

A comparison of squeeze-film theory with measurements on a microstructure

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Abstract

Mathematical formulations of the effects of squeezed films of gas have been available for some years. We compare the theoretical predictions for two isolated rectangular plates oscillating normal to each other with measurements on a silicon microstructure that approximates such plates. A range of pressure from vacuum to atmospheric, and frequencies from d.c. to 50 kHz are employed, representing squeeze numbers between zero and 1000, and flow regimes from molecular, through transition to substantially viscosity dominated. Generally the agreement with predictions is good, though there is a small but significant difference in effective plate separation between low frequencies (<10 kHz) and high frequencies (>10 kHz). Attention is drawn to the high degree of gas trapping between the plates at resonance, for all pressures investigated, the possibility of using this effect as a pressure sensor is noted. Phase measurements at low frequency provide a simple measurement of gas viscosity.

Introduction

Small oscillating structures can be profoundly influenced by the gas which surrounds them. The significance of these effects becomes greater as micromachined structures decrease in size. Following work on the resonating gate transistor [1], Newell [2] discussed the effect of the surrounding air on the Q -value of a resonator. He observed that the ever-present damping due to the ambient air would be increased when the oscillator was near a second surface due to the pumping action of the gas between the surfaces. Howe and Muller [3] constructed a resonating beam device that contained perforations to minimize damping caused by the air trapped between the beam and the substrate. In this work, we present a detailed comparison between the predictions based on the theory of squeezed gas films, and experimental measurements on a microstructure that approximates two isolated plates oscillating normal to each other. It will be seen that in general the agreement is quite good though there are discrepancies.

Prior to the advent of microstructures, an extensive literature has been developed relating to gas-film lubrication [4], which had application in bearings and levitation systems. Most of this work

related to the parallel motion of the containing surfaces, with less attention being given to the 'squeeze film' effect caused by normal motion of the surfaces (e.g., Salbu [5]). The realization that squeeze films could have a significant effect on bearing stability prompted a more detailed analysis [6] and experiments to test the theory.

Traditionally, it is the damping provided by squeeze films that provides their technological interest. Thus, Griffin *et al* [7] investigated the use of a squeezed gas film as a controlled damper for pneumatic machines. Blech [8] refers to their use to tailor the frequency response of seismic accelerometers, and more recently their use in tailoring the response of micromachined sensors has been described by Allen *et al* [9], and in more detail by Starr [10]. Seidel *et al* [11] have published details of an accelerometer whose resonance frequency rose slightly and whose damping increased with increasing air pressure. The results were modelled quantitatively by the empirical assumption of a pressure-dependent phase lag in the damping force. More recently, the frequency response of a micromachined accelerometer has been modelled in terms of a fixed damping parameter at atmospheric pressure [12].

A pressure-dependent rise in the resonance frequency of structures containing a trapped film of

gas and a decrease in the relative importance of damping were recognized in the early work Salbu [5], for example, describes how with increasing drive frequency, the region of viscous damping due to parallel flow out of the trapping structure will be increasingly confined to the edges of the structure, leaving a growing region at the centre dominated by compression effects. In their investigation of the viscous damper, Griffin *et al* [7] noted that at sufficiently high frequency, the damper acts as a spring and fails to perform its damping function. This can be made use of in capacitor microphone structures, where the stiffness of the trapped gas can be used to raise the structure resonance frequency [13].

Clearly, the relative importance of damping and compressibility effects is frequency dependent, and any analysis must take this into account. A solution of the Reynolds equation governing gas films trapped between plates was given in convenient form by Blech [8]. In the following, we measure the damping and spring coefficients of a microstructure consisting of two oscillating plates separated by a small gap as functions of pressure and frequency. The results are compared with the predictions of Blech. In general the agreement between theory and experiment is good, but there are some discrepancies in the fitting parameters deduced at high and low frequencies. Despite the fact that the highest frequency used is 50 kHz, there is little evidence that the compressions in the 2 μm gap structure are other than isothermal.

Theoretical background

Consider a plate of area A and mass m oscillating in air close to a second fixed plate of equal size, driven by a periodic forcing function F and subject to a restoring force that is linear with displacement y and a damping force that is proportional to the plate velocity. The system can be described by the second-order differential equation

$$my + by + ky = F \quad (1)$$

The parameters b and k will consist of a combination of the inherent stiffness and damping of the mechanical structure, with the restoring and dissipative forces arising from the gas film. For small-amplitude oscillations, b and k can be expected to be constant, but for large amplitudes, such as

might occur near resonance, they could be amplitude dependent and produce a non-linear response to the drive. To predict the system frequency response, these frequency-dependent components must be evaluated and added to the frequency-independent structural parameters.

At a given frequency the response of the system is periodic and can be expressed in terms of an amplitude y_0 and phase ϕ , i.e.,

$$y_0 = \frac{F}{m} \left(\frac{1}{(\omega_0^2 - \omega^2)^2 + b^2 \omega^2 / m^2} \right)^{1/2} \quad (2)$$

$$\phi = -\tan^{-1} \frac{b\omega}{m(\omega_0^2 - \omega^2)} \quad (3)$$

where b and k are the values appropriate to the frequency and ω_0 is defined by $\omega_0^2 = k/m$.

The behaviour of the gas between the plates is described by the compressible-gas-film Reynolds equation, a single expression which relates pressure, density and surface velocity for the specific geometry of a bounded film (e.g., [4], p. 38). This equation assumes that inertial forces are small compared to viscous forces, that shear velocities over a mean free path are small compared to the thermal velocity, and that the gap is large compared to the mean free path ([4], p. 33, p. 38, [10]). While the first two conditions are valid over the range explored here, the last, which is a measure of the breakdown of continuum behaviour of the gas, is seriously violated and is discussed later.

Under the assumption of isothermal conditions, Blech [8] has derived solutions for the pressure within the gap between two oscillating rectangular plates. The pressure has two components: one in phase with the drive, which represents the spring-like behaviour of the gas, and one in quadrature (i.e., in phase with the velocity), which represents damping. The integrals of these pressures over the plates (eqns (25) and (26) in [8]) lead directly to expressions for the air spring and damping contributions. For square plates the air spring coefficient is given by

$$k_a = \frac{64\sigma^2 P_a A}{\pi^8 d} \sum_{m,n \text{ odd}} \frac{1}{(mn)^2 \{ [m^2 + n^2]^2 + \sigma^2 / \pi^4 \}} \quad (4)$$

and the air damping coefficient by

$$b_a = \frac{64\sigma P_a A}{\pi^6 \omega d} \sum_{m,n \text{ odd}} \frac{m^2 + n^2}{(mn)^2 \{ [m^2 + n^2]^2 + \sigma^2 / \pi^4 \}} \quad (5)$$

where P_a is the ambient pressure and d the plate separation. Note that in these solutions, an ideal

boundary is assumed in which the gas pressure falls to ambient and there is no turbulence associated with the plate edges

The central parameter in these expressions is the non-dimensional squeeze number, σ , defined by

$$\sigma = 12\mu a^2 \omega / P_a d^2$$

where μ is the gas viscosity and a the plate side length. A relevant interpretation of the squeeze number in the present context is the ratio of the gas velocity needed to be set up between the plates to ensure no compression, to the velocity which can be achieved under viscous flow. Low squeeze numbers means gas escapes readily, high values mean that gas is trapped in the structure by its viscosity.

The behaviour of the microstructure will be determined largely by the squeeze number. At sufficiently high pressure μ is constant, and the squeeze number will rise with frequency and fall with pressure. At lower pressures the gas behaviour in the gap is not collision dominated and the effective viscosity becomes proportional to pressure [14], so that while the squeeze number is still proportional to frequency, it is not a function of pressure. The value of the pressure at which this transition occurs will be considered later.

There are three limiting cases of eqns (4) and (5). At low squeeze numbers, the air is squeezed from the gap without compression, hence $k_a \rightarrow 0$. The oscillation time is sufficiently long that the air behaves as if it were incompressible. The damping coefficient becomes

$$\begin{aligned} b_a &= 0.035 P_a A \sigma / \omega d \\ &= 0.42 A^2 \mu / d^3 \end{aligned} \quad (6)$$

At high squeeze number, the air spring constant simply converges to

$$k_a = P_a A / d \quad (7)$$

The damping coefficient does not converge to a limit, but falls approximately as $1/\sigma^{0.4}$. Curves presented by Blech [8] show that for low σ , the damping force exceeds the spring force, the damping reaches a maximum for $\sigma \approx 10$, while the spring force continues to rise and become the dominant force. This cross-over is the cut-off of the pneumatic damper mentioned in the Introduction.

Two questions arise in applying these expressions. The first is the extent to which the underlying

assumption of isothermal conditions applies, considering that frequencies of up to 50 kHz will be used in the experiments. The second relates to the pressure below which collective molecular behaviour fails in the essentially two-dimensional films explored here. At very low pressures, gas collisions within this film are unimportant and momentum is transferred between gas and surface by ballistic trajectories and wall collisions. In this regime, momentum transfer is proportional to gas pressure [14]. Thus while the true gas viscosity is not pressure dependent, an effective viscosity that is proportional to pressure can be defined.

Between this molecular-flow regime and the high pressure constant-viscosity regime there is no theoretical description of the gas-flow behaviour. As pressure is decreased from high values, a decrease in momentum transfer between gas and walls below the value expected for a constant viscosity is observed. The effect is described in terms of slip as molecules suffer decreasing numbers of collisions in crossing the structure. Knudsen, quoted in ref. 14, Table 2.8, gives values of capillary pipe conduction as a function of pressure. When the Knudsen number falls below 0.01 (i.e., a molecule would experience more than 100 collisions in crossing the pipe), the flow is almost entirely viscosity dominated. When the pipe size equals the mean free path, viscous flow accounts for only 14% of the observed flow as a result of interactions with the system walls being more important than gas-gas collisions. This can be interpreted as a pressure-dependent fall in the effective viscosity, bridging the region between the pressure-proportional effective viscosity of the molecular region and the constant-viscosity, high-pressure, viscous-flow region. In this work we have assumed that the effective viscosity rises linearly with pressure from zero, reaching the viscous-flow value at a pressure of 250 Torr. While there is no reason to suppose that pipe flow data should be identical to data for flow in a gap, this gives an approximate fit to the Knudsen data at the pressures used here. At 250 Torr, air molecules would experience 10 collisions in crossing the gap of the structure used here.

Experiments

The test structure used is shown in Fig. 1. It consists of a mass-loaded silicon plate supported

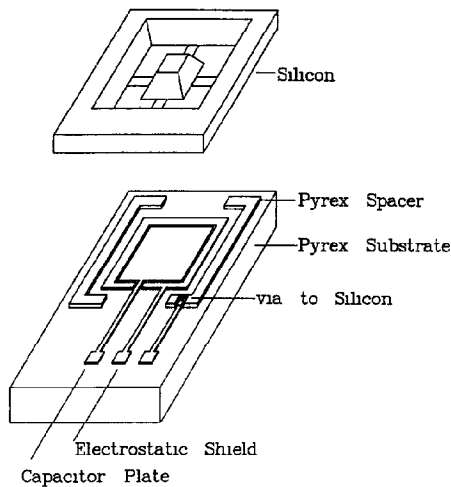


Fig 1 Beam-supported mass-loaded plate and Pyrex substrate comprising the test microstructure

by four beams, each $7\ \mu\text{m}$ thick, $50\ \mu\text{m}$ wide and $190\ \mu\text{m}$ long. The chip to which these beams are anchored is electrostatically bonded to a Pyrex base on which a system of $3\ \mu\text{m}$ high spacers has been etched. The metal pattern on the diaphragm is $1\ \mu\text{m}$ thick, providing an air gap of $2\ \mu\text{m}$ between the silicon plate and the base. The surfaces of the movable silicon plate and the beam are n^+ -doped to provide high conductivity. A side effect of this doping is to produce a tensile stress, which raises the resonance frequency above that expected for undoped silicon. The movable plate and metallization form a variable capacitor. The gaps between the support beam allow air to escape when the plates are brought together, so that the system is a reasonable approximation to the isolated pair of oscillating plates described in the previous Section.

The device was shaken by a small a.c. voltage superimposed on a larger d.c. offset. Since the force between the plates is proportional to $(\text{voltage})^2$, this combination produces a drive proportional to the product of the d.c. and a.c. voltages at the drive frequency, together with a small second-harmonic term. Plate motion was detected via capacitance changes measured using a Boonton model 72AD meter, modified to extend its useful range to $15\ \text{kHz}$. Capacitance fluctuations were detected synchronously in the Boonton output using a Princeton Applied Research Lock-in Analyser (model 5204). The phase and amplitude response of the Boonton/Princeton

combination was calibrated using a variable capacity diode, and this response has been subtracted from the measurements presented. Measurements of resonance frequency at higher pressures were made using an HP impedance analyser. At resonance, the plate amplitude rises and more work is done against the ambient air. Since this work must be supplied by the analyser, a peak is seen in the real part of the admittance.

Comparison with squeeze-film theory requires that the parameters of the structure, i.e., the plate separation, silicon spring constant and mass of the oscillating plate, be known. Of these, the separation is critical since the damping varies as d^3 (eqn (6)). The measured capacitance C_0 and the accurately known plate area imply a plate separation of $1.8\ \mu\text{m}$, compared with the design value of $2.0\ \mu\text{m}$. Since C_0 will include a lead-in contribution, we regard $1.8\ \mu\text{m}$ as a lower limit. Weighing a number of broken plates and masses yielded a value of $0.4\ \text{mg} \pm 10\%$ for the oscillating mass.

Information on the remaining structural parameter, k_{si} , can be obtained in three ways. The ratio k_{si}/m is found from the vacuum resonance frequency

$$\omega_0^2 = k_{\text{si}}/m \quad (8)$$

The measurement is highly accurate provided that the mode of oscillation at resonance is the simple parallel motion measured at d.c. and low frequencies. This has not been proved, but two observations suggest that it is probable. First, the measured frequency is close to the expected one, and computer modelling indicates that higher modes would have significantly different frequencies. Secondly, these devices have been shocked into resonance by a short electrostatic pulse. The existence of a smoothly decaying wavetrain (detected capacitively) indicates that one mode is dominant. In other devices, this test has produced beats in the decay train, indicating that multiple modes could be excited.

A d.c. value for k_{si}/m can be obtained from the capacitance change ΔC when the structure is inverted. It is readily shown that

$$k_{\text{si}}/m = \frac{2\epsilon g A}{\Delta C d^2} \quad (9)$$

Finally, a third measurement comes from the capacitance change seen when a d.c. or low-frequency electric field is applied, giving a value of

TABLE 1 Comparison of measured and predicted vacuum resonance frequencies

Method	Vacuum oscillation	Gravity	Electric force
Resonance frequency (kHz)	9.0	10.4	12.0
Error source	oscillation mode	$\sqrt{\Delta C}$, d	$\sqrt{m}$, d^2

dC/dV The electrostatic force between the plates is given by

$$\begin{aligned}
 F &= V^2 \frac{\epsilon A}{2d^2} \\
 k_{st} &= -\frac{dF}{dd} \\
 &= \frac{dF}{dV} \frac{dV}{dC} \frac{dC}{dd} \\
 &= \frac{\epsilon^2 V A^2}{d^4} \frac{dV}{dC} \quad (10)
 \end{aligned}$$

While in principle k_{st} can be found independently of m from eqn (10), the accuracy can be compromised by the d^4 term

Table 1 compares the vacuum resonance frequency predicted by eqns (9) and (10) with that measured, assuming $d = 1.8 \mu\text{m}$

Since the errors in both ΔC and m are about 10%, it is tempting to conclude that the value of d is too low by about 10%. In the squeeze-film measurements to follow, it will be seen that at frequencies below 10 kHz, a gap $d = 2.1 \mu\text{m}$ gives an excellent agreement between predictions and measurements. At higher frequencies, however, a gap of $2.1 \mu\text{m}$ leads to significant discrepancies between theory and experiments, and the lower value gives much better agreement.

Measurements of amplitude and phase have been made at pressures of 50 mTorr, 10 Torr, 80 Torr and 760 Torr, for frequencies from dc to 15 kHz. Vacuum data (50 mTorr) showed a constant amplitude and phase to a relatively narrow resonance peak, as expected. Amplitude and phase data for pressures of 10, 80 and 760 Torr are shown in Figs 2–4, along with responses computed by combining the squeeze-film expressions (4) and (5) with the silicon parameters in the response functions (2) and (3). Since the data in these Figures are obtained at constant pressure, the frequency axis can be rescaled in terms of

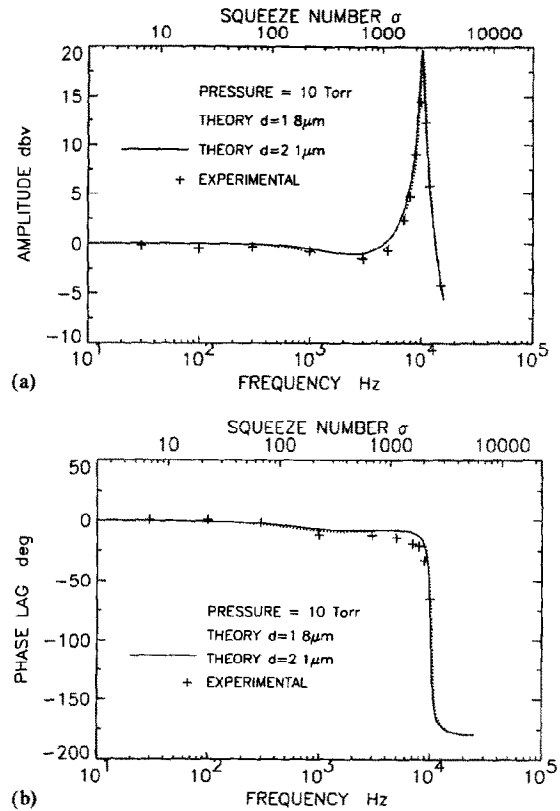
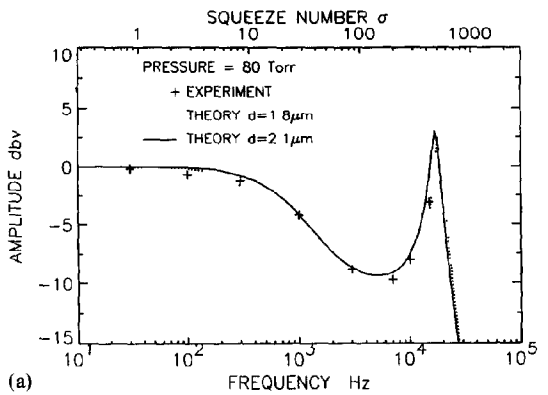


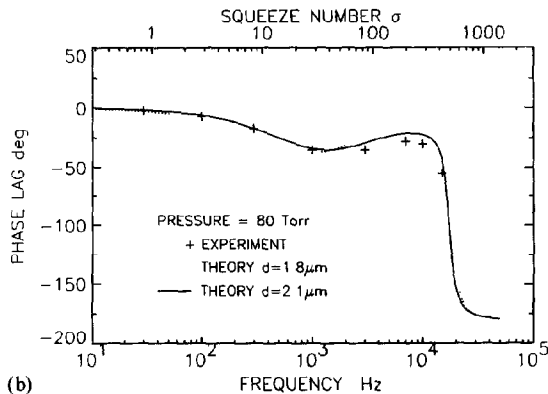
Fig 2 (a) Amplitude and (b) phase response of the structure at 10 Torr pressure. The lines are the predicted response for the two values of gap discussed. From these data and Figs 3 and 4, best agreement is found for a spacing of $2.1 \mu\text{m}$. The squeeze numbers shown correspond to a $2 \mu\text{m}$ gap, approximately midway between the two values of spacing used in the calculations.

squeeze number. An alternative presentation is to use the amplitude and phase measurements to extract the underlying spring and damping coefficients. These are compared with the computed values in Figs 5 and 6 for a pressure of 80 Torr. It was confirmed experimentally that the parameters derived were not amplitude dependent over the range of excitation used.

Finally, the resonant region at frequencies above 15 kHz, which was not accessible by direct capacitive measurement, was explored by the impedance analyser to give a measure of the high-frequency spring constant via the system losses. Figure 7 shows typical measurements, again plate motion at the drive frequency is induced by superimposing a small ac on a larger dc voltage. The plate drive is proportional to the product of the two, and the losses measured by the analyser

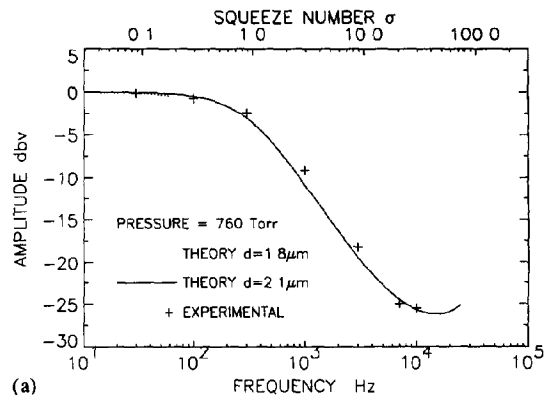


(a)

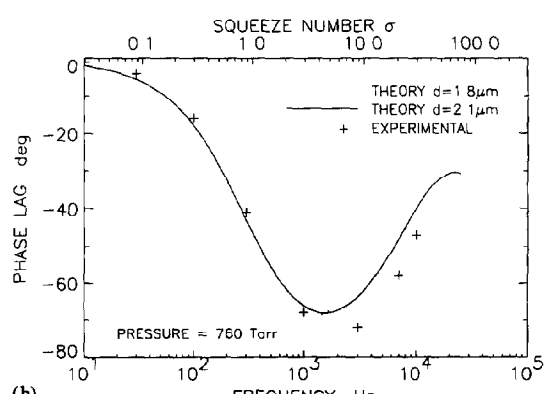


(b)

Fig 3 As for Fig 2, pressure is 80 Torr



(a)



(b)

Fig 4 As for Fig 2, pressure is 1 atm

indicate this Resonance frequencies were derived in this way to an accuracy of 2 kHz for pressures up to 1 atm. They are compared in Fig 8 with predicted values. Data are plotted as $(\text{frequency})^2$ versus pressure, since eqn (7) indicates that at high squeeze numbers, there should be a linear relation between these quantities.

Discussion

The data in Figs 2–6 show a good agreement with the theory over a wide range of pressure if the plate separation is taken to be $2.1 \mu\text{m}$. The gap value of $1.8 \mu\text{m}$, determined from static capacitance measurements and expected to be a lower limit, clearly seems to be too small. This conclusion is in agreement with that drawn from the data of Table 1. The range of operation of this structure goes from molecular flow, through the transition region and almost into the viscosity-dominated region. It is perhaps surprising that the Reynolds equation, modified only by an empirical

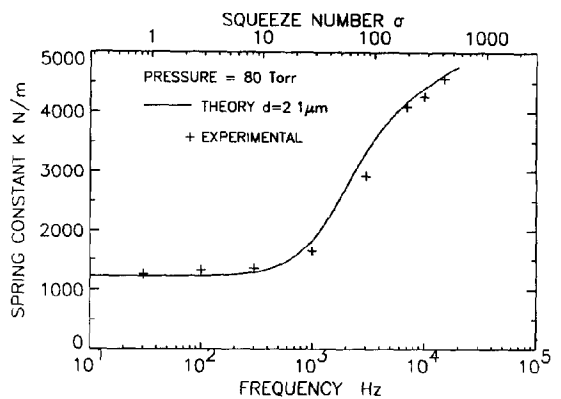


Fig 5 Comparison of the measured and theoretical spring coefficients as a function of frequency for a pressure of 80 Torr. The value at low squeeze number corresponds to the spring constant of the silicon structure alone.

slip correction to viscosity derived from capillary flow, gives such good agreement.

A value of the air damping coefficient b_a can be found rather easily at low frequencies from the variation of phase with frequency (eqn (3)), when

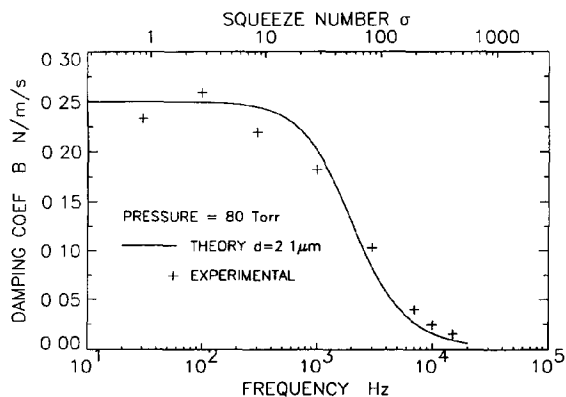


Fig 6 Measured and theoretical air (squeeze film) damping as a function of frequency for a pressure of 80 Torr

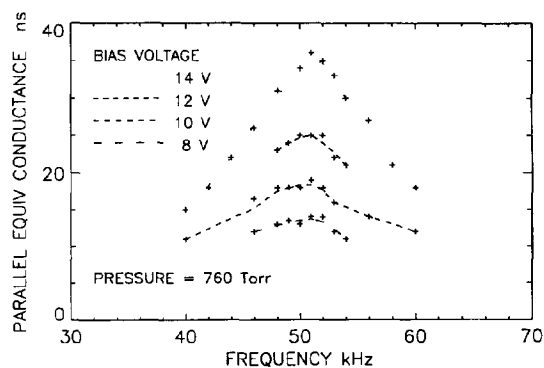


Fig 7 Resonance effects seen in measurements of electrical loss as the structure is shaken electrostatically. The amplitude of the motion (and hence the loss) rises with the d.c. bias

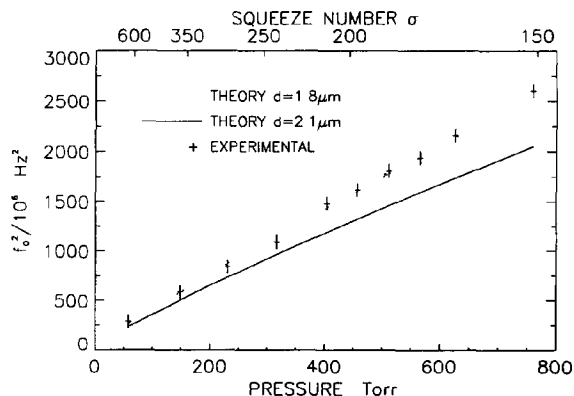


Fig 8 The square of the device resonance frequency as a function of pressure. The two lines are the predicted values for two assumed values of the air gap. Unlike the low-frequency data, better agreement is obtained if the gap is taken to be the lower limit (1.8 μm). The squeeze-number scale corresponds to an air gap of 2 μm

this is combined with eqn (6), a value of effective gas viscosity is readily obtained. Below 300 Hz, for example, the 760 Torr phase data yield a value of air viscosity equal to $15.3 \times 10^{-6} \text{ kg/m/s}$ at 20°C . According to the Knudsen-corrected data [14], the effective viscosity for air in a 2 μm diameter pipe is 0.85 times its true value, or $15.6 \times 10^{-6} \text{ kg/m/s}$, which is in rather good agreement with the deduced value. A microstructure shaken at low frequency could be used in this way to explore the variation in effective viscosity through the transition region. With calibration, it could be used to measure the viscosity of a gas, or perhaps to distinguish one gas from another. As a device to measure viscosity, the present structure could be scaled to produce full viscous flow at atmospheric pressure. This could be done by increasing the gap to 7 μm , while increasing the plate side dimension to 2500 μm to achieve the same conveniently measurable phase shifts below 1 kHz. With these dimensions, the Knudsen number at 1 atm pressure would be 0.01.

The low-frequency approximation (eqn (6)) indicates that b is independent of frequency. This is true only up to a squeeze number of about five, which occurs at frequencies of several hundred hertz in these experiments. The damping actually falls rapidly with frequency and the fall in amplitude response as the frequency is increased is due to the rapid stiffening of the air spring constant. This is seen particularly clearly in the 80 Torr pressure data (Figs 3(a), 5 and 6). Any modelling of such a structure in terms of frequency-independent parameters is unlikely to be very accurate and is physically unsound. The concepts of 'maximally flat response' or 'critically damped', which derive from eqn (1) with constant coefficients, would seem to have little relevance to squeeze-film damped systems. The present structure at atmospheric pressure, for example, has an amplitude response which falls from d.c. and has a 3 dB point at 400 Hz. It continues to fall for a number of frequency octaves, and is therefore 'overdamped'. However, judged from the impedance measurements, it still shows a small resonance peak near 50 kHz with a Q -value, defined by

$$Q = f_{\text{res}}/\Delta f$$

where Δf is the local 3 dB bandwidth, of four or five. The response of this system to a step function may be aperiodic on long time scales, but on a

short time frame ringing effects from the resonance can be detected

The data of Fig 8 are interesting in that they are little affected by uncertainties in the silicon spring constant at the upper end of the frequency axis at atmospheric pressure the silicon contributes only 4% to the combined stiffness. Obviously the assumed gap of $2.1\text{ }\mu\text{m}$, a value which yielded a good fit to the experimental data at lower frequencies, now gives a significantly low resonance frequency. However, the lower limit to the gap, $1.8\text{ }\mu\text{m}$, provides an excellent fit. The reason for this discrepancy is not clear. Damping in this particular structure is quite high at high pressures. The effect of damping on the resonance frequency is to lower it (eqn (2)) and this effect is included in the theoretical data of Fig 8. The data all correspond to high squeeze numbers, when approximation (7) for the spring constant is beginning to apply. The air spring constant is simply related to the inverse of the gap, and the simplest explanation of the 15% discrepancy in spring constant is to assume that at high squeeze numbers an effective gap must be used, which is smaller than the physical gap.

However, the Blech analysis is for isothermal conditions, which historically appear to be the dominant mode of bearing structures [4]. Griffin *et al* [7] analysed squeeze films using a polytropic gas law, i.e., the possibility of adiabatic behaviour was admitted. The result, in the limit of large σ , is simply to multiply the spring constant in eqn (7) by γ , the specific heat ratio, which is 1.4 for air. An alternative explanation of the greater air stiffness at high frequencies is that the effective gap remains $2.1\text{ }\mu\text{m}$, but the behaviour of the gas film is partially adiabatic. A strong argument against this comes from data presented elsewhere [15] for different gases (and different values of γ), which indicate that the behaviour is in fact isothermal even at high frequencies.

The resonant data of Fig 8 all correspond to high values of σ . As the pressure rises, so does the resonance frequency. Thus over a wide range of pressure, the parallel-plate structure maintains a high squeeze number and at resonance, therefore, the plates are effectively confining the gas. This has been made the basis of a wide-range and accurate pressure sensor [15].

Conclusions

The characteristics of gases within cavities in microstructures are relevant to a number of recent micromachined devices. The squeezing of air between close-spaced plates leads to highly frequency-dependent behaviour. We have taken expressions for the spring and damping effects caused by this air, originally developed in connection with gas-film lubrication, and compared them with measurements on a silicon-based microstructure. Squeeze numbers ranged between zero and 1000. Qualitatively, the expressions given by Blech [8] give excellent descriptions of the data, and an expression for the damping at low frequencies is given. The goodness of fit of the squeeze-film expressions depends critically on the air-gap thickness. Below 10 kHz, a gap value slightly above that expected, but within the uncertainty of the plate spacing, gives good quantitative agreement between theory and experiment, but the same value underestimates the air stiffness at high frequencies. In this regime, a gap close to the lower limit of the plate spacing is indicated. The reason for the discrepancy could relate to the failure of isothermal behaviour at higher frequencies, but other evidence indicates that this explanation is unlikely.

Two applications are suggested from this work. At low frequencies, the phase characteristics of a driven microstructure have been shown to have potential for measuring gas viscosity reasonably accurately. More significantly, the well-behaved form of the resonance frequency versus pressure curve for closely spaced plates oscillating normal to each other shows that the gas is effectively trapped (i.e., the squeeze number remains large) at pressures from vacuum to atmospheric pressure. The possibility of a resonant pressure sensor based on this principle is mentioned.

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