Progress of Theoretical Physics, Vol. 22, No. 3, September 1959

Electrical Resistance of Ferromagnetic Metals

Isao MANNARI

Department of Physics, Kyoto University, Kyoto

(Received February 18, 1959)

We use a model for ferromagnets such that the electrical current is carried by $4s$ (or $6s$) electrons which are assumed to be described in band scheme and the unpaired electrons ($3d$ or $4f$) are assumed to be localized on the lattice points. In the temperature region far below the Curie point the spin-disorder in spin orientation, which can be naturally described in terms of spin waves, gives rise to the scattering of conduction electrons through the s-d interaction and accordingly contributes to the anomalous part of the resistivity of type T^2, which agrees fairly well with the experimental results both in order of magnitude and in temperature dependence. The effects of the lattice vibration are also discussed and turn out to give only the small additional contribution of type T^5.

§ 1. Introduction

It has long been recognized that the electrical resistivity of the transition metals and of rare earth metals consists of two parts[1),2)]; the one is the resistivity coming from the usual electron-phonon interaction, and the other part is the anomalous resistivity, which is usually larger than the former part up to the Curie point. This anomalous part has been discussed by several authors[1),2),3),4),5)] whose models are classified generally into two types: The one is spin-disorder resistivity[1),2),4),6)] and the other is the so-called s-d transition mechanism[3),7)]. Taking the former point of view, we discuss the anomalous part of the resistivity of ferromagnetic metals in the temperature region far below the Curie point.

The s-d interaction is written as[6)]

$$V=-\sum_{kk'}\sum_{n} J(\boldsymbol{k}'-\boldsymbol{k}) \exp[i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{R}_n]\{(a^*_{k'\uparrow}a_{k\uparrow}-a^*_{k'\downarrow}a_{k\downarrow})S^z_n + a^*_{k'\uparrow}a_{k\downarrow}S^-_n + a^*_{k'\downarrow}a_{k\uparrow}S^+_n\}, \tag{1}$$

where $a^*_{k\uparrow}$ and $a_{k\uparrow}$ are usual creation and annihilation operators for the conduction electron with spin upwards moving through the lattice with wave vector $\boldsymbol{k}$, and $a^*_{k\downarrow}$, $a_{k\downarrow}$ refer to the electron having spin downwards. $J(\boldsymbol{k}'-\boldsymbol{k})$ is given by[6)]

$$J(\boldsymbol{k}'-\boldsymbol{k})=\iint \varphi^*_n(1)\phi_{k'}{}^*(2)\frac{e^2}{r_{12}}\varphi_n(2)\phi_k(1)\,d\boldsymbol{r}_1\,d\boldsymbol{r}_2 \exp[i(\boldsymbol{k}'-\boldsymbol{k})\cdot\boldsymbol{R}_n] \tag{2}$$

and is easily shown to be independent of $\boldsymbol{R}_n$, where $\varphi_n(\boldsymbol{r})$ and $\phi_k(\boldsymbol{r})$ are the wave functions of unpaired electron localized on the n-th lattice point and conduction electron of wave vector $\boldsymbol{k}$, respectively. $\boldsymbol{S}_n$ is the spin operator of the unpaired electron localized at the n-th lattice point, and if the Hund-rule is effective in the unfilled shell on each lattice point the above operator $\boldsymbol{S}_n$ may be considered to represent the total spin angular momentum at the n-th lattice point and then the above quantity $J(\boldsymbol{k}'-\boldsymbol{k})$ must be understood to represent some relevant mean of J over the states occupied in each unfilled shell.

We now classify the types of spin-disorders by which the mean free path of conduction electron is limited. Confining ourselves to the case of regular lattice, we have the two types of spin-disorders, i.e., in spin orientation and in its location. The former type of disorder can be described in terms of spin waves[8] (magnons) in the low temperature region far below the Curie point and the latter in terms of lattice waves (phonons).

In the low temperature region only the magnons of long wave length are effective in determining the mean free path of conduction electron. Through a single scattering a conduction electron diminishes its wave vector by a small amount of order $k_f\cdot\kappa^2/k_f^2$, and on the average, k_f^2/κ^2 collisions are necessary for a conduction electron to lose its wave vector completely, or to travel a distance of order of its mean free path. On the other hand, the probability of occurrence of a single scattering process is proportional to the number of magnons and the magnitude of coupling constant in V, or, explicitly speaking, is proportional to the quantity

$$\int d\boldsymbol{\kappa} n_{\kappa}|J(\boldsymbol{\kappa})|^2/\kappa \propto T,$$

where $J(\boldsymbol{\kappa})$ is assumed to tend to a constant value with decreasing $\boldsymbol{\kappa}$, and n_{κ} stands for the number of magnons with $\boldsymbol{\kappa}$. Further, we have $(\kappa^2/k_f^2)_{\mathrm{AV}}\propto T$ and the mean free path is proportional to T^{-2}. Then we can expect to have the resistivity of type const.$\times T^2$. In § 3 these and some allied problems are discussed. In the next section as to these scattering processes a more quantitative discussion is given in detail, and the effects of lattice vibration are also discussed.

§ 2. Transition probabilities for the various processes and the electrical resistivity

a) *Transition probability in terms of spin correlation and resistivity*

The transition probability for the scattering process $|\boldsymbol{k}\downarrow\rangle \longrightarrow |\boldsymbol{k}'\uparrow\rangle$ is[9]

$$P(\boldsymbol{k}\downarrow\rightarrow\boldsymbol{k}'\uparrow)=\frac{N}{\hbar^2}|J(\boldsymbol{k}'-\boldsymbol{k})|^2 f_{k}^-(1-f_{k'}^+)\int e^{i(\boldsymbol{\kappa}\cdot\boldsymbol{r}-\omega t)}\sum_n G_n(\boldsymbol{r},t)\gamma_n^{+-}(t)\,d\boldsymbol{r}\,dt \quad (3)$$

with $\boldsymbol{\kappa}=\boldsymbol{k}-\boldsymbol{k}'$, $\hbar\omega=E_k^- - E_{k'}^+$ and

$$G_n(\boldsymbol{r},t)=\left\langle\int d\boldsymbol{r}'\,\delta(\boldsymbol{r}+\boldsymbol{R}_0(0)-\boldsymbol{r}')\,\delta(\boldsymbol{r}'-\boldsymbol{R}_n(t))\right\rangle \quad (4)$$

$$\gamma_n^{+-}(t)=\langle S_0^+(0)\ S_n^-(t)\rangle, \tag{5}$$

where $\langle B\rangle$ stands for the statistical average of the operator B.

In order to observe the essential features of the resistivity determined by the disorder of spins in their orientations, in the first place we discuss the case of rigid lattice; $G_n(\boldsymbol{r},t)=\delta(\boldsymbol{r}-\boldsymbol{R}_n)$. In the low temperature region the spin-disorder in orientation can be described in terms of the free spin waves[8),10)]. Then Eq. (3) becomes

$$\begin{aligned}
P(\boldsymbol{k}\downarrow\to\boldsymbol{k}'\uparrow)&=\frac{4\pi SN}{\hbar^2}|J(\boldsymbol{\kappa})|^2 f_k^-(1-f_{k'}^+)\,(n_{k-k'}+1)\,\delta(\omega_1-\omega_{k-k'})\\
P(\boldsymbol{k}\uparrow\to\boldsymbol{k}'\downarrow)&=\frac{4\pi SN}{\hbar^2}|J(\boldsymbol{\kappa})|^2 f_k^+(1-f_{k'}^-)\,n_{k'-k}\delta(\omega_{-1}+\omega_{k'-k})\\
P(\boldsymbol{k}\updownarrow\to\boldsymbol{k}'\updownarrow)&=\frac{4\pi SN}{\hbar^2}|J(\boldsymbol{\kappa})|^2 f_k^\pm(1-f_{k'}^\pm)\,(NS/2-\sum_\lambda n_\lambda)\,\delta_{kk'}\,\delta(\omega_0)
\end{aligned} \tag{6}$$

in which $\hbar\omega_{\pm1}=E_k^\mp-E_{k'}^\pm$, $\hbar\omega_0=E_k^\pm-E_{k'}^\pm$ and $\hbar\omega_\kappa$ is the energy quantum of spin wave $\boldsymbol{\kappa}$.

Next we determine the distribution function $f_k^\pm$ of the conduction electron when a uniform external electric field $\boldsymbol{F}$ is present in the direction of the x-axis by the use of the steady state equation[11)]

$$df_k^\pm/dt)_F+df_k^\pm/dt)_{\text{coll}}=0, \tag{7}$$

and then we calculate the resistivity. In what follows we make the following assumptions.

(1) $J(\boldsymbol{\kappa})$ depends only on the magnitude of $\boldsymbol{\kappa}$.

(2) In the low temperature region most effective spin waves to the scattering processes are of long wave length and the energy spectrum of these spin waves is $\varepsilon_\kappa=\hbar^2\kappa^2/2\mu$, μ being the effective mass of magnon.

(3) The phonon spectrum is assumed to be $\hbar\omega_q=\hbar sq$, where s is the sound velocity.

The scattering process leaving the spin orientation unchanged (corresponding to the last member of Eqs. (6)) does not bring the change of $\boldsymbol{k}$ in the low temperature region. Hence this type of process can be neglected in determining the resistivity and we have

$$df_k^+/dt)_{\text{coll}}=\frac{4\pi SN}{\hbar}\sum_{\substack{\kappa\\(k'=k+\kappa)}}|J(\boldsymbol{\kappa})|^2\{f_{k'}^-(1-f_k^+)\,(n_\kappa+1)-f_k^+(1-f_{k'}^-)\,n_\kappa\}$$
$$\times\delta(E_{k'}^--E_k^+-\varepsilon_\kappa). \tag{8}$$

Because the electrons near the Fermi surface are only effective to these scattering processes, we put as usual

$$f_k^{\pm}=f_k^0-F\Phi^{\pm}\, df_k^0/dE_k^{\pm}\cdot k_x. \tag{9}$$

Leaving the terms up to the first order in Φ's, we have that

$$\int_0^{\kappa_0} n(\boldsymbol{\kappa})|J(\boldsymbol{\kappa})|^2 \kappa d\kappa \Big\{\Phi^+(E_k^+)\frac{df^0(E_k^+)}{dE_k^+}\exp\left(\frac{\varepsilon_\kappa}{k_BT}\right)\frac{f^0(E_k^+ +\varepsilon_\kappa)}{f^0(E_k^+)}$$
$$-\Phi^-(E_k^+ +\varepsilon_\kappa)\frac{df^0(E_k^+ +\varepsilon_\kappa)}{d(E_k^+ +\varepsilon_\kappa)}\frac{f^0(E_k^+)}{f^0(E_k^+ +\varepsilon_\kappa)}\left[1-\frac{(1-m/\mu)\kappa^2}{2k^2}\right]\Big\} \tag{10}$$
$$+\frac{\pi\hbar^4 ek}{m^2N^2Sv_0}\frac{df^0(E_k^+)}{dE_k^+}=0$$

where v_0 stands for the volume of unit cell. Similarly we have that for the electron with spin downwards

$$\int_0^{\kappa_0} n(\boldsymbol{\kappa})|J(\boldsymbol{\kappa})|^2 \kappa d\kappa \Big\{\Phi^-(E_k^-)\frac{df^0(E_k^-)}{dE_k^-}\frac{f^0(E_k^- -\varepsilon_\kappa)}{f^0(E_k^-)}$$
$$-\Phi^+(E_k^- -\varepsilon_\kappa)\frac{df^0(E_k^- -\varepsilon_\kappa)}{d(E_k^- -\varepsilon_\kappa)}\exp\left(\frac{\varepsilon_\kappa}{k_BT}\right)\frac{f^0(E_k^-)}{f^0(E_k^- -\varepsilon_\kappa)}\left[1-\frac{(1+m/\mu)\kappa^2}{2k^2}\right]\Big\}$$
$$+\frac{\pi\hbar^4 ek}{m^2N^2Sv_0}\frac{df^0(E_k^-)}{dE_k^-}=0. \tag{11}$$

As the function $-df^0(E)/dE$ has sharp maximum at E_f and Φ's would be very slowly varying function of E, one may regard Φ's are constant. Integrating (10), (11) with respect to $E_k^{\pm}$ and combining the resulting equations, we have

$$A+\frac{m}{\mu}B+\frac{4\pi\hbar^4 ek_f^3}{m^2Sv_0J_2}=0$$
$$-\frac{m}{\mu}A+\frac{4k_f^2J_1-J_2}{J_2}B=0, \tag{12}$$

where

$$A=\Phi^+(E_f)+\Phi^-(E_f),\qquad B=\Phi^+(E_f)-\Phi^-(E_f)$$
$$J_s=N^2\int_0^{\kappa_0} n(\kappa)|J(\kappa)|^2\frac{x}{1-e^{-x}}\kappa^{2s-1}\,d\kappa \tag{13}$$
$$x=\hbar^2\kappa^2/2\mu k_BT.$$

Upon solving Eq. (12) one finally has (cf. (9)) as resistivity

$$\rho=\frac{3\pi m^2Sv_0}{2e^2\hbar^3k_f^6}\cdot J_2\cdot\left(1+\left(\frac{m}{\mu}\right)^2\frac{J_2}{4k_f^2J_1-J_2}\right). \tag{14}$$

J_1 reveals a logarithmic divergence at the lower limit of the integration. However, in reality, the actual sample is of finite dimension so the mode $\kappa=0$ has to be excluded from the collective excitations. Moreover, this mode does not have any contribution to limit the mean free path of conduction electron and if one tentatively use the values

$$\kappa_{\min} \cong (Nv_0)^{-1/3} \sim 1\text{cm}^{-1},\ \mu/m=100,\ T=40^0K,$$

one has that as order of magnitude

$$4k_f^2 J_1/J_2 \simeq 60/\zeta(2)\cdot E_f/k_B T\cdot m/\mu \gg 1.$$

So one can neglect the second term in the parenthesises of Eq. (14) and obtains

$$\rho_{\text{spin wave}} = \frac{3\pi^5}{16}\ \frac{S}{e^2}\left(\frac{\mu}{m}\right)^2 \frac{k_B^2 T^2 N^2 J^2(0)}{E_f^4}\ \frac{\hbar}{k_f}. \tag{15}$$

In Ni, adopting the following values[2)],

$$NJ(0)=0.48\text{ eV},\ E_f=3\text{ eV},\ S=1/2 \text{ and } \mu/m=38,$$

one has that

$$\rho_{\text{spin wave}} = 11\times 10^{-6}\times T^2 \text{ (microhm-cm)}. \tag{16}$$

b) *Effects of the lattice vibration*

As has been shown in § 1, $J(\boldsymbol{k}'-\boldsymbol{k})$ is independent of $\boldsymbol{R}_n$. Therefore if it is assumed that the electronic wave functions relevant to the quantity $J(\boldsymbol{k}'-\boldsymbol{k})$ are rigid, J can be regarded merely as a coupling parameter independent of phonon coordinates. As to the inner shell electrons such as $3d$ or $4f$ this may be the case but as to the conduction electrons this assumption is rather serious. However, making this assumption throughout the present calculation, we investigate the effects of lattice vibration whose degrees of freedom are then contained in the quantity G_n only.

In the usual Debye approximation[9)] one has

$$G_n(\boldsymbol{r},t) = \left(\frac{1}{2\pi}\right)^3 \int d\boldsymbol{\kappa}\, e^{-i\boldsymbol{\kappa}\cdot(\boldsymbol{r}-\boldsymbol{R}_n)} \exp\left\{-\sum_{\alpha\beta}^{x,y,z}[M_{\alpha\beta}(0,0)-M_{\alpha\beta}(\boldsymbol{R}_n,t)]\kappa_\alpha\kappa_\beta\right\} \tag{17}$$

with

$$M_{\alpha\beta}(\boldsymbol{R}_n,t) = \frac{\hbar}{2MN}\sum_{qj}\frac{e_{qj}^\alpha e_{qj}^\beta}{\omega_{qj}^p}\Big\{(n_{qj}^p+1)\exp[-i(\boldsymbol{q}\cdot\boldsymbol{R}_n-\omega_{qj}^p t)] + n_{qj}^p \exp[i(\boldsymbol{q}\cdot\boldsymbol{R}_n-\omega_{qj}^p t)]\Big\} \tag{18}$$

where $\boldsymbol{e}_{qj}$ is the polarization vector, j specifies three independent modes of lattice waves and the quantities with superscript p are referred to phonon. At low temperatures far below the Debye temperature Θ

$$\Big|\sum_{\alpha\beta} M_{\alpha\beta}(\boldsymbol{R}_n,t)\kappa_\alpha\kappa_\beta\Big| \lesssim \sum_{\alpha\beta} M_{\alpha\beta}(0,0)\kappa_\alpha\kappa_\beta \simeq \frac{3m}{2M}\left(\frac{\kappa}{k_f}\right)^2 \frac{E_f}{k_B\Theta}\left(1+4\zeta(2)\frac{T^2}{\Theta^2}\right) \tag{19}$$

$$\ll 1,$$

then we can expand $G_n(\boldsymbol{r}, t)$ in terms of these quantities and obtain

$$P(\boldsymbol{k}\downarrow\to\boldsymbol{k}'\uparrow)=\frac{4\pi SN}{\hbar}|J(\boldsymbol{\kappa})|^2 f_k^{\pm}(1-f_{k'}^{\pm})\delta(\hbar\omega)\delta_{\varkappa,0}\, e^{-W}\left(\frac{NS}{2}-\sum_\lambda n_\lambda\right)$$
$$+\frac{2\pi S}{M}|J(\boldsymbol{\kappa})|^2 f_k^{\pm}(1-f_{k'}^{\pm})e^{-W}\frac{(e_\varkappa\cdot\boldsymbol{\kappa})^2}{\omega_\varkappa^p}\left(\frac{NS}{2}-\sum_\lambda n_\lambda\right)$$
$$\times\{(n_k^p+1)\delta(\hbar\omega-\hbar\omega_\varkappa^p)+n_{-\varkappa}^p\delta(\hbar\omega+\hbar\omega_\varkappa^p)\} \qquad (20)$$

with

$$\hbar\omega=E_k-E_{k'},\quad \boldsymbol{\kappa}=\boldsymbol{k}-\boldsymbol{k}',\quad W=\sum_{\alpha\beta}M_{\alpha\beta}(0,0)\kappa_\alpha\kappa_\beta. \qquad (21)$$

Confining ourselves to the lowest order scattering process, hence making use of the second term of Eq. (20), we have the following equation,

$$df_k^+/dt)_{\text{phonon}}=\frac{2\pi S}{M}\left(\frac{NS}{2}-\sum_\lambda n_\lambda\right)\sum_\varkappa|J(\boldsymbol{\kappa})|^2 e^{-W}\frac{\kappa^2}{\omega_\varkappa^p}$$
$$\times\{[f_{k'}^+(1-f_k^+)(n_{-\varkappa}+1)-f_k^+(1-f_{k'}^+)n_{-\varkappa}]\delta(E_{k'}^+-E_k^+-\hbar\omega_\varkappa^p)$$
$$+[f_{k'}^+(1-f_k^+)n_\varkappa-f_k^+(1-f_{k'}^+)(n_\varkappa+1)]\delta(E_{k'}^+-E_k^++\hbar\omega_k^p)\}. \qquad (22)$$

At low temperatures one may put $J(\boldsymbol{\kappa})$ and e^{-W} as constants $(J(\boldsymbol{\kappa})=J(0)$, $e^{-W}=1$; cf. (19), (21)). Upon solving the steady state equation we have[12]

$$f_k^+=f_k^0-F\phi^+(E_k)df^0_k/dE_k\cdot k_x,$$
$$(\phi^+)^{-1}=-\frac{2\pi S}{M}\left(\frac{NS}{2}-\sum_\lambda n_\lambda\right)\frac{Nv_0}{4\pi^2}\frac{m^2}{\hbar^3 es}\frac{\kappa_0^5}{k_f^3}\left(\frac{T}{\theta}\right)^5 J^2(0)\int_0^{\Theta/T}\frac{x^5dx}{(e^x-1)(1-e^{-x})}. \qquad (23)$$

We can similarly determine f_k^- and obtain

$$\rho_{\text{phonon}}=\frac{3\pi Nv_0m^2S\kappa_0^5J^2(0)}{2\hbar^2e^2sMk_f^6}\left(\frac{NS}{2}-\sum_\lambda n_\lambda\right)\left(\frac{T}{\theta}\right)^5\int_0^{\Theta/T}\frac{x^5dx}{(e^x-1)(1-e^{-x})}. \qquad (24)$$

With Eqs. (15) and (24) we finally arrive at

$$\rho=\rho_{\text{spin wave}}(1+\delta), \qquad (25)$$
$$\delta=\frac{2^{2/3}}{\zeta(2)}\frac{mS}{M}\frac{E_f}{k_B\theta}\left(\frac{\theta}{\Theta}\right)^2\left(\frac{T}{\theta}\right)^3\left(\frac{M(T)}{M(0)}\right)^2\int_0^{\Theta/T}\frac{x^5dx}{(e^x-1)(1-e^{-x})}, \qquad (26)$$

where $M(T)$ is the magnetization at T, $k_B\theta=\hbar^2\kappa_0^2/2\mu$.

We now estimate the magnitude of ρ_{phonon} relative to $\rho_{\text{spin wave}}$, that is Eq. (26). θ is determined by the experimental value[13),14)] of the coefficient of the term $T^{3/2}$ in $M(T)$ and is listed in Table I with other constants. Using these values we obtain values of δ. From Table II we see that the lattice vibration considered here has only a small contribution to resistivity.

Table I

	S	E_f(eV)[2]	θ(^{0}K)	Θ(^{0}K)
Ni	1/2	3·14	8670*	456
Gd	7/2	4·4	433**	152§

* θ is determined from the coefficient of $T^{3/2}$ term in $M(T)$ given in Ref. 14).

** Determined from Kurti's data[15]) on the magnetic part of the specific heat of Gd at very low temperatures which has temperature dependence $T^{3/2}$ and is to be attributed to spin waves. If one uses the magnetic data[13]) one has $\theta=467^0$K.

§ This is estimated from Kurti's data[15]) on the lattice part of specific heat which has temperature dependence T^3 at very low temperatures and is to be ascribed to phonon.

Table II Values of δ.

Ni		Gd	
T(^{0}K)	δ	T(^{0}K)	δ
0	0.0000	0	0.0000
22.80	0.0020	7.6	0.0005
35.07	0.0072	11.69	0.0017
45.60	0.0150	15.2	0.0037
57	0.0255	19	0.0062
76	0.0420	25.33	0.0102
91.2	0.0514	30.4	0.0123
114	0.0585	38	0.0139_3
152	0.0594	50.66	0.0138_9
228	0.0493	76	0.0110
304	0.0391	101.3	0.0082

§ 3. Discussions

Electrical resistivities due to the spin-disorders in spin orientation and in its location have the temperature dependences of type T^2 and T^5, respectively, in the low temperature region. This type of difference can be ascribed mainly to the difference in the energy spectra between that of magnon and that of phonon. Then if one has the metallic antiferromagnets in which the *s-d* interaction is effective, one can easily expect that the spin-disorders in orientation and in location give the resistivities of type T^4 and T^5, respectively. This is because the energy spectrum of spin wave in antiferromagnetics is proportional to k. Hence, in an antiferromagnet, the lattice vibration may have effects of much more importance than in a ferromagnet.*

* In the case of inelastic scattering of slow neutrons by ferro- or antiferromagnet, one may expect at least qualitatively that lattice vibration plays a more important role in antiferromagnet than it does in ferromagnet (magneto-vibrational effect [16])).

Now from the view-point of the s-d transition[3] one can obtain the anomalous resistivity proportional to $\exp(-\theta_E/T)$. θ_E is related to such minimum wave vector $q_{\min}$ as it enables an s-d transition to occur.* Further[2], this view-point does not seem to be able to explain satisfactorily the abrupt change in the resistivity at the Curie point, and to explain why the rare earth metal, whose unfilled shell does not seem to constitute a band, does have an anomalous resistivity.

In § 2-(b) we have confined ourselves to the discussion of spin-disorder in spin location and neglected the effect of the deformation of electronic wave functions by the presence of lattice waves, that is, the deformation effects of $J(\boldsymbol{\kappa})$. Indeed, this type of effect must exist but the resulting resistivity might be expected to be of the same order of magnitude of or even smaller than the resistivity due to the usual electron-phonon interaction. This latter type of resistivity, when determined from Potter's experimental data[12], is of order of the magnitude given in § 2-(b). Thus one can conclude that the electrical resistivity of ferromagnet is almost due to the spin-disorder in spin orientation and is given by Eq. (15). In Ni Eq. (16) agrees fairly well with the experimental results[17] both in the order of magnitude and in temperature dependence. As was shown in Potter's experiment, in Ni and Fe the electrical resistivity deviates very much from the one given by Grüneisen formula and this deviation, namely the anomalous part of resistivity, is proportional to $T^n (2 \leq n \leq 2.2)$ up to the room temperatures.

Acknowledgements

The author would like to thank Professor K. Tomita under whose stimulating discussions and continual encouragement the present investigation was carried out. His thanks are also due to the members of Solid State Physics Group in Kyoto University for many stimulating discussions.

* It is suggested by Wilson that if the energy surfaces are not spheres it is possible for s- and d-surfaces to cut one another in some directions in $\boldsymbol{k}$-space, and in the neighbourhood of these directions long lattice waves may be able to participate in the s-d transition. $q_{\min}$ is then zero. These circumstances, however, seem to occur only in a very special case, and in general there is a non-zero value of wave vector for the phonon which is required to produce an s-d transition.

References

1) B. R. Coles, Phil. Mag. Supplement **7** (1958), 40.
2) T. Kasuya, Prog. Theor. Phys. **16** (1956), 58.
3) N. F. Mott, Proc. Roy. Soc. **A153** (1936), 699; **A156** (1936), 368.
A. H. Wilson, Proc. Roy. Soc. **A167** (1938), 580.
4) K. Yosida, Phys. Rev. **107** (1957), 396.
5) P. G. de Gennes and J. Friedel, J. Phys. Chem. Solids **4** (1958), 71.
6) T. Kasuya, Prog. Theor. Phys. **16** (1956), 45.
7) N. F. Mott and K. W. H. Stevens, Phil. Mag. **2** (1957), 1364.
8) T. Holstein and H. Primakoff, Phys. Rev. **58** (1940), 1098.
9) L. van Hove, Phys. Rev. **95** (1954), 249.
10) L. van Hove, ibid., 1374.
11) A. Sommerfeld and H. Bethe, *Handbuch der Physik* (J. Springer, Berlin) **24**-2 (1933), p. 517.
12) Ref. 11), p. 509.
13) J. F. Elliott, S. Legvold and F. H. Spedding, Phys. Rev. **91** (1953), 28.
14) C. Kittel, *Introduction to Solid State Physics* (John Wiley & Sons, Inc.), (1956), p. 406.
15) N. Kurti and R. S. Safrata, Phil. Mag. **3** (1958), 780.
16) R. J. Elliott, Proc. Roy. Soc. **235** (1956), 289.
R. D. Lowde, ibid., 305.
17) H. H. Potter, Proc. Phys. Soc. **49** (1937), 671.
R. M. Bozorth, *Ferromagnetism* (Van Nostrand, New York) (1951), p. 270.

Note added in proof:

Recently White and Woods (Phil. Trans. Roy. Soc. (London) **A251** (1959), 273) reported the detailed data about the electrical conductivity for twenty transition elements and showed that ρ is proportional to T^2 at very low temperatures in Mn, Fe, Co, Ni etc. They ascribed T^2-term to Baber's mechanism (Proc. Roy. Soc. **158** (1937), 383), but his mechanism only is not sufficient to explain the abrupt change of ρ at Curie point.