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ABSORPTION AND EMISSION OF ELECTROMAGNETIC WAVES BY TWO-DIMENSIONAL PLASMONS

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Optical activity of plasma oscillations in 2D electron systems is theoretically investigated. A set of various situations is considered including a uniform and spatially density modulated plasma, single- and multi-component systems, lateral and multilayer superlattices. Possible non-linear optical phenomena caused by concentrational non-linearity of the 2D plasma oscillations are also discussed.

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Contents

1. Introduction

During the past few years considerable interest is maintained to the far infrared (FIR) investigating of the collective oscillations of two-dimensional (2D) plasmas. Though the interaction between electromagnetic and plasma waves is by no means brand new in physics, its 2D version possesses a number of rather interesting and attractive peculiarities. The main point is the possibility to vary nearly all characteristic parameters of 2D plasmons over a wide range, whereas the most essential parameter, which is the areal charge density, can be varied over a few orders of magnitude. Hence, the frequency of 2D plasma waves can be continuously tuned over a range more than ten times wider than the frequency itself. Thus FIR optics is a very interesting tool for resonant experiments, and various physical and technical applications are expected to be possible.

The purpose of this paper is to survey the theory of processes in which optical activity of 2D plasmons plays an essential role. The appropriate experiments (if available) are described only in principle, without technical details.

2. Plasma oscillations of 2D electron gas

There is a rather large number of theoretical papers devoted to the derivation of the dispersion law of 2D plasmons (for a bibliography, see Ando et al. [1]). Here a simple and clear way is used that allows one to track the arising 2D plasmons as a limiting case of the usual 3D Langmuir oscillations. To this aim one has to consider the eigenmodes of a plasma slab of finite thickness L. The slab is supposed to be embedded in a dielectric medium with permeability ϵ and confined by the planes $z = \pm L/2$. If retardation effects are negligible (the criterion will be given below), the problem becomes purely electrostatic: we have to solve the Poisson equation for the potential $\phi(z)$ in three regions: $z < -L/2$, $|z| < L/2$, $z > L/2$ and to match properly ϕ and $\partial\phi/\partial z$ at $z = \pm L/2$. In the region $|z| < L/2$ the dielectric constant ϵ for the frequency ω is equal to that of a free isotropic plasma

$$\epsilon = \epsilon_0 - \omega_0^2/\omega^2, \tag{2.1}$$

where $\omega_0^2 = 4\pi e^2 N_v/m$ is the 3D plasma frequency, N_v is the bulk density of electrons with mass m and charge e. By making use of the Fourier transformation of $\phi(z)$ with respect to z and y we get

$$d^2\phi_k/dz^2 - k^2\phi_k = 0, \tag{2.2}$$

where $\boldsymbol{k}$ is the two-component wave vector of a plasmon.

The solution of eq. (1.2) may be chosen in the form

$$\phi_k(z) = A \exp(-k|z + L/2|) + B \exp(-k|z - L/2|), \tag{2.3}$$

with A and B being arbitrary constants. After the boundary conditions are satisfied we obtain the dispersion equation

$$(\epsilon+\epsilon_0)^2=(\epsilon-\epsilon_0)^2\exp(-2kL), \tag{2.4}$$

which gives two branches of frequencies $\omega(k)$: the symmetrical branch ($A=B$)

$$\omega_+^2(k)=(\omega_0^2/2\epsilon_0)(1-e^{-kL}), \tag{2.5a}$$

and the antisymmetrical branch ($A=-B$)

$$\omega_-^2(k)=(\omega_0^2/2\epsilon_0)(1+e^{-kL}). \tag{2.5b}$$

In the limiting case of an infinitely thin slab ($kL\to 0$) the symmetrical branch goes over to the dispersion relations of 2D plasmons

$$\omega_+^2=2\pi e^2N_sk/m\epsilon_0, \tag{2.6}$$

where $N_s\equiv N_vL$ is the areal density of electrons.

This result is based on formula (2.1) and therefore the spatial dispersion is not taken into account. In other words, eq. (2.6) is valid only for sufficiently small wave vectors k. A more general result may be obtained by quantum field theoretical methods. If the Coulomb interaction may be treated perturbatively the problem is reduced to a calculation of the polarization operator to lowest order in e^2. The result is [2]

$$\omega_p^2(k)=\frac{2\pi\tilde{e}^2N_sk}{m}\frac{(2+ka_0)^2}{4+ka_0}, \tag{2.7}$$

where a_0 is the effective Bohr radius, $a_0=\hbar^2/m\tilde{e}^2$, and $\tilde{e}^2=2e^2/(\epsilon_1+\epsilon_2)$, the plasma layer is supposed to separate two half-spaces with electrical permeabilities ϵ_1 and ϵ_2. One can see from eq. (2.7) that eq. (2.6) corresponds to the approximation $ka_0\ll 1$. Formula (2.7) is derived under the condition $k\ll k_0$, where $\hbar k_0$ is the Fermi momentum of a degenerate electron gas. For $ka_0\gg 1$ eq. (2.7) gives a dispersion relation of zero-sound $\omega=kv_0$, where v_0 is the Fermi velocity. In the long-wavelength limit, $ka_0\ll 1$, the two leading terms in the expansion of eq. (2.7) are

$$\omega_p^2(k)=2\pi\tilde{e}^2N_sk/m+\tfrac{3}{4}(kv_0)^2. \tag{2.8}$$

This result was first derived by Stern [3].

In the case of inversion layers in MOS structures, the electron–electron interaction is modified by the presence of conducting boundaries. If the gate electrode is treated as an ideal metal, and the semiconductor as a half-infinite dielectric with permeability ϵ_s, the 2D plasmon dispersion law is given by eq. (2.7) with the substitution

$$\tilde{e}^2=2e^2[\epsilon_s+\epsilon_{ox}\coth(k\Delta)]^{-1},$$

where ϵ_{ox} and Δ are the electrical permeability and the thickness of the oxide, respectively. In the long-wavelength limit ($ka_0 \ll 1$) we get [4]

$$\omega_p^2 = \omega_{MOS}^2 = \frac{4\pi e^2 N_s k}{m[\epsilon_s + \epsilon_{ox} \coth(k\Delta)]}. \tag{2.9}$$

The 2D electron gas can be created also at the interface of two semiconductors with different energy gaps. The heterojunction GaAs–n-$Al_xGa_{1-x}As$ is being investigated most extensively. Electrons, transferred from shallow donors on the "dielectric side" of the heterojunction (AlGaAs) into the energetically lower conduction band of GaAs, form the 2D gas. Usually one or two levels of the transversal (with respect to the interface) part of the energy are populated. The plasma frequency in such a system is given by:

$$\omega_p^2 = \omega_{hj}^2 = \frac{2\pi e^2 N_s k}{m\epsilon}\left(1 + \frac{\epsilon - 1}{\epsilon + 1}\, e^{-2k\Delta}\right). \tag{2.10}$$

Here Δ is again the thickness of the dielectric layer AlGaAs and we neglected the difference in electrical permeability between AlGaAs and GaAs. One can see from eqs. (2.9) and (2.10) that the heterojunction plasma frequency is larger than that of an equivalent MOS structure by the factor

$$\left(\frac{\epsilon \coth(k\Delta) + 1}{\epsilon + 1}\right)^{1/2}$$

(for the "equivalent MOS structure" one has to put $\epsilon_{ox} = \epsilon_s = \epsilon$, in eq. (2.9)).

Retardation effects are negligible if the inequality $\omega_p \ll ck$ is satisfied (see ref. [3]), where c is the speed of light. Hence, the wave vector k must be larger than $2\pi\tilde{e}^2 N_s/mc^2$, which is usually fulfilled in real experiments. By making use of the dispersion laws (2.9) or (2.10) one can formulate the same condition on the ω-scale: retardation effects are negligible if $\omega \gg 2\pi\tilde{e}^2\sqrt{\epsilon}\, N_s/mc$ (the right-hand side of this inequality is called sometimes "the surface plasma frequency").

In such a retardationless approximation the electric field accompanying the plasma wave is purely potential: $\boldsymbol{E} = -\nabla\phi$; ϕ must be proportional to $\exp(i\boldsymbol{k}\cdot\boldsymbol{\rho} - k|z|)$, where $\boldsymbol{\rho}$ is the 2D vector in the electron-sheet plane $z = 0$, to satisfy the Laplace equation outside the plane. Hence, the electric field has longitudinal ($\boldsymbol{E} \| \boldsymbol{k}$) and transversal ($E_z$) components, and there is a phase shift between them, $\delta = \pi/2$. We see that the spatial distribution of the 2D plasmon electric field is reminiscent of the wavefunction of the bound state in a short-range potential well: the penetration length of the field is much larger than the plasma layer thickness.

The same structure of the electric field also holds for magnetoplasmons, but the dispersion law changes. In a magnetic field perpendicular to the electron sheet the plasma oscillations are governed by the parameter kv_0/ω_c, where ω_c is the cyclotron frequency. In the long-wavelength limit $kv_0 \ll \omega_c$ where the velocity distribution of electrons may be neglected, the magnetoplasmon frequency is given by [4]:

$$\omega_{\mathrm{mp}}^2 = \omega_{\mathrm{c}}^2 + \omega_{\mathrm{p}}^2.$$

This formula has an obvious physical sense: the magnetoplasmon is nothing but a charged oscillator in a magnetic field perpendicular to the direction of its vibration. For an arbitrary value of the parameter kv_0/ω_c the problem becomes much more complicated (see ref. [5]). The spatial dispersion is essential in this case, and the harmonics of the cyclotron frequency together with the so-called "geometric resonances" are displayed in the magnetoplasmon dispersion law (see below, section 3.2).

3. Excitation of 2D plasmons by FIR radiation in a single plasma layer

3.1. How to couple plasmons with FIR radiation; plasmon absorption

A free plasmon, both in two and three dimensions, in a homogeneous infinite plasma is a non-radiative eigenmode. Its phase (and group) velocity does not exceed the speed of light at any k. To rigorously prove this conclusion one should, of course, use the dispersion relations allowing for retardation effects. In the simplest situation (2.6) a substitution

$$k \to \left(k^2 - \epsilon_0\omega^2/c^2\right)^{1/2}$$

is to be done. After that we see that the maximum of phase and group velocities of 2D plasmons is achieved for $k \to 0$ and equals $c/\sqrt{\epsilon_0}$. To couple the 2D plasmon with an electromagnetic wave the latter has to be transformed in a proper way in order to generate a spatially modulated electric field of the type of a plasma wave accompanying field

$$E \sim \exp\left[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{\rho} - \omega t)\right].$$

This can be done by means of a special grating structure (see fig. 1) fabricated on the gate electrode of the MOS system [6] or on the "dielectric side" of the heterojunction [7,8]. The strips of the grating structure are made of a highly conductive metal (e.g., Al) and are non-transparent for FIR radiation. A different possibility has been demonstrated by Mackens et al. [9] on Si–SiO_2 MOS capacitors: a linear periodic structure has been prepared in which the thickness of the insulating oxide varies with a certain periodicity. Both these methods result in the desirable field transformation. If the FIR wave falling onto the system is polarized along the direction of periodicity (say, the x-axis) the electric field acting upon electrons is represented by the Fourier series:

$$E(x, t) = \sum_n E_n \exp\left[\mathrm{i}(2\pi nx/a - \omega t)\right],$$

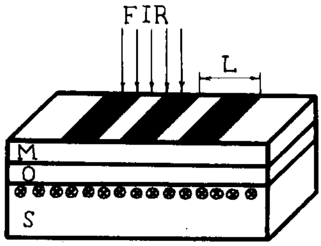

Fig. 1. MOS structure with a grating structure upon the gate electrode; (⊗) represent 2D electrons.

where a is the period of the grating structure, $n = 0, 1, 2, \dots$. Thus the quantities a and n determine the wave vector of 2D plasmons $k = 2\pi n/a$ while the FIR frequency ω or N_s can be tuned. When ω, k and N_s are connected by the 2D plasmon dispersion relation, the resonance of absorption arises in the system. This was first observed by Allen et al. [6] on Si–SiO_2 MOS structure (earlier Grimes and Adams [10] observed 2D plasmons in a sheet of electrons on liquid helium; they measured radio-frequency standing-wave resonances in a rectangular cell).

An elementary theory of FIR-absorption by 2D plasmons is based on the Boltzmann kinetic equation in the relaxation time approximation (τ approximation), which has to be solved together with the Poisson equation. We have:

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\frac{\partial f}{\partial \boldsymbol{r}} + e(\boldsymbol{E}_{ex} + \boldsymbol{E}_{sc})\,|_{z=0}\frac{\partial f}{\partial \boldsymbol{p}} + \frac{f - \langle f \rangle}{\tau} = 0, \tag{3.1}$$

$$\operatorname{div} \epsilon(z)\boldsymbol{E}_{sc} = -4\pi e\tilde{N}_s\delta(z), \tag{3.2}$$

$$\tilde{N}_s = 2g_v\int \tilde{f}(\boldsymbol{p})\,d^2p/(2\pi\hbar)^2, \quad \tilde{f} \equiv f - f_0. \tag{3.3}$$

Here f is the electron distribution function, f_0 the Fermi distribution, $\tilde{f}$ the non-equilibrium part of the total distribution, connected with the plasma wave,

$$\langle f \rangle = (2\pi)^{-1}\int f\,d\varphi,$$

where φ is the azimuthal angle in the plane of electron momenta $\boldsymbol{p}$. The field term in the kinetic equation (3.1) contains both the external (exciting) FIR field $\boldsymbol{E}_{ex}$ and the self-consistent contribution $\boldsymbol{E}_{sc}$ generated by the plasma oscillations. To close the system of equations one has to express the non-equilibrium part of the electron areal density $\tilde{N}_s$ via $\tilde{f}(\boldsymbol{p})$; this is given by eq. (3.3); g_v is the valley degeneration factor. The spin and valley splitting are neglected, i.e. $f(\boldsymbol{p})$ is supposed to be independent of spin and valley indices.

When considering the absorption of the spatially modulated exciting field

$$\boldsymbol{E}_{ex} = \boldsymbol{E}_0 \exp[i(kx - \omega t)],$$

$\boldsymbol{E}_0$ has only an x-component, and all values in eqs. (3.1)–(3.3) spatially depend only on x. From eq. (3.2) one can easily obtained for a MOS structure

$$\boldsymbol{E}_{sc}|_{z=0} = -i \frac{4\pi e \tilde{N}_s \boldsymbol{k}}{k\left[\epsilon_s + \epsilon_{ox} \coth(k\Delta)\right]}.$$

Then by making use of (3.1) and (3.3), after some obvious algebra, we find the current density and the absorption power per unit area

$$Q = \tfrac{1}{2} \operatorname{Re}(\boldsymbol{j} \cdot \boldsymbol{E}^*_{ex}).$$

The result is of the form:

$$Q = \frac{E_0^2 \sigma_0}{2} \frac{(\omega/\tau)^2}{\left[\omega^2 - \omega_p^2(k)\right]^2 + (\omega/\tau)^2}. \tag{3.4}$$

Here σ_0 is the 2D static conductivity, $\sigma_0 = N_s e^2 \tau / m$, $\omega_p(k)$ is ω_{MOS} from eq. (2.9).

The experimentally measured total absorption is a series of contributions from all possible values of k; the Drude background comes from $k = 0$, the first plasmon resonance from $k = 2\pi/a$, the second from $k = 4\pi/a$, and so on. Theis [11] succeeded in observing all the plasmon absorption peaks up to $k = 10\pi/a$. Note that, if τ tends to infinity, Q becomes proportional to $\delta(\omega - \omega_p)$. This means there exists a collisionless absorption of FIR radiation; it is quite similar to the interaction of electromagnetic waves with optical phonons in an ion crystal.

3.2. *Plasmon absorption in a magnetic field*

FIR absorption by 2D plasmons in a perpendicular magnetic field was first observed experimentally on Si–SiO_2 MOS structures by Theis et al. [12]. The magnetoplasmon peak position agreed reasonably well with elementary theory ($\omega_{mp}^2 = \omega_p^2 + \omega_c^2$), ignoring non-local effects. However, both in this work and in more recent papers by Mohr and Heitmann [13] and by Batke et al. [14] an additional structure was observed, i.e. a coupling of the magnetoplasmon with the harmonics of the cyclotron resonance.

It is well known that, usually, a cyclotron resonance in a spatially uniform high-frequency field can be accompanied by the harmonic structure only due to collisions of electrons. The reason is quite evident: a harmonic oscillator, subjected to the action of a monochromatic force which does not depend on coordinates, can absorb only its eigenfrequency. The interaction of electrons in cyclotron orbits with impurities violates the harmonic motion so that absorption at the frequencies $n\omega_c$ ($n = 2, 3, \dots$) becomes possible. This scatterer-induced origin of harmonics has been investigated theoretically by Ando [15] in connection with the observations by Theis et al. [12].

There exists, however, a different mechanism for the origin of harmonics, i.e. geometric resonances. The high-frequency electric field does depend on the coordinates in this case and the parameter kv_0/ω_c governs the effect under consideration. In other words, the non-locality becomes essential if kv_0/ω_c is non-negligibly small. This kind of resonances and the closely connected cyclotron Landau damping are well established in a gas discharge plasma (see for example, ref. [16]) and in a 3D plasma of solids [17]. Now one may state that the non-local effects are observed in the dynamic 2D conductivity of AlGaAs–GaAs heterostructures [14]. The experimental situation was such ($N_s = 6.5 \times 10^{11}$ cm^{-2}, $B \approx 1.5$ T) that the number of populated Landau levels was sufficiently high (≈ 8), and the quasiclassical approximation should be well applicable.

Considering the problem in the framework of a classical description of electrons we have to introduce the term

$$\frac{e}{c}[\boldsymbol{V}\cdot\boldsymbol{B}]\frac{\partial f}{\partial \boldsymbol{p}}$$

in the left-hand side of the Boltzmann kinetic equation (3.1). By making use of polar coordinates in the p-plane we obtain for the non-equilibrium part of the distribution function $\tilde{f}$ the following equations:

$$\tilde{f}(\boldsymbol{r},\boldsymbol{p},t) = \tilde{f}_1(v,\varphi)\exp[\mathrm{i}(kx-\omega t)], \tag{3.5a}$$

$$\omega_c\frac{\partial \tilde{f}_1}{\partial\varphi} + \mathrm{i}\left(\omega - kv\cos\varphi + \frac{\mathrm{i}}{\tau}\right)\tilde{f}_1 = e(E_{ex}+E_{sc})\cos\varphi\frac{\partial f_0}{\partial E} - \frac{\langle\tilde{f}_1\rangle}{\tau}, \tag{3.5b}$$

with $v \equiv p/m$. Eqs. (3.5) can easily be solved:

$$\tilde{f}_1(v,\varphi) = -\mathrm{e}^{\mathrm{i}z\sin\varphi}\int^{\infty}\mathrm{e}^{\mathrm{i}\gamma\psi - \mathrm{i}z\sin(\varphi+\psi)}Z(\varphi-\psi)\,\mathrm{d}\psi, \tag{3.6}$$

where Z stands for the right-hand side of eq. (3.5b), $z \equiv kv/\omega_c$, $\gamma \equiv (\omega + \mathrm{i}/\tau)/\omega_c$. Then, as before in section 3.1, we calculate $\tilde{N}_s$ and the current j_s which is now a series of Bessel functions $J_n(z_0)$, where $z_0 = kv_0/\omega_c$. The absorbed power per unit area is of the form

$$Q = \mathrm{Im}\left\{\frac{e^2 m\omega g_v E_{ex}^2}{2\pi k^2}\left[\frac{4g_v}{ka_0^*(k)} + \frac{\omega_c\tau - \mathrm{i}M}{\omega_c\tau - (\mathrm{i}+\omega\tau)M}\right]\right\}, \tag{3.7}$$

where

$$M = \sum_{m=-\infty}^{+\infty}\frac{J_n^2(z_0)}{\gamma+n}, \quad a_0^*(k) = \frac{\hbar^2[\epsilon_s + \epsilon_{ox}\coth(k\Delta)]}{2me^2}.$$

In the absence of electron scattering the series M has poles at $\omega = n\omega_c$. So one may expect that eq. (3.7) describes oscillations in absorbed power, Q, with maxima at the positions of the cyclotron harmonics [18].

Batke et al. [14] have thoroughly investigated the vicinity of the first harmonic ($n = 2$) in magnetoplasmon absorption of the GaAs–GaAlAs heterostructure. Good quantitative agreement with eq. (3.7) was established both for the resonance positions ω_{res} and for the excitation amplitude, i.e. the peak absorption Q_{max}.

4. Absorption of FIR by plasma oscillations in multicomponent 2D systems

Up to now we assumed that an electron gas is in an ultraquantum state with respect to the motion which is transversal to the layers, i.e. that the electrons occupy the lowest transversal quantization level. However, modern technology of preparation of thin films and layer structures makes it possible to vary easily the parameters of such systems. The number of layers, their thickness, level of doping, and – consequently – the carrier density can all be varied within very wide limits. Therefore, it is interesting to consider systems which are essentially two-dimensional but exhibit to some extent a transversal degree of freedom. In the case of a quantum film this means that more than one transversal level is populated and that transitions between them have to be allowed for. In the case of a layer structure one may have to allow for tunneling between layers.

This section is devoted to 2D plasma waves in films and layer structures in those cases when the electron plasma has several components. The components are groups of electrons differing either in respect of the quantum number of transversal motion (film, or inversion channel, or a single heterojunction) or in respect of the number of layers (layer structure). The spectrum of plasma waves in such systems has a number of interesting features. New oscillation branches appear and some of them resemble ion-acoustic waves in a gas plasma, whereas others are analogous to excitons. The Landau damping in such systems and the optical absorption spectra of such systems also have special properties [19–21]. It is shown in these works, that the number of oscillation branches is generally $\frac{1}{2}n(n+1)$, where n is the number of plasma components. Moreover, there are exactly n branches with a 2D plasmon-like gapless dispersion law, whereas the other branches are of intersubband exciton character. In what follows the FIR absorption by the plasmon-like branches in a two-component plasma is considered.

4.1. Spatially separate plasma layers

The simplest case of two-component 2D plasma is represented by a system of two quantum films separated by an insulating gap. In each film only the lowest level is assumed to be occupied by electrons and tunneling across the insulator is negligible. Thus, the films are connected only "electrically" (by the electric fields of the plasma waves) but not "electronically". Allowance for tunneling makes the system fully equivalent to a film with two populated levels, which is considered in the next subsection.

As shown in ref. [19], there exists a situation where both plasmon branches of the system under consideration have no Landau damping for sufficiently small k. From the dispersion equation for oscillations of two infinitesimally thin plasma layers, it follows that

$$\left[1+\frac{ka_{01}R_1}{2(R_1-1)}\right]\left[1+\frac{ka_{02}R_2}{2(R_2-1)}\right]=e^{-2kd}, \tag{4.1a}$$

$$R_{1,2}\equiv\left(1-k^2v_{01,2}^2/\omega^2\right)^{1/2}, \tag{4.1b}$$

where a_{01} and a_{02} are the effective Bohr radii, v_{01} and v_{02} are the Fermi velocities of the electrons in the layers, and d is the distance between the layers. If ka_{01}, ka_{02}, $kd \ll 1$, one of the branches represents the in-phase oscillation of particles in both plasmas and it is characterized by the usual (for 2D plasmons) square-root dispersion law

$$\omega_+^2=\frac{2\pi e^2k}{\epsilon}\left(\frac{N_1}{m_1}+\frac{N_2}{m_2}\right), \tag{4.2}$$

where $N_{1,2}$ and $m_{1,2}$ are the surface densities and the effective masses of the particles in the layers, respectively.

In the same limit, $k\to 0$, the second branch $\omega_-(k)$ describes anti-phase oscillations of electrons in the layers and exhibits the acoustic dispersion law $\omega_-=sk$. One obtains from eqs. (4.1) a real and positive value of s if (at $v_{01}>v_{02}$)

$$d>d_0\equiv\frac{1}{4}\frac{a_{02}\left(v_{01}^2-v_{02}^2\right)^{1/2}}{v_{01}-\left(v_{01}^2-v_{02}^2\right)^{1/2}}. \tag{4.3}$$

Thus, if condition (4.3) is satisfied, both "optical" $\omega_+(k)$ and "acoustic" $\omega_-(k)$ plasmons have no Landau damping for ka_{01}, ka_{02}, $kd\ll 1$ since their phase velocities exceed the maximal Fermi velocity v_{01}. If $d\gg a_{01,2}$ the dispersion equation is easily solved since in this case $s\gg v_{01,2}$. It is then found that

$$\omega_-^2=\frac{4\pi e^2 d}{\epsilon}\frac{N_1N_2k^2}{N_1m_2+N_2m_1},\quad kd\ll 1. \tag{4.4}$$

Qualitatively, the branches $\omega_\pm$ are depicted in fig. 2.

We now discuss the question of optical activity of the described oscillations. It is clear that the branch ω_+ is fully analogous to a plasmon in a one-component plasma and that it interacts resonantly with a longitudinal inhomogeneous electric field $E_0\exp[i(kx-\omega t)]$. The branch ω_- is also, in principle, optically active (for the same fields), but the amplitude and width of the

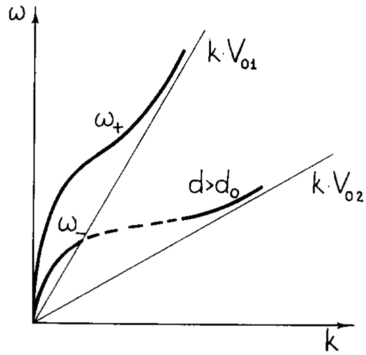

Fig. 2. Dispersion curves of plasma oscillations in a system of two spatially separated layers.

absorption resonance differ considerably from the corresponding parameters in the ω_+ case.

By excluding transitions of electrons between layers one makes it possible to introduce collision frequencies for each layer ν_1 and ν_2. The absorption coefficient will be defined as the ratio of the power dissipated per unit area Q to the surface energy density of a plasma wave amounting to $P = E_0^2/8\pi k$. In the case of constant frequencies ν_1 and ν_2 the calculations are elementary. The resonance values of Q and of the linewidth are given by the following expressions.

For the ω_+ branch, we have

$$\frac{Q_+^{\text{res}}}{P} = \frac{\omega_+^4}{\omega_1^2\nu_1 + \omega_2^2\nu_2}, \quad \Gamma_+ = \frac{\nu_1\omega_1^2 + \nu_2\omega_2^2}{\omega_1^2 + \omega_2^2}. \tag{4.5}$$

For the ω_- branch, we obtain

$$\frac{Q_-^{\text{res}}}{P} = \frac{1}{4}\frac{\omega_-^4}{\omega_1^2\nu_2 + \omega_2^2\nu_1}\left(\frac{\omega_1}{\omega_2} - \frac{\omega_2}{\omega_1}\right)^2, \quad \Gamma_- = \frac{\nu_1^2\omega_2^2 + \nu_2\omega_1^2}{\omega_1^2 + \omega_2^2}. \tag{4.6}$$

The resonance frequencies $\omega_\pm$ occurring in eqs. (4.5) and (4.6) are given in the $kd \ll 1$ region by eqs. (4.2) and (4.4), respectively; ω_1 and ω_2 are the frequencies of 2D plasmons in the layers. The expression for Q_-^{res} given above is valid for

$$\nu_1\nu_2\left(\omega_1^2 + \omega_2^2\right) \ll \left(\omega_1^2 - \omega_2^2\right)^2.$$

We can easily show that in the case of a symmetric structure ($\omega_1 = \omega_2$, $\nu_1 = \nu_2$)

$$\omega_-(k) = kv_0\frac{2d + a_0}{\left(4da_0 + a_0^2\right)^{1/2}}, \tag{4.7}$$

and the value of Q_-^{res} is proportional to ν in the frequency range $\omega \sim \omega_-$, Outside this interval the absorption decreases in accordance with the law

$\nu\omega_-^2/\omega^2$. Thus, in the case of a symmetric structure the optical activity of the ω_- branch is anomalously weak and the absorption differs only slightly from that due to the Drude background of free carriers. This is due to the fact that the branch ω_- corresponds to the anti-phase motion of like charges in the layers. The total current in the system at resonance differs from zero only because of electron scattering. As one can see from the analysis given above the possibility of observing experimentally FIR absorption by acoustic plasmons ω_- seems to be rather small.

4.2. Quantum film (quantum well) or inversion layer

In this situation the components of the system under consideration are connected both electrically and electronically due to intersubband electron scattering. Strictly speaking one should use in this case the quantum theory of non-equilibrium processes instead of the Boltzmann kinetic equation. We shall assume, however, that the scattering is sufficiently small, so that the usual conditions for describing the longitudinal effects in a quantum film by means of the classical kinetic equation are satisfied (see e.g. refs. [22,23]). Besides, we assume also that quantum effects associated with the energy of the appropriate photons $\hbar\omega \sim \hbar\omega_p$ are negligible ($\hbar\omega_p \ll mv_0^2$, ΔW, where ΔW is the intersubband energy interval). This permits the use of the Boltzmann equation when a high-frequency external field is present.

Nevertheless, even after accepting the simplifications mentioned above the problem remains rather complicated, mainly due to the necessity to solve a system of kinetic equations with integral Stoss-terms. The phenomenological parameters ν_α (α is the subband index) are not sufficient for solving the problem under consideration where the spatial dispersion ($V\partial f/\partial z$ in eq. (3.1)) as well as intersubband transitions are very important. In this respect our problem differs from the one considered by Siggia and Kwok [24] for a uniform external field. Below an approximate theory is developed using the small parameter $kv_0/\omega_p \sim \sqrt{ka_0}$ [20].

4.2.1. General equations

The electrostatic potential of a plasma wave is governed by the equation

$$\epsilon(z)\left(\frac{d^2\phi}{dz^2} - k^2\phi\right) = -4\pi e \sum_\alpha \Psi_\alpha^2(z)\tilde{N}_\alpha. \tag{4.8}$$

Here $\Psi_\alpha(z)$ is the wavefunction of the transversal motion for the αth level (subband), $\tilde{N}_\alpha$ is the non-equilibrium part of the areal density of electrons in level α; $\epsilon(z)$ has different values in the two regions of a MOS structure (ϵ_{ox} and ϵ_s), but for a quantum film the values of ϵ in the film and in the surrounding medium are assumed to be identical (e.g. AlGaAs–GaAs–AlGaAs). In the field-term of the kinetic equation the force acting on the electrons has to be averaged over the z-coordinate with Ψ_α^2 as a weight function. After separating the factor $\exp[i(kx - \omega t)]$ one has

$$-\mathrm{i}(\omega - \boldsymbol{k}\cdot\boldsymbol{v})\tilde{f}_\alpha(\boldsymbol{v}) + f'_{0\alpha}(\boldsymbol{E}_{\mathrm{ex}\,\alpha} - \mathrm{i}\boldsymbol{k}\phi_\alpha)\boldsymbol{v} + \sum_{\beta v'} W_{\alpha\beta}(\boldsymbol{v}, \boldsymbol{v}')\cdot\left[\tilde{f}_\alpha(\boldsymbol{v}) - \tilde{f}_\beta(\boldsymbol{v}')\right] = 0. \tag{4.9}$$

Here

$$\phi_\alpha = \int \phi(z)\Psi_\alpha^2(z)\,\mathrm{d}z, \quad E_{\mathrm{ex},\alpha} = \int E_{\mathrm{ex}}(z)\Psi_\alpha^2(z)\,\mathrm{d}z,$$

$f_{0\alpha}$ is the equilibrium distribution function for the αth subband, $W_{\alpha\beta}(\boldsymbol{v}, \boldsymbol{v}')$ is the transition probability $\alpha\boldsymbol{v} \to \beta\boldsymbol{v}'$, averaged with respect to the position of the impurity centres. Obviously,

$$\tilde{N}_\alpha = \sum_v \tilde{f}_\alpha(\boldsymbol{v}).$$

The ϕ_α can be expressed via a formal solution of eq. (4.8) by making use of its Green function:

$$\phi_\alpha = -4\pi e \sum_{\beta v} \int G_k(z, z_0)\Psi_\alpha(z)\Psi_\beta(z_0)\,\mathrm{d}z\,\mathrm{d}z_0. \tag{4.10}$$

The Green function is

$$G_k(z, z_0) = -\frac{1}{2\epsilon k}\mathrm{e}^{-k|z - z_0|}$$

in the case of a quantum film, and

$$G_k(z, z_0) = -\frac{1}{2k\epsilon_\mathrm{s}}\left[\mathrm{e}^{-k|z-z_0|} + \frac{\epsilon_\mathrm{s} - \epsilon_\mathrm{ox}\coth(k\Delta)}{\epsilon_\mathrm{s} + \epsilon_\mathrm{ox}\coth(k\Delta)}\mathrm{e}^{-k(z+z_0)}\right] \tag{4.11}$$

in the case of MOS structure. Eqs. (4.9) and (4.10) form a closed system from which one can find the functions $\tilde{f}_\alpha$ and then calculated the dissipated power.

4.2.2. Short-range scatterers

In this case the probability $W_{\alpha\beta}$ depends on the velocities only via a δ-function providing for energy conservation. It follows from eq. (4.9) that the $\tilde{f}_\alpha(\boldsymbol{v})$ are linearly connected with their zero cylindric harmonics

$$\tilde{f}_\alpha^0(v) = (2\pi)^{-1}\int_0^{2\pi} f_\alpha(v, \varphi)\,\mathrm{d}\varphi,$$

$$\tilde{f}_\alpha(v) = \frac{\mathrm{i}e(\phi_{0\alpha} + \phi_\alpha)\boldsymbol{k}\cdot\boldsymbol{v}f'_{0\alpha} + \Sigma_{\beta v'}W_{\alpha\beta}(v, v')\tilde{f}_\beta^0(v')}{\nu_\alpha - \mathrm{i}\omega + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}}. \tag{4.12}$$

The summation in eq. (4.12) runs only over the modulus v',

$$\phi_{0\alpha} = \mathrm{i}E_{\mathrm{ex}\,\alpha}/k, \quad \nu_\alpha = \sum_{\beta v'} W_{\alpha\beta}(v, v').$$

Thus, the values of ν_α determine the conductivity in a uniform constant field:

$$\sigma_0 = (e^2/m)\sum_\alpha N_\alpha/\nu_\alpha,$$

where N_α is the equilibrium concentration of electrons in the αth subband. By integrating eq. (4.12) from 0 to 2π over φ one obtains a system of equations for $\tilde{f}_\alpha^0(v)$. On the right-hand sides of these equations the summation over v' with the energy δ-function gives $\tilde{f}_\beta^0$ at the arguments

$$v_{\alpha\beta}^2 = v^2 + (2/m)(T_\alpha - T_\beta),$$

where T_α are the transversal energy levels. To close the system one has to make the argument v on the left-hand sides equal to a series of appropriate values $v_{\alpha\beta}$. Thus, for n levels occupied one gets a system of n^2 equations. In what follows, results are given for the case $n = 2$.

$$Q = \frac{\omega\epsilon_{\text{eff}}E_0^2}{4\pi k}\operatorname{Im}\frac{L(\omega) - \omega_{p1}^2\omega_{p2}^2[\lambda(I) - \mu(I)](\omega + 2iV_{12})^{-1}}{L(\omega) - \omega(\omega + i\nu_1)(\omega + i\nu_2)}. \tag{4.13}$$

Here ω_{p1}, ω_{p2} are the 2D plasmon frequencies corresponding to the equilibrium concentrations of the particles in subbands 1 and 2; $\lambda = I_{11} + I_{22} - 2I_{12}$, $\mu = I_{11}I_{22} - I_{12}^2$,

$$I_{\alpha\beta} = -2k\int\Psi_\alpha^2(z)G_k(z, z_0)\Psi_\beta^2(z_0)\,dz\,dz_0,\quad V_{12} = \sum_{v'} W_{12}(v, v'),$$

ϵ_{eff} equals ϵ for the quantum film and $(\epsilon_s + \epsilon_{ox}\coth k\Delta)/2$ for the MOS structure. The result (4.13) for Q is a leading term allowing only for the first term in the numerator of eq. (4.12). The contribution of the second term is small in the parameter $(kv)^2\nu/\omega^3$. The frequency dependence of $Q(\omega)$ has a typical resonant shape. In the available experiments, the condition $kL \ll 1$ is well satisfied, where L is the thickness of the film (or inversion layer). Then the resonance positions are given by the formulae

$$\omega_+^2 = I_{11}\omega_{p1}^2 + I_{22}\omega_{p2}^2, \tag{4.14a}$$

$$\omega_-^2 = \frac{\omega_{p1}^2\omega_{p2}^2}{\omega_{p1}^2 + \omega_{p2}^2}\mu(I), \tag{4.14b}$$

with $\mu \approx kL$. The values of I_{11} and I_{22} tend to 1 when kL tends to zero. The peak absorbed power and the widths of the appropriate resonances are

$$(Q_+)_{\max} = \frac{\epsilon_{\text{eff}}E_0^2}{4\pi k}\frac{\omega_+^2}{\Gamma_+},\quad \Gamma_+ = \frac{\nu_1\omega_{p1}^2 + \nu_2\omega_{p2}^2}{\omega_{p1}^2 + \omega_{p2}^2}, \tag{4.15a}$$

$$(Q_-)_{\max} = \frac{\epsilon_{\text{eff}}E_0^2}{4\pi k}\frac{\omega_-^2}{\Gamma_-}\frac{\mu(I) - \lambda(I)}{\mu(I)} \approx (Q_+)_{\max}\left(\frac{\omega_-}{\omega_+}\right)^4, \tag{4.15b}$$

$$\Gamma_- = 2V_{12} + (\nu_2\omega_{p1}^2 + \nu_1\omega_{p2}^2)/\omega_{p1}^2 + \omega_{p2}^2,\quad \mu - \lambda \approx (kL)^2. \tag{4.15c}$$

In eqs. (4.13), (4.15a) and (4.15b) the substitution $E_{\mathrm{ex}\,1} = E_{\mathrm{ex}\,2} = E_0$ has been made; this is valid with a relative accuracy kL. The characteristics of the ω_- branch depend not only on ν_1 and ν_2 but also on the probability V_{12}. This means that all scattering matrix elements are represented in the final results independently in contrast with the uniform field case, where the conductivity can be expressed via the scattering frequencies ν_1 and ν_2 only (see ref. [24]). For typical MOS structures $kL \approx 10^{-2}$, $\omega_-/\omega_+ \approx 10^{-1}$, $Q_-/Q_+ \approx 10^{-4}$.

4.2.3. Arbitrary scattering potential

Let the scattering probability $W_{\alpha\beta}(\boldsymbol{v}, \boldsymbol{v}')$ depend only on the angle difference $\varphi - \varphi'$. By expanding $W_{\alpha\beta}$ in a series of cylindrical harmonics one gets after integration the Stoss term in eq. (4.9) over $\mathrm{d}v'$:

$$\tilde{f}_\alpha(v, \varphi) = \frac{\mathrm{i}e(\phi_{0\alpha} + \phi_\alpha) f'_{0\alpha} kv \cos\varphi + \Sigma_{\beta n} A^n_{\alpha\beta}(v) \tilde{f}^n_\beta(v_{\alpha\beta})\, \mathrm{e}^{\mathrm{i}n\varphi}}{\nu_\alpha - \mathrm{i}\omega + \mathrm{i}kv\cos\varphi}, \tag{4.16}$$

where

$$A^n_{\alpha\beta}(v) = (S/2\pi^2\hbar^2) \int W_{\alpha\beta}(v, v', \psi)\, \mathrm{e}^{\mathrm{i}\psi} m^2 v' \, \mathrm{d}v' \, \mathrm{d}\psi,$$

$$\tilde{f}^n_\alpha(v) = (2\pi)^{-1} \int \tilde{f}_\alpha(v, \varphi)\, \mathrm{e}^{\mathrm{i}n\varphi} \, \mathrm{d}\varphi,$$

S is the area of a specimen. For our purposes we need only the zeroth and the first harmonics of the functions $\tilde{f}_\alpha$. One can see from eq. (4.16) that the contribution of higher harmonics to $\tilde{f}^0_\alpha$ and $\tilde{f}^1_\alpha$ decreases at $kv \ll \omega$ as $(kv/\omega)^{|n|}$ since

$$\int (\nu - \mathrm{i}\omega + \mathrm{i}kv\cos\varphi)^{-1}\, \mathrm{e}^{\mathrm{i}n\varphi}\, \mathrm{d}\varphi \approx 2\pi\mathrm{i}(kv)^{|n|}(\omega + \mathrm{i}\nu)^{-|n|-1}.$$

We consider now the two-level case and we expand both sides of eq. (4.16) in a series of cylindrical harmonics. By keeping only the leading terms with respect to the parameter kv/ω one can obtain a closed system of four equations for the functions $\tilde{f}^1_{1,2}$, determining the current density. The number of equations is doubled because the unknown quantities occur at two magnitudes of their argument. Then the functions $\tilde{f}^1_{1,2}$ have to be substituted in the system of four equations for $\tilde{f}^0_{1,2}$, and, at last, one has to find ϕ_1, ϕ_2 from eq. (4.10). After rather cumbersome calculations we obtain the following results for a degenerate system with two levels occupied:

$$\Gamma_+ = \frac{\omega^2_{\mathrm{p}1}\nu_{1\mathrm{tr}} + \omega^2_{\mathrm{p}2}\nu_{2\mathrm{tr}}}{\omega^2_{\mathrm{p}1} + \omega^2_{\mathrm{p}2}} + A^1_{12} \frac{(\omega_{\mathrm{p}1} - \omega_{\mathrm{p}2})^2}{\omega^2_{\mathrm{p}1} + \omega^2_{\mathrm{p}2}}, \tag{4.17a}$$

$$\Gamma_- = \frac{\omega^2_{\mathrm{p}1}\nu_{2\mathrm{tr}} + \omega^2_{\mathrm{p}2}\nu_{1\mathrm{tr}}}{\omega^2_{\mathrm{p}1} + \omega^2_{\mathrm{p}2}} + 2A^0_{12} + A^1_{12} \frac{(\omega_{\mathrm{p}1} + \omega_{\mathrm{p}2})^2}{\omega^2_{\mathrm{p}1} + \omega^2_{\mathrm{p}2}}. \tag{4.17b}$$

The absorbed power at resonance $(Q_\pm)_{\max}$ is given again via $\Gamma_\pm$ and $\omega_\pm$ by eqs. (4.15a) and (4.15b). As could be expected for an arbitrary scattering potential, the transport collision frequencies

$$\nu_{\alpha\mathrm{tr}} = \sum_{\beta v'} \left(A^0_{\alpha\beta} - A^1_{\alpha\beta} \right)$$

appeared in the formulae for $\Gamma_\pm$ and $Q_\pm$. Besides, it is worthy of note that now the absorption spectral parameters of both branches ω_+ and ω_- contain all the scattering matrix elements and not only $\nu_{\alpha\mathrm{tr}}$. Of course, if only one level is occupied the result can be expressed via a single relaxation time $(\nu_{1\mathrm{tr}})^{-1}$ and the appropriate formulae follow from eq. (4.17a) by putting $\omega_{\mathrm{p}2} = 0$, $A^0_{12} = A^1_{12} = 0$.

To calculate corrections to the obtained results one has to keep the second term in the sum over n in eq. (4.16) and furthermore the $n = 2$ harmonic in expanding $\tilde{f}_\alpha$. It turns out that the correction of $\tilde{f}^0_\alpha$ is of the order of $(kv/\omega)^4$ whereas for $\tilde{f}^1_\alpha$ one gets $(kv/\omega)^3$. Thus, in both cases the obtained results are valid with a relative accuracy $(kv/\omega)^2 \approx ka_0$.

The experiments available up to now concern the ω_+ branch. It follows from the results of the present subsection that when the Fermi level crosses the bottom of the second subband, the second maximum in the absorption curve $Q(\omega)$ will appear at $\omega = \omega_-$. Moreover, the parameters of the main resonance are changed: in eq. (4.17a) the values of $\nu_{\alpha\mathrm{tr}}$ exhibit a jump, and A^1_{12} has a discontinuity in its first derivative as a function of N_s. If one sweeps N_s and tunes ω in such a way as to keep the resonance condition satisfied $(\omega = \omega_+)$, the width Γ_+ increases with jump at a certain N_s value whereas Q_+ correspondingly decreases. One may note an analogy between this phenomenon and the conductivity oscillations in a quantum film when the thickness of the film is changed.

5. Electromagnetic generation of 2D plasma waves in superlattices

Man-made periodic structures, or superlattices, are of great interest in the up-to-date physics of 2D electron systems. The plasma oscillations in such structures exhibit all the general features of wave processes in periodic systems and, first of all, the band spectrum of plasmons. The interaction of FIR radiation with electrons in superlattices results in both collisional absorption and plasmon excitation.

5.1. Multilayer superlattices

There is a rather extensive literature devoted to plasma oscillations in multilayer superlattices. The simplest model was proposed by Visscher and

Falicov [25]: the superlattice is treated as an infinite pile of equidistant electron sheets submerged in an insulating medium. Tunneling across the insulating gaps is negligibly small, and the electrons are free to move only in the planes $z = nd$ ($n = 0, \pm 1, \pm 2$, d is the superlattice period). The Poisson equation for the considered system coincides with eq. (4.8), where $\delta(z - nd)$ has to replace $\psi_\alpha^2(z)$ and the summation over n (instead of α) goes from $-\infty$ to $+\infty$. The solution of the type of travelling wave is

$$\phi = \sum_n A_n \exp(\mathrm{i}kx - \mathrm{i}\omega t - k|z - nd|),$$

where the coefficients A_n are determined by the equations

$$\left.\frac{\partial\phi}{\partial z}\right|_{nd+0} - \left.\frac{\partial\phi}{\partial z}\right|_{nd-0} = -\frac{4\pi e}{\epsilon}\tilde{N}_n. \tag{5.1}$$

In the collisionless approximation $\tilde{N}_n$ can be expressed via the polarization operator (see e.g. ref. [2]). For zero temperature we have

$$\tilde{N}_n = -\frac{me}{\pi}\phi(z = nd)\left[1 - \left(1 - \frac{k^2 v_0^2}{\omega^2}\right)^{-1/2}\right]. \tag{5.2}$$

In eq. (5.2) the momentum k is assumed to be much less than the Fermi momentum mv_0. After substituting (5.2) into (5.1) one gets the following system of equations for the coefficients A_n:

$$A_n - \frac{2}{ka_0}\left[1 - \left(1 - \frac{k^2 v_0^2}{\omega^2}\right)^{-1/2}\right]\sum_m A_m \,\mathrm{e}^{-kd|m|} = 0. \tag{5.3}$$

The dispersion law which follows from eq. (5.3) has the form (cf. eq. (2.7))

$$\omega^2(k, q) = \frac{2\pi e^2 N_s k}{m\epsilon}\,\frac{[2F(k, q) + ka_0]^2}{4F(k, q) + ka_0}, \tag{5.4a}$$

$$F(k, q) = \frac{\sinh(kd)}{2[\sinh^2(\frac{1}{2}kd) + \sin^2(\frac{1}{2}kd)]}. \tag{5.4b}$$

Thus there exists a band of plasma oscillations $\omega(k, q)$ and the plasmons are described by the quasimomentum q in the z-direction (perpendicular to the layers) and by the momentum k in two others directions. The maximal frequency of the plasmon corresponds to the case $q = 0$, and in the most probable experimental situation, $kd \ll 1$, it equals $\omega_{max} = \omega_0$, i.e. the 3D plasma frequency at the average bulk density of electrons

$$\omega_0^2 = \frac{4\pi e^2 N_s}{\epsilon m d}.$$

The first and second leading terms in the expansion of $\omega^2(k, q)$ from eq. (5.4) in the region $kd_0 \ll 1$ give a result obtained earlier by Fetter [26] and Apostol

[27]. At still longer wavelength ($k, q \ll d^{-1} \ll d_0^{-1}$) a dispersion law for 3D anisotropic plasmons arises:

$$\omega^2 = \omega_0 k(k^2 + q^2)^{-1/2}.$$

Finally, in the middle part of the plasmon band ($qd \approx 1$) at $kd \ll 1$ we have

$$\omega \approx ku \operatorname{cosec}(\tfrac{1}{2}qd),$$

where the velocity u equals $(\pi e^2 dN_s/\epsilon m)^{1/2}$.

An experimental observation of the band dispersion law $\omega(q, k)$ could, in principle, be realized by means of FIR absorption measurements using the same grating structure as described before. In this case one has to consider a semi-infinite set of electron sheets occupying the half-space $z > 0$. The exciting field has the form

$$E_0 \exp(-kz + \mathrm{i}kx - \mathrm{i}\omega t);$$

this is the kth Fourier component of the FIR field having passed through the grating structure (as usual, $k = 0, \pm 2\pi/\alpha, \dots$).

5.1.1. Resonant excitation of the plasmons in multilayer superlattices

To allow for the exciting field we have to substitute

$$\phi \to \phi + (\mathrm{i}E_0/k)\,\mathrm{e}^{-knd}$$

in eq. (5.2). Then in the region $\omega \gg kv$ one obtains for $\tilde{N}_n$

$$\tilde{N}_n = \frac{eN_s k}{m\omega(\omega + \mathrm{i}\nu)}\left[k\phi(z = nd) + \mathrm{i}E_0\,\mathrm{e}^{-kdn}\right], \tag{5.5}$$

where ν is the phenomenological collision frequency of electrons (identical for all the sheets). The solution $\phi(x, z)$ in the form written above leads to the following semi-infinite chain of equations for the coefficients A_n:

$$A_{n \geqslant 0} = \frac{\omega_p^2}{\omega(\omega + \mathrm{i}\nu)}\left(\sum_{m=0}^{\infty} A_m\,\mathrm{e}^{-kd|m-n|} + \frac{\mathrm{i}E_0}{k}\,\mathrm{e}^{-kdn}\right). \tag{5.6}$$

The system (5.6) is a discrete version of the Wiener–Hopf integral equation. It can be solved by making use of the methods described e.g. by Rogozhin [28] and Gilinskii and Syltanov [29]. The result is

$$A_{n \geqslant 0} = \frac{2\mathrm{i}E_0}{k}\,\frac{\omega_p^2}{\omega(\omega + \mathrm{i}\nu)}\,\frac{\sinh(kd)}{\exp(\mathrm{i}qd) - \exp(-kd)}\exp(-\mathrm{i}qnd). \tag{5.7}$$

The plasmon quasimomentum for fixed ω and k is given by the dispersion equation

$$\cos(qd) = \cosh(kd) - \frac{\omega_p^2}{\omega(\omega + \mathrm{i}\nu)}\sinh(kd). \tag{5.8}$$

An interesting formula is obtained for the x-component of the *total* electric field $E_{tot}(n)$ acting upon electrons in the nth layer:

$$E_{tot}(n) = E_0 \exp(-nkd) - ik\phi(z = nd) = \frac{2E_0 \sinh(kd) \exp(-iqdn + ikx)}{\exp(iqd) - \exp(-kd)}. \tag{5.9}$$

In the collisionless approximation ($\nu = 0$), $E_{tot}(n)$ decreases exponentially when n increases if the frequency ω lies beyond the plasmon band determined by eq. (5.8). For the experimentally most realistic case, $kd \ll 1$, the band boundaries are $\omega_{max} = \omega_p(2/kd)^{1/2}$ and $\omega_{min} = \omega_p(kd/2)^{1/2}$. In the vicinity of the upper threshold $\omega \gtrsim \omega_{max}$ the spatial decrement of the wave (5.9) equals

$$\operatorname{Im} q = q'' = -k\sqrt{2(\omega - \omega_{max})/\omega_{max}}, \tag{5.10a}$$

whereas at $\omega \lesssim \omega_{min}$

$$q'' = -(2/d)\sqrt{2(\omega_{min} - \omega)/\omega_{min}}, \tag{5.10b}$$

within the plasmon band, when collisions are neglected, q'', of course, equals zero. This allows us, in a sense, to speak about resonant excitation of the plasma waves in superlattices: in the region $\omega_{min} < \omega < \omega_{max}$ the total electric field in the nth sheet does not vanish at $n \to \infty$ though the exciting field attenuates as $\exp(-kdn)$. The frequency dependence of the transmitted wave intensity is given by

$$|E_{tot}|^2 = E_0^2(e^{2kd} - 1)\omega^2/\omega_p^2, \quad \omega_{min} < \omega < \omega_{max}. \tag{5.11}$$

If the scattering of electrons is allowed for, an additional damping arises. For example, in the middle of the band ($\omega = \omega_{max}/2$) one has $q'\Delta \ll 1$ (where $q' = \operatorname{Re} q$), $q'' = -k\nu/\sqrt{2}\,\omega_{max}$. An estimation for the GsAs–GaAlAs structure gives the following results: $d = 2 \times 10^{-6}$ cm, $\nu = 3 \times 10^{11}$ s^{-1}, $N_s = 10^{12}$ cm, $m = 6 \times 10^{-29}$ g, $\epsilon = 12.5$, the grating structure period $a = 3 \times 10^{-4}$ cm; then $kd = 4 \times 10^{-2}$, $\omega_{max} = 4 \times 10^{13}$ s, $|q''| = 0.5 \times 10^{-2}k$.

Thus, if the number of layers is larger than 60, the exciting field decreases by more than one order of magnitude, i.e. the transmission of the "diffracted" field through the superlattice is provided mainly by a transformation into plasma waves.

5.1.2. The absorption band shape

The total work done by the electric field on the electrons per unit of time is the sum over all the layers:

$$Q = \tfrac{1}{2} \operatorname{Re} \sum_{n=0}^{\infty} E_0 \exp(-kdn)\, j_n,$$

$$j_n = E_{tot}(n)\sigma, \quad \sigma = ie^2N_s/m(\omega + i\nu).$$

By making use of eq. (5.9) we obtain the following (rather cumbersome) expression

$$Q=\frac{E_0^2e^2N_s}{m\omega}\frac{\omega^2\sinh(kd)}{\omega^2+\nu^2}\frac{e^{q''d}\cos(q'd)}{A}$$
$$\times\frac{e^{q''d}(1-e^{-2kd})\sin(q'd)+(\nu/\omega)\sinh(kd)(1-e^{2q''d-2kd})}{\left[1-2e^{q''d-kd}\cos(q'd)+e^{2q''d-2kd}\right]^2},\tag{5.12}$$

where

$$A=\cosh(kd)-\frac{\omega_p^2}{\omega^2+\nu^2}\sinh(kd),\quad B=\frac{\nu}{\omega}\frac{\omega_p^2\sinh(kd)}{\omega^2+\nu^2},$$
$$C=\frac{1-A^2-B^2}{2}.$$

The frequency dependence of $Q(\omega)$ is contained also in q' and q'' determined by eq. (5.8):

$$\sin^2(qd)=C+\sqrt{B^2+C^2},\quad e^{q''d}=\frac{A}{\cos(q'd)}-\frac{B}{\sin(q'd)}.\tag{5.13}$$

At the band edges $Q(\omega)$ is characterized by simple asymptotic laws (we assume $kd\ll 1$, $q''\ll k$):

$$Q=Q_0\frac{\nu}{\left[2(\omega-\omega_{max})\omega_{max}\right]^{1/2}},\qquad \omega-\omega_{max}\gg\nu,\tag{5.14a}$$

$$Q=Q_0(\nu/2\omega_{max})^{1/2},\qquad |\omega-\omega_{max}|\ll\nu,\tag{5.14b}$$

$$Q=Q_0\left[2(\omega_{max}-\omega)/\omega_{max}\right]^{1/2},\qquad \omega_{max}-\omega\gg\nu.\tag{5.14c}$$

Here

$$Q_0=E_0^2e^2N_skd/2m\omega_{max};$$

in the vicinity of the lowermost threshold $Q(\omega)$ is described by a similar formula, only a substitution has to made: $\omega_{max}\to\omega_{min}$,

$$Q_0=E_0^2e^2N_s(kd)^2/m\omega_{min}.$$

In the inner part of the plasmon band, when $|\omega-\omega_{max}|$, $|\omega-\omega_{min}|\gg\nu$, electron scattering becomes unimportant. The work of the exciting field is spent mainly for generating plasma waves. The absorption is described by

$$Q(\omega)=\frac{E_0^2e^2N_s}{4m\omega}(e^{2kd}-1)\frac{\omega^2}{\omega_p^2}\left[2\frac{\omega^2}{\omega_p^2}\coth(kd)-1-\frac{\omega^4}{\omega_p^4}\right]^{1/2}.\tag{5.15}$$

The $Q(\omega)$ curve depicted in fig. 3 is calculated from eq. (5.12) for $kd=4\times 10^{-2}$, $\nu/\omega_p=0.05$.

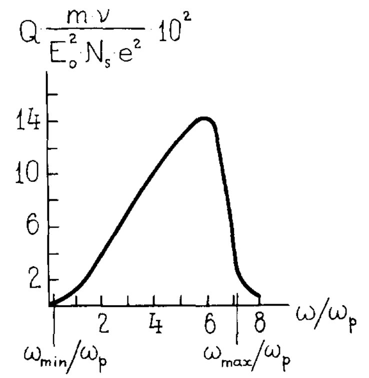

Fig. 3. Absorption curve for a multilayer superlattice. The middle part of the curve corresponds to transformation of the external FIR wave into 2D plasmons.

Thus, in the multilayer superlattice there occurs a transformation of FIR radiation into plasma waves travelling through the system. Therefore, the superlattice may be regarded as a delay line. The characteristic parameters of the line are illustrated by the following numerical example. Let the amplitude of the exciting field be slowly varying according to the Gaussian law

$$E_0(t) = E_0 \exp(-t^2/T^2)$$

and $\omega T \gg 1$. Then the transmitted pulse is also a Gaussian and its width T^* at $z = nd$ is

$$T^* = T\sqrt{1 + 4\left(\frac{\partial^2 q}{\partial \omega^2}\right)^2 \frac{(nd)^2}{T^4}} . \tag{5.16}$$

We shall consider the frequency region in which qd, $kd \ll 1$ but $q \gg k$. Then the dispersion equation gives $\omega \approx \omega_{max} k/q$. For the values of characteristic parameters used in the preceding subsection and for $k/q \sim 0.1$ we get $\omega \sim 4 \times 10^{12}\ \mathrm{s}^{-1}$, $\omega/\nu \approx 12$, the group velocity of the pulse $v_g = 2 \times 10^7$ cm/s. The relative broadening of the pulse for a number (n) of layers of $\approx 10^3$ equals $(10^{-11}/T)^4$, where T is measured in seconds. The given estimate corresponds to the frequency region $\omega \sim \sqrt{\omega_{max}\omega_{min}}$. Just this region is optimal for obtaining small group velocities. In the vicinity of a band edge one has to allow for electron scattering. In the region $|\omega_{max} - \omega| \ll \nu$ the calculations give

$$v_g = \sqrt{2\nu\omega_{max}}/k \approx 2.5 \times 10^8\ \mathrm{cm/s}$$

(for the same ω_{max}, ν and k as used above). The lower most band boundary is hardly of interest because in this case collisional damping in the superlattice is too high: $\omega_{min} \sim \nu$.

5.2. Lateral superlattices

Independently of how the lateral superlattice is fabricated its most important feature is the periodic spatial modulation of the equilibrium electron density $N_s(x) = N_s(x+l)$. We consider here a one-dimensional superlattice with period l and confine ourselves to purely classical plasma oscillations. Strictly speaking there is also a quantum effect of this periodicity on the electron spectrum (minibands and minigaps), so that the electron plasma consists of a number of components. As is clear from what was written above, the optically most active branch of the plasma waves corresponds to in-phase oscillations of all the electron groups. This branch is insignificantly affected by distortions in the electron dispersion law due to superlattice effects. However the plasmon dispersion relation displays all the features characteristic of wave processes in a periodic structure.

It is convenient to write the equilibrium plasma density in the form $N_s(x) = N_0 + n(x)$, where $n(x)$ is a periodic function with an average value of zero. The equations of motion of a cold plasma ($\omega \gg kv$) together with the Poisson equation give a relation between the Fourier components of the non-equilibrium part of the electron density $\tilde{N}_s(q)$ and the Fourier components of the potential $\phi(q)$:

$$\tilde{N}_s(q) = \frac{e}{2\pi m\omega^2}\left[N_0\phi(q) + \sum_j n(q-g_j)\phi(g_j)\right], \tag{5.17}$$

where $g_j = 2\pi j/l$ ($j = \pm 1, \pm 2, \dots$) are the reciprocal lattice vectors. After satisfying the boundary conditions an equation is obtained that determines $\phi(q)$:

$$\phi(q) = \frac{\omega_p^2(q)}{\omega^2 - \omega_p^2(q)} \sum_j \frac{n(g_j)}{N_0}(1 - g_j/q)\phi(q-g_j); \tag{5.18}$$

here $\omega_p(q)$ is the average plasma frequency for an "appropriate" situation. For example, in MOS structure

$$\omega_p^2(q) = 4\pi e^2 N_0 q/m\left[\epsilon_s + \epsilon_{ox}\coth(|q|\Delta)\right].$$

The spectrum $\omega(q)$ that follows from eq. (5.18) is a typical band spectrum. If $|n| \ll N_0$ one may solve the problem by making use of an approximation quite similar to the nearly free electron approximation in the quantum theory of solids. The plasmon dispersion curve is now broken by forbidden gaps situated at $q_j = 2\pi j/l$. In the vicinity of the jth forbidden gap one has the dispersion equation

$$N_0^2 = G_\omega(q)G_\omega(q-g_j)n^2(g_j), \quad G_\omega(q) = \frac{\omega_p^2(q)}{\omega^2 - \omega_p^2(q)}. \tag{5.19}$$

The boundaries of the forbidden gap $\omega_{\pm}$ are determined by the formula [30]:

$$\omega_{\pm}^2 = \frac{4\pi^2 e^2 j\left[N_0 \pm |n(g_j)|\right]}{ml\left[\epsilon_s + \epsilon_{ox}\coth(\pi j\Delta/l)\right]}. \tag{5.20}$$

Evidently this expression, obtained in a macroscopic hydrodynamic approximation, is valid only if the inequality $N_0 l^2 \gg 1$ is satisfied. This means that even within one period of the superlattice the plasma may be considered as a continuous liquid, but not as a collection of discrete particles.

Thus, in the reduced zone scheme there is a set of plasmon frequencies for each quasimomentum k within the first Brillouin zone $\omega_p(k+g_j)$. The external field $E_0\exp[i(kx-\omega t)]$ excites, in principle, all the plasmon branches but, of course, with a different efficiency. To characterize this efficiency quantitatively one may calculate the electric field amplitude E of the appropriate plasma wave at the resonance point $\omega_j = \omega_p(k+g_j)$ [31]. Simple calculations give

$$E(\omega_j) = E_0\frac{n(g_j)}{N_0}\left[\omega_p(k+g_j)\tau\right] e^{i(k+g_j)x - i\omega t}, \tag{5.21}$$

where τ is the electron relaxation time. If the momentum k of the exciting field coincides with one of the reciprocal lattice vectors g_j a splitting of the plasmon resonances must be observed in accordance with eq. (5.20). Such a splitting has been reported recently by Mackens et al. [9] who fabricated a MOS capacitor with a modulated oxide thickness of submicrometer periodicity. The splitting, $\Delta\omega_p^2 = \omega_+^2 - \omega_-^2$, increased nearly linearly with the gate voltage, $V_g - V_t$, just what one should expect from eq. (5.20), because $V_g - V_t$ is proportional to the areal density of 2D free electrons.

6. Emission of electromagnetic waves by 2D plasmons

Experiments on resonant FIR absorption by 2D plasmons deal with a system of an open-type resonator. A grating structure fabricated upon the gate electrode defines the momentum of the excited plasmons. When a plasma wave is already excited the grating structure acts as an emitting antenna and transforms the plasmon energy into electromagnetic radiation. This means the plasma waves in such structures have an additional damping mechanism, i.e. radiative decay. This effect has been observed experimentally by Tsui et al. [32] and Höpfel et al. [33]. On the other hand, it has been pointed out [34] that the width of the plasmon resonance exceeds the scattering rate of electrons. A question arises whether this can be explained by the radiative decay of plasmons.

A surprising circumstance has not been commented upon in publications from the very first experiments with 2D plasmons: why is the observed dispersion relation of the plasma waves in good agreement with a formula (eq. (2.9)) derived for a closed resonator, i.e. for a system with a uniform gate electrode? After all there is no small parameter in the problem under consideration: the plasmon wavelength equals the period of the grating structure, the sizes of the transparent and opaque regions differ not too strong. Thus, we are dealing with waves in a strongly modulated periodic structure, and the wave vector is equal to the reciprocal lattice vector. Hence, the continuous-medium approximation is totally inapplicable. To settle this contradiction one has to allow for the influence of the grating structure both on the imaginary (radiative decay) and on the real part of the plasmon frequency. In what follows a model of the system under consideration is proposed for which a solution can be found analytically [35].

6.1. Radiative damping and dispersion law of 2D plasmons in an open resonator

The model proposed is depicted in fig. 4. The 2D plasma occupies the plane P ($z = \Delta$), the grating structure lies in the plane M ($z = 0$). One can treat the M-plane as also being occupied by 2D plasma with a periodically modulated equilibrium density $N(y) = N(y + a)$. The electrical permeabilities are: 1 for $z < 0$, ϵ_{ox} for $0 < z < \Delta$, ϵ_s for $z > \Delta$.

Of course, posing the problem as a diffractional one seems to be more natural: appropriate boundary conditions for the fields $\boldsymbol{E}$ and $\boldsymbol{H}$ are to be satisfied on metal strips of the grating structure. However, in such an approach even the easier purely electromagnetic problem of diffraction of a plane wave on a periodic system of metal strips can be solved only for $t/a = \frac{1}{2}$, where t is the width of the transparent parts of the grating structure (see e.g. ref. [36]). The model proposed here allows us to get an approximate solution and retains the main features of the real experimental situation.

We have to solve self-consistently the Maxwell equations and the equation of motion of the plasma. A special feature of our problem is the strong correlation between the spatial spectrum of the plasma waves and that of the

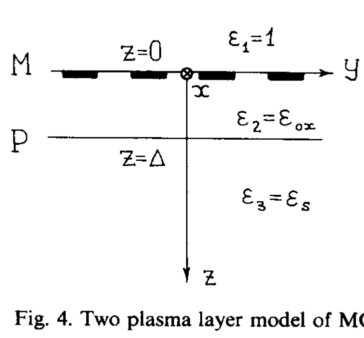

Fig. 4. Two plasma layer model of MOS structure or heterojunction with grating structure.

external force acting upon the plasma: both are determined by the same periodic function $N(y)$. The spatial Fourier components of the external force are non-zero only for waves numbers equal to $2\pi n/a$ ($n = 0, \pm 1, \pm 2, \ldots$) whereas the gaps in the plasma wave spectrum occur at the plasmon quasi-momenta $n\pi/a$. Hence, in FIR absorption experiments one observes the plasmons in the vicinity of forbidden gaps of *even* ordinal number. This means that in the reduced band scheme we deal with the centre of the first Brillouin zone and have to find a solution of the Maxwell equations which is periodical in y with period a.

As is clear from fig. 4, the non-zero components are E_y and E_z for the electric field and H_x for the magnetic one. The latter is chosen in the form:

$$H_x^{(1)} = \sum_n D_n \exp(ikyn + \kappa_{n1} z), \qquad \text{for} \quad z < 0, \qquad (6.1\text{a})$$

$$H_x^{(2)} = \sum_n A_n \exp(ikyn + \kappa_{n2} z) + B_n \exp(ikyn - \kappa_{n2} z), \qquad \text{for} \quad 0 < z < \Delta, \quad (6.1\text{b})$$

$$H_x^{(3)} = \sum_n C_n \exp(ikyn - \kappa_{n3} z), \qquad \text{for} \quad z > \Delta, \qquad (6.1\text{c})$$

where $k = 2\pi/a$, $\kappa_{nj}^2 = q^2 n^2 - \epsilon_j \omega^2/c^2$, $j = 1, 2, 3, \ldots$ (see fig. 4). The tangential component of the electric field is given by the equation

$$E_y^{(j)} = \frac{ic}{\omega \epsilon_j} \frac{\partial H_x^{(j)}}{\partial z}. \qquad (6.2)$$

The boundary conditions are

$$E_y^{(1)} = E_y^{(2)}, \quad H_x^{(2)} - H_x^{(1)} = \frac{4\pi}{c} j_{\text{surf}}, \quad j_{\text{surf}} = \frac{ie^2 N(y)}{m\omega} E_y, \qquad (6.3\text{a})$$

at $z = 0$, and

$$E_y^{(2)} = E_y^{(3)}, \quad H_x^{(3)} - H_x^{(2)} = \frac{4\pi}{c} \frac{ie^2 N_s}{m^* \omega} E_y, \qquad (6.3\text{b})$$

at $z = \Delta$. The surface currents in eqs. (6.3a) and (6.3b) are written in the cold collisionless plasma approximation; N_s is the equilibrium plasma density in the P-plane ($z = \Delta$), m and m^* are the effective masses of electrons in the grating structure and inversion layer, respectively. By making use of eqs. (6.1)–(6.3b) one can obtain a secluded equation for the coefficients D_n. In the most general case (three parts of the space with different electrical permeabilities) this equation is rather cumbersome. Two small parameters allow us to get an essentially simple equation. These parameters are ω/ck and $\omega\Delta/c$; usually $\omega/ck \sim a\omega/2\pi c < 10^{-2}$; $\omega\Delta/c \sim 10^{-3}$. Then we have

$$\frac{D_n \alpha_n F_n}{M_n} - \frac{4\pi e^2}{m^* \omega^2} \sum_l N_{n-l} \kappa_{l1} D_l = 0. \qquad (6.4)$$

In eq. (6.4) the following notation is used:

$$\alpha_{n\neq 0}=\frac{\epsilon_{ox}\left[\epsilon_s \coth(k\Delta|n|)+\epsilon_{ox}\right]+\epsilon_{ox}\coth(k\Delta|n|)+\epsilon_s}{\epsilon_s+\epsilon_{ox}\coth(k\Delta|n|)},$$

$$\alpha_0=\epsilon_s+1,\quad F_n=1-\omega_n^2/\omega^2,\quad M_n=1-\bar{\omega}_n^2/\omega^2,$$

$$\kappa_{l1}=k|l| \quad \text{if } l\neq 0, \quad \text{and } \kappa_{l1}=-\mathrm{i}\omega/c \quad \text{for } l=0;$$

N_n are the Fourier components of $N(y)$, ω_n and $\bar{\omega}_n$ are the 2D plasmon frequencies in the system without metal electrode and with a uniform ideal conductive electrode, respectively:

$$\omega_n^2=\frac{4\pi e^2 N_s k|n|\left[\epsilon_{ox}\coth(k\Delta|n|)+1\right]}{m^*\left\{\epsilon_{ox}\left[\epsilon_s\coth(k\Delta|n|)+\epsilon_{ox}\right]+\epsilon_{ox}\coth(k\Delta|n|)+\epsilon_s\right\}}, \tag{6.5a}$$

$$\bar{\omega}_n^2=\frac{4\pi e^2 N_s k|n|}{m^*\left[\epsilon_s+\epsilon_{ox}\coth(k\Delta|n|)\right]}. \tag{6.5b}$$

If the modulation $N(y)$ is relatively small ($N_{\pm 1}, N_{\pm 2}, \ldots \ll N_0$) one may solve the system (6.4) in the "weak-coupling" approximation (cf. section 5.2).

The main plasmon resonance corresponds to the plasmon momenta $\pm 2\pi/a$. Hence, the coefficients $D_{\pm 1}$ are the most important in eq. (6.4). The coupling between D_1 and D_{-1} is provided by the Fourier components $N_{\pm 2}$. We have to keep also the coefficient D_0 in as much as we are going to catch the effect of electromagnetic wave emission, because only for $l=0$ does the parameter κ_{l1} become purely imaginary which just corresponds to emission. The coupling $D_0 \rightleftarrows D_{\pm 1}$ is provided by the components $N_{\pm 1}$ and has an additional small parameter $\omega a/2\pi c$ as compared with $D_1 \rightleftarrows D_{-1}$ coupling. Thus the system (6.4) is reduced to three equations for D_0, D_1 and D_{-1}, and we obtain the following dispersion equation

$$\left(\frac{\alpha_1 F_1}{M_1}\frac{m^*\omega^2}{4\pi e^2 N_0 k}-1+\mathrm{i}\frac{4\pi e^2 N_1 N_{-1}}{m^* c(\epsilon_s+1)\omega N_0}\right)^2$$
$$=\left(\frac{N_2}{N_0}-\mathrm{i}\frac{4\pi e^2 N_1^2}{m^* c(\epsilon_s+1)N_0}\right)\left(\frac{N_{-2}}{N_0}-\mathrm{i}\frac{4\pi e^2 N_{-1}^2}{m^* c(\epsilon_s+1)N_0}\right). \tag{6.6}$$

First let us neglect the emission, i.e. we omit the imaginary terms in eq. (6.6). Moreover, in accord with the experimental situation, one may put $N_0 \gg N_s$. Then the roots of the dispersion equation (6.6) are close to the zeros of M_1 and two plasmon frequencies are obtained for the momentum $k=2\pi/a$:

$$\omega_\pm^2=\bar{\omega}_1^2\left[1-\frac{\alpha_1 m^*}{4\pi e^2 k N_0}\left(\omega_1^2-\bar{\omega}_1^2\right)\left(1\pm\frac{|N_2|}{N_0}\right)\right]. \tag{6.7}$$

The gap in the plasmon spectra is

$$\omega_+ - \omega_- \approx \frac{\omega_1 \alpha_1 m^* (\omega_1^2 - \bar{\omega}_1^2) \mid N_2 \mid}{4\pi e^2 k N_0^2} \sim \bar{\omega}_1 \frac{N_s \mid N_2 \mid}{N_0^2}. \tag{6.8}$$

One can see from eq. (6.8) that the gap is small, not only due to the weak modulation ($N_2 \ll N_0$) but also because the ratio N_s/N_0 is very small. N_0 is the average areal density of electrons in the grating structure and for typical N_s this ratio does not exceed 10^{-3}. For rectangular modulation of the gate electrode transparency

$$N_n/N_0 = a \sin(\pi n t/a)[n\pi(a-t)]^{-1},$$

and if $t \ll a$ we have $N_n/N_0 \sim t/a$. Now it is clear why the experimentally observed dispersion law of the plasmon corresponds to the situation with a continuous ideal conductive gate electrode ($\omega = \bar{\omega}$) despite the presence of the grating structure. The solution (6.7) and (6.8) has been obtained for a weakly modulated transparency of the gate electrode, whereas in a real experimental situation the modulation is strong. However, it is shown that the distortions in the dispersion law have an additional small parameter N_s/N_0 which is not connected with the weakness of the modulation. It is plausible that in a real situation this parameter provides for the close agreement of the observed dispersion relation to that of a closed resonator.

Now we can calculate the radiative damping $\Gamma_R = -\operatorname{Im} \omega$. Let us suppose, for the sake of simplicity, the function $N(y)$ to be symmetrical with respect to the middle of the metal stripe of the grating structure. Then the damping of the upper branch ω_+ is

$$\Gamma_R = \omega_+ \frac{\omega}{ck} \left(\frac{N_1}{N_0} \right)^2 \frac{\alpha_1}{\epsilon_s + 1} \frac{\omega_1^2 - \bar{\omega}_1^2}{\omega_+^2}. \tag{6.9}$$

In the region $k\Delta \ll 1$ the emission increases when Δ decreases:

$$\Gamma_R = \omega_+ \frac{\omega}{ck} \left(\frac{N_1}{N_0} \right)^2 \frac{\epsilon_{ox}}{(\epsilon_s + 1) k\Delta}. \tag{6.10}$$

Eqs. (6.9) and (6.10) are derived under the supposition $\Gamma_R \ll \omega_+$; in other words Γ_R is an imaginary correction to a certain root of the dispersion equation (6.6) and it must, of course, be much less than the interval between two various roots. If, for example, Δ tends to zero the root ω_+ vanishes, and for the plasma that forms the grating structure itself one obtains

$$\omega^2 = \omega_{p0}^2 [1 - 2i(\omega/ck) N_1^2/N_0^2],$$

where ω_{p0} is the plasma frequency for the areal density N_0 of the electrons.

In accord with eq. (6.10) the radiative contribution to the total width of the plasmon resonance is rather small. In experiments by Heitmann et al. [34] the characteristic parameters are: $\Delta = 5 \times 10^{-6}$ cm, $a = 5 \times 10^{-5}$ cm, $t/a = 0.25$, $\omega \approx 2 \times 10^{13}$ s^{-1}; then one obtains $\Gamma_R/\omega_+ \sim 3 \times 10^{-4}$. This is two orders of magnitude smaller than the collisional damping. Probably additional broadening observed in the experiment is caused by some other reasons.

In the opposite limiting case $k\Delta \gg 1$ the damping becomes exponentially small: $\Gamma_R \sim \exp(-2k\Delta)$. One can see this from eqs. (6.5) and (6.9); the difference $\omega_1^2 - \overline{\omega}_1^2$ tends to zero which corresponds to an exponential decrease of the plasmon electric field when the distance between the inversion layer and the grating structure increases.

The branch ω_- (the lower edge of the gap in the plasmon dispersion relation) has no radiative damping in the considered case of a symmetric $N(y)$. This can easily be understood from the spatial dependence of the field E_y. In the ω_+ branch E_y has maxima in the centres of the metal stripes, whereas in the ω_- branch there are knots of E_y at the same points.

Formulae (6.7) and (6.9) define, respectively, the frequency and the intensity of spontaneous radiation of the 2D plasma. $2\Gamma_R$ is the amount of energy emitted per second by the plasma mode (k, ω), or, in other words, Γ_R characterizes the grating structure as an emitting antenna. It is interesting to note that in the most real case, $k\Delta \ll 1$, the radiative width does not depend on the geometrical parameters $k = 2\pi/a$ and Δ but is governed only by N_s and the degree of modulation

$$\Gamma_R = \left[4\pi e^2 N_s/mc(\epsilon_s + 1)\right](N_1/N_0)^2.$$

Thus, the transformation coefficient of 2D plasmons into FIR radiation contains only a small kinematic parameter, i.e. the ratio of the plasmon phase velocity to the speed of light. As there is no smallness of the type N_s/N_0, we see that in the system under consideration the imaginary part of the eigenfrequency is much larger as compared with the distortion of its real part. This circumstance is rather important for understanding experiments on electromagnetic radiation of 2D plasmons.

6.2. *Non-equilibrium plasmons in a 2D electron gas*

Electromagnetic radiation corresponding to radiative decay of 2D plasmons appears when a constant electric field is applied to a MOS structure or heterojunction parallel to the interface. In the available experiments the electric field causes the electron to drift at a velocity u_0 of the order of $(1-10) \times 10^5$ cm/s. The radiation frequency does not depend on the applied field and is determined, in accord with the plasmon dispersion law, by the momentum k and by the areal charge density N_s. The effect therefore cannot be attributed to the onset of transition radiation when the charge moves near the lattice of metallic electrodes. On the other hand, the attained drift velocities are much lower than the phase velocity of the generated plasmons ($\omega \sim 10^{13}$

s^{-1}, $k \sim 10^4$ cm^{-1}), so that Čerenkov emission of plasma waves is likewise impossible. It seems quite evident that the observed emission occurs due to a heating of the electron gas in the electric field [33].

The physical mechanism of 2D plasmon excitation, when the electron gas is being heated up, is bremsstrahlung of plasma waves by electrons. Plasmon emission and absorption are allowed, irrespective of the electron velocity, if they are accompanied by electron scattering from impurities, phonons, etc. These bremsstrahlung effects produce a plasmon gas described in the thermodynamic equilibrium state by a Planck distribution function $n_0(\omega)$. The radiative decay of the equilibrium plasmons contributes to the background ("black") radiation. The temperature of this radiation equals the temperature of the electron gas (not the lattice!). This has been proved in experiments by Höpfel et al. [33].

The electric field, however, not only heats the electrons, but also upsets the equilibrium of the plasmon subsystem due to electron drift. If the non-equilibrium distribution function of the plasmons is such that $n_k > n_0$ in the mode "followed" by the experiment, an excess of electromagnetic radiation must be observed at the frequency $\omega(k)$. The "followed" mode is one of the eigenmodes of the resonator formed by the grating structure, i.e. $k = 2\pi/a$, $4\pi/a, \ldots$. The intensity of the excess radiation is obviously proportional to $\delta n_k = n_k - n_0$. The problem is, thus, reduced to the calculation of the non-equilibrium distribution function of the plasmons [21].

6.2.1. Basic equations

The plasma oscillations of electrons located in the $z = 0$ plane are described by the equations

$$\Delta\phi = -4\pi e\tilde{N}_s(\rho)\delta(z), \quad \tilde{N}_s = -N_s \operatorname{div} \boldsymbol{R}(\rho), \tag{6.11}$$

where $\boldsymbol{R}(\rho)$ is the particle-displacement vector. We expand all quantities in Fourier series and introduce the normal coordinates $\boldsymbol{Q}_k$ in standard fashion:

$$\boldsymbol{R} = \sum_{\boldsymbol{k}} \boldsymbol{Q}_k \, e^{i\boldsymbol{k}\cdot\boldsymbol{\rho}} + \text{c.c.}$$

The Fourier component of the potential in the $z = 0$ plane is then

$$\phi_k = -2\pi i e N_s(\boldsymbol{k}\cdot\boldsymbol{Q}_k)/k,$$

and the Hamiltonian of the free plasmon field is

$$H = \tfrac{1}{2}\sum_{\boldsymbol{k}} \left\{ \frac{\boldsymbol{p}_k \cdot \boldsymbol{p}_{-k}}{mN_s} + m\omega^2(k) N_s \boldsymbol{Q}_k \cdot \boldsymbol{Q}_{-k} \right\}, \tag{6.12}$$

where $\omega^2 = 2\pi e^2 N_s k/m$, and $\boldsymbol{P}_k$ is the momentum operator. The interaction of the electrons with the plasmons is described by the operator

$$V_{\text{int}} = 2\pi i e^2 N_s \sum_k \frac{\boldsymbol{k}\cdot\boldsymbol{Q}_k}{k} e^{i\boldsymbol{k}\cdot\boldsymbol{\rho}} + \text{c.c.} \tag{6.13}$$

As already mentioned, first-order processes are forbidden by the energy–momentum conservation laws, so that, in the transport equation for the function n_k, it is necessary to take into account plasmon creation and annihilation processes described by V_{int}, with simultaneous scattering of the electrons by impurities or by phonons. We denote by $W(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{k})$ the probability of a process in which the transition of an electron from a state $\boldsymbol{p}$ into $\boldsymbol{p}'$ is accompanied by creation of a plasmon having a momentum $\boldsymbol{k}$.

We write down the transport equation for n_k:

$$\frac{\partial n_k}{\partial t} = \sum_{pp'} W(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{k})\left[n_k(f_p - f_{p'}) + f_p(1 - f_{p'})\right], \tag{6.14}$$

$$p^2 - (p')^2 = 2m\omega, \quad \hbar = 1.$$

f_p is the electron-momentum distribution function.

In the experiments the crystal lattice was at liquid-helium temperature, and the electron gas heated up substantially. It is known that at the drawing fields used (10–30 V/cm) the temperature of the two-dimensional electrons in the Si–SiO_2 system is of the order of 10–20 K at a Fermi energy of the order of $E_F \sim 200$ K. Without determining here the true form of the electron distribution function, we substitute in (6.14) a function f_p that describes a drift with specified velocity $\boldsymbol{u}_0$. We then obtain n_k from the stationarity conditions. The simplest procedure is to put $f_p = f_0(\boldsymbol{p} - \boldsymbol{p}_0)$, where f_0 is the equilibrium distribution function at a temperature T different from the lattice temperature. $\boldsymbol{p}_0 = m\boldsymbol{u}_0$.

We note that in principle the plasma oscillations can be excited by another mechanism connected with the development of instability. The instability sets in if the coefficient of n_k in the right-hand side of eq. (6.14) becomes positive. We shall show below that this possibility is realized at drift velocities much higher than in the experiments reported,and can therefore have no bearing on the effect observed.

6.2.2. Impurity scattering

We begin with calculating $W(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{k})$ in the lowest Born approximation in the electron–impurity interaction. Within the framework of standard perturbation theory, account must be taken of two processes, in which the plasmon emission either precedes or follows the scattering by the impurity. We obtain for the probability W averaged over the impurity locations

$$W(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{k}) = \frac{8\pi N_i m^4\omega^3}{k^2}\left|\frac{U(\boldsymbol{p} - \boldsymbol{p}' - \boldsymbol{k})}{p^2 - (\boldsymbol{p}' + \boldsymbol{k})^2} + \frac{U(\boldsymbol{p}' - \boldsymbol{p} - \boldsymbol{k})}{(\boldsymbol{p}')^2 - (\boldsymbol{p} - \boldsymbol{k})^2}\right|^2$$
$$\times\,\delta\left(p^2 - (p')^2 - 2m\omega\right), \tag{6.15}$$

where $U(\boldsymbol{p})$ is the Fourier component of the impurity potential and N_i is the impurity density. We simplify (6.15) by neglecting $\boldsymbol{k}$ in the argument of $U(\boldsymbol{p}-\boldsymbol{p}')$, as well as the terms $\boldsymbol{k}\cdot\boldsymbol{p}/m$ compared with $\omega(k)$. The validity of these simplifications follows from the condition for the existence of weakly damped plasma waves ($\omega \gg kv$, $k \ll mv_0$). We have

$$W=\frac{2\pi N_i}{\omega(k)}|U(\boldsymbol{p}-\boldsymbol{p}')|^2\left[\frac{\boldsymbol{k}\cdot(\boldsymbol{p}-\boldsymbol{p}')}{k}\right]^2\delta\left(p^2-(p')^2-2m\omega\right). \tag{6.16}$$

It follows from the energy conservation law that at $T=0$ the plasmon frequency cannot exceed $2k_F u_0$, where k_F is the Fermi wave number. Estimates show that $\omega/2k_F u_0 \approx 3$–$4$ in the experiments, i.e., the radiation is due only to thermal smearing of the Fermi distribution. We consider first the region $k_F u_0 \ll \omega$, T. The distribution function $f_0(\boldsymbol{p}-\boldsymbol{p}_0)$ should be expanded in this case in powers of $\boldsymbol{p}_0$. Obviously, when eq. (6.16) is used for the transition probability, all the odd terms of this expansion vanish. A contribution linear in $\boldsymbol{p}_0$ would appear if corrections of order $\boldsymbol{k}\cdot\boldsymbol{p}/m\omega$ were retained in eq. (6.15). The corresponding increment to $n_{\boldsymbol{k}}$ is obviously proportional to $\boldsymbol{k}\cdot\boldsymbol{p}_0$ and cannot contribute to the observed radiation, since the total number of plasmons with specified frequency $\omega(k)$ does not change.

We obtain the stationary solution of eq. (6.14) by substituting $W(\boldsymbol{p},\boldsymbol{p}',\boldsymbol{k})$ from (6.15), and $f_{\boldsymbol{p}}$ and $f_{\boldsymbol{p}'}$ in the form of expansion up to and including terms of order p_0^2. It is easy to verify that for any isotropic impurity potential (i.e. $U(\boldsymbol{p}-\boldsymbol{p}')$ depends only on the angle between $\boldsymbol{p}$ and $\boldsymbol{p}'$) the stationary distribution function of plasmons is of the form

$$n_{\boldsymbol{k}}=n_0(\omega)+\text{const.}\times p_0^2(\cos^2\alpha+\tfrac{1}{2}), \tag{6.17}$$

where α is the angle between the vector $\boldsymbol{k}$ and the drift velocity. The actual value of the constant in (6.17) depends on the form of the potential and on the relations between T, ω and E_F. In the Boltzmann case, when it is natural to assume also that $\omega \ll T$, scattering by short-range impurities and two-dimensional Rutherford scattering lead to the same result

$$\delta n_{\boldsymbol{k}}=\left(p_0^2/2m\omega\right)\left(\cos^2\alpha+\tfrac{1}{2}\right),\quad p_0^2\ll mT. \tag{6.18}$$

It can be shown that for scattering by short-range impurities eq. (6.18) is valid in a larger region, namely $p_0^2 \ll mT^2/\omega$. For degenerate electrons scattered by charged impurities we have

$$\delta n_{\boldsymbol{k}}=\frac{p_0^2k_F^2}{2m^2T^2}e^{-\omega/T}\left(\cos^2\alpha+\tfrac{1}{2}\right),\quad E_F\gg T,\quad \omega\gg T, \tag{6.19a}$$

$$\delta n_{\boldsymbol{k}}=\frac{p_0^2k_F^2}{12m^2\omega T}\left(\cos^2\alpha+\tfrac{1}{2}\right),\quad E_F\gg T,\quad \omega\ll T, \tag{6.19b}$$

where E_F is the Fermi energy. For short-range impurity centres the results differ from (6.19b) by a factor of 3/2.

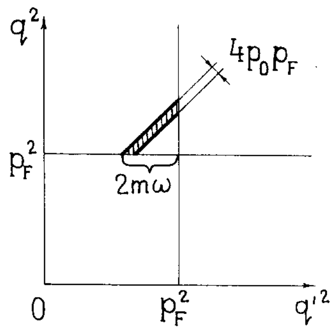

Fig. 5. Region of integration in eq. (6.14).

We consider now a region in which $k_F u_0/T$ is arbitrary, but in accord with the experimental conditions we have $k_F u_0 \ll \omega$, $T \ll \omega \ll E_F$; to calculate the integrals in (6.14) it is convenient to transform to the variables $\boldsymbol{q} = \boldsymbol{p} - \boldsymbol{p}_0$, $\boldsymbol{q}' = \boldsymbol{p}' - \boldsymbol{p}_0$. In the $(q^2, (\boldsymbol{q}')^2)$ plane the effective integration region is then a strip of width $4p_0 p_F$ and length $2^{3/2} m\omega$ (see fig. 5). The following approximations are valid in this region

$$f_0(\boldsymbol{q}) - f_0(\boldsymbol{q}') = -1, \quad f(\boldsymbol{q})(1 - f(\boldsymbol{q}')) \approx \exp\left(\frac{(q')^2 - q^2}{2mT}\right).$$

The calculations lead to the following results:
for short-range impurities

$$n_k = e^{-\omega/T}\{ I_0^2(\gamma) + I_1^2(\gamma) + \cos(2\alpha)\left[I_0(\gamma) I_2(\gamma) + I_1^2(\gamma)\right]\}, \tag{6.20a}$$

for Coulomb centres

$$n_k = e^{-\omega/T}\left[I_0^2(\gamma) + \cos(2\alpha) I_1^2(\gamma)\right], \tag{6.20b}$$

where $\gamma = u_0 k_F/T$ and $I_{0,1,2}$ are Bessel functions of imaginary argument.

At $\gamma \ll 1$ eqs. (6.20a) and (6.20b) reduce to the earlier results for low drift velocities. If, however, $\gamma \gg 1$ then

$$n_k = \frac{A}{\pi\gamma}\cos^2\alpha \exp\left(\frac{2k_F u_0 - \omega}{T}\right); \tag{6.21}$$

$A = 2$ in the case of (6.20a) and $A = 1$ for (6.20b). Thus, with increasing drift velocity the anisotropy of the effect becomes stronger (a factor $\cos^2\alpha$ in place of $\cos^2\alpha + \frac{1}{2}$). We present the value of the anisotropy parameter η, defined as the ratio of the radiation intensity at $\boldsymbol{k} \parallel \boldsymbol{u}_0$ to the analogous quantity at $\boldsymbol{k} \perp \boldsymbol{u}_0$ for scattering by Coulomb centres

γ:	0.5	0.8	1	1.2	1.5	2
η:	3.06	3.17	3.25	3.36	3.58	4.03

The temperature dependence of n_k as expected, is activation-governed in the region $\omega \gg T$, with an activation energy equal to the excess energy of the emitted plasmons above the threshold value $2k_F u_0$.

If both types of scattering are simultaneously present, it is obvious as a result of the random distribution of the impurities that the total probability W is made up additively of the quantities W_C and W_n, which describe, respectively, the contribution of the Coulomb and neutral centres. In the absence of degeneracy and at drift velocities much lower than thermal, we again obtain eq. (6.18) for δn, this being obvious beforehand, since both types of impurity give the same result in this case. The situation is more complicated for degenerate electrons. We denote by X the ratio of the integrated probabilities of plasmon emission in scattering by two types of impurity:

$$X = \sum_{pp'} W_n(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{k}) / \sum_{pp'} W_C(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{k}).$$

At low drift velocities (in the sence of $\gamma \ll 1$) we then have

$$n_k - n_0 = \frac{2+3X}{2+2X} \delta n,$$

where δn is taken from (6.19a) or (6.19b) depending on the ratio of ω and T. If however, $\gamma \gg 1$, then

$$n_k = \frac{\cos^2\alpha}{\pi\gamma} \frac{1+2X}{1+X} \exp\left(\frac{2k_F u_0 - \omega}{T}\right);$$

see (6.21). Thus, in both limiting cases the dependence of n_k on α and u_0 remains the same as before, and all that depends on X, is the total radiation intensity. At arbitrary γ the formula obtained is more cumbersome, but the minimum of the effect corresponds as before to $\alpha = \pi/2$, while the maximum is reached at $\alpha = 0$ and $\alpha = \pi$.

6.2.3. Scattering by phonons

Under the experimental conditions (lattice temperature 4 K, electron temperature not higher than 10–20 K) we can neglect electron scattering by optical phonons. It suffices to consider interaction with acoustic phonons via the deformation potential, and in the case of GaAs, also via the piezoeffect. In can easily be seen that owing to the low sound speed $s \ll v_0 \ll \omega/k$, the electrons lose energy mainly on plasmons and momentum on phonons, i.e. $|\boldsymbol{p} - \boldsymbol{p}'| \sim \omega/v_0 \sim q$, where q is the phonon momentum, $sq \sim s\omega/v_0 \ll \omega$. We can therefore neglect the plasmon momentum k and the phonon energy sq in the δ-functions that express the energy and momentum conservation laws. In the transport equation for n_k we must now take into account contributions of four processes, creation and annihilation of a plasmon accompanied by emission or absorption of a phonon. Taking the foregoing into account, we obtain for the total probability of the process $\boldsymbol{p} \to \boldsymbol{p}' \cdot \boldsymbol{k}$:

$$W = \sum_{\boldsymbol{q},\sigma} g^2(q)(2N_q + 1)\left\{[\boldsymbol{k}\cdot(\boldsymbol{p}-\boldsymbol{p}')]^2/(m\omega_k^2)^2\right\}$$
$$\times \delta(\boldsymbol{p}-\boldsymbol{p}'+\sigma\boldsymbol{q})\delta(E_p - E_{p'} - \omega); \quad \sigma = \pm 1. \tag{6.22}$$

Here $g(q)$ is the electron–phonon interaction constant and N_q is the distribution function of the phonons and can be regarded as close to equilibrium, at least if the electron drift is slower than the longitudinal sound (8×10^5 cm/s in Si and 5×10^5 cm/s in GaAs). In the essential region of q, the phonon energy can easily be estimated to be much lower than the lattice temperature T_L, i.e., we can put $N_q \approx T_L/sq \gg 1$. Substituting in (6.22) the corresponding expressions for $g(q)$ we find that $W \sim [\boldsymbol{k}\cdot(\boldsymbol{p}-\boldsymbol{p}')]^2$ for interaction via the deformation potential and

$$W \sim [\boldsymbol{k}\cdot(\boldsymbol{p}-\boldsymbol{p}')]^2(\boldsymbol{p}-\boldsymbol{p}')^{-2}$$

for the piezoelectric coupling. The former case reduces to scattering by short-range impurities, and the latter to two-dimensional Rutherford scattering. Therefore the interaction with the phonons does not call for a separate treatment.

6.2.4. Exact allowance for Coulomb scattering

The wavefunction that describes scattering by a Coulomb centre in the two-dimensional case takes the form

$$\psi_p = \pi^{-1/2}\exp(\pi e^2 m/2p)\Gamma(\tfrac{1}{2} - \mathrm{i}me^2/p)\exp(\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{\rho})$$
$$\times \phi[\mathrm{i}me^2/p, \tfrac{1}{2}; \mathrm{i}(p\rho - \boldsymbol{p}\cdot\boldsymbol{\rho})], \tag{6.23}$$

where ϕ is a confluent hypergeometric function. The Born approximation considered above corresponds to the limit $me^2 \ll p$, $\phi \approx 1$. The matrix element $M(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{k})$ that describes plasmon emission can be calculated in the dipole approximation ($k\rho \ll 1$), as in non-relativistic bremsstrahlung theory, since the electron velocity is much lower than the plasmon phase velocity. We shall not present here the rather long calculations, which are perfectly analogous to the aforementioned problem of bremsstrahlung of an electron in a nucleus (see e.g. ref. [37]). The result is

$$|M(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{k})|^2 = |\langle \boldsymbol{p}', \boldsymbol{k}|\boldsymbol{k}\cdot\boldsymbol{\rho}|\boldsymbol{p}\rangle|^2$$
$$= \left(\frac{4\pi}{m\omega}\right)^2 \frac{\exp(-2\pi me^2/p)}{(p-p')^2(p^2-(p')^2)^2\cosh(\pi me^2/p)\cosh(\pi me^2/p')}$$
$$\times \left| me^2(\boldsymbol{k}\cdot\boldsymbol{p} - \boldsymbol{k}\cdot\boldsymbol{p}')F(z) + \mathrm{i}\frac{(\boldsymbol{p}-\boldsymbol{p}')^2}{(p-p')^2}\boldsymbol{k}\cdot(p\boldsymbol{p}' - p'\boldsymbol{p})\frac{\mathrm{d}F}{\mathrm{d}z}\right|^2. \tag{6.24}$$

Here

$F(z) \equiv F(ime^2/p, ime^2/p'; \frac{1}{2}; z)$

is a complete hypergeometric function and

$z = 2(\boldsymbol{p}\cdot\boldsymbol{p}' - pp')/(p-p')^2.$

We consider only the case $m\omega \ll k_F^2$ for which a simple result can be obtained ($m\omega/k_F^2 \sim 0.1 - 0.2$ in the experiments). Putting $p \approx p' \approx k_F$, $p - p' \approx m\omega/k_F$ in (6.24) and using the asymptotic expansion of $F(z \gg 1)$, we obtain for the probability W:

$$W(p, p', k) = W_0 C\left(\frac{me^2}{k_F}\right)\left\{1 + \left(\frac{2me^2}{k_F}\right)^2 \ln^2 \frac{2k_F^4[1-\cos \sphericalangle(\boldsymbol{p}, \boldsymbol{p}')]}{m^2\omega^2}\right\}, \tag{6.25}$$

where W_0 is the Born probability of the process and $C(me^2/k_F)$ stands for all the factors in (6.24) that do not depend on the directions of the vectors $\boldsymbol{p}$, $\boldsymbol{p}'$ and $\boldsymbol{k}$. Thus, in the approximation considered, the exact probability differs only by a logarithmic factor from the perturbation theory result. At low drift velocities ($\gamma \ll 1$), as already stated, any probability of the form

$W_0 f[\cos \sphericalangle(\boldsymbol{p}, \boldsymbol{p}')]$

leads to a relation

$\delta n_k \sim p_0^2(\cos^2\alpha + \frac{1}{2}).$

At $\gamma \gg 1$ (and as before $\omega \gg T$), the ratio n_k/n_0 is exponentially large. The main contribution to n_k is made by the region of integration over the directions of $\boldsymbol{p}$ and $\boldsymbol{p}'$, a region in which $\cos \sphericalangle(\boldsymbol{k}, \boldsymbol{p}) \approx 1$, $\cos \sphericalangle(\boldsymbol{k}, \boldsymbol{p}') \approx -1$. We can therefore put in (6.25) $\cos \sphericalangle(\boldsymbol{p}, \boldsymbol{p}') = -1$, after which we obtain eq. (6.20b).

6.2.5. *Possibility of instability development*

We have so far taken into account in the transport equation (6.14) only the electronic plasmon relaxation mechanism. In a real situation there are also other mechanisms, one of which is the observable radiative decay. If all the "non-electronic" relaxation processes are described by a phenomenological time τ, it is necessary to add to the right-hand side of (6.14) a term $(n_0 - n)/\tau$. We then easily obtain for δn_k:

$$\delta n_k = \delta n_k^{(0)}(1 + 1/\Omega\tau)^{-1}; \quad \Omega \equiv \sum_{pp'} W(\boldsymbol{p}, \boldsymbol{p}', \boldsymbol{k})(f_{p'} - f_p). \tag{6.26}$$

Ω is positive in all the cases considered above. If the electron gas is degenerate, Ω does not depend on either the magnitude or the direction of the drift in the region $k_F u_0 \ll \omega$, which is most important for the experiment. All the derived relations remain therefore in force, and only the absolute value of δn_k decreases.

Instability sets in at $\Omega < -1/\tau$. It turns out that for the model considered by us, in which we assume $f_p = f_0(\boldsymbol{p} - \boldsymbol{p}_0)$, the onset of instability depends significantly on the electron-scattering mechanism. This is easiest to verify with a non-degenerate electron gas as an example. We calculate the first term in the right-hand side of (6.14) without assuming p_0 to be small. It turns out that for short-range impurities we have $\Omega > 0$ at all drift velocities, i.e., no instability sets in.

In scattering by charged impurities we have

$$\Omega \sim \int_0^{\omega/T} e^{-x} I_0(2\sqrt{\beta x})\, dx - \frac{\omega}{T}\cos(2\alpha)\int_{\omega/T}^{\infty} e^{-x} I_2(2\sqrt{\beta x})\, dx/x, \qquad (6.27)$$

with $\beta \equiv p_0^2/2mT$. It is easy to verify that in the region $\omega \ll T$, $\beta(\omega/T)^{1/2} \ll 1$, a sign reversal of Ω takes place when β is the root of the equation

$$(\exp\beta - \beta - 1)\cos(2\alpha) = \beta. \qquad (6.28)$$

Thus, instability is possible in the sector $0 < \alpha < \pi/4$ and $3\pi/4 < \alpha < \pi$ if $\beta > \beta_{min}$. At $\alpha = 0$ we have $\beta_{min} \approx 1.59$, which does not contradict the condition $\beta(\omega/T)^{1/2} \ll 1$.

In the case of degenerate electrons, the calculations become much more cumbersome, but it can be stated that there is no instability up to drift velocities of the order of the electron Fermi velocity. For short-range centres and at $T = 0$ it is possible to calculate Ω for any ratio of $p_0 k_F$ and $m\omega$ by assuming only that the conditions $p_0 \ll k_F$ and $m\omega \ll k_F^2$ are satisfied. It turns out that $\Omega > 0$ in the entire indicated region, i.e., no instability can develop. It is worthwhile to note here that there exists a quite different possibility to get an instability (and, consequently, an amplification) of the 2D plasma waves. The physical mechanism of this amplification has no connection with the bremsstrahlung but is a version of the negative Landau damping in periodic structures [38]. The most suitable systems for realizing this effect seem to be the lateral superlattices occurring at high-index surfaces of monocrystals (vicinal surfaces).

If a MOS structure is fabricated on such a surface and a 2D electron gas is formed, the plasma is affected by a 1D periodic potential with a period L much larger than the crystal lattice constant. Typical values L are of the order of 100 Å. We shall consider the Landau damping and amplification of the plasma oscillations due to electron drift along the superlattice.

The problem is to calculate the work done by the monochromatic electric field

$$E = E_0 \exp[i(kx - \omega t)]$$

on an electron moving in the periodic potential $U_0 \cos(x/L)$. The amplitude U_0 is assumed to be small as compared with the kinetic energy of the electron.

In the available MOS structures with superlattices the Fermi energy usually is one order of magnitude higher than U_0. To first order in U_0, the motion of the electron along the x-direction is given by the relation

$$x(t) = x(0) + vt + \left(U_0 L/mv^2\right) \sin(vt/L), \tag{6.29}$$

where v is the average velocity of the over-barrier motion (locking of particles in the potential wells is excluded by the condition $U_0 \ll mv^2$). The electric field wave distorts the law of motion (6.29). The perturbation $\delta x(t)$ in the linear (in E_0) approximation and at $U_0 \ll mv^2$ obeys the equation

$$\frac{\mathrm{d}^2\delta x}{\mathrm{d}t^2} = \frac{eE_0}{m} \exp\left\{\mathrm{i}\left[(kv - \omega - \mathrm{i}\gamma)t - \frac{U_0 kL}{mv^2}\sin\left(\frac{vt}{L}\right)\right]\right\}, \quad \gamma \to +0. \tag{6.30}$$

From eq. (6.30) one can obtain

$$\delta x = -\frac{eE_0}{m} \sum_{n=-\infty}^{+\infty} \frac{J_n(\alpha) \exp\{\mathrm{i}[(k+g_n)vt - (\omega + \mathrm{i}\gamma)t]\}}{\left[(k+g_n)v - \omega - \mathrm{i}\gamma\right]^2}, \tag{6.31}$$

with $\alpha \equiv U_0 kL/mv^2$. Here, $g_n = n/L$, the J_n are Bessel functions. The term $n = 0$ in eq. (6.31) corresponds to the spatially uniform case, the remaining terms describe umklapp processes.

The time-averaged energy absorbed by the particle is

$$q(v) = \frac{e}{2}\langle \mathrm{Re}\frac{\mathrm{d}}{\mathrm{d}t}(x + \delta x)E^*(x, t)\rangle = -\frac{(eE_0)^2}{2m}\mathrm{Im}\left(\sum_n J_n^2(\alpha)\frac{\omega + \mathrm{i}\gamma}{b_n}\right), \tag{6.32}$$

where b_n stands for $(k + q_n)v - \omega - \mathrm{i}\gamma$. To derive formula (6.32) one has to allow for two contributions to the velocity of the particle which are proportional to U_0 and E_0. By taking the limit $\gamma \to 0$ one obtains the total absorbed power Q:

$$Q = \int q(v) f(v - u)\mathrm{d}v, \tag{6.33a}$$

$$q(v) = \frac{\pi(eE_0)^2}{2m} \sum_n J_n^2(\alpha) \frac{\mathrm{d}}{\mathrm{d}v}\left[v\delta(\omega - kv - g_n v)\right]. \tag{6.33b}$$

Here u is the plasma drift velocity in the superlattice, f is the distribution function of the x-component of the velocity (it goes without saying that only the projection of the velocity on the drift direction is essential in the problem under consideration). For 2D degenerate electrons we have

$$f(v) = (m/\pi\hbar)^2\left(v_0^2 - v^2\right)^{1/2}, \quad \text{if } v < v_0,$$

$$f(v) \equiv 0, \quad \text{otherwise.}$$

As known, in a spatially uniform Fermi plasma the Landau damping equals zero due to the inequality $\omega > kv_0$. Accordingly, the term with $n = 0$ does not contribute to the sum in eq. (6.33). However, the umklapp processes result in a finite damping for a degenerate plasma in an external periodic potential. Usually one may assume $kL \ll 1$ and $\alpha < 1$, so that the leading term in the damping Γ is defined by the terms $n = \pm 1$. Γ, which is the imaginary part of the plasma wave frequency, equals $Q/8\pi k$:

$$\Gamma = \frac{e^2 k}{m\omega L^2}\left(\frac{U_0}{\hbar\omega}\right)^2\left[\frac{4\left(v_0^2 - u_-^2\right)^{1/2} + 4\left(v_0^2 - u_+^2\right)^{1/2}}{\omega L} + \frac{u_-}{\left(v_0^2 - u_-^2\right)^{1/2}} + \frac{u_+}{\left(v_0^2 - u_+^2\right)^{1/2}}\right]; \quad u_\pm = u \pm \omega L. \tag{6.34}$$

Formula (6.34) is valid until all values standing under the square roots are positive: otherwise the corresponding terms are to be omitted. Moreover, the condition $kLU_0 \ll m\omega^2 L^2$ is assumed to be satisfied, which corresponds to the inequality $\alpha \ll 1$. This is realized for the most typical experimental conditions $k \sim 10^4$ cm^{-1}, $N_s \sim 10^{12}$ cm^{-2}, $\omega \sim 10^{12}$–10^{13} s^{-1}. If there is no drift the damping has a minimum at $\omega \sim v_0/L$ and increases both for small ω and for $\omega \to v_0/L$

$$\Gamma = \frac{2e^2 k}{m\omega^2 L^3}\left(\frac{U_0}{\hbar\omega}\right)^2 \frac{4v_0^2 - 3\omega^2 L^2}{\left(v_0^2 - \omega^2 L^2\right)^{1/2}}. \tag{6.35}$$

Amplification of the wave occurs for $\Gamma < 0$; this happens in a certain interval of the drift velocities:

$$\left[v_0^2 + (\omega L/8)^2\right]^{1/2} + \tfrac{7}{8}\omega L < u < v_0 + \omega L \tag{6.36}$$

(the last two terms in (6.34) are zero in this case). In the vicinity of the upper edge of the interval (6.36), where $u = u_{max} = v_0 + \omega L$, the increment of amplification increases as $(u_{max} - u)^{-1/2}$. Thus, the amplification of 2D plasma waves can take place at drift velocities $u \sim v_0$, i.e. $u \sim 10^7$ cm s^{-1} for typical structures with a 2D electron gas. This rather high value seems, nevertheless, to be experimentally attainable. On the other hand, the amplification in a spatially uniform system (for example, in the case of two separated plasma layers) is possible only if the Čerenkov criterion, $v > \omega/k$, is satisfied. It demands much higher drift velocities since ω/k is typically of the order of 10^8–10^9 cm s^{-1}.

7. Non-linear optical phenomena associated with 2D plasmons

The variability of 2D electron systems and, especially, the possibility of tuning the plasmon frequency over a wide range is very attractive from the viewpoint of "active spectroscopy". On the other hand, during the past few years important progress has been achieved in the construction of powerful tunable sources of FIR radiation (first of all, the free electron laser). This opens new possibilities for investigating 2D plasmas affected by a strong electromagnetic field when various plasma instabilities become essential. Two examples are considered in this section to demonstrate the non-linear optical properties of 2D plasmons connected with so-called "concentrational non-linearity".

7.1. Synthesis of combination frequencies

Our purpose is to calculate a current of frequency $\omega_1 \pm \omega_2$ induced in the 2D electron gas by two FIR fields, $\boldsymbol{E}_1 \cos(\omega_1 t)$ and $\boldsymbol{E}_2 \cos(\omega_2 t)$. The self-consistent system of equations for the distribution function $f(\boldsymbol{r}, \boldsymbol{v}, t)$ and the electrostatic potential $\phi(\boldsymbol{r}, t)$ is of the form (the scattering of electrons is neglected for a while):

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\frac{\partial f}{\partial \boldsymbol{r}} + \frac{e}{m}\left[\boldsymbol{E}(\boldsymbol{r}, t) - \nabla\phi\right]\frac{\partial f}{\partial \boldsymbol{v}} = 0, \tag{7.1a}$$

$$\epsilon(z)\Delta\phi = -4\pi e\delta(z)\int(f - f_0)\,\mathrm{d}^2\boldsymbol{p}/2\pi^2, \tag{7.1b}$$

where $\boldsymbol{E}(\boldsymbol{r}, t)$ is the total external field created by the two incident FIR waves. The grating structure described above provides a spatial modulation of the field $\boldsymbol{E}$ identically for both waves. By expanding all the functions of $\boldsymbol{r}$ in Fourier integrals and by making use of a formal solution of eq. (7.1b) with the Green function (4.11), one can find the qth Fourier component of the electrostatic potential in the plane $z = 0$, where the electrons are supposed to be located:

$$\phi_q(z=0) = \frac{4\pi e}{q\left[\epsilon_s + \epsilon_{ox}\coth(q\Delta)\right]}\int\tilde{f}\,\mathrm{d}^2\boldsymbol{p}/2\pi^2, \quad \tilde{f} \equiv f - f_0. \tag{7.2}$$

By substituting eq. (7.2) in eq. (7.1a) one gets a closed non-linear equation for $\tilde{f}_{\boldsymbol{k}}(\boldsymbol{v}, t)$:

$$\frac{\partial\tilde{f}_{\boldsymbol{k}}}{\partial t} + \mathrm{i}\boldsymbol{k}\boldsymbol{v}\tilde{f}_{\boldsymbol{k}} + \frac{e}{m}\frac{\partial f_0}{\partial \boldsymbol{v}}\left[\boldsymbol{E}_{\boldsymbol{k}}(t) - \mathrm{i}\frac{4\pi e\boldsymbol{k}}{k\left[\epsilon_s + \epsilon_{ox}\coth(k\Delta)\right]}\int\tilde{f}_{\boldsymbol{k}}\frac{\mathrm{d}^2\boldsymbol{p}}{2\pi^2}\right]$$
$$+\frac{e}{m}\sum_{\boldsymbol{q}}\frac{\partial\tilde{f}_{\boldsymbol{k}-\boldsymbol{q}}}{\partial \boldsymbol{v}}\left[\boldsymbol{E}_{\boldsymbol{q}} - \mathrm{i}\frac{4\pi e\boldsymbol{q}}{q\left[\epsilon_s + \epsilon_{ox}\coth(q\Delta)\right]}\int\tilde{f}_{\boldsymbol{q}}\frac{\mathrm{d}^2\boldsymbol{p}}{2\pi^2}\right] = 0. \tag{7.3}$$

We shall solve eq. (6.3) iteratively, assuming the external field $\boldsymbol{E}_{\boldsymbol{q}}$ to be sufficiently small. The criterion of this approximation will be clear from what follows. Let the external field consist of two waves with frequencies $\omega_{1,2}$ and amplitudes $E_{1,2}$. The spatial dependence of the fields in a plane occupied by the plasma can be expressed by the Fourier series of $\cos(2\pi nx/a)$ (both fields are polarized along the x-direction). We shall keep only the first spatial harmonic, so we put

$$E = \left[E_1 \exp(\omega_1 t) + E_2 \cos(\omega_2 t) \right] \cos(k_1 x), \quad k_1 = 2\pi/a. \tag{7.4}$$

The second iteration gives an equation for $\tilde{f}^{(2)}$ which is bilinear in the E_1, E_2 term of the distribution function expansion. Evidently, the spatial dependence of $\tilde{f}^{(2)}$ corresponds to the momenta $k = 2k_{1'} - 2k_1$ and 0, while the frequency must be $2\omega_1$, $2\omega_2$ or $\omega_1 \pm \omega_2$. To be specific we put $\omega = \omega_1 + \omega_2$, $k = 2k_1$:

$$(\omega_1 + \omega_2 - 2k_1 v_x)\tilde{f}^{(2)}_{2k_1} + \frac{4\pi e^2 v_x f_0'}{\epsilon_s + \epsilon_{ox} \coth(2k_1 \Delta)} \int \tilde{f}^{(2)}_{2k_1} \frac{d^2 p}{2\pi^2}$$
$$= \frac{e^2 \omega_1^2 \omega_2^2 E_1 E_2 (\omega_1 + \omega_2 - 2k_1 v_x) d(f_0' v_x)/dv_x}{m(\omega_1 - kv_x)(\omega_2 - kv_x)\left[\omega_1^2 - \omega_p^2(k_1)\right]\left[\omega_2^2 - \omega_p^2(k_1)\right]}. \tag{7.5}$$

This equation can easily be solved and the total current j_{comb} of the combination frequency $\omega_1 + \omega_2$ equal to $j(2k_1) + j(-2k_1) + j(0)$ has the form [39]:

$$j_{\text{comb}} = -\frac{i e^2 E_1 E_2 N_s k_1 \sin(2k_1 x)(\omega_1 + \omega_2)(\omega_1^2 + 4\omega_1\omega_2 + \omega_2^2)}{m^2\left[(\omega_1 + \omega_2)^2 - \omega_p^2(2k_1)\right]\left[\omega_1^2 - \omega_p^2(k_1)\right]\left[\omega_2^2 - \omega_p^2(k_1)\right]}. \tag{7.6}$$

As expected, $j_{\text{comb}} = 0$ at $\omega_1 + \omega_2 = 0$: a direct current cannot exist in a system which has an inversion centre in the presence of an external field (just that case we have: $E_{1,2} \sim \cos(k_1 x)$. For different values of N_s three resonant frequencies are possible: ω_1, ω_2 and $\omega_1 + \omega_2$. Close to resonance, electron scattering should be taken into account, which gives in the denominator of eq. (7.6) $\omega_1(\omega_1 + i\nu)$ instead of ω_1^2, etc. The magnitude of the optical non-linearity in the considered system can be estimated as the ratio of the current j_{comb} quadratic in E_1 and E_2, to the linear current j_0. At resonance, one gets

$$j_{\text{comb}}/j_0 \sim ek_1 E/m\nu\omega_p \sim E\omega_p/e\nu N_s$$

(we put $E_1 \sim E_2 = E$ in order of magnitude). In the available experiment $\nu \sim 0.1\omega_p$, and for $N_s \sim 10^{12}$ cm^{-2} and an intensity of the incident wave of ~ 1 W/cm^2, one obtains $j_{\text{comb}}/j_0 \sim 10^{-3}$; the amplitude of the current j_{comb} is of the order of 10^{-5}–10^{-6} A/cm.

The absolute maximum of j_{comb} occurs at a double resonance: ω_1 (or ω_2) $= \omega_p(k_1)$, $\omega_1 + \omega_2 = \omega_p(2k_1)$. There is only one tunable parameter N_s, so that these two conditions can be simultaneously satisfied only if a certain relation between ω_1 and ω_2 is fulfilled. For example, let $\omega_1 > \omega_2$. A very simple result is obtained in the case $k_1\Delta \gtrsim 1$, when $\coth(k_1\Delta) \approx 1$. Then the relation mentioned above is $\omega_2 = \omega_1(\sqrt{2} - 1)$.

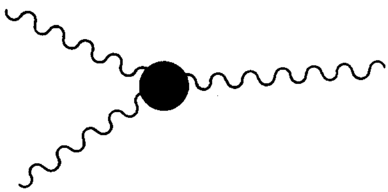

Fig. 6. Three-wave diagram for synthesis of harmonics and parametric division of frequency.

In a magnetic field perpendicular to the plane $z = 0$ an additional degree of freedom arises. The resonances are now given by the conditions: $\omega_1 + \omega_2$ or $\omega_{1,2} = [\omega_C^2 + \omega_p^2(k)]^2$, where $k = 2k_1$ or k_1. By tuning the parameters N_s and ω_C one can obtain a double resonance for any ω_1 and ω_2 obeying the inequality $\omega_2 \leqslant \omega_1(\sqrt{2} - 1)$. At a double resonance j_{comb} increases additionally by a factor of ω/ν.

One of the possible ways of observing a non-linear transformation of FIR radiation by a 2D plasma could be the detection of the first harmonic of the incident wave ($\omega_1 + \omega_2 = 2\omega$) emitted by a "MOS-plus-grating" structure when subjected to a sufficiently intense exciting field. Thus, MOS structures seem to be rather promising non-linear optical systems, allowing frequency tuning over a range of 10^{12}–10^{13} s^{-1}.

7.2. Parametric generation of 2D plasma oscillations

The process investigated in the preceding subsection can be graphically depicted as is shown in fig. 6. The wavy lines correspond to plasmons, the vertex (black point) is the non-linear interaction of the plasmons. This three-wave process may be read also "from right to left", and then it represents the frequency division according to the mechanism of parametric resonance.

The treatment of this problem on the basis of kinetic equations leads to very complicated calculations. Luckily, actual plasmons satisfy the condition $\omega \gg kv$ so that a more simple hydrodynamic description of the plasma is applicable. Then spatial dispersion may be neglected and one may use the continuity equation and the equations of motion of a cold plasma. With allowance for effects of the concentration non-linearity the continuity equation may be written as

$$\frac{\partial N(x, t)}{\partial t} + \operatorname{div}\left[(n_s + \tilde{N})\boldsymbol{u}(x, t)\right] = 0, \tag{7.7}$$

where $\boldsymbol{u}$ is the hydrodynamical velocity of the plasma. The equation of motion including the plasma wave electric field is

$$\dot{\boldsymbol{u}} + \nu\boldsymbol{u} + (e/m)\nabla\phi(x, t)\,|_{z=0} = (e/m)\boldsymbol{E}(x, t). \tag{7.8}$$

Together with the Poisson equation, eqs. (7.7) and (7.8) form a closed system. One can obtain from it a system of equations for the Fourier components of the velocity $\boldsymbol{u}_k(t)$ and the areal density $\tilde{N}_k(t)$:

$$\dot{\tilde{N}}_k + \mathrm{i}\boldsymbol{k}\cdot\boldsymbol{u}N_s + \mathrm{i}\boldsymbol{k}\cdot\sum_{\boldsymbol{q}}\boldsymbol{u}_q\tilde{N}_{k-q}(t) = 0, \tag{7.9a}$$

$$\dot{\boldsymbol{u}}_k + \nu\boldsymbol{u}_k + \mathrm{i}\frac{2\pi e^2\boldsymbol{k}}{km\epsilon(k)}\tilde{N}_k(t) = \frac{e}{m}\boldsymbol{E}_k(t). \tag{7.9b}$$

Here $\epsilon(k)$ stands for

$\frac{1}{2}[\epsilon_s + \epsilon_{ox}\coth(k\Delta)]$.

We are interested in exciting the principal mode of plasma oscillations with the momentum $k_1 = 2\pi/a$. The parametric generation is the three-wave process: $(\omega, 2k_1) \to 2(\omega/2, k_1)$. Hence, the pumping in the right-hand side of eq. (7.9b) must be the second spatial harmonic of the incident FIR wave (its amplitude is denoted by E_2):

$$\boldsymbol{E}_k(t) = \tfrac{1}{2}\boldsymbol{E}_2(\delta_{k,2k_1} + \delta_{k,-2k_1})\cos(\omega t). \tag{7.10}$$

The first iteration of the system (6.9a), (6.9b) with respect to the pumping field gives a correction to the density in the form:

$$\tilde{N}_k^{(1)} = -\frac{\mathrm{i}eN_s\boldsymbol{k}\boldsymbol{E}_k}{2m}\left[\frac{\exp(\mathrm{i}\omega t)}{\omega_p^2(k) - \omega^2 + \mathrm{i}\nu\omega} + \text{c.c.}\right]. \tag{7.11}$$

In what follows k and E stands for the absolute values of the vectors $\boldsymbol{k}$ and $\boldsymbol{E}$ because of the one-dimensional character of the problem under consideration.

As is shown, the parametric resonance originates from terms of the type $\tilde{N}^{(1)}\tilde{N}^{(2)}$ occurring in the third iteration. One can see from eqs. (7.10) and (7.11) that the components $\tilde{N}^{(2)}_{k\pm 2k_1}$ combine with $\tilde{N}^{(1)}_{k\pm 2k_1}$ and with the external field that also contains the momenta $\pm 2k_1$. An infinite chain of coupled equations arises from which one can find the region of the principal parametric resonance (the first zone), if one puts $k = \pm k_1$ and neglects all the higher harmonics $\tilde{N}_q$ starting from $q = \pm 3k_1$. It is convenient to introduce the function

$$\Pi(k, t) = \int_{-\infty}^{t}\tilde{N}^{(2)}(k, \tau)\,\mathrm{e}^{\nu\tau}\,\mathrm{d}\tau.$$

Then one gets for $\Pi_+ \equiv \Pi(k_1, t)$ and $\Pi_- \equiv \Pi(-k_1, t)$:

$$\ddot{\Pi}_+ - \nu\dot{\Pi}_+ + \omega_p^2(k_1)\Pi_+ + (ek_1E_2/m)\exp(\mathrm{i}\omega t)$$
$$\left[\mathrm{i}\omega_p^2(k_1)\Pi_- - \tfrac{1}{2}\omega\dot{\Pi}_-\right]\left[\omega_p^2(2k_1) - \omega^2 + \mathrm{i}\nu\omega\right]^{-1} = 0, \tag{7.12}$$

and a similar equation for Π_-, wherein Π_+ and Π_- have to be interchanged and the coefficients have to be replaced by their complex conjugates. The

solution of these two coupled equations has the evident form
$\Pi_{\pm} \sim \exp[(s \pm \frac{1}{2}\mathrm{i}\omega)t]$,
where s is defined by the characteristic equation

$$(s^2 - 2\epsilon\omega_0 - \nu s)^2 + 4\omega_0^2(s - \tfrac{1}{2}\nu)^2 = \frac{(2e\omega_0 k_1 E_2/ms)^2}{[\omega_p^2(2k_1) - \omega^2]^2 + \nu^2\omega^2}. \tag{7.13}$$

Here $\omega_0 \equiv \omega_p(k_1)$, $\epsilon \equiv \omega/2 - \omega_p(k_1)$ and the condition $\epsilon \ll \omega_0$ is used. Again making use of this condition we solve eq. (7.13) approximately and find the boundaries of the instability region [40]

$$\epsilon^2 + \tfrac{1}{4}\nu^2 < \frac{(ek_0 E_2\omega_0/m)^2}{[\omega_p^2(2k_1) - \omega^2]^2 + \nu^2\omega^2}. \tag{7.14}$$

Two different cases can be realized depending on the plasmon dispersion relation: the square-root regime $\omega_p(k_1) \sim \sqrt{k_1}$ which occurs at $k_1\Delta \gg 1$ (thick dielectric layer), and the linear regime which could take place, for example, in a MOS structure with a thin oxide $k_1\Delta \ll 1$. In the square-root regime the threshold field providing the parametric instability equals $E_2^{(0)} = m\omega_0\nu/ek_1$ which can also be expressed via the mobility μ of 2D electrons: $E_2^{(0)} = \omega_0/\mu k_1$. For the record mobility reported in GaAs–GaAlAs heterojunctions ($\sim 10^6$ $\mathrm{cm^2/V \cdot s}$) we estimate ($a = 1 \times 10^{-4}$ cm, $\omega_0 = 2 \times 10^{12}\ \mathrm{s^{-1}}$) $E_2^{(0)} \sim 90$ V/cm which corresponds to a FIR intensity of $\sim 20\ \mathrm{W/cm^2}$. However, in the linear regime the threshold field estimate is more optimistic since the conditions $\omega = 2\omega_p(k_1)$ and $\omega = \omega_p(2k_1)$ can be satisfied simultaneously. Then the threshold field is decreased by a factor of ν/ω_0, which is 4×10^{-3} for the same magnitude of the mobility. This gives a threshold intensity as low as 3×10^{-4} $\mathrm{W/cm^2}$. As is shown in section 6.1 the grating structure acts as a nearly uniform metal gate electrode at least as far as the plasmon dispersion law is concerned. Hence, the linear regime can be realized in a GaAs–GaAlAs heterojunction with, for example, an aluminium grating structure upon the GaAlAs layer if the thickness of the layer is small compared with the period of the grating structure.

To define the amplitude of oscillations in the parametric resonance regime one has to keep non-linear terms in the chain of equations for the $\Pi_k(t)$:

$$\ddot{\Pi}_k - \nu\dot{\Pi}_k + \omega_p^2\Pi_k + \frac{k}{N_s}\sum_q \frac{\omega_p^2(q)}{q}\,\mathrm{e}^{-\nu t}\Pi_q\dot{\Pi}_{k-q} = R(k, t). \tag{7.15}$$

The right-hand side of eq. (7.15), $R(k, t)$, contains all the terms obtained by the first iteration of the system (7.9a), (7.9b) which are linear in the pumping field E_2. Hence, $R(k, t) \equiv 0$ if $k \neq \pm k_1$, whereas for $k = \pm k_1$ one obtains

$$R(\pm k_1) = \frac{ek_1E_1}{2m}\,\frac{\omega\dot{\Pi}(\mp k_1) \mp 2\mathrm{i}\omega_0^2\Pi(\mp k_1)}{\omega_p^2(2k_1) - \omega^2 \pm \mathrm{i}\nu\omega}\,\mathrm{e}^{\pm\mathrm{i}\omega t}. \tag{7.16}$$

By selecting again the essential harmonics, i.e. putting in eq. (7.15) $k = k_1$, and $q = -k_1,\ 2k_1$, or $k = -k_1$, and $q = k_1,\ -2k_1$ we arrive at a system of four equations for $\Pi(\pm k_1)$, $\Pi(\pm 2k_1)$. The solutions of these equations are searched for in the form

$$\Pi(\pm k_1) = a_\pm \exp[(\pm \mathrm{i}\omega/2 + \nu)t],$$
$$\Pi(\pm 2k_1) = A_\pm \exp[(\pm \mathrm{i}\omega + \nu)t],$$

which corresponds to a constant amplitude of the oscillations in the concentration $\tilde{N}^{(2)}(\pm k_1)$. It follows from eqs. (7.15) and (7.16) that the system of equations is invariant with respect to a transposition $a_+ \to a_-^*$, $A_- \to A_-^*$. This essentially simplifies the solution. By using the conditions ν, $\epsilon \ll \omega$ and by excluding, consequently, A_+ and A_- one obtains:

$$a\Big\{\big[\omega_\mathrm{p}^2(k_1) - \tfrac{1}{4}\omega^2 + \mathrm{i}\nu\omega_\mathrm{p}(k_1)\big]\big[\omega_\mathrm{p}^2(2k_1) - \omega^2 + 2\mathrm{i}\nu\omega_\mathrm{p}(k_1)\big] - \big[\omega_\mathrm{p}^4(k_1)/N_\mathrm{s}^2\big]\,|a|^2\big[4\omega_\mathrm{p}^2(k_1) + \omega_\mathrm{p}^2(2k_1)\big]\Big\} = -\big[2\mathrm{i}ek_1E_2\omega_\mathrm{p}^2(k_1)/m\big]a^*, \tag{7.17}$$

where $a = a_+ = a_-^*$. There is, of course, a trivial solution of eq. (7.17), $a = 0$, but besides there are non-zero solutions defining the amplitude of resonant oscillations as a function of the resonance defect ϵ and other parameters. For the most preferential linear regime, $\omega_\mathrm{p}(2k_1) = 2\omega_\mathrm{p}(k_1)$, the result is

$$|a|^2 = \big(4N_\mathrm{s}^2/\omega^4\big)\Big\{8\epsilon^2 - \nu^2 \pm \big[(ek_1E_2/m)^2 - 36\nu^2\epsilon^2\big]^{1/2}\Big\}. \tag{7.18}$$

The amplitude of the alternative part of the concentration equals $\frac{1}{2}\omega|a|$. Formula (7.18) is applicable if the pumping field exceeds the threshold magnitude which corresponds to the case $\epsilon = 0$, namely $E_2 > m\nu^2/ek_1$. The two branches in the dependence of $|a|^2$ on ϵ described by eq. (7.18) link up at $\epsilon_\mathrm{max} = ek_1E_2/6m\nu$; the upper branch (+ sign in front of the square root in eq. (7.18)) corresponds to stable oscillations (see fig. 7). The threshold field depends on the resonance defect ϵ according to the formula

$$E_2^{(0)}(\epsilon) = (m/ek_1)\big[(16\epsilon^2 + \nu^2)(4\epsilon^2 + \nu^2)\big]^{1/2}, \tag{7.19}$$

and increases as ϵ^2 in the region $\epsilon \gg \nu$. The behaviour of $|a|^2$ at small ϵ is defined by the parameter $\lambda = ek_0E_2/m\nu^2$. If $\lambda \leqslant 9/4$ there is a maximum at $\epsilon = 0$, whereas for $\lambda > 9/4$ there occurs a minimum for $\epsilon = 0$ and two maxima at

$$\epsilon = \pm(\nu/6)\big[\lambda^2 - (\tfrac{3}{2})^4\big]^{1/2}.$$

The unstable branches of $|a|^2$ are depicted in fig. 7 by dashed lines. If the pumping field exceeds the threshold value so that $(\lambda - 1) \sim 1$, one can estimate the amplitude of concentration oscillations as $|\tilde{N}^{(2)}| \sim N_\mathrm{s}(\nu/\omega)$ all over the interval $|\epsilon| < \epsilon_\mathrm{max}$.

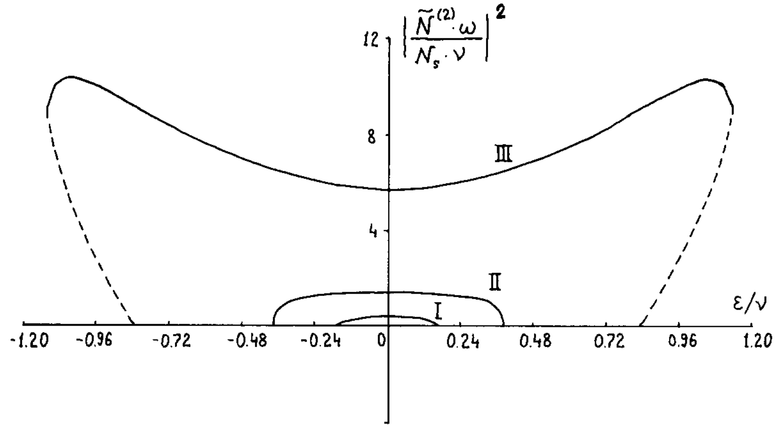

Fig. 7. Dependence of squared amplitude of resonance oscillations on the resonance defect $\epsilon = \omega/2 - \omega_0$; I: $\lambda = 5/4$, II: $\lambda = 9/4$, III: $\lambda = 27/4$.

The experimental manifestation of the considered effect is an emission arising from the 2D electron system at a frequency $\frac{1}{2}\omega$, if the latter is excited by radiation of frequency ω. Eq. (7.18) gives the dependence of the $\frac{1}{2}\omega$-emission intensity on that of the incident wave. Probably in experiments it is more convenient to tune the defect ϵ by sweeping N_s at a fixed frequency of the incident FIR radiation.

8. Conclusion

In conclusion, I tried to demonstrate various possibilities of 2D charge carrier systems as objects for investigation by FIR physics methods. Some of the possible experiments have already been done, others are still waiting for their turn. Among the latter, plasma waves in multilayer superlattices and non-linear effects seem to be most interesting. It is hoped that the rapid progress of semiconductor technology will lead to the discovery and investigation of these and many other phenomena in the nearest future.

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