# Optical implementation of Boltzmann machine for travelling salesman problem

**N.Yu.Kulakov, V.A.Ivanov, B.S.Kiselyov**
Russian Academy of Science,
Institute of Optical Neural Technologies,
App.55, 44-2, Vavilov St., Moscow, 117333, Russia
Tel.: (095) 943 - 4179, E-mail: iont@glas.apc.org,
Fax: (095) 925 - 5972, box K-03

## Abstract

The Boltzmann machine (BM) for travelling salesman problem is considered. The BM modification having only the distance connections between neurons and using the "column replacement" rule is proposed. Computer simulation results are presented. The optoelectronic hardware of this BM is discussed.

## 1.Introduction

The improvement of traditional methods for solving NP-complete combinatorial optimization problems has been successfully made over the past 30 years. Now these approaches have exhausted substantially, because non-traditional methods using parallel computing have attracted much attention lately.

It was J. Hopfield who showed how a feedback neural network could be used for solving optimization problems [1], in particular the travelling salesman problem (TSP) [2]. Later on, the stochastic "simulated annealing" method [3] was added to Hopfield's approach and the BM was proposed by D.Ackley, G.Hinton and T.Sejnowski [4].

The BM was originally used in creating associative memory or classifier. E.Aarts and J. Korst argued that computer architectures based on BM may also provide computational power required for carrying out the complex calculations imposed by e. g. combinatorial search and combinatorial optimization [5]. The approach is based on the observation that the structure of many combinatorial optimization problems can be mapped directly onto the structure of the connections between the BM computer units.

It should be noted that the BM has a massively parallel architecture so the solution process is the fastest if multi processor computer systems are used. Among such systems the neurocomputer and the optical neurocomputer hardware are more attractive. In this paper we discuss the modification of BM for TSP and its optoelectronic implementation.

## 2. Boltzmann machine

The BM being one of the neural paradigms is a connectionist model in which the information is represented as strengths of connections between individual computing units. The BM consists of a network of simple computing units (neurons) having two discrete states, viz. "1" and "0". The neurons are connected in some way and have effect on states of each other. The neurons try to reach a maximum consensus as to their individual states. A stochastic mechanism controls state transitions of neurons.

 *0-8194-1778-5/94/$6.00*

Let the BM consists of N neurons $V_i$ linked by connections $S_{ij}$. The energy of the system is defined as

$$E = \sum_{i,j}^{N} S_{ij} V_i V_j . \tag{1}$$

The energy variation $\Delta E_k$ due to a transition of neuron $k$ $\Delta V_k = V_k^{'} - V_k$ is:

$$\Delta E_k = \Delta V_k \left( \sum_{j}^{N} S_{ij} V_j + S_{kk} \right). \tag{2}$$

The probability of transition is:

$$P = 1 / (1 + e^{-\Delta E_k / T}), \tag{3}$$

where parameter $T$ denotes the temperature of the system. To achieve the global or near global energy minimum the simulated annealing algorithm is applied with a temperature starting from some high value and decreasing sufficiently slowly, so that the system reaches thermal equilibrium at each temperature.

## 3. Travelling salesman problem

The TSP is a famous combinatorial optimization problem that has attracted much attention over the years. It can be formulated as follows. Let N be the number of cities and $\{d_{ij}\}$ be the distance matrix whose entries $d_{ij}$ denote the length of the shortest path from city $i$ to city $j$. Then TSP is the problem of finding a tour of minimal length, visiting each of the N cities exactly once.

Let binary variable $x_{ip}$ 1 or 0 indicates whether or not the tour visits city $i$ at $p$-th position. The TSP is formulated as:

minimize
$$F = \sum_{i,j,p,q}^{N-1} a_{ijpq} x_{ip} x_{jq} , \tag{4}$$

subject to
$$\sum_{i}^{N} x_{ip} = 1, \qquad p = 1, \ldots, N, \tag{5}$$

$$\sum_{p}^{N} x_{ip} = 1, \qquad i = 1, \ldots, N, \tag{6}$$

where
$$a_{ijpq} = \begin{cases} d_{ij} & if \quad q = (p+1) \bmod N \\ 0 & otherwise \end{cases}, \tag{7}$$

i.e. $d_{ij}$ only contributes to the cost function if the tour goes directly from city $i$ to city $j$.

## 4. TSP on a Boltzmann machine

The approach to use a BM for solving TSP is based [5]:

- a structure of a BM is chosen such that an instance of the TSP can be directly mapped on to this structure;
- the energy of the BM represents the cost function ( tour length ) in the TSP.

Let the 2-D arrangement of binary neurons $V_{ip}$ be used to define the variables $x_{ip}$. Let $S(V_{ip}, V_{jq})$ be the strength of the connection between $V_{ip}$ and $V_{jq}$ neurons. The energy of the system is defined in accordance with the equation (1).

$$E = \sum_{i,j,p,q}^{N} S(V_{ip}, V_{jq}) V_{ip} V_{jq} \quad . \tag{8}$$

Consider the structure of BM for TSP in detail. The state of neuron $V_{ip}$ denotes whether or not the tour visits city $i$ at $p$ position. The state matrix $\{V_{ip}\}$ determining one of possible configurations of neuron states is shown in Fig.1.

$$\begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{matrix}$$

Fig.1. The state matrix determining the tour configuration.

Aarts and Korst use tree types of connections [5]:

- distance connections for $i \neq j \cap q = (p+1) mod N;$
- inhibitory connections for $(i = j \cap p \neq q) \cup (i \neq j \cap p = q);$
- bias connections for $i = j \cap p = q.$

The distance connections link only neurons determining the visit of two cities when the tour directly goes to one city from another one. The strength of distance connection between neurons $V_{ip}$ and $V_{jq}$ is equal to the distance between cities $i$ and $j$. The inhibitory connections connect all neurons of the same row or column of the state matrix. These connections prevent from visiting every city more than one time and visiting some cities at the same position in tour. The bias connections link every neuron with itself and cause visiting every city.

The inhibitory connections contribute inversely to the bias connections. They should be chosen such that local minima of system energy are corresponded only with the BM's configurations in which every row and column of the state matrix contains exactly one unity. We will call such BM's configurations as tour configurations. All the other configurations where the number of non-zero neuron states in each row (or column) is excess of or less than one are unstable, and the system tends to evolve to a tour configuration.

The process of the BM convergence to a configuration with global ( or near global ) energy minimum appears to be strongly dependent on the strengths of the inhibitory and bias connections. Computer simulations show that the system can reach a tour configuration only when operating in the serial mode. In the parallel mode, the BM slides down to a non-controllable process of updating states of several neurons whose number grows, so that the process finally involves every neuron. This can be avoided by reducing the number of neurons that can be simultaneously switched over and increasing the strengths of the inhibitory and bias connections as compared to the strengths of the distance connections. On the one hand, this produces a positive effect: evolution of neuron states always results in tour configurations. However, as the distance connection strengths contribute less to the system energy, the difference between energy minima corresponding to tours of different length vanishes. The global minimum slightly differs from other local minima, and the algorithm for finding the shortest tour becomes low- effective.

## 5. The Boltzmann machine with "column replacement"

To avoid the above disadvantages we propose to modify BM:

- removing the inhibitory and bias connections;

- using the "column replacement" rule to update the state matrix leading to transition of four neurons in parallel. It should be noted that new tour configuration of BM can be obtained by exchanging two columns (or rows) of the state matrix $\{V_{ip}\}$. It results in changing the states of four neurons. In our modification only the distance connections exist, so the energy of the system is defined as:

$$E = \sum_{i,j,p}^{N} d_{ij} V_{ip} V_{jp+1} . \tag{9}$$

The energy change due to transition of one neuron $V_{kr}$ is:

$$\Delta E_{kr} = \sum_{j \neq k}^{N} d_{jk} (V_{jr-1} + V_{jr+1}) \Delta V_{kr} . \tag{10}$$

Because $\Delta V_{kr}$ = +1 or -1, define the action force driving the state of neuron $V_{kr}$.

$$F_{kr} = \sum_{j \neq k}^{N} d_{jk} (V_{jr-1} + V_{jr+1}) . \tag{11}$$

Let two columns $r$ and $s$ must be exchanged. Assume that in the initial moment $V_{kr}$ and $V_{ms}$ are on. After rearrangement of columns $r$ and $s$, these neurons will be off and neurons $V_{mr}$ and $V_{ks}$ will be on. The action force driving the states of four neurons is:

$$F_{rs} = ( F_{mr} - F_{kr} + F_{ks} - F_{ms} ) . \tag{12}$$

The probability of setting neuron $V_{mr}$ and $V_{ks}$ to 1 is:

$$P = \exp(-F_{rs} / T) . \tag{13}$$

## 6.Simulation results

We have carried computer simulations of the our BM modification for different problem instances, i.e. with 10, 20, 30 cities. Simulations were implemented on PC-386 computer. The parameters of cooling schedule for TSP with 30 cities were follows:

- the initial temperature is $T_0 = 15$;

- the temperature decreasing rule $T = T_0 0.9999^n$, where $n$ is the number of iteration.

Fig.2 shows the tour of shortest length obtained by computer simulations. The average number of column replacements were 285187. The worthiest tour had length of 149.3 while the length of the best determined tour was 134.3. Thus the relative difference in tour lengths was about 11%. This is good result because the cooling was not slow enough.

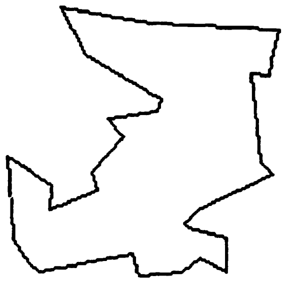

Fig.2. The best tour determined by computer simulations.

## 7. Optical implementation

Below we discuss the practical implementation of the BM algorithm using parallel computing structures. These very structures can provide for the fastest computing speed. The need in parallel operations arises as soon as the energy increment is computed due to a transition of the state of the neuron $V_{ip}$.
Introduce a modified matrix $\{V^*_{ip}\}$ determining for the $p$-th position of visiting the $i$-th town, the previous $(p-1)$th and next $(p+1)$th positions.

$$.V^*_{ip} = V_{ip-1} + V_{ip+1}. \tag{14}$$

Fig.3 shows the matrix $\{V^*_{ip}\}$ corresponding to the state matrix $\{V_{ip}\}$. Using (14) transform the expression (11) to determine the action force $F_{ip}$.

$$\begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{matrix}$$

Fig.3. The modified state matrix

$$F_{ip} = \sum_{j \neq i}^{N} d_{ji} V^*_{jp} \ . \tag{15}$$

These computations can be effectively performed by means of a vector-matrix multiplier depicted in Fig.4. Here a linear array of light emitters LMA forms optical signals $V^*_p$ corresponding to the $p$-th matrix column $\{V^*_{ip}\}$. By using an anamorphic lens system (omitted for the sake of simplicity) light beams are smeared in the horizontal plane and come to a spatial light modulator SLM that sets the distance matrix $d_{ji}$. Then, the beams passed through the SLM cells and attenuated in accordance with the $d_{ji}$ values are collected by another anamorphic lens system (too, omitted in the figure for simplicity) in the vertical plane, and come to the PDA. In the result, electric signals are formed at the outputs of the PDA cells, corresponding to $F_{ip}$.

In our algorithm, the above-described operation should be performed for two columns $r$ and $s$ and the state matrix. The simplest solutions is either to double the device shown in Fig.4, or to perform these operations sequentially. However, the first assumes two identical SLMs, which is not only advisable, but also problematic. The second increases the time of computing twice, which is quite undesirable.

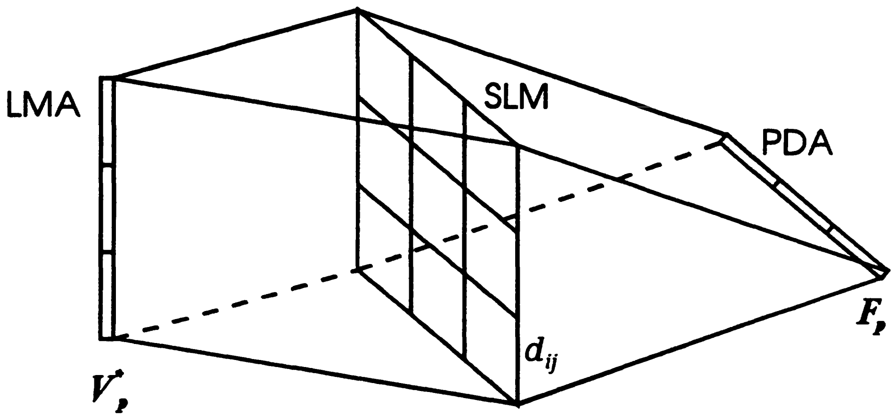


Fig.4. The scheme of the vector-matrix multiplier.

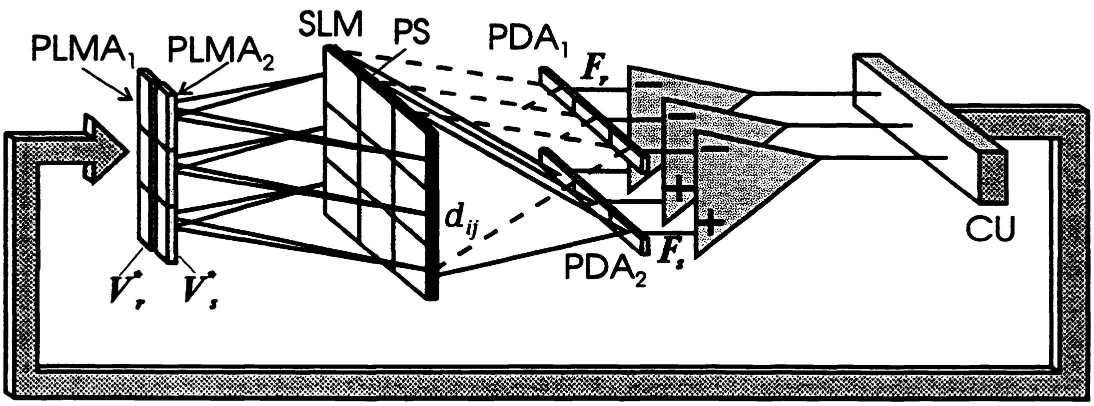


Fig.5. The optoelectronic implementation of BM for TSP.

To multiply simultaneously two vectors by the same matrix, we propose to use the polarization properties of the light, as described in [6]. Fig.5 shows illustrates the optoelectronic multiplier scheme that computes the possible energy increment due to exchange of any two state matrix columns. Two light polarizing modulators $PLMA_1$ and $PLMA_2$ form optical signals of orthogonal polarization, which correspond to the values of two columns $V^*_r$ and $V^*_s$ of the modified state matrix. As in the previous scheme, beams of each modulator pass through the SLM, but they are collected on two linear photodector arrays $PDA_1$ and $PDA_2$. This is achieved by using a polarising beam splitter (PS), e.g. Wollaston prism placed after the SLM. The output signals of the PDAs $F_r$ and $F_s$ are parity subtracted and go to the control unit (CU). The CU computes the action force $F_{rs}$ driving the arrangement of columns $r$ and $s$ in accordance with the equation (11). This can be readily implemented by electronic methods, so not to encumber the explanation by secondary details, we will omit the CU scheme and, instead, confine ourselves by description of its functions. The control unit forms the vector signals $V^*_r$ and $V^*_s$ for controlling the $PLMA_1$ and $PLMA_2$, computes the action force and determines the probability of column replacement . Depending on the result, the columns $r$ and $s$ of the state matrix $\{V_{ip}\}$ are either exchanged or not. To select randomly the columns $r$ and $s$, as well as generate the random value for performing stochastic process, a random number generator is used, implemented by software.

## 8. Conclusion

We have proposed an improved version of the Boltzmann Machine as applied for solving the TSP and its optoelectronic implementation. This BM operates in a series-parallel mode. We sequentially exchange the state matrix columns, 2N neurons exchanging the states. In fact, only four neurons can change their states. The parallelism can be augmented. When two columns are exchanged, the energy of the system is affected, in the general case, by four others columns, two by each of the columns to be exchanged. Hence, N/6 column pairs can be exchanged in one clock, i.e. one third of the neurons will be used. This is the highest parallelism which is possible.

## 9.Acknowledgment

The research described in this publication was made possible in part by Grant # MG7000 from the International Science Foundation and the Fundamental Research Foundation of the Russian Academy of Sciences. The authors wish to thank S.Glebov and L.Shevtsova for technical assistance.

## References.

1. J.J.Hopfield, "Neural Networks and Physical systems with emergent collective computational abilities", *Proc. Nat. Academy Sciences,* 79, 2554-2558, (1982).

2. J.J.Hopfield, D.W.Tank, "Neural Computation of Decision in Optimization Problems", *Biological Cybernetics*, 52, 141-152, (1985).

3. S.Kirkpatrick, C.D.Gelatt, M.P.Vecchi, "Optimization by simulated annealing", *Science*, 220, 671-680, (1983).

4. D.H.Ackley, G.E.Hinton, T.J.Sejnowski, "A Learning Algorithm for Boltzmann Machines", *Cognitive Science*, 9, 147-169, (1985).

5. E.H.L.Aarts, J.H.M.Korst, "Boltzmann Machines for Travelling Salesman Problems", *European Journal of Operational Research*, 39, 79-95, (1989).

6. A.Mikaelian, B.Kiselyov, N.Kulakov, V.Shkitin, V.Ivanov, "Optical Implementation of High-Order Associative Memory", *Optical Computing*, 1, 89-92, (1990).