

with

$$\overrightarrow{E}=\sum_m A_m(x)E_m(y,z)e^{i(\omega t-\beta_m x)}, \qquad (5\text{—}6)$$

where A_r, A_t are the mode amplitudes, β_r, β_t are the wave vectors, and

$$C_{rt}^{(m,n)}=\frac{\omega}{4}\langle r|\varepsilon_{m,n}|t\rangle=\frac{\omega}{4}\int \overrightarrow{E_r^*}(y,z)\ \varepsilon_{m,n}\overrightarrow{E_t}(y,z)dydz, \qquad (5\text{—}7)$$

$$\varepsilon_{m,n}=\Delta\varepsilon(y,z)A_{m,n}\ . \qquad (5\text{—}8)$$

Here the coefficient $C_{rt}^{(m,n)}$ reflects the magnitude of coupling between the rth and tth modes due to the (m, n)th Fourier component of the dielectric perturbation. So we call it a coupling coefficient. The larger the $C_{rt}^{(m,n)}$ is, the stronger the coupling is.

Eq.(5—5) constitutes a set of coupled linear differential equations. In principle, an infinite number of mode amplitudes are involved. However, in practice, especially near the condition of resonant coupling, only two modes are strongly coupled and Eq.(5—5) reduces to two equations for the two mode amplitudes. We designate the two coupled modes as 2 and 3. Neglecting interaction with any of the other modes, we get

$$\frac{dA_2}{dx} = -i\frac{\beta_2}{|\beta_2|} C_{23}^{(m,n)} A_3 e^{i\Delta\beta x} ,$$

$$\frac{dA_3}{dx} = -i\frac{\beta_3}{|\beta_3|} C_{32}^{(-m,-n)} A_2 e^{-i\Delta\beta x} , \qquad (5\text{—}9)$$

with

$$\Delta\beta = \beta_2 - \beta_3 - G_{m,n} . \qquad (5\text{—}10)$$

If the dielectric tensor $\varepsilon(x,y,z)$ of Eq.(5—1) is a function of x only, the normal modes of the unperturbed medium are plane waves and the $\varepsilon_{m,n}$ is constant. The coupling coefficients $C_{rt}^{(m,n)}$ for this case become

$$C_{rt}^{(m,n)} = \frac{\omega^2 \mu}{2\sqrt{|\beta_r \beta_t|}} \overrightarrow{P_r^*} \, \varepsilon_{m,n} \overrightarrow{P_t} , \qquad (5\text{—}11)$$

where μ is the permeability, $\overrightarrow{P_r}$ and $\overrightarrow{P_t}$ are unit polarization vectors of the plane waves.

B.Special case

The theory just developed above will be applied to a more special case, an FOS. Its schematic diagram is shown in Fig.1—3.

In order to make the normal modes coupled together under the action of an electric field, it is necessary that the electrodes be on the y surfaces or the x surfaces of the

FOS[1]. Here we choose the y surfaces to be the electrodes and the x axis to be the propagating direction of the light beams.

The geometrical arrangement of our system is shown in Fig.5—1. The polarizer has its transmission axis parallel to the y axis, and the analyzer parallel to the z axis. In the absence of an electric field, the FOS is homogeneous to the propagation of light and the direction of its principal axes are along the x, y, z axes, respectively(see Fig.5—1). The dielectric tensor in the principal coordinate of the FOS

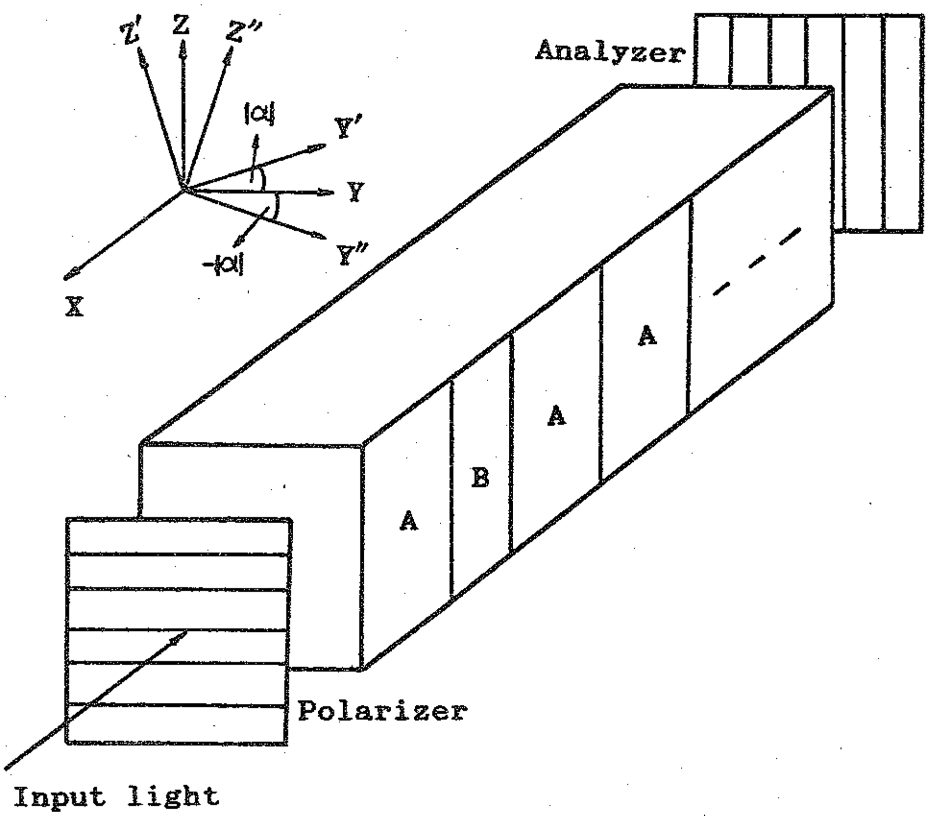

Fig.5—1.Geometrical configuration of the Fibonacci optic superlattice. X, Y, Z denote the principal axes of unperturbed dielectric tensor and X, Y', Z'(X,Y'',Z'') the

principal axes of perturbed dielectric tensor.

is

$$\tilde{\varepsilon}_0=\varepsilon_0\begin{pmatrix} n_o^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_e^2 \end{pmatrix}, \qquad (5—12)$$

where ε_0 is the dielectric constant of the vacuum.

In the presence of an electric field, because of the electrooptic effect, the FOS becomes inhomogeneous to the propagation of light. If the magnitude of the field is moderate(say about 10^6 v/cm), the change of the dielectric tensor is very small[1] and the modulation can be taken for a perturbation. Using the fact that the perturbed terms are much smaller than the unperturbed ones, we get

$$\varepsilon=\tilde{\varepsilon}_0+\Delta\varepsilon, \qquad (5—13)$$

with

$$\Delta\varepsilon=-\varepsilon_0 r_{42}E_2 n_o^2 n_e^2\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} f(x), \qquad (5—14)$$

Here $\Delta\varepsilon$ can be treated as a small dielectric perturbation.

At the same time, we get the rotation angle between the perturbed principal axes,(Y',Z' or Y'',Z'') due to the electrooptic effect and the unperturbed principal axes(Y,Z)(see Fig.5—1)

$$\tan 2\alpha=\frac{2r_{42}E_2 f(x)}{(n_o^2)^{-1}-(n_e^2)^{-1}} . \qquad (5—15)$$

Obviously, the angle rocks back and forth from $+|\alpha|$ to $-|\alpha|$ along the x direction from one domain to the next. In this way the function of the FOS is similar to a Solc filter with quasiperiodicity.[5]

Because $\Delta\varepsilon$ is a Hermitian dielectric tensor, it is easy to prove from Eq.(5—5) that

$$C_{23}^{(m,n)}=(C_{32}^{(-m,-n)})^{*}. \qquad (5—16)$$

According to the geometrical arrangement of our system, we have

$$\vec{P}_r=\vec{P}_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$\vec{P}_t=\vec{P}_3=\begin{pmatrix}0\\0\\1\end{pmatrix}. \qquad (5—17)$$

Therefore when $l_A=\tau l_B$, we have

$$K=C_{23}^{(m,n)}=-\frac{1}{2}\,\frac{\omega}{c}\,\frac{n_o^2 n_e^2}{\sqrt{n_o n_e}}\,r_{42}E_2A_{mn}, \qquad (5—18)$$

where $G_{m,n}=2\pi(m+n\tau)/D=\pi(m+n\tau)/((1+\tau)l)$.

Since the coupled modes are propagating in the same direction, the sign factors $\beta_2/|\beta_2|$ and $\beta_3/|\beta_3|$ are both equal to 1. Thus Eqs.(5—9) become

$$\frac{dA_2}{dx}=-iKA_3e^{i\Delta\beta x},$$

$$\frac{dA_3}{dx}=-iK^*A_2e^{-i\Delta\beta x}. \quad (5\text{—}19)$$

The initial condition at x=0 which is determined by the polarizer is given by

$$A_2(0)=1,$$

$$A_3(0)=0. \quad (5\text{—}20)$$

The solution of the coupled equations then is

$$A_2(x)=e^{i\frac{1}{2}\Delta\beta x}\left[\cos(sx)-i\frac{\Delta\beta}{2s}\sin(sx)\right],$$

$$A_3(x)=e^{-i\frac{1}{2}\Delta\beta x}(-iK^*)\frac{\sin(sx)}{s}, \quad (5\text{—}21)$$

where s is given by

$$s^2=K^*K+(\Delta\beta/2)^2. \quad (5\text{—}22)$$

At the analyzer (z-polarized) x=L(which is directly related to the block number N), A_2 is extinguished. The transmission for the z-polarized light is thus given by

$$T=|K|^2\frac{\sin^2 sL}{s^2}. \qquad (5—23)$$

This is the same as the well known pendellosung or pendulum solution of Ewald[6]. The energy is passed back and forward between the incident and diffracted beams as they travel in the FOS. The coupling is provided in this case by the scattering from one beam to the other due to the existence of an perturbed dielectric tensor.

In the following we will discuss some interesting phenomena in the FOS. All the calculations have been made by the use of Jones matrix method[1].

§5—2.Phase matching concept and dynamical effect

When one discusses interaction among waves in a dielectric medium, one often meets with the phase matching concept. The phase matching plays an important role in many processes such as the nonlinear optic, electrooptic and acoustooptic processes. If the phase matching condition is satisfied, the energy conversion can be complete; if not, the conversion will be inefficient. In homogeneous dielectric media and periodic dielectric media, such concept has already been established[1,7]. In recent years, researches on quasiperiodic superlattice have been made much progress. But to our knowledge, there is no such concept proposed for quasiperiodic superlattices up to now.

From Eq.(5—23), we find that for significant mode coupling to take place between modes 2 and 3, three conditions must be fulfilled. The first is a kinematic condition which is

$$\Delta\beta=0. \qquad (5\text{—}24)$$

Eq.(5—24) will be referred to as phase matching condition. This condition is a counterpart of the one for periodic structures[1]. In both cases, the reciprocal vector plays an important role. It is the reciprocal vector that compensates the birefringence and makes the energy coupling process proceed efficiently.

We have calculated the dependence of transmission on the block number N of the FOS for some $G_{m,n}$. In the calculations, l is kept constant. We find that for some wavelength, there exists a corresponding reciprocal vector $G_{m,n}$ by which the phase matching condition is satisfied. For example, for $\lambda=0.8000\mu m$ and $\lambda=0.5478\mu m$, the corresponding reciprocal vectors are $G_{1,1}$ and $G_{1,2}$, respectively, which make the phase matching condition satisfied. Figs.5—2 show the results. For comparison, the transmission of the y-polarized light is also shown. It can be seen that when Eq.(5—24) is valid, the energy conversion can be complete. In Fig.5—3, a wavelength is chosen such that the phase matching condition is not satisfied, clearly the energy conversion is very low. Alternatively, if λ is kept constant, we also find that for some value of l, there exists a corresponding reciprocal vector $G_{m,n}$ by which the phase matching condition is satisfied. The results are the same. Here we also present the results for the periodic

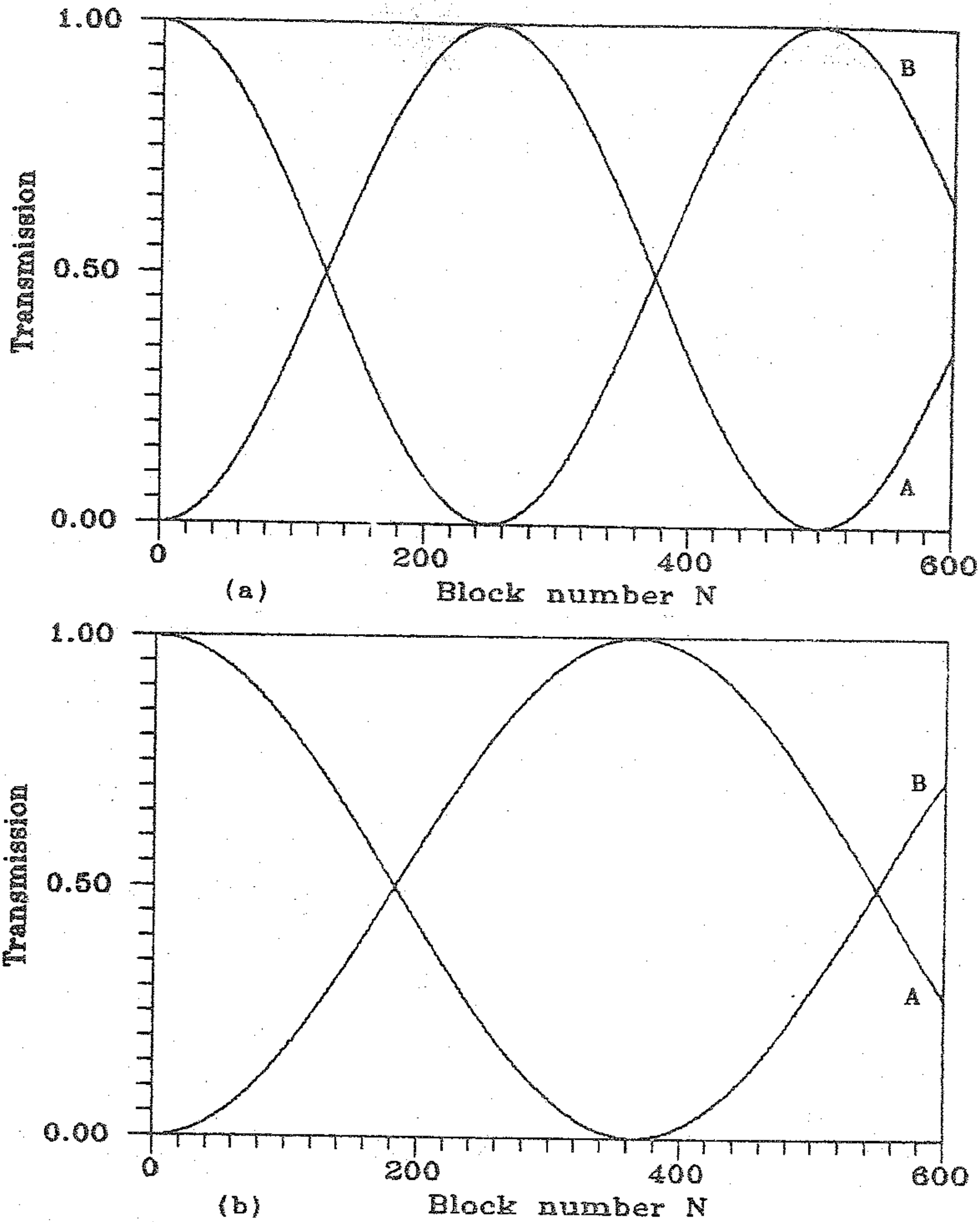

Fig.5—2.Dependence of the transmission of the z-polarized light on the block number N with phase matched and l=5.0441μm(curves A). For comparison, the transmission of the y-polarized light is also shown(curves B).(a)λ=0.8000μm, $\beta_2-\beta_3-G_{1,1}=0$;(b)$\lambda$=0.5478$\mu$m, $\beta_2-\beta_3-G_{1,2}=0$.

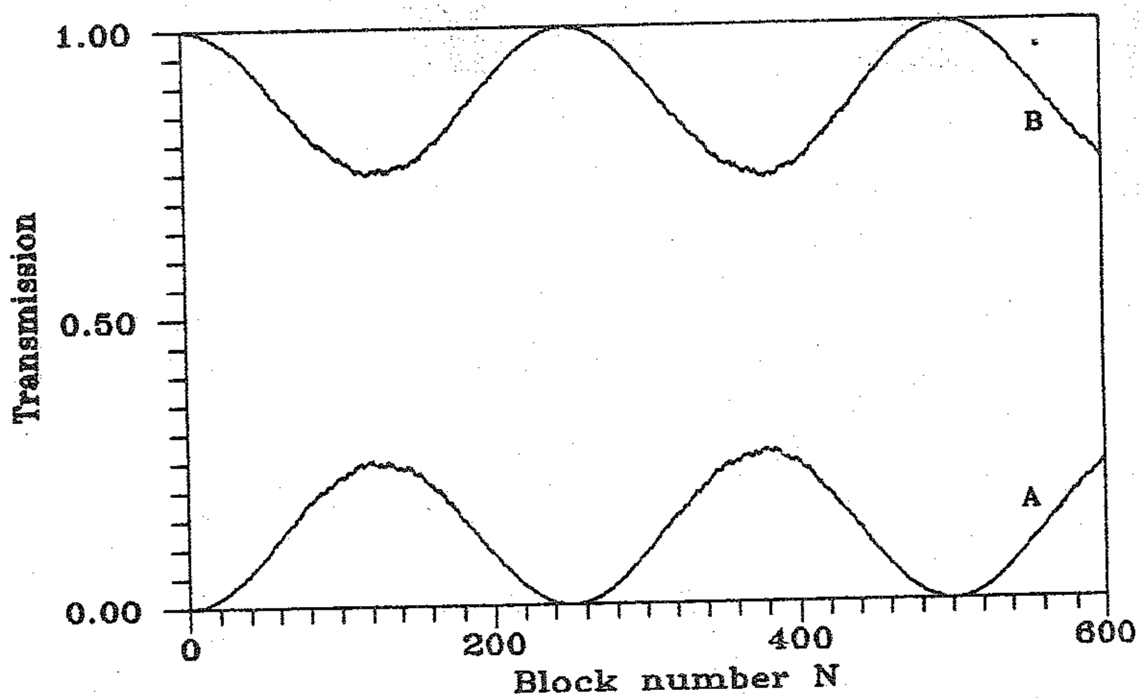

Fig.5—3.Dependence of the transmission of the z-polarized light on the block number N with phase unmatched: l=5.0441μm, λ=0.798μm(curve A). For comparison, the transmission of the y-polarized light is also shown(curve B).

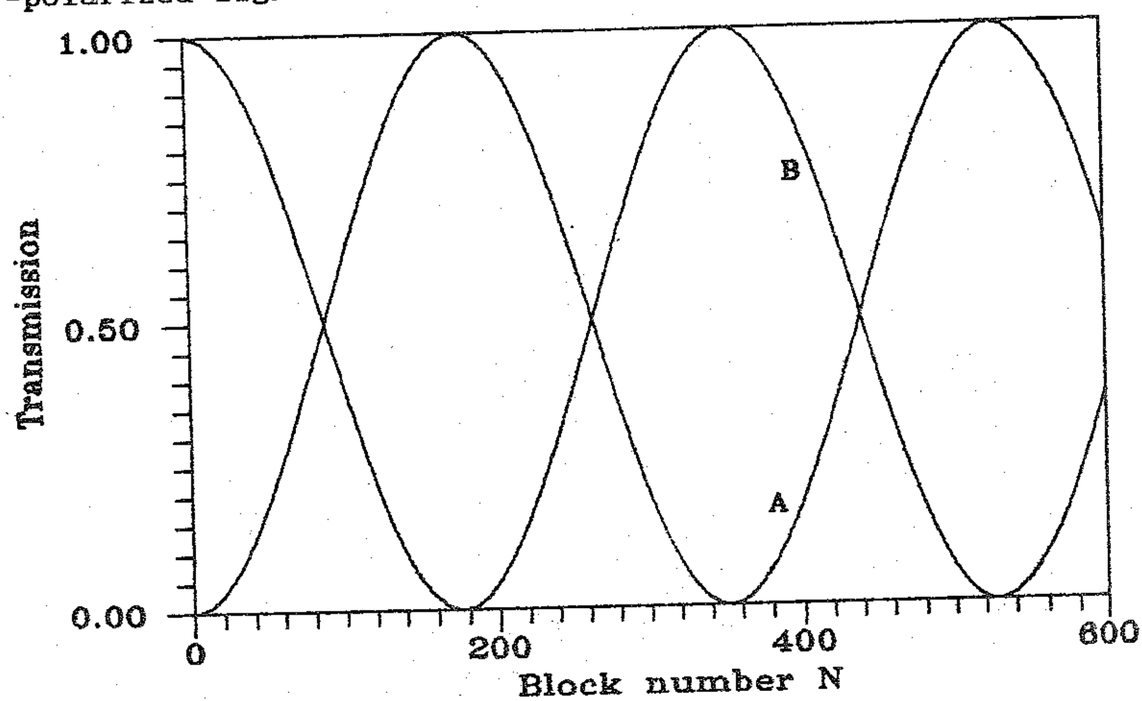

Fig.5—4.Dependence of the transmission of the z-polarized

light on the block number N for periodic structure with phase matched: l=8.1614μm, λ=1.224μm, $\beta_2-\beta_3-G_1=0$(curve A). For comparison, the transmission of the y-polarized light is also shown(curve B).

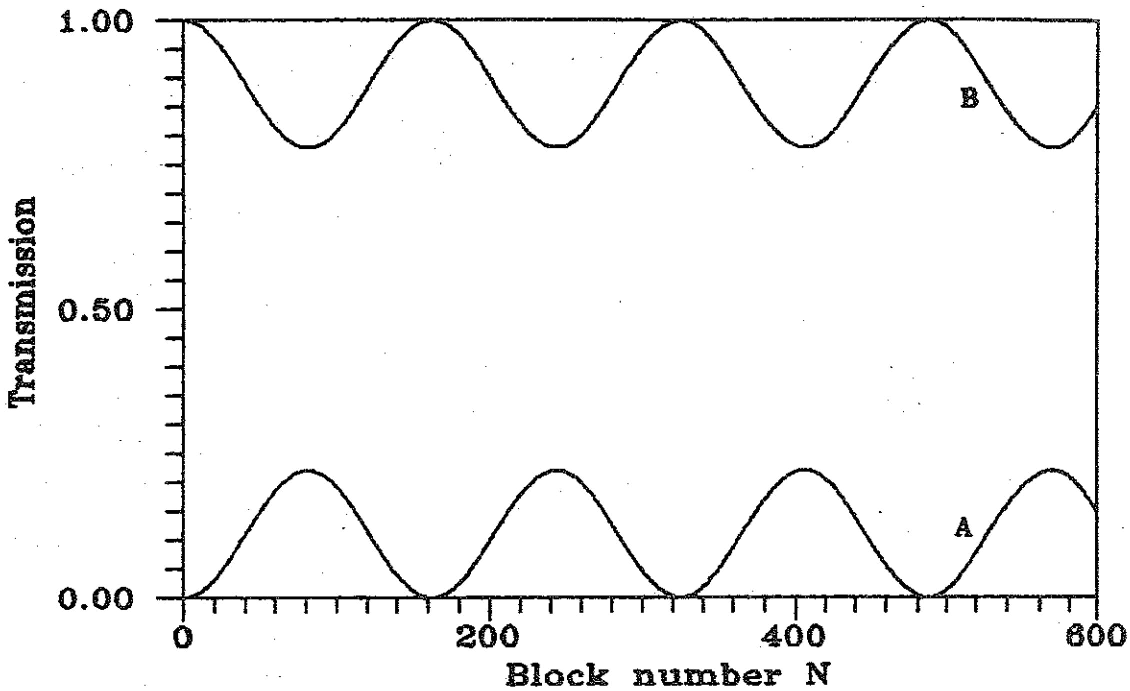

Fig.5—5.Dependence of the transmission of the z-polarized light on the block number N for periodic structure with phase unmatched: l=8.1614μm, λ=1.218μm(curve A).For comparison, the transmission of the y-polarized light is also shown(curve B).

structure calculated numerically. Fig.5—4 shows the dependence of transmission on the block number with phase matched, while Fig.5—5 is for the phase unmatched. Clearly, the main features of quasiperiodic structure bear a strong resemblance to those of periodic structure.

The second is

$$|K|L=(2u+1)\pi/2, u=0,1,2,\ldots, \tag{5—25}$$

which is a dynamical condition. If m and n are fixed, then K is a constant. Eq.(5—25) thus indicates that for some value of L, i.e., for some value of N, the block number, the transmission will be at its maximum. And for some other value of N, the transmission will be zero if

$$|K|L=\nu\pi, \nu=1,2,3,\ldots, \tag{5—26}$$

is satisfied. It is this dynamical effect that makes the transmission oscillate almost sinusoidally with the block number N, which can be seen in Fig.5—2. Analogous to x-ray diffraction and electron diffraction,[6,8] here $|K|^{-1}$ can be defined as the extinction distance.

The third is that K must not vanish. If it does, then the transmission will be zero. We call it the extinction phenomenon which will be discussed in §5—3. Here we will point out that the extinction phenomenon is totally determined by its structure parameters not by the block number N. Whereas the extinction distance is determined by N and is a dynamical effect. These two different phenomena should not be confused.

§5—3. Transmission spectrum

$\Delta\beta$ in Eq.(5—23) is of much significance not only because it determines the degree of energy conversion, but also because it determines the characteristics of the transmission spectrum. Let us rewrite Eq.(5—24) in its explicit form

$$\frac{1}{\lambda}(n_o-n_e)=\frac{m+n\tau}{2(1+\tau)l},$$

or

$$\left(\frac{1}{\lambda}\right)_{m,n}=\frac{m+n\tau}{2(1+\tau)(n_o-n_e)}. \qquad (5—27)$$

Here n_o and n_e are the refractive indices of the ordinary and extraordinary lights, respectively. Eq.(5—27) is much similar to the one obtained for the second harmonic generation in an FOS.[9] Both come from the energy coupling between waves and in both cases the dispersion of the refractive indices must be taken into account. So the discussion will be to some extent parallel to that of second harmonic generation.

In order to gain an insight into the main features of the spectrum, numerical calculations have been made which are valid only under room temperature. Because of lack of sufficient data, the dispersion of the electrooptic coefficient is not considered. This will only affect the peak heights and do not affect the peak positions as can be seen in Eqs.(5—23) and (5—24).

According to the discussion in §5—2, for some values of N, the block number, some peaks will be missing due to $|K|L=\nu\pi$, ν is an integer. Therefore in the calculations, we have avoided these values of N. Below only the results with N=200 have been presented.

There are two situations which can be considered here. The first one is to keep the wavelength constant and change the structure parameter l, thus change the magnitude of reciprocal vectors. Therefore, the dispersion of the refractive indices of n_o and n_e has no effect on the spectrum. Eq.(5—27) can be rewritten as

$$l_{m,n}=\frac{(m+n\tau)\lambda}{2(1+\tau)(n_o-n_e)}. \qquad (5—28)$$

For those peaks with n, m being successive Fibonacci numbers, Eq.(5—28) becomes

$$l(1,p)=\frac{\lambda}{2(1+\tau)(n_o-n_e)}\tau^p, \qquad (5—29)$$

here p is an integer.

Fig.5—6 shows the dependence of the transmission on the structure parameter l with the input light of $\lambda=0.8000\mu m$ under the condition $l_A=\tau l_B$(η=0.34). We note here the relation $l(1,p+1)=l(1,p)+l(1,p-1)$ holds. This shows that the transmission spectrum reflects the structural self-similarity of the FOS. More generally, we have calculated the transmission under conditions $l_A\neq\tau l_B$. Results show the peak positions remaining the same, except for their heights, which conform to the literature[2,9].

The second one is to keep the structure parameter l unchanged and vary the wavelength of the input light. In this case, the dispersive effect of the refractive indices on the transmission spectrum must be taken into account.

Eq.(5—27) can be rewritten as

$$\left(\frac{1}{\lambda}\right)_{m,n}=\frac{m+n\tau}{2(n_o(\lambda)-n_e(\lambda))(1+\tau)l} \quad (5—30)$$

Here $n_o(\lambda)$ and $n_e(\lambda)$ are functions of λ[10].

Fig.5—7 shows the dependence of the transmission on the wavelength with $l=l_c=\pi/\Delta\beta_0$. Here $\Delta\beta_0=\beta_2-\beta_3=2\pi(n_o-n_e)/\lambda_0$ with $\lambda_0=0.8000\mu m$. Though, compared with Fig.5—6, the structure of the spectrum seems to be the same, the positions of the peaks shift a lot due to the dispersion. For those peaks with n, m being the successive Fibonacci

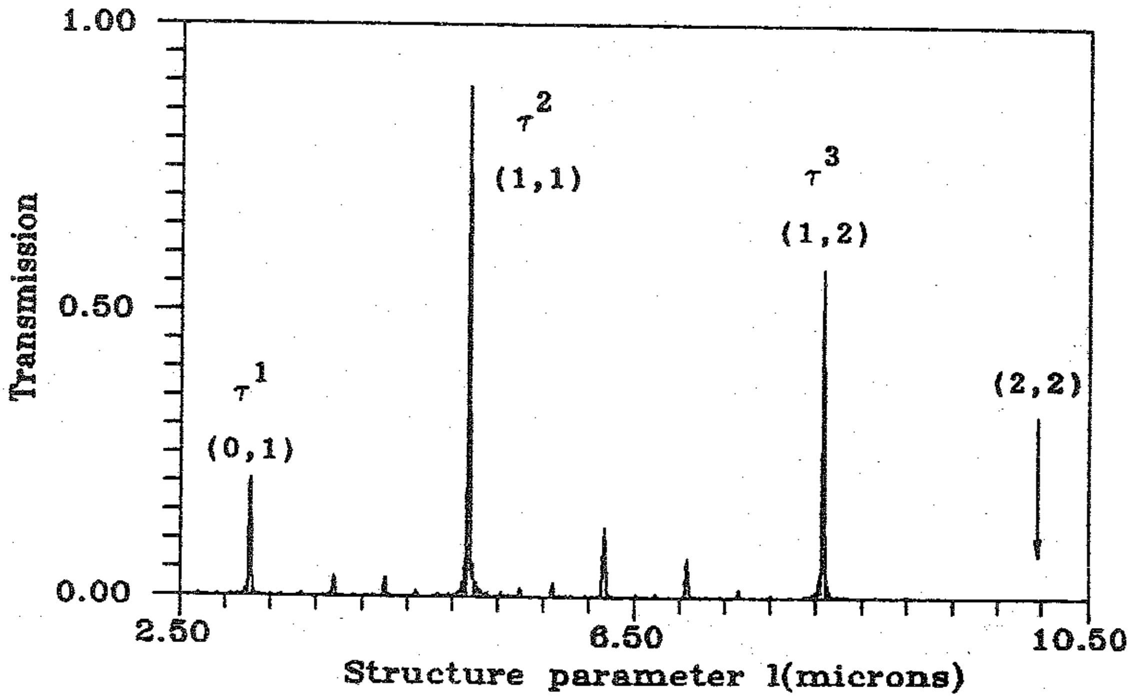

Fig.5—6.Dependence of the transmission on the structure parameter l with $\lambda=0.8000\mu m$. Note that $l(1,p+1)=l(1,p)+l(1,p-1)$. The transmission spectrum reflects the self-similarity of the FOS structure.

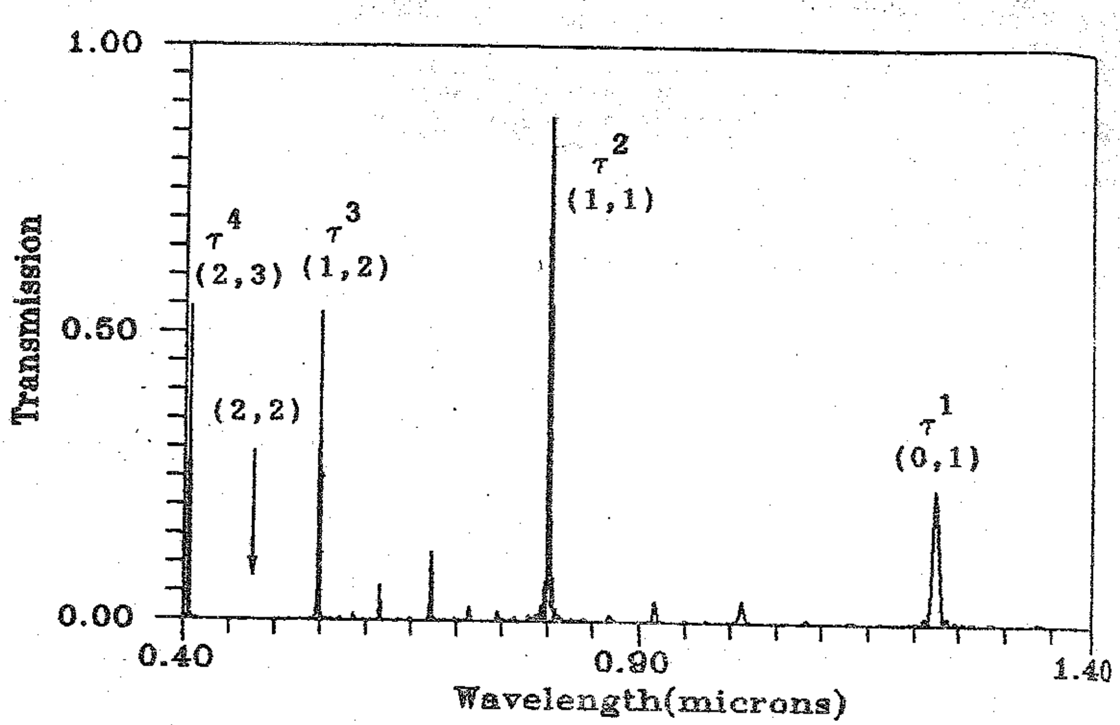

Fig.5—7.Dependence of the transmission on the wavelength with l=5.0441μm. Note that $(1/\lambda)_{1,p+1} \neq (1/\lambda)_{1,p} + (1/\lambda)_{1,p-1}$.

numbers, Eq.(5—30) becomes

$$\left(\frac{1}{\lambda}\right)_{1,p} = \frac{\tau^p}{2(n_o(\lambda)-n_e(\lambda))(1+\tau)l}. \tag{5—31}$$

The relation

$$\left(\frac{1}{\lambda}\right)_{1,p+1} = \left(\frac{1}{\lambda}\right)_{1,p} + \left(\frac{1}{\lambda}\right)_{1,p-1}, \tag{5—32}$$

is no longer valid because of the dispersion. So here the spectrum is non-self-similar. In other words, the spectrum does not reflect the self-similarity of the reciprocal space and does not reflect the symmetry of the quasiperiodic structure.

In the second harmonic generation of an FOS, we have discussed an extinction phenomenon. Here this phenomenon also exists which can be seen in both Figs.5—6 and 5—7. In §5—2, we have already mentioned this phenomenon. The general extinction rule can be derived directly from Eq.(5—28). We can see from it that the transmission depends on K as well as on $\Delta\beta$. The extinction occurs when K=0. From Eqs.(5—4) and (5—18), this happens when

$$\sin\frac{1}{2}G_{m,n}l=0. \qquad (5\text{—}33)$$

That is $l=2j(\pi/G_{m,n})=2j(\pi/(\beta_2-\beta_3))$, or

$$(m,n)=(2j,2j). \qquad (5\text{—}34)$$

In deriving Eq.(5—34), the phase matching condition has been used. Here $\pi/(\beta_2-\beta_3)$ is the thickness of a $\lambda/2$ wave plate. Namely, all peaks with indices (m,n)=(2j,2j) are absent in the spectrum provided the structure parameter l equals an even number times the thickness of a $\lambda/2$ wave plate.

Finally, please notice that the whole set of the reciprocal vectors $G_{m,n}$ possesses some invariant property. From $G_{m,n}=\pi(m+n\tau)/((1+\tau)l)$, we can derive that if l changes by τ^p times, the whole set of the reciprocal vectors $G_{m,n}$ only transforms into a new set of $G_{m',n'}$, which is equivalent to reindex the old one. The derivation is as follows. For example, l changes by τ^{-1} times, we have

$$G_{m',n'}=\frac{\pi(m'+n'\tau)}{(1+\tau)\tau^{-1}l}=\frac{\pi(m+n\tau)}{(1+\tau)l}=G_{m,n} \quad . \qquad (5—35)$$

Where m=n' and n=m'+n'. Because m and n run over all the integers, the $G_{m',n'}$ is just the same as the $G_{m,n}$. Fig.5—8 reflects this invariance.

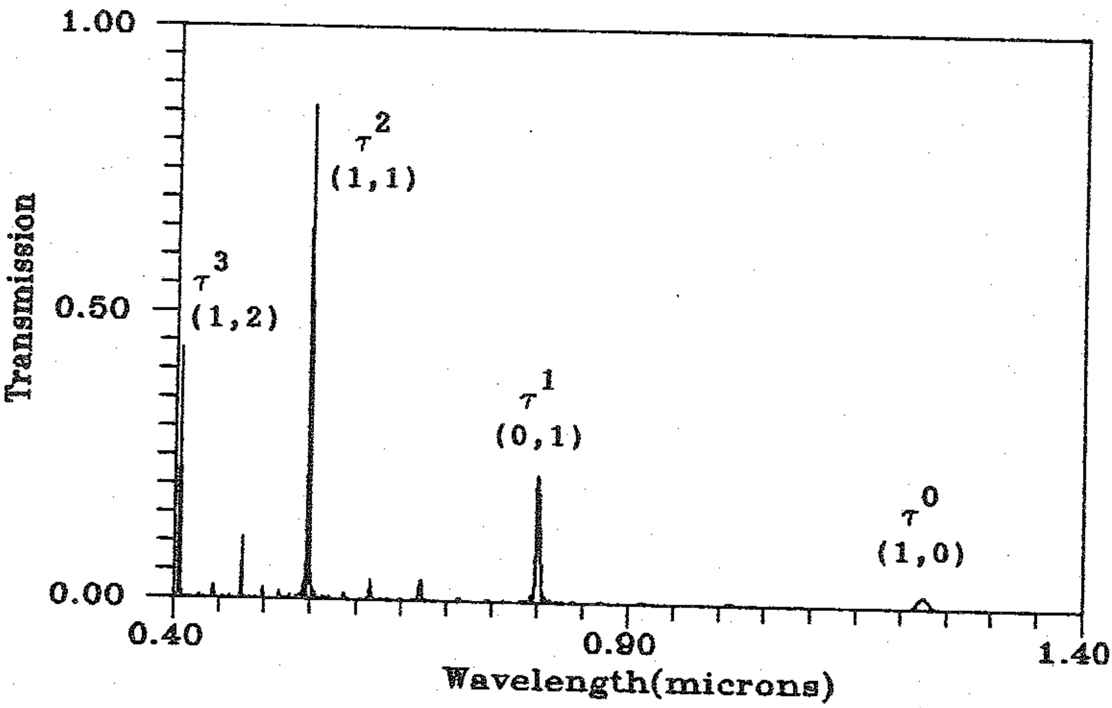

Fig.5—8.Dependence of the transmission on the wavelength with l=5.0441μm/τ=3.1175μm. Note that there exists a one-to-one correspondence of the peak positions between Fig.5—7 and Fig.5—8 which is equivalent to reindexing the peaks of Fig.5—7 despite of the peak height and extinction.

§5—4.Summary

We have analyzed theoretically the electrooptic effect in an FOS. Both coupled mode theory and Jones matrix method have been used. The phase matching concept has been presented for the first time for the FOS. The transmission spectrum shows non-self similarity due to the dispersion of the refractive indices. The extinction phenomenon has been discussed. An invariant property of the reciprocal vectors $G_{m,n}$ under the inflation transformation of the structure parameter *l* has also been discussed.

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CHAPTER 6

Reflection of light and acoustic waves by micron superlattice

In the previous chapters, we have discussed the piezoelectric, nonlinear optic and electrooptic effects of the micron superlattice made of a crystal with its symmetry of 3m point group. All of them are related to the tensors of physical quantity with odd-rank. Generally speaking, in the absence of an external electric field, the even-rank tensors are the same in positive domains and negative domains. So the micron superlattice is homogeneous to the propagation of light and acoustic waves. But in the presence of an external electric field, the situation is different. As we have already seen in chapter 5 that the dielectric tensor, being a second-rank one, is modulated by an electric field through the electrooptic effect. Alshits et al.[1] have proved, both theoretically and experimentally, that not only the dielectric tensor but also the elastic tensor can be modulated by an external electric field through nonlinearity or coupled physical effects such as electroacoustic and electrooptic effects, which are associated with the odd-rank tensors. Therefore, in this case, the micron superlattice will be inhomogeneous to the propagation of light and acoustic waves, and is similar to a conventional superlattice composed of two different materials.

In addition, in introduction, we have mentioned that apart from the 180^0 ferroelectric domain superlattice, there may be others, e.g., the non-180^0 ferroelectric domain superlattice, the laminar twin superlattice, the polar-inversion superlattice, etc.. Under certain conditions, all of them will be inhomogeneous to the propagation of classical waves, and are similar to the conventional superlattice. To make the discussion suitable to all of these superlattices, here, in this chapter, we will study the propagation of light and acoustic waves in a conventional superlattice composed of two materials. In this case, tensors both with odd-rank and with even-rank may change their magnitudes or their signs or both regularly. Many researchers have studied the transmission spectra of light and acoustic phonons in the conventional superlattice[2—4]. In their works, they did not take into account the effect of dispersion of the light velocity (or rather the refractive index) and the velocity of acoustic phonons on the spectra (transmission). They did not notice that there exist two kinds of interfaces, one with its reflection coefficient positive and the other with its reflection coefficient negative. Owing to these two reasons, they did not obtain the exact analytical expression for the transmission of waves in the superlattice (as for light, the analytical expression was not given). Hence they did not predict the extinction phenomenon as that predicted in chapter 4 and 5. And more, they did not discuss the phase matching concept. These will be the topics of this chapter.

§6—1. Theory

Fig.6—1(a) shows a conventional Fibonacci superlattice made of two different materials. There are two building blocks, each composed of two different materials. The thicknesses of the first layers of the blocks are equal. Let n_1, v_1, ρ_1 be the refractive index, the sound velocity and the density of the first layer (material 1), respectively, and n_2, v_2, ρ_2 be the corresponding parameters of the second layer (material 2). Without the loss of generality, we assume that $n_2^{-1} > n_1^{-1}$ and $v_2 > v_1$. For simplicity, we consider that the incident wave is normal to the surface of the structure, we define this direction as the x axis. The

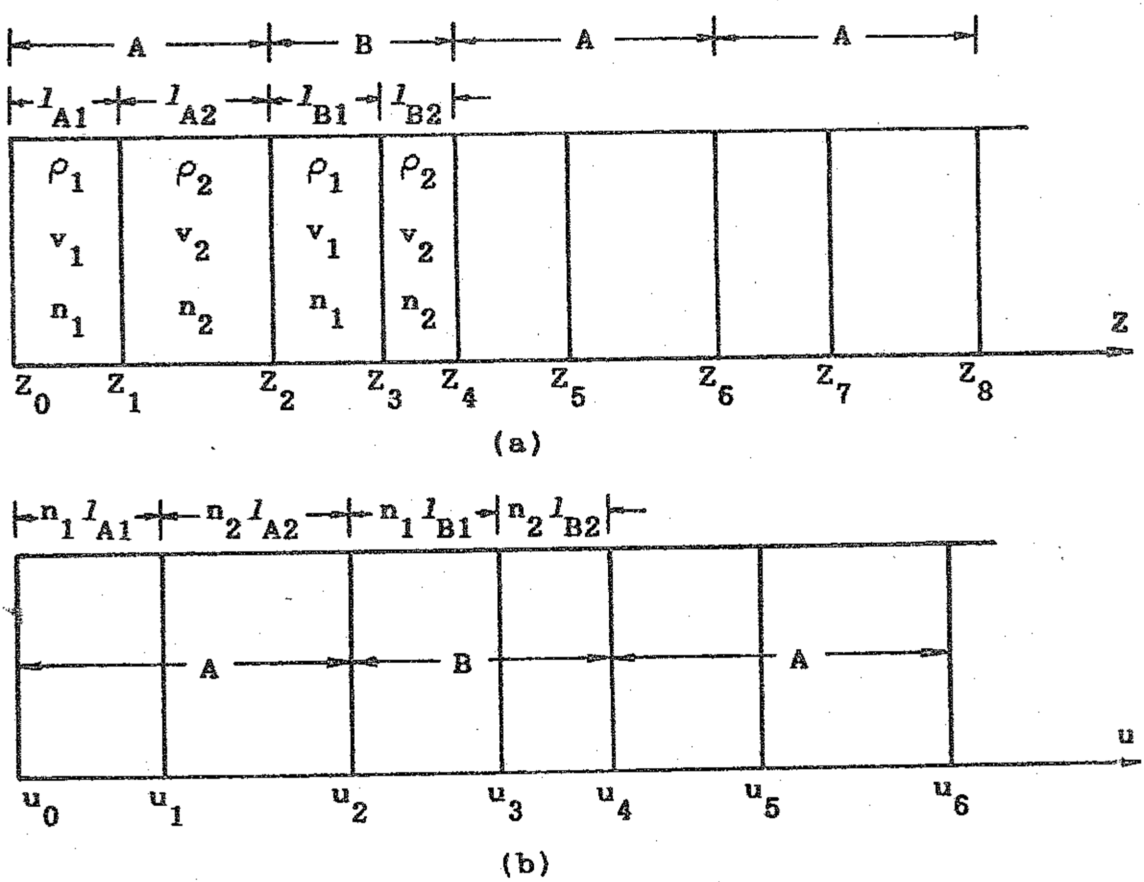

Fig.6—1.(a)Schematic diagram of a conventional Fibonacci superlattice. (b)Transformed from (a) for light waves with the thicknesses of layers altered.

incident wave will be reflected by the interfaces. The reflection coefficient for the ith interface is given by[5]

$$r=\frac{y_{i+1}-y_i}{y_{i+1}+y_i}\ , \tag{6—1}$$

where , for light waves, $y_i=n_i^{-1}$ and for acoustic waves, $y_i=\rho_i v_i$ (i=1,2). When a wave travels from material 1 to material 2, the reflection coefficient is positive. While a wave travels from material 2 to material 1, the reflection coefficient is negative, or the reflected wave undergoes a phase change of π with respect to the incident one. Obviously, there are two kinds of interfaces. The position coordinates in Fig.6—1(a) with their subscripts being even numbers constitute one set, others form the second set.

Below we will take the light waves as an example. For convenience, we replace the real thicknesses of the layers with the corresponding optical path, i.e.,

$$L_{A1}=n_1 l_{A1}=n_1 l,$$

$$L_{A2}=n_2 l_{A2}\ ,$$

$$L_{B1}=L_{A1}=n_1 l\ ,$$

$$L_{B2}=n_2 l_{B2}\ , \tag{6—2}$$

where we have set $l_{A1}=l_{B1}=l$. In other words, here we have a new superlattice with the thicknesses of the layers expressed by Eq.(6—2), which is shown in Fig.6—1(b). The properties of interfaces are as the old one. When a light wave travels in such a structure, its wavelength remains constant. By the same analysis as that in x-ray diffraction in solid-state physics, the structure factor can be expressed as[6]

$$S_F(\Delta k)=\sum_j e^{i\Delta k u_{2j+1}} + e^{i\pi}\sum_j e^{i\Delta k u_{2j}} , \qquad (6—3)$$

where $\Delta k=\vec{k}_{in}-\vec{k}_{ref}=2k$, $\vec{k}_{in}$ represents the wave vector of the incident wave, $\vec{k}_{ref}$ that of the reflected wave and $k=2\pi/\lambda$ with λ being the wavelength in vacuum.

If the condition $L_{A1}+L_{A2}=\tau(L_{B1}+L_{B2})$ is satisfied Eq.(6—3) can be written in the form[7,8]

$$S_F(\Delta k)\propto e^{i\frac{1}{2}\Delta k L_{A1}}\sin\frac{1}{2}\Delta k L_{A1}\sum_{m,n}\frac{\sin X_{m,n}}{X_{m,n}}e^{-iX_{m,n}}\,\delta\left(\Delta k-\frac{2\pi(m+n\tau)}{D}\right) \qquad (6—4)$$

with

$$D=\tau(L_{A1}+L_{A2})+L_{B1}+L_{B2}.$$

For acoustic waves, the corresponding expression is

$$S_F(\omega)\propto e^{i\omega\frac{l_{A1}}{v_1}}\sin\omega\frac{l_{A1}}{v_1}\sum_{m,n}\frac{\sin X_{m,n}}{X_{m,n}}e^{-iX_{m,n}}\,\delta\left(\omega-\frac{\pi(m+n\tau)}{D'}\right) \qquad (6—5)$$

with

$$D'=\tau\left(\frac{l_{A1}}{v_1}+\frac{l_{A2}}{v_2}\right)+\frac{l_{B1}}{v_1}+\frac{l_{B2}}{v_2} .$$

Eq.(6—4)(or Eq.(6—5)) is much the same as those obtained in previous chapters. Likewise, here the most significant peaks in reflectivity occur for those $k=k_{m,n}$ for which m and n are successive Fibonacci numbers. These reflections are commonly labeled τ^p.

In order to see the features of the spectrum more clearly, the matrix method is used which is convenient for numerical calculations. Again we take the light waves as an example. The electric field of the light wave within each homogeneous layer can be expressed as a sum of an incident and a reflected plane wave. The complex amplitudes of these two waves constitute the components of a column vector. The electric field in layer α (α=1,2) of the nth block can be represented by a column vector

$$\begin{pmatrix} a_n^{(\alpha)} \\ b_n^{(\alpha)} \end{pmatrix} ,\alpha=1,2 \qquad (6—6)$$

And so the electric field distribution in the same layer can be written

$$E(z)=a_n^{(\alpha)}e^{ik_n^{(\alpha)}(z-z_{n-1})}+b_n^{(\alpha)}e^{-ik_n^{(\alpha)}(z-z_{n-1})} . \qquad (6—7)$$

By imposing the boundary conditions at $z=z_{n-1}+l_{A1}$ and $z=z_{n-1}+l_{A1}+l_{A2}$, the matrix for block A can be obtained which is

$$\begin{pmatrix} a_n^{(1)} \\ b_n^{(1)} \end{pmatrix} = T^A \begin{pmatrix} a_{n+1}^{(1)} \\ b_{n+1}^{(1)} \end{pmatrix}, \qquad (6\text{—}8)$$

where

$$T_{11}^A = e^{-ik_1 l_{A1}} \left[\cos k_2 l_{A2} - \frac{i}{2}\left(\frac{n_2}{n_1} + \frac{n_1}{n_2}\right) \sin k_2 l_{A2} \right],$$

$$T_{12}^A = -\frac{i}{2} e^{-ik_1 l_{A1}} \left(\frac{n_2}{n_1} - \frac{n_1}{n_2}\right) \sin k_2 l_{A2},$$

$$T_{21}^A = \frac{i}{2} e^{ik_1 l_{A1}} \left(\frac{n_2}{n_1} - \frac{n_1}{n_2}\right) \sin k_2 l_{A2},$$

$$T_{22}^A = e^{ik_1 l_{A1}} \left[\cos k_2 l_{A2} + \frac{i}{2}\left(\frac{n_2}{n_1} + \frac{n_1}{n_2}\right) \sin k_2 l_{A2} \right],$$

$$k_1 = 2\pi n_1/\lambda,$$

$$k_2 = 2\pi n_2/\lambda. \qquad (6\text{—}9)$$

The matrix for block B is given by the same expression with l_{A2} replaced by l_{B2}.

Thus by iterated substitution, the column vectors in the Nth block are related to that of the zeroth block by

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = T \begin{pmatrix} a_N \\ b_N \end{pmatrix}, \tag{6—10}$$

with $T = T^A T^B T^A T^A T^B \ldots$.

The same expression can be obtained for acoustic waves except that in matrix elements T_{ij}, $k_1 = \omega/v_1$, $k_2 = \omega/v_2$, and n_1 is replaced by $\rho_1 v_1$, n_2 by $\rho_2 v_2$.

The reflection coefficient is given by

$$r_N = \left(\frac{b_0}{a_0}\right)_{b_N=0} . \tag{6—11}$$

That is, it is the ratio of the complex amplitude b_0 at the input to the incident amplitude a_0, subject to the boundary condition that to the right of the structure there is no wave incident on it(i.e., $b_N=0$).

From Eq.(6—10), we have

$$r_N = \frac{T_{21}}{T_{11}} . \tag{6—12}$$

The reflectivity is obtained by taking the absolute square of r_N , that is

$$|r_N|^2 = \left|\frac{T_{21}}{T_{11}}\right|^2 . \tag{6—13}$$

§6—2. Discussions

The δ-function in Eq.(6—4) is very important in the analysis of the problem. The condition

$$\Delta k=\frac{2\pi(m+n\tau)}{D}=G_{m,n} \quad (6\text{—}14)$$

may be called the Bragg condition or rather the phase-matching condition. When this condition is satisfied, the reflected waves are in phase, and therefore they interfere constructively. If, on the other hand, the condition is not observed, the reflected waves are out of

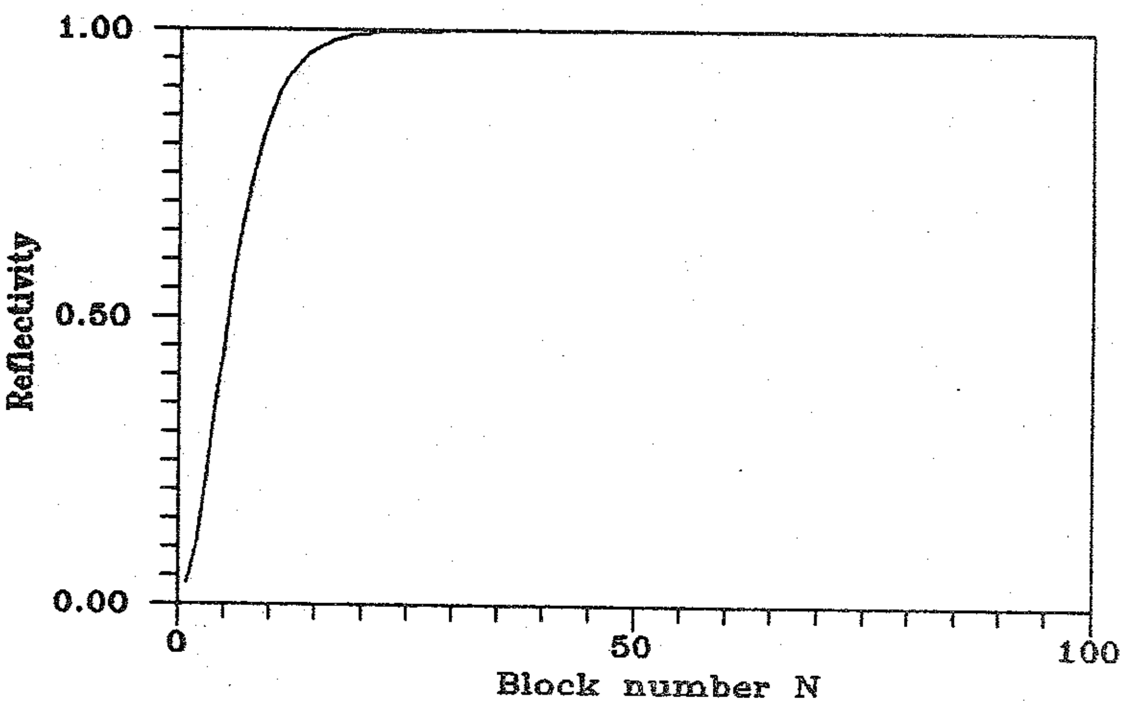

Fig.6—2.Reflectivity vs.block number for light waves with $\Delta k=G_{1,1}$ and $n_1=2.5$, $n_2=2.0$.

phase, and they interfere destructively. In nonlinear optic effect and electrooptic effect, we have met the same problem. Clearly, in interaction processes between waves,