Resonant states and transmission coefficient oscillations for potential wells and barriers

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The oscillatory behavior of the transmission coefficient T as a function of energy is examined for an attractive square well and a rectangular barrier. We calculate T using resonant state boundary conditions and demonstrate that the maxima in T are correlated with the broad resonances generated by these potentials. For barrier potentials the maxima signify resonances occurring at energies above the barrier height. It is shown that the resonance position and width can also be generated from the complex poles of the amplitude of the transmitted plane wave. We also explain the relation between the positions of the resonances generated by the square well and the rectangular barrier to the energy eigenvalues of the corresponding rigid box with the same range. We show for a potential with an attractive well and a repulsive barrier that T exhibits oscillations when the particle energy is below the barrier, implying that in many cases the simple WKB type barrier penetration expression for T is not adequate. These features of T are likely to hold for most attractive potentials and flat repulsive barriers. We also discuss the attractive modified Poschl–Teller type potential for which T does not show oscillations as a function of energy.
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I. INTRODUCTION

Transmission and tunneling across potential barriers in one dimension are important topics in undergraduate quantum mechanics courses. Most textbooks on introductory quantum mechanics consider only transmission and tunneling across potential barriers and calculate the transmission coefficients using WKB type approximations,[1] but the physical features of the transmission across attractive potentials are not considered in detail. Similarly resonance tunneling is illustrated for potentials with well separated barriers.[2,3] As a result, students might gain the impression that transmission across attractive potential wells or across a barrier when the incoming particle energy is greater than the barrier height is not of much physical interest.

The advent of nanotechnology and electronic devices based on resonant tunneling has made quantum mechanical transmission and tunneling in one dimension an area of much interest in the applied sciences.[4–12] The goal of this paper is to explore the subtle and interesting features involved in the transmission across attractive square wells and repulsive barriers and to provide an interpretation of transmission as a function of energy in terms of broad resonances generated by the potential.

In three-dimensional systems a potential with an attractive well followed by a barrier is useful for analyzing bound states, sharp resonances generated by potential pockets, and broader resonant states generated by wide barriers.[13] In contrast, in one dimension we usually study a set of two or more well separated potential barriers.[2,3] The gap between two adjacent potential barriers provides a pocket for the formation of sharp resonant states, which are quasibound states and are related to reflectionless transmission. These states generate sharp peaks in the transmission coefficient T. When the barrier is wide, resonances can occur at above barrier energies. We term these resonances as barrier top resonant states or above barrier resonances. A detailed analysis of such narrow quasibound and broader barrier states generated by twin barriers enclosing a pocket in between and their correlation with the positions and widths of T peaks when the energy is both less and greater than the barrier energies is given in Ref. 3. The motivation of the present paper is to explore the broader resonances and related subtle features of the transmission across purely attractive wells and corresponding repulsive barriers to obtain a deeper understanding of transmission across a potential in one dimension.

In Sec. II we study the resonant states and transmission for an attractive square well and relate the oscillations in T to the broad resonance states. Section III does a similar analysis for above barrier oscillations of T. In Sec. IV we describe transmission for a potential with a combination of a well and barrier and give a counterexample of an attractive potential for which T does not exhibit any oscillations generated by resonances. Our results are summarized in Sec. V.

II. TRANSMISSION ACROSS ATTRACTIVE POTENTIALS

The equation governing the bound states, resonant states, and transmission and reflection is the time independent Schrödinger equation for a particle of mass m with potential $U(x)$ and total energy E,

$$\frac{d^2\psi(x)}{dx^2} + (k^2 - V(x))\psi(x) = 0, \tag{1}$$

where $k^2=2mE/\hbar^2$ and $V(x)=2mU(x)/\hbar^2$. For convenience we choose units $2m=1$ and $\hbar^2=1$ so that k^2 is the energy E. We assume $V(x)$ is symmetric in x so that the wave functions have definite parity. In particular, we consider an attractive square well of width $2a$ given by

$$V(x) = \begin{cases} -V_0 & (|x| < a) \\ 0 & (|x| > a). \end{cases} \tag{2}$$

For the study of T and the reflection coefficient R, we seek a solution $\psi(x)$, which satisfies the conditions

$$\psi(x) \to \begin{cases} Ae^{ikx} + Be^{-ikx} & (x \to -\infty) \\ Fe^{ikx} & (x \to \infty), \end{cases} \quad (3)$$

such that $T = |F/A|^2$ and $R = |B/A|^2$. For evaluation of the resonant states, we seek a solution with complex energy $k_R^2 = (k_r - ik_i)^2 = (k_r^2 - k_i^2) - i2k_r k_i = E_R = E_r - i\Gamma_r/2$ with $E_r > 0$ and $\Gamma_r > 0$ such that the wave function behaves asymptotically as

$$\psi(x) \to \begin{cases} e^{i(k_r - ik_i)x} & (x \to \infty) \\ e^{-i(k_r - ik_i)x} & (x \to -\infty). \end{cases} \quad (4)$$

This behavior implies a positive energy state with width Γ_r and lifetime $\hbar/\Gamma_r$, which diverges exponentially as $e^{k_i|x|}$ but decays exponentially in time because the time-dependent part of the solution of the Schrödinger equation is $e^{-iEt/\hbar}$. For potentials vanishing as $|x| \to \infty$, the bound state energies are negative. In particular, for short-range potentials satisfying plane wave or free particle asymptotic boundary conditions, the bound state wave function has the asymptotic behavior

$$\psi(x) \underset{|x| \to \infty}{\to} e^{-k_b|x|}, \quad (5)$$

and the corresponding bound state energy E_b is given by $E_b = (ik_b)^2 = -k_b^2$. We see that the bound states can be thought of as negative energy states with zero width. If the widths are very narrow, resonant states behave similarly to bound states in the vicinity of the potential and may be considered quasi-bound.

We first study the bound states, resonant states, and transmission and reflection generated by the potential in Eq. (2). For this potential T and R are readily calculated. We denote $\alpha^2 = V_0 + k^2$ and write the results as[10]

$$\frac{F}{A} = \frac{4\alpha k e^{-2ika}}{(\alpha + k)^2 e^{-2i\alpha a} - (\alpha - k)^2 e^{2i\alpha a}}, \quad (6)$$

$$\frac{B}{A} = -2i\frac{(k^2 - \alpha^2)}{4k\alpha}\sin(2\alpha a)e^{2ika}\left(\frac{F}{A}\right), \quad (7)$$

$$\frac{B}{A} = \frac{2i[(\alpha^2 - k^2)/4k\alpha]\sin(2\alpha a)}{e^{2ika}[e^{-2i\alpha a}(\alpha + k)^2 - e^{2i\alpha a}(\alpha - k)^2]}, \quad (8)$$

$$\frac{B}{F} = \frac{2i(\alpha^2 - k^2)\sin(2\alpha a)}{4k\alpha}, \quad (9)$$

$$T = \left|\frac{F}{A}\right|^2 = \frac{(4\alpha k)^2}{(4k\alpha)^2 + 4(\alpha^2 - k^2)^2\sin^2(2\alpha a)}, \quad (10)$$

$$R = \left|\frac{B}{A}\right|^2 = \frac{4(\alpha^2 - k^2)^2\sin^2(2\alpha a)}{(4k\alpha)^2 + 4(\alpha^2 - k^2)^2\sin^2(2\alpha a)} = 1 - T. \quad (11)$$

In Fig. 1 we show the variation of T with energy for a set of potential parameters. As $E \to \infty$, $T \to 1$ as it should. In our numerical calculations we use angstrom as the unit of length and Å^{-2} as the unit of energy and choose parameters such that the numerical results and figures show the physical features we seek to highlight.

The oscillatory behavior of T is due to the sine term in Eq. (10). It is more interesting to see if these peaks are related to the resonance states generated by the potential in Eq. (2). The resonance state positions and widths can be evaluated by solving Schrödinger's equation [Eq. (1)] with boundary conditions given by Eq. (4). Because we have chosen the potential to be symmetric with respect to the origin, we expect both even and odd wave functions. The former behave as $\cos(\alpha x)$ near the origin and the latter as $\sin(\alpha x)$. Thus for even resonant states, the matching of the wave function and its derivative at $x = a$ leads to the condition $\alpha \tan(\alpha a) = -ik$. As a result the even resonance state energy and widths are obtained from the complex roots k_n of the equation

$$k\cos(\alpha a) - i\alpha\sin(\alpha a) = 0. \quad (12)$$

The roots are just below the real k-axis in the complex k-plane such that Re $k_n > |\text{Im } k_n|$. This condition signifies a positive energy resonant state with finite width. The corresponding equation for the energies and widths of odd states is

$$k\sin(\alpha a) + i\alpha\cos(\alpha a) = 0. \quad (13)$$

By incorporating the asymptotic condition (5), a similar procedure can be used to obtain the bound state energy eigenvalues E_n. Before examining the results based on these calculations, it is instructive to examine the correspondence between the bound and resonant states generated by the potential in Eq. (2) and the negative and positive energy states of a particle confined in a one-dimensional box such that

$$V(x) = \begin{cases} -V_0, & |x| < a \\ \infty, & |x| > a. \end{cases} \quad (14)$$

This potential generates the eigenvalues

$$E_n = -V_0 + n^2\pi^2/(2a)^2, \quad n = 1, 2, \ldots . \quad (15)$$

Because the potential in Eq. (14) can be considered to be infinitely repulsive for $|x| > a$, we expect that the states for $E < 0$ have higher energies than the corresponding square well given by Eq. (2). In Table I we summarize the numerical results obtained using $V_0 = -3$ and $a = 5$. In this case the square well has six bound states, and the corresponding box potential has only five negative energy states. We also give the positions of the peaks of T. From Table I it is clear that the peaks positions of T are related to the energies of the corresponding resonant states, even though there is a small difference in the numerical values. This difference is primarily due to the large widths associated with the resonances of the attractive well.

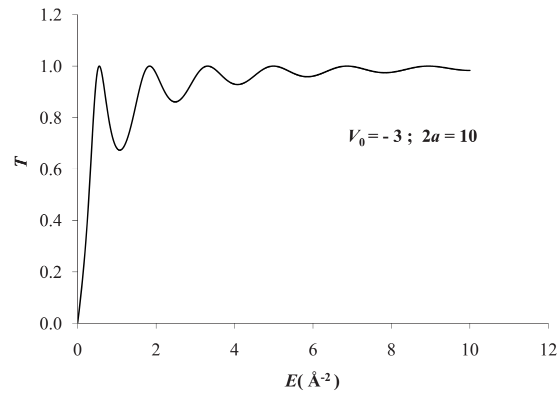

Fig. 1. Variation of the transmission coefficient T as a function of the energy E for the attractive square well in Eq. (2) with $V_0 = -3$ and $a = 5$.

Table I. Transmission across a square well of depth $V_0 = -3$ and range $a = 5$ [see Eq. (2)]. In all tables and figures the energy unit is $Å^{-2}$ and length unit is angstrom. The positive energy box states (E_n) coincide with the maxima of T.

Sequence number n	Energy levels E_n	Peak energies of T	Bound states and resonance state energies	Half-width $\Gamma_n/2$ of states
1	−2.90		−2.92	0
2	−2.61		−2.68	0
3	−2.11		−2.29	0
4	−1.42		−1.75	0
5	−0.53		−1.08	0
6	0.55		−0.39	0
7	1.84	0.55	0.46	0.30
8	3.32	1.84	1.72	0.62
9	4.99	3.32	3.18	0.91
10	6.87	4.99	4.84	1.20
11	8.94	6.87	6.70	1.50
12	11.21	8.94	8.75	1.81

In Fig. 2 we demonstrate a correlation between the bound and resonance states of the attractive square well and the corresponding box energy eigenvalues given by Eq. (15). Along the x-axis we give the sequence number of the resonant states and bound states generated by the box, as they occur, starting with the lowest energy state. These numbers can be interpreted as quantum numbers for the bound states. From Table I we see that the sets of positive energy states generated by the potential in Eq. (14) and the peak positions of T for the corresponding well are the same, but the results for negative energy bound states for the potential given by Eq. (2) and the corresponding results for the box potential differ. We defer the explanation of these results to Sec. III.

It is well known that the bound states and resonant states can be associated with the poles of the S-matrix in complex k or, equivalently, complex-E plane.[14] For transmission across a potential in one dimension, it is natural to examine whether the resonant states we have described correspond to the complex poles of the transmission amplitude F/A. From Eq. (7) we see that the pole structure of F/A and the reflection amplitude B/A are the same. The only condition to be satisfied is that for real positive energies $R+T=1$, which means that whenever T has a peak R has a minimum. If we use Eq. (7) for F/A and a convenient numerical procedure such as an iterative method, we can calculate the zeros of the denominator of Eq. (6), which will generate the poles of F/A and B/A. From this calculation we can verify that all the resonance states listed in Table I correspond to the complex poles of F/A.

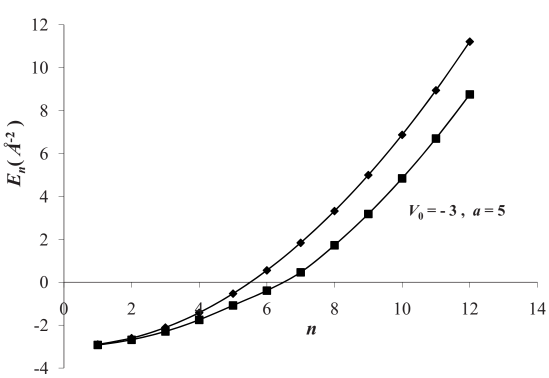

Fig. 2. Comparison of bound and resonant states of the attractive square well in Eq. (2) with $V_0 = -3$ and $a = 5$ to the corresponding states for the box potential (14). The abscissa is the sequence number of the states.

We next discuss the reason why the complex poles of F/A are related to resonances. The amplitude A/F can be understood as the coefficient of an incoming incident wave e^{ikx} as $x \to -\infty$ when the outgoing transmitted wave e^{ikx} for $x \to \infty$ has amplitude one. Similarly, B/F can be understood as the coefficient of the reflected wave as $x \to -\infty$ for the same condition. If $A/F = 0$ for a complex $k = k_r - ik_i$ in the lower half of k-plane with $k_r > k_i$, the resulting wave function satisfies the asymptotic condition given in Eq. (4) signifying a resonant state. However, $A/F = 0$ implies a corresponding pole in F/A, signifying that F/A has complex poles corresponding to a resonance. This correspondence is analogous to three-dimensional potential scattering for which the complex zeros $k = k_r - ik_i$ of the coefficient of the incoming spherical wave component of the regular solution of the scattering problem represent the resonant states and consequently are identified as the pole position of the corresponding partial wave S-matrix S_l.[14] This result demonstrates that resonances have similar interpretations in one and three dimensions. If let $A/F = 0$, we obtain the condition satisfied by the zeros of A/F,

$$\frac{\alpha + k}{\alpha - k} = \pm e^{2i\alpha a}, \tag{16}$$

which corresponds to the complex poles of the transmission amplitude F/A. It is straightforward to verify that Eq. (16) implies Eqs. (12) and (13), which generate the resonance energies and widths. This verification and the numerical example we have described demonstrate that the complex poles of the transmission amplitude F/A for unit incident wave signify resonances and generate the peaks of the oscillations of T for an attractive square well potential.

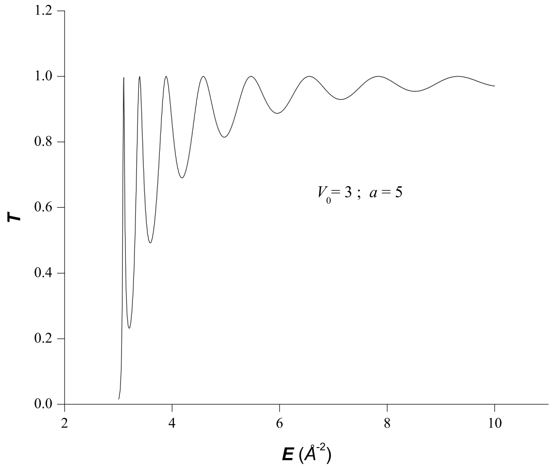

Fig. 3. Variation of T with E for the repulsive rectangular barrier [Eq. (17)] with V_0=3 and a=5.

III. OSCILLATIONS OF T ABOVE THE RECTANGULAR BARRIER

Transmission across a rectangular barrier is the most commonly studied example in introductory quantum mechanics. In Fig. 3 we show the variation of T with energy for the potential barrier

$$V(x) = \begin{cases} V_0 > 0 & (|x| < a) \\ 0 & (|x| > a). \end{cases} \tag{17}$$

We take V_0=3 and a=5 so we can consider the repulsive counterpart of the attractive square well studied in Sec. II. Our primary interest is in the interpretation of the oscillations of T for energies $E > V_0$. Based on our interpretation of the oscillations in T generated by an attractive square well, we can understand these oscillations in a similar manner. Resonances generated by a reasonably flat barrier are well studied for nucleus-nucleus collisions.[13,15] Barrier top resonances are broader states compared to the very narrow resonant states that are generated by the potential pockets sandwiched between wide barriers. However if the barrier is flatter and wider, a number of narrower resonance states are generated for energies above the barrier. This condition is satisfied in our example of the rectangular barrier with the choice a=5. Unlike the attractive well, there are no bound states associated with the rectangular barrier in Eq. (17).

We used the conditions given in Eq. (4) to search for resonance states generated by the barrier at energies above V_0. These states along with the maxima of T are listed in Table II. The expression for T for $E < V_0$ is given by Eq. (10) with $\alpha^2 = k^2 - V_0$. It is clear that the peaks of the oscillations of T at above barrier energies correspond to the barrier top resonances. As before, there is a small difference between the peak positions of T and the corresponding resonance energies due to the fact that we are dealing with broad states. Note that the numerically computed T has contributions from background terms in addition to pole terms. When the imaginary part of the pole position of transmission amplitude is small, the resonance contribution dominates T in the vicinity of resonance energy, and the peak position of T and the corresponding resonance position are closer. However, when a resonance is broad, the background term becomes more significant and there is a shift of the peak position of T with respect to the corresponding resonance energy. The numerical results shown in Tables I and II demonstrate this effect.

Table II. Comparison of transmission peaks and barrier resonance energies for the square barrier potential [Eq. (17)] with V_0=3 and a=5.

Sequence number	Energy levels	Peak energy of T	Resonance energies E_n	Resonance half-width $\Gamma_n/2$
1	3.10	3.10	3.10	0.02
2	3.40	3.40	3.38	0.09
3	3.89	3.89	3.86	0.19
4	4.58	4.58	4.53	0.33
5	5.47	5.47	5.40	0.50
6	6.55	6.55	6.46	0.70
7	7.84	7.84	7.72	0.92
8	9.32	9.32	9.19	1.16

To make the comparison between the attractive well and the corresponding barrier more complete, we examine the variation of the barrier top resonance position E_n with n and compare it with the corresponding positive energy eigenvalues

$$E_n = n^2\pi^2/(2a)^2 + 3, \tag{18}$$

generated by the box potential

$$V(x) = \begin{cases} V_0 = 3 & (|x| < a) \\ \infty & (|x| > a). \end{cases} \tag{19}$$

In Fig. 4 we compare these with the energies of the barrier top resonance state energies. The correlation between box states and barrier top states is close as in the case for the attractive well. Figure 2 looks different because of the single sequencing of bound and resonant states generated by the well. The positive energy states generated by Eq. (17) and the corresponding peak position of T match very well. The reason for this common feature can be understood as follows.

The expression for T in Eq. (10) holds for an attractive square well with $\alpha^2 = k^2 + V_0$. The same expression is valid for the barrier for $k^2 > V_0$ with $\alpha^2 = k^2 - V_0$. We have taken the width $2a$ of the well and the barrier to be large (a=5) to

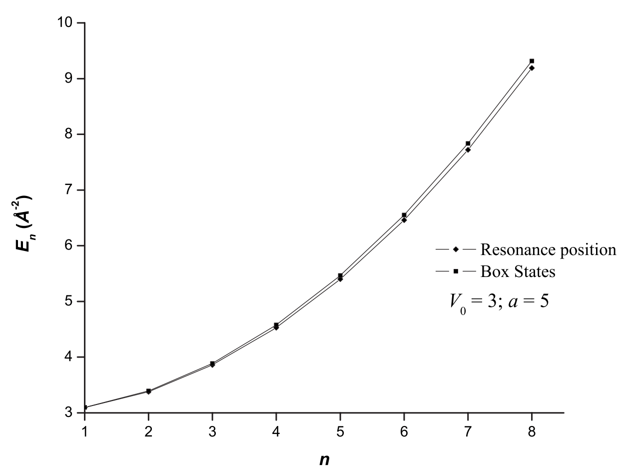

Fig. 4. Comparison of the barrier top resonant state energies of the repulsive rectangular potential (17) with V_0=3 and a=5 to the corresponding states for the box potential (19). The abscissa is the sequence number of the states.

generate a greater number of states. For both the well and the barrier, the maxima of T are governed by the zeros of $\sin(2\alpha a)$. Thus for the attractive well, the peaks of T are at positive energies, signifying that the resonances occur at $E_n=(n^2\pi^2/(2a)^2)-V_0>0$. Such a close correlation is not present for the negative energy states, as is clear from Table I. For the barrier the peaks of T are at $E_n=(n^2\pi^2/(2a)^2)+V_0$ for $n=1,2,\ldots$. Both sets of E_n are eigenvalues of the corresponding box potentials. In contrast, resonant state energies and widths are generated from the complex poles of T, and their positions are slightly shifted from the maxima of T because the resonances are broader. Tables I and II summarize these results.

IV. EFFECT OF ADJACENT WELL ON BARRIER TRANSMISSION

We study here the effect of an attractive well on the tunneling probability across a repulsive barrier for incoming particle energies below the barrier. The most common procedure for studying tunneling across a potential barrier at energies below the barrier is the WKB expression for the transmission coefficient.[10] The WKB expression for T is given by

$$T=\left(\Theta+\frac{1}{4\Theta}\right)^{-2}\approx\left(\frac{1}{\Theta}\right)^{2}$$

$$=\exp\left[-2\int_{x_1}^{x_2}\sqrt{(V(x)-k^2)}dx\right], \tag{20}$$

where the Gamow factor Θ^{-2} is given by

$$\Theta=\exp\left[\int_{x_1}^{x_2}\sqrt{(V(x)-k^2)}dx\right]. \tag{21}$$

Here x_1 and $x_2>x_1$ are the turning points, which define the classically forbidden region $x_1<x<x_2$ of the barrier at $E=k^2$.

This semiclassical approach to tunneling incorporates the potential between the turning points at a given energy but ignores the potential elsewhere resulting in the expression for T given by Eq. (20), which is independent of $V(x)$ in the domains $x<x_1$ and $x>x_2$. In the light of our discussion on the transmission across a well, it is interesting to see the difference that an adjacent well makes in tunneling across a barrier. For this purpose we consider the potential

$$V(x)=\begin{cases}0 & (x<-a)\\ -V_0 & (-a<x<a)\\ V_1 & (a<x<b)\\ 0 & (x>b).\end{cases} \tag{22}$$

This potential is a combination of an attractive well followed by a barrier $(V_0,V_1>0)$. In Fig. 5 we show the variation of T for energies below and above the barrier for $V_0=3$, $a=5$, $V_1=6$, and $b-a=0.5$. We have kept $b-a$ small to reduce the excessive damping of T by the barrier at energies below V_0. Figure 5 also gives the variation of T generated by only the attractive well [Eq. (2)] and only the repulsive barrier with width $(b-a)$. The variation of T for the potential given by Eq. (22) generates oscillatory structures related to the resonant states of the well. If we used a WKB approach, we would have obtained a smoothly increasing curve up to

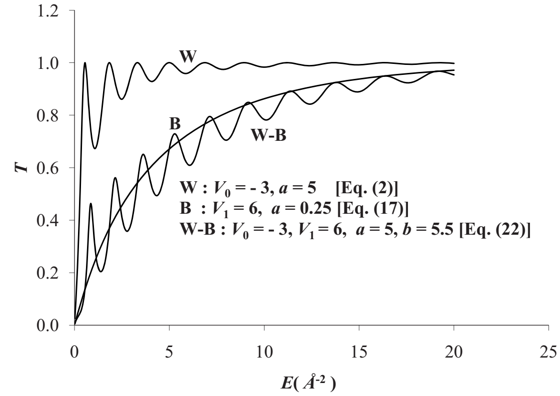

Fig. 5. Plot of $T(E)$ for the attractive square well (2) with $V_0=-3$, $a=5$ as indicated by W; the repulsive rectangular barrier [Eq. (17)] with $V_0=6$ and $a=0.25$ is indicated by B, and the combination of attractive square well and repulsive barrier (22) with $V_0=-3$, $V_1=6$, $a=5$, and $b=5.5$ is indicated by W-B.

$E=V_0$ and missed the oscillatory features. However, T for a barrier only gives the overall variation of T, implying that the WKB approach provides a reasonable approximation of T but does not incorporate finer details. The variation of T by only a potential well deviates farther from the results obtained by using Eq. (22).

V. TRANSMISSION ACROSS A WELL WITH NO OSCILLATIONS

We may be tempted to ask if T for an attractive well with bound states always shows oscillations with E. Reference 13 studied the nature of the resonances generated by the potential pocket sandwiched between well separated barriers and also the barrier top resonances in three-dimensional scattering for rectangular type potentials and for more smoothly varying potentials, and demonstrated that the nature of the results in both cases is similar. To have a sharper resonance above the barrier, it is necessary to have a broader barrier. In the light of this result we might expect that the general pattern of resonances we have found for the attractive rectangular well and rectangular barrier is applicable even when they are replaced by smoother potentials. However there can be exceptions. Such an exception is given by the behavior of T for the modified Poschl–Teller type potential,[16]

$$V(x)=-\beta^2\frac{\lambda(\lambda-1)}{\cosh^2\beta x}. \tag{23}$$

This potential has bound states given by

$$E_n=-\beta^2(\lambda-1-n)^2 \quad (n\leq\lambda-1). \tag{24}$$

The expression for T is given by

$$T=\frac{p^2}{1+p^2}, \tag{25}$$

where

$$p=\frac{\sinh(\pi k/\beta)}{\sin(\pi\lambda)}. \tag{26}$$

An interesting feature of this potential is that $T=1$ for integer $\lambda\geq1$. Our interest here is to examine T for a typical set of λ and β. In Fig. 6 we show the variation of T for three

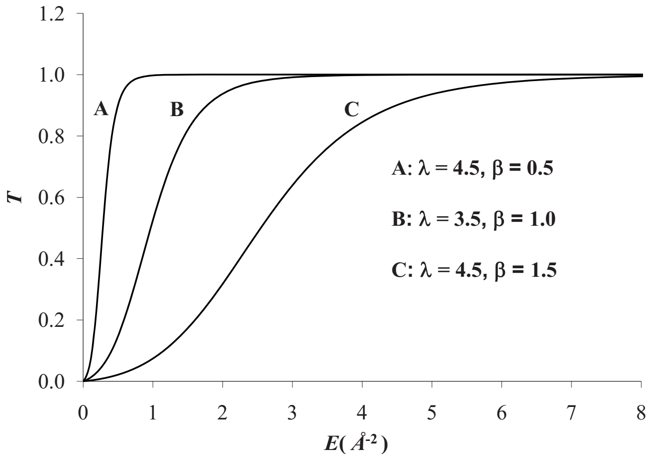

Fig. 6. Plot of $T(E)$ for the Poschl–Teller potential (23).

sets of λ and β. No oscillatory structure is present, which implies that no resonant states are generated by this attractive potential. This property is also evident from Eq. (25). There also is the important case of the attractive Coulomb potential $U(r)=-\gamma/r$, with $\gamma>0$, which generates an infinite number of bound states but no resonant states. In this case the bound states are associated with the poles of the S-matrix for the ℓth partial wave given by (Ref. 14, p. 429)

$$S_l = \frac{\Gamma(l+1-i\eta)}{\Gamma(l+1+i\eta)}. \tag{27}$$

Here $\eta=\gamma/2k$ is the Rutherford parameter associated with the Coulomb potential. The repulsive Coulomb potential with $\gamma<0$ also is an example that does not generate resonant states.

VI. SUMMARY AND CONCLUSIONS

For the attractive square well and its repulsive counterpart, we have shown that the oscillations in T as a function of energy correspond to the broad resonant state energies obtained by using boundary conditions satisfied by the resonant state wave function. We found that the resonant energies and their widths can be obtained in terms of the complex poles of the transmission amplitude or reflection amplitude in the lower half of the complex E-plane in the vicinity of the real axis. This result is similar to the corresponding behavior of S-matrix poles in three-dimensional scattering and hence provides a unified understanding of transmission in one dimension and potential scattering in three dimensions in terms of the pole structure of the corresponding amplitudes. For an attractive square well and the rectangular barrier, the positions of the maxima of T are the same as the energies of the states of the corresponding one-dimensional box potentials. The resonance state energies are also close to the peak positions of T. For a potential that is a combination of an attractive well and a repulsive barrier, T exhibits oscillations even at energies below the barrier, in contrast to the WKB type barrier expression for T, which does not manifest these features. Hence simple WKB type expressions for T may not be reasonable approximation if the total potential has physical features such as an attractive well outside the barrier region. The oscillations of T across an attractive potential are likely to be found for most potentials. But these features can have exceptions such as was shown for the attractive modified Poschl–Teller type potential.

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[1]See, for example, V. K. Thankappan, *Quantum Mechanics*, 2nd ed. (New Age, New Delhi, 1996), pp. 250–253.

[2]Hiroki Hamaguchi, Hiroaki Yamamoto, and Norifumi Yamada, “Theoretical study on tunneling phenomena in asymmetrical rectangular double-barrier structures with acceleration well,” Electron. Commun. Jpn., Part 2: Electron. **87**, 1–11 (2004).

[3]A. Uma Maheswari, S. Mahadevan, P. Prema, and C. S. Shastry, “Quasi-bound states, resonance tunneling, and tunneling times generated by twin symmetric barriers,” Pramana, J. Phys. **73**, 969–988 (2009).

[4]Mohsan Razavy, *Quantum Theory of Tunneling* (World Scientific, Singapore, 2003).

[5]M. C. Goorden, Ph. Jacquod, and J. Weiss, “Superconductivity-induced macroscopic resonant tunneling,” Phys. Rev. Lett. **100**, 067001-1–4 (2008).

[6]V. S. Olkhovsky, E. Recami, and G. Salesi, “Superluminal tunneling through two successive barriers,” Europhys. Lett. **57**, 879–884 (2002).

[7]S. A. Rakityansky, “Unified treatment of bound, scattering, and resonant states in one-dimensional semiconductor nanostructures,” Phys. Rev. B **68**, 195320-1–17 (2003).

[8]A. P. Barbero, H. E. Hernández-Figueroa, and E. Recami, “Propagation speed of evanescent modes,” Phys. Rev. E **62**, 8628–8635 (2000).

[9]E. P. F. Lyngdoh, I. Jamir, and C. S. Shastry, “Schrödinger equation with complex potentials: The phenomenon of absorption,” Phys. Educ. **15**, 145–153 (1998).

[10]S. Mahadevan, A. Uma Maheswari, P. Prema, and C. S. Shastry, “Quantum mechanical transmission with absorption,” Phys. Educ. **23**, 13–21 (2006).

[11]S. Mahadevan, P. Prema, S. K. Agarwalla, B. Sahu, and C. S. Shastry, “Resonance-like tunneling across a barrier with adjacent wells,” Pramana J. Phys. **67**, 401–413 (2006).

[12]Zhi-an Shao, Wolfgang Porod, Craig S. Lent, and David J. Kirkner, “An eigenvalue method for open-boundary quantum transmission problems,” J. Appl. Phys. **78**, 2177–2186 (1995).

[13]P. Susan, B. Sahu, B. M. Jyrwa, and C. S. Shastry, “Pocket and barrier resonances in potential scattering and their application to heavy-ion reactions,” J. Phys. G **20**, 1243–1256 (1994).

[14]R. G. Newton, *Scattering Theory of Waves and Particles* (McGraw-Hill, New York, 1966).

[15]W. A. Friedman and C. J. Goebel, “Barrier-top resonances and heavy ion reactions,” Ann. Phys. (N.Y.) **104**, 145–183 (1977).

[16]Siegfried Flügge, *Practical Quantum Mechanics* (Springer-Verlag, New York, 1994), pp. 94–100.