

## RETARDATION EFFECTS ON INTRA- AND INTERSUBBAND PLASMONS IN QUASI-ONE-DIMENSIONAL QUANTUM-WELL WIRES

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The spectrum of the intra- and intersubband plasmon-polaritons of a quasi-one-dimensional quantum-well wire is investigated. The polarization tensor is treated in detail within the random-phase approximation. The dispersion curves of the collective excitations are calculated and presented in graphical form.

A broad range of fundamental research and novel applications in many fields of semiconductor physics was initiated by the progress in crystal growth techniques of the last decade which made it possible to fabricate layered semiconductor heterostructures precise in atomic-scale. These novel systems have unique physical properties which arise from the quasi-two-dimensional (Q2D) behaviour of the carriers. One of the challenging topics of current interest involves systems of further reduced dimensionality, namely Q1D quantum-well wires (QWW) and Q0D quantum dots (QD). Q1D and Q0D systems have been prepared by starting from a Q2D system employing high-resolution nanometer lithographic techniques or by using novel growth techniques. The study of these low-dimensional quantum-confined systems has gained a great deal of attention in the past few years.

The most prominent collective excitation of semiconductor nanostructures is the plasmon. In a QWW the confinement potential acts in two spatial directions and hence, caused by size-quantization the single-

particle and collective excitations of the quasi-one-dimensional electron gas (Q1DEG) are splitted into intra- and intersubband excitations.<sup>1-11</sup> These excitations are measured with far-infrared (FIR) spectroscopy<sup>11-13</sup> and Raman-scattering.<sup>14</sup>

The aim of this paper is to investigate the retardation effects on the dispersion relation of intra- and intersubband plasmons in parabolic QWW's. We study the QWW by a model in which the electrons are confined in a zero thickness  $x-y$  plane along the  $z$ -direction at  $z = 0$ . In the  $y$ -direction a parabolic quantum well is assumed. Following in the effective-mass approximation the one-electron envelope wave function  $\xi_L(y)$  and the corresponding subband energy  $\mathcal{E}_L$  are given by an one-dimensional effective Schrödinger equation, where the electron is confined in an effective potential  $V_{eff}(y)$ . This lateral effective confinement potential is a sum of the bare initial potential  $V_0(y)$ , the Hartree-potential  $V_H(y)$  and the exchange-correlation potential  $V_{xc}(y)$ . In general it is

necessary to distinguish two different cases for  $V_{eff}(y)$ : (i) The first is realized in semiconductor nanostructures with electrostatic confinement. In this case the resulting  $V_{eff}(y)$  is parabolic for small electron densities. (ii) The second case is realized in semiconductor nanostructures with a bare initial ideal parabolic confinement potential  $V_o(y) = m\Omega^2 y^2/2$ . Electrons which arise from donor impurities located away from the quantum well, enter the well and screen this bare initial parabolic potential changing the shape of the resulting potential  $V_{eff}(y)$ . Here we are concerned with case (i).

The current-response of the Q1DEG gives the dispersion relation of the collective excitations including retardation effects, the intra- and intersubband plasmon-polaritons. Plasmon-polaritons are coupled collective excitations formed of the collective excitations of the solid and photons. The dispersion relation reads<sup>18</sup>

$$\det[\delta_{\alpha\gamma}\delta_{LL'}\delta_{L'L_2} - \mu_o D_{\alpha\beta}^{L_1 L_2 L' L'}(q_x, \omega) \chi_{\beta\gamma}^{L L'}(q_x, \omega)] = 0. \quad (1)$$

$D_{\alpha\beta}^{L_1 L_2 L_3 L_4}(q_x, \omega)$  are the matrix elements of the electrodynamic Green's tensor of the inhomogeneous wave equation

$$D_{\alpha\beta}^{L_1 L_2 L_3 L_4}(q_x, \omega) = \int_{-\infty}^{\infty} dx e^{-iq_x(x-x')} \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dy' * \\ * \zeta_{\alpha}^{L_1 L_2}(y) D_{\alpha\beta}(\vec{x}_{\parallel}, \vec{x}'_{\parallel} | \omega) \zeta_{\beta}^{L_3 L_4}(y') \quad (2)$$

where  $\vec{x}_{\parallel} = (x, y, 0)$  is the position vector in the  $x-y$  plane ( $\alpha, \beta = x, y, z$ ) and

$$\zeta_{\alpha}^{L L'}(y) = \delta_{\alpha x} \eta_{LL'}(y) + \delta_{\alpha y} g_y^{L L'}(y), \quad (3)$$

with

$$\eta_{LL'}(y) = \xi_L(y) \xi_{L'}(y) \quad \text{and} \\ g_y^{L L'}(y) = \xi_L(y) \frac{d\xi_{L'}(y)}{dy} - \xi_{L'}(y) \frac{d\xi_L(y)}{dy}. \quad (4)$$

The electrodynamic Green's tensor of the inhomogeneous wave equation is given by

$$D_{\alpha\beta}(\vec{x}_{\parallel}, \vec{x}'_{\parallel} | \omega) = \frac{c^2}{\epsilon_s \omega^2} \frac{\partial^2}{\partial x_{\alpha} \partial x_{\beta}} G(\vec{x}_{\parallel}, \vec{x}'_{\parallel} | \omega) + \\ + \delta_{\alpha\beta} G(\vec{x}_{\parallel}, \vec{x}'_{\parallel} | \omega), \quad (5)$$

with

$$G(\vec{x}_{\parallel}, \vec{x}'_{\parallel} | \omega) = \frac{e^{i\sqrt{\epsilon_s} \frac{c}{\omega} |\vec{x}_{\parallel} - \vec{x}'_{\parallel}|}}{4\pi |\vec{x}_{\parallel} - \vec{x}'_{\parallel}|} \quad (6)$$

neglecting image effects. Further,  $\chi_{\alpha\beta}^{L L'}(q_x, \omega)$  of Eq.(1) is the matrix polarisation tensor of the Q1DEG defined by<sup>15</sup>

$$\chi_{\alpha\beta}^{L L'}(q_x, \omega) = \\ = \begin{cases} P_{\alpha\beta}^{L L'}(q_x, \omega) & \text{if } L = L' \\ & \text{and } \alpha = \beta = x, \\ P_{\alpha\beta}^{L L'}(q_x, \omega) + P_{\alpha\beta}^{L' L}(q_x, \omega) & \text{if } L \neq L' \\ & \text{and } \alpha = \beta, \\ P_{\alpha\beta}^{L L'}(q_x, \omega) - P_{\alpha\beta}^{L' L}(q_x, \omega) & \text{if } L \neq L' \\ & \text{and } \alpha \neq \beta, \\ 0 & \text{else} \end{cases} \quad (7)$$

where

$$Re P_{xx}^{L L'}(q_x, \omega) = \frac{e^2}{4\pi m q_x} \left\{ 4q_x(k_F^L + k_F^{L'}) + \frac{8m}{\hbar q_x} * \right. \\ * (\omega - \Omega_{LL'})(k_F^{L'} - k_F^L) + \frac{4m^2}{\hbar^2 q_x^2} (\omega - \Omega_{LL'})^2 * \\ * \left. \ln \left| \frac{k_+ k'_+}{k_- k'_-} \right| \right\},$$

$$Re P_{yy}^{L L'}(q_x, \omega) = \frac{e^2}{4\pi m q_x} \left\{ \ln \left| \frac{k_+ k'_+}{k_- k'_-} \right| \right\} - \frac{\hbar e^2 k_F^{L'}}{\pi m^2 \Omega_{LL'}}, \quad (9)$$

$$Im P_{xy}^{L L'}(q_x, \omega) = -Im P_{yx}^{L L'}(q_x, \omega) = \frac{e^2}{2\pi m q_x} * \\ * \left\{ 2(k_F^L - k_F^{L'}) - \frac{m}{\hbar q_x} (\omega - \Omega_{LL'}) \ln \left| \frac{k_+ k'_-}{k_- k'_+} \right| \right\}, \quad (10)$$

with  $k_{\pm} = k_F^L \pm \left[ \frac{q_x}{2} + \frac{m}{\hbar q_x} (\omega - \Omega_{LL'}) \right]$  and  $k'_{\pm} = k_F^{L'} \pm \left[ \frac{q_x}{2} - \frac{m}{\hbar q_x} (\omega - \Omega_{LL'}) \right]$ . Outside of the single-particle continua,  $Re P_{xy}^{L L'}(q_x, \omega) = Re P_{yx}^{L L'}(q_x, \omega) = Im P_{xx}^{L L'}(q_x, \omega) = Im P_{yy}^{L L'}(q_x, \omega) = 0$  is valid. In Eqs.(2)-(10)  $m$  is electron effective mass and  $\epsilon_s$  is static dielectric constant of the semiconductor containing the Q1DEG,  $k_F^L = [2m(E_F - \mathcal{E}_L)/\hbar^2]^{1/2}$  if  $E_F > \mathcal{E}_L$  and zero for  $E_F \leq \mathcal{E}_L$ .  $\Omega_{LL'} = (\mathcal{E}_L - \mathcal{E}_{L'})/\hbar$  is the subband separation frequency and  $E_F$  is the Fermi energy.

Due to the spatial symmetry of the effective confinement potential  $V_{eff}(y)$  the system of algebraic equations (1) splits into the dispersion relation of symmetric intra- and intersubband plasmons and the dispersion relation of antisymmetric intersubband plasmons. Here we are mainly interested in the investigation of the retardation effects on the plasmons. Hence, to solve the complicated algebraic dispersion relation

we restrict ourself to the diagonal approximation and, further, we assume that only the lowest subband is occupied. In the diagonal approximation the coupling between the different plasmon-polariton modes is neglected.

For numerical work we have chosen a  $GaAs - Ga_{1-x}Al_xAs$  QWW with  $\hbar\Omega = 2$  meV:  $\epsilon_s = 12.87$  and  $m = 0.06624m_0$ .

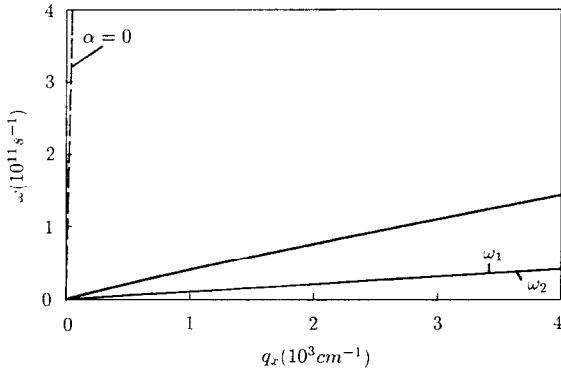


Fig.1 Dispersion relation of the intrasubband plasmon-polariton (heavy solid lines) where one subband is occupied ( $n_{1DEG} = 3.7 \cdot 10^5 cm^{-1}$ ). The thin solid lines are the boundaries  $\omega_1$  and  $\omega_2$  of the single-particle intrasubband continuum. The dashed line is the light line  $\alpha = 0$ .

Figure 1 shows the full RPA dispersion curve of the intrasubband plasmon-polariton. The thin solid lines in the  $\omega - q_x$  plane are the boundaries of the region where the single-particle intrasubband excitations exist. Notice that for such small wave vectors as plotted this region is very narrow. Inside of this region  $ImP_{xx}^{00}$ ,  $ImP_{yy}^{00}$  and  $ReP_{xy}^{00}$  are nonzero and, hence, the intrasubband plasmon-polaritons are Landau-damped. From Figure 1 it is seen that the intrasubband plasmon-polariton starts for  $q_x = 0$  at  $\omega = 0$ . This is also true if one neglects the retardation. The corresponding collective excitation is the intrasubband plasmon. Our result is that the retardation effect on the intrasubband plasmon is very small and, hence, the dispersion curves of the intrasubband plasmon and of the intrasubband plasmon-polariton are practically

indistinguishable. The retardation effect causes the intrasubband plasmon-polariton to approach the light line  $\alpha = (q_x^2 - \epsilon_s \omega^2 / c^2)^{1/2} = 0$ . Further, this dispersion curve is always located to the right of this light line in the  $\omega - q_x$  plane. For small wave vectors  $q_x$  and small  $\alpha$  the dispersion relation, Eq.(1), reads

$$\omega = \omega_s |q_x| l_\Omega \sqrt{\frac{\ln(\alpha l_\Omega)}{\epsilon_s \omega_s^2 l_\Omega \ln(\alpha l_\Omega) - 1}} \quad (11)$$

with  $\omega_s = [e^2 k_F^0 / (\pi^2 \epsilon_0 \epsilon_s m l_\Omega^2)]^{1/2}$  and  $l_\Omega = [\hbar / (m\Omega)]^{1/2}$

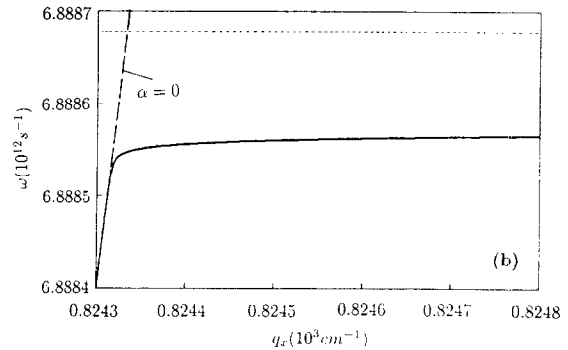
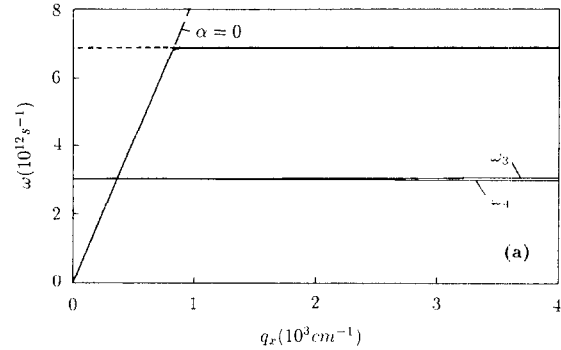


Fig.2 Dispersion relation of the (1 - 0) intersubband plasmon-polariton (heavy solid line) and the corresponding unretarded intersubband plasmon (dotted line) where one subband is occupied ( $n_{1DEG} = 3.7 \cdot 10^5 cm^{-1}$ ). The thin solid lines are the boundaries  $\omega_3$  and  $\omega_4$  of the single-particle intersubband continuum. The dashed line is the light line  $\alpha = 0$ . (b) shows the dispersion relation of the intersubband plasmon-polariton and the corresponding intersubband plasmon near the resonance splitting.

is the effective width of the parabolic quantum well. For  $c \rightarrow \infty$  Eq.(11) results in the well-known long-wavelength approach of the dispersion relation of the Q1D intrasubband plasmon.

In Figure 2 the full RPA dispersion curve of the intersubband plasmon-polariton is plotted. The thin solid lines in the  $\omega - q_x$  plane are the boundaries of the single-particle intersubband continuum. It is seen that the dispersion curve is again always located to the right of the light line  $\alpha = 0$  in the  $\omega - q_x$  plane. In this region the difference between the plasmon-polariton and the plasmon is very small but more pronounced than in the case of the intrasubband modes. The region where the intersubband plasmon crosses the light line  $\alpha = 0$  is plotted in Figure 2(b). It is seen that the retardation shifts the dispersion curve to lower frequencies. But this is valid only in the very near vicinity of the line  $\alpha = 0$ .

Using the diagonal approximation we have shown that all plasmons of a Q1D QWW are influenced by retardation and, hence, are plasmon-polaritons. Following, all dispersion curves are located to the right of the light line  $\alpha = 0$ . Comparing the obtained results of the here used simple model with those using a more complete model to include the coupling between the modes, but neglecting retardation<sup>8</sup>, we can conclude: the retardation should influence the modes in the same manner, as if one considers the coupling between the modes and the occupation of more than one subband. In general the retardation influences the antisymmetric modes more than the symmetric ones.

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