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Residual Stresses in Metal/Ceramic Bonded Strips

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Residual stresses that develop during cooling of a metal/ceramic strip are calculated analytically. It is shown that the metal may behave elastically or plastically (with full or partial plasticity) depending on the mechanical properties, the thickness of the two constituents, and the mismatch in thermal expansion. Residual stresses are also calculated for a sequence consisting of constrained undercooling, removal of the constraint, and reheating. It is demonstrated that reheating, which results in elastic stress relaxation, may be used to eliminate the residual stress. The optimum undercooling and reheating conditions needed to produce a stress-free strip, at the operational temperature, are calculated, and specific results are presented for the system Cu/Al_2O_3.

I. Introduction

A number of applications in microelectronics involve combinations of metal and ceramic constituents. These constituents are subject to residual stress due to thermal expansion mismatch. The stresses that develop depend on the configuration of the system. A metal cylinder imbedded in an infinite ceramic matrix has been previously analyzed,[1] as appropriate for a conducting element in a microelectronics package.[2] A metal/ceramic strip, pertinent to a metallized substrate in hybrid power electronics,[3] has also been analyzed,[4] but only for a nonhardening metal. The present paper extends the stress analysis for the strip configuration to include work hardening and to identify the existence of an important partially plastic condition. Furthermore, a method of eliminating the residual stresses in the strip configuration is presented and analyzed.

Residual stress elimination can be achieved if the metal/ceramic strip is undercooled and reheated. Stress elimination is further facilitated if the strip is constrained from bending during cooling, allowed to undercool in the constrained state, and then reheated to the operational temperature. Judicious selection of the undercooling temperature permits the strip to be stress free at the operating temperature. The requisite undercooling is calculated in the present study.

The evolution of the residual stresses in the metal/ceramic strip is calculated, subject to the premise that bonding is conducted at elevated temperatures, where the longitudinal stresses are fully relaxed. Specific stresses and curvatures are calculated for the technologically important system Cu/Al_2O_3. For purposes of stress elimination, the strip is constrained during cooling, whereupon spatially uniform stresses of opposite sign develop in the metal and ceramic. After cooling the constraint is released, bending occurs due to the asymmetric cooling stress, and spatially varying residual stresses result. Subsequent heating then straightens the strip and relaxes the residual stresses.

II. Residual Stresses

The residual stresses are determined incrementally upon cooling by invoking the following analytic logic for each temperature decrement. The two constituents experience an unconstrained differential shrinkage, as depicted in Fig. 1. Uniform tensile and compressive stresses are then imposed on the metal and the ceramic, respectively, to achieve displacement compatability, while the total forces still remain zero. Finally, bending is allowed to occur to balance the bending moment induced by the asymmetric stresses in the previous step. Naturally, these processes occur simultaneously in the actual strip.

The bending strains in both materials are prescribed, by bending theory, as being proportional to the distance from the neutral axis and inversely proportional to the radius of curvature.[5] The strains, ε, in a strip can thus be expressed (Fig. 1) by

$$\varepsilon = (x - t_n)/r + c \tag{1}$$

where t_n is the position of the neutral axis, r is the radius of curvature, and c is a constant.

The bending strains tend to reduce the stress in the metal. However, the reduction is appreciably larger at the metal surface than at the metal/ceramic interface. This behavior may induce partial yield in a strip adjacent to interface (Fig. 1), i.e., when the effective stress in this region exceeds the yield strength. Three deformation characteristics of the metal must, therefore, be considered: elastic, fully plastic, and partially plastic. These three conditions are evaluated in the subsequent analysis.

The stress, σ_c, in the linear elastic *ceramic* can be invariably related to the strain by simply applying the relations

$$\sigma_c = E_c(\varepsilon - \alpha_c \Delta T) \qquad (0 \le x \le t_c)$$

or

$$\sigma_c = E_c[\varepsilon_0 + (\alpha_m - \alpha_c)\Delta T] + E_c(x - t_n)/r \tag{2}$$

where t_c is the thickness and E_c is the Young's modulus of the ceramic, $\alpha_c \Delta T$ and $\alpha_m \Delta T$ are the thermal strains in the ceramic and metal, respectively, and ε_0 is a constant ($\varepsilon_0 = c - \alpha_m \Delta T$).

In the *metal*, plastic strains may also be involved. The metal is assumed to satisfy the Von Mises criterion,[6] such that yield occurs when the effective stress equals the yield stress, σ_y. Furthermore,

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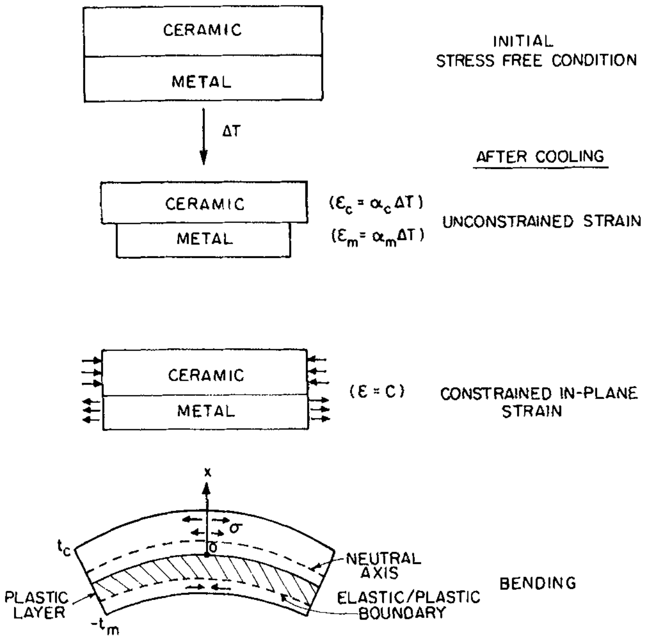

Fig. 1. Metal and ceramic strips in stress-free initial condition. After cooling, strips would exhibit unconstrained differential shrinkage. However, residual stresses allow displacement compatibility. Finally, bending stresses develop to balance bending moment.

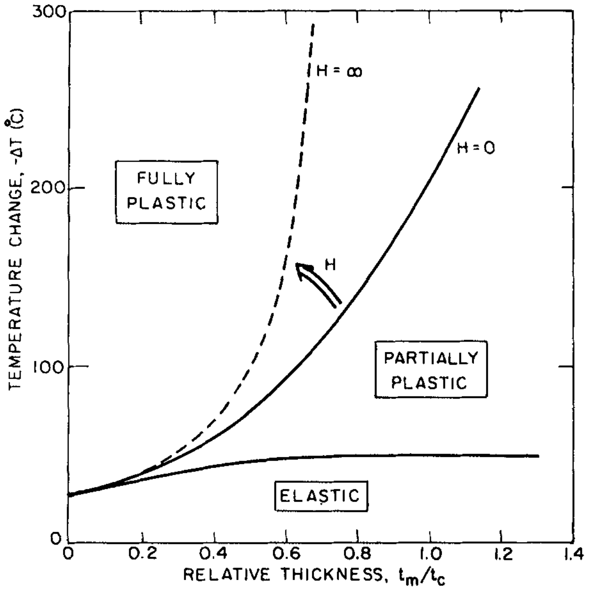

Fig. 2. Temperature regimes for elastic, partially plastic, and fully plastic deformation of Cu in Cu/Al_2O_3 strip.

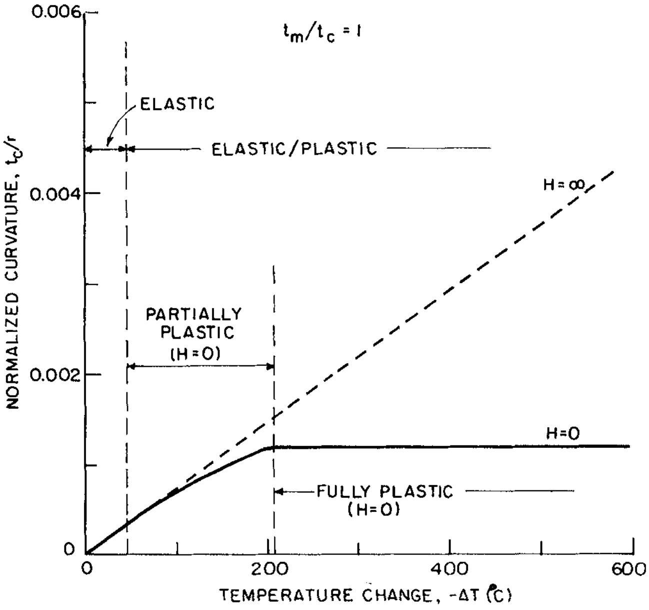

Fig. 4. Curvature of Cu/Al_2O_3 strip as a function of temperature change, $-\Delta T$, for $t_m/t_c = 1$.

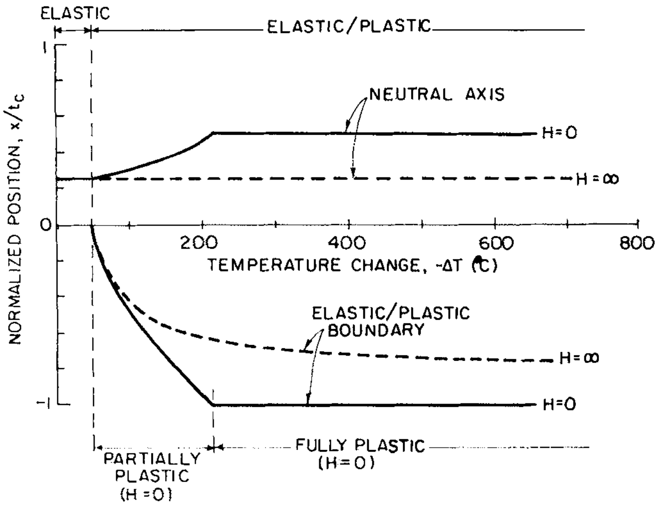

Fig. 3. Position of neutral axis and elastic/plastic boundary of Cu/Al_2O_3 strip as a function of temperature change, $-\Delta T$, for $t_m/t_c = 1$.

linear work hardening is assumed, whereupon the plastic strain, ε_p, satisfies the Prandtl-Reuss relation.[6] For the present geometry

$$\varepsilon_p = (\sigma_m - \sigma_y)/H \tag{3}$$

where H is the work-hardening rate and σ_m is the stress in the metal. Hence, for *fully plastic* deformation of the metal

$$\sigma_m = E_m\left(\varepsilon_0 + \frac{\sigma_y}{H} + \frac{x - t_n}{r}\right)\left(1 + \frac{E_m}{H}\right)^{-1} \tag{4}$$

whereas, for conditions of *partial plasticity* the stresses in the elastic region are

$$\sigma_m = E_m[\varepsilon_0 + (x - t_n)/r] \qquad (-t_m \leq x \leq -t_y) \tag{5a}$$

while in the plastic region

$$\sigma_m = E_m\left(\varepsilon_0 + \frac{\sigma_y}{H} + \frac{x - t_n}{r}\right)\left(1 + \frac{E_m}{H}\right)^{-1} \qquad (-t_y \leq x \leq 0) \tag{5b}$$

where t_m is the thickness of the metal and $x = -t_y$ is the plane of the elastic/plastic boundary.

The stresses are contingent upon the magnitudes of the constants ε_0, t_n, and r (and t_y for partial yield), which can be determined from the following boundary conditions.* The sum of the bending stresses (terms involving $(x - t_n)/r$) is zero:

$$\int_{-t_m}^{-t_y} E_m(x - t_n)r^{-1}\,dx + \int_{-t_y}^{0} (x - t_n)\left[\left(\frac{1}{E_m} + \frac{1}{H}\right)r\right]^{-1} dx + \int_0^{t_c} E_c(x - t_n)r^{-1}\,dx = 0 \tag{6}$$

The stress at the elastic/plastic boundary equals the yield strength (partial yield):

$$E_m[\varepsilon_0 - (t_y + t_n)/r] = \sigma_y \tag{7}$$

The net stress is zero:

$$\int_{-t_m}^{-t_y} E_m\varepsilon_0\,dx + \int_{-t_y}^{0}\left(\varepsilon_0 + \frac{\sigma_y}{H}\right)\left(\frac{1}{E_m} + \frac{1}{H}\right)^{-1} dx + \int_0^{t_c} E_c[\varepsilon_0 + (\alpha_m - \alpha_c)\Delta T]\,dx = 0 \tag{8}$$

The sum of the bending moments with respect to the neutral axis ($x = t_n$) is zero:

$$\int_{-t_m}^{-t_y} E_m[\varepsilon_0 + (x - t_n)/r](x - t_n)\,dx + \int_{-t_y}^{0}[\varepsilon_0 + \sigma_y/H + (x - t_n)/r](x - t_n)\left(\frac{1}{E_m} + \frac{1}{H}\right)^{-1} dx + \int_0^{t_c} E_c[\varepsilon_0 + (\alpha_m - \alpha_c)\Delta T + (x - t_n)/r](x - t_n)\,dx = 0 \tag{9}$$

General solutions can be obtained for both the elastic and fully plastic cases by evaluating the constants, ε_0, t_n, and r from the boundary conditions (Appendix A). However, for partial yield, analytic solutions of the four simultaneous equations (Eqs. (6) to (9)) are too complex. Essential trends, are thus elucidated (Appen-

*Equations (6) to (9) are explicitly formulated for a metal subject to partial plasticity.

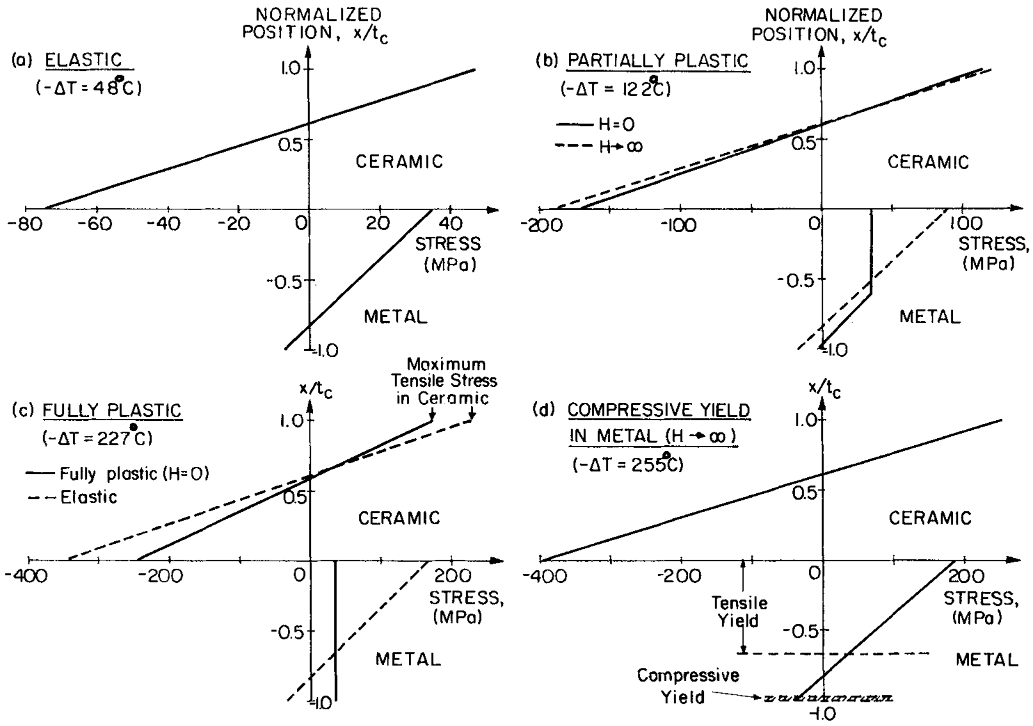

Fig. 5. Stress distribution within Cu/Al_2O_3 strips for three deformation responses of Cu: (*a*) elastic at $-\Delta T$ = 48.3°C, (*b*) partially plastic at $-\Delta T$ = 122°C, and (*c*) fully plastic for $H = 0$ (perfect plasticity) at $-\Delta T$ = 227°C. Also shown (*d*) compressive yield occurring at metal surface for $H \rightarrow \infty$ at $-\Delta T$ = 225°C, for $t_m/t_c = 1$.

dix A) for two limiting cases: $H = \infty$ (zero plastic strain) and $H = 0$ (perfect plasticity). Specific residual stresses are computed for the system Cu/Al_2O_3 (Figs. 2 to 5) by substituting the constants ε_0, t_n, r, and t_y into Eqs. (2), (4), and (5) and using the following material parameters[2]: $E_m = 1.2 \times 10^5$ MPa, $E_c = 3.5 \times 10^5$ MPa, $\alpha_m = 17 \times 10^{-6}$ °C^{-1}, $\alpha_c = 6.5 \times 10^{-6}$ °C^{-1}, and $\sigma_y = 35$ MPa.

The conditions of temperature and thickness ratio that determine elastic, partially plastic, and fully plastic behavior in the metal are summarized in Fig. 2. Note that the fully plastic condition is suppressed as the work-hardening rate increases or as the metal layer thickness increases. This trend can be appreciated by recognizing that bending generates compressive stresses on the metal surface, which reduce the tensile stresses induced by the thermal mismatch.

The locations of the neutral axis and the elastic/plastic boundary are plotted on Fig. 3, as a function of the temperature change, $-\Delta T$. For a metal and ceramic of equal thickness, the neutral axis is observed to be always located within the ceramic, since the ceramic has the higher Young's modulus. Furthermore, for a metal with a high work-hardening rate, note that the elastic/plastic boundary never reaches the metal surface and hence, conditions of partial plasticity always occur.

Trends in the curvature with the temperature change are shown in Fig. 4. For a perfectly plastic metal ($H = 0$), once the metal becomes fully plastic, it complies with the strains within the ceramic and the curvature does not alter with further decrease in the temperature.

Some trends in the stress distributions are plotted in Fig. 5 for the elastic, partially plastic, and fully plastic cases. It is evident from Fig. 5(*b*) that while partial yielding conditions apply, the stress in the ceramic is relatively unaffected by the plasticity in the metal, being similar for $H = 0$ (no work hardening) and $H = \infty$ (rapid work hardening). However, once fully plastic conditions develop, the plasticity of the metal exerts a profound influence on the stress in the ceramic (Fig. 5(*c*)), as also apparent from the curvature (Fig. 4). Most importantly, the maximum tensile stress in the ceramic is considerably smaller for the nonhardening metal. Brittle fracture of the ceramic is thus suppressed by using a nonhardening metal constituent. Finally, it is also noted that for a metal with high work-hardening rate, fully plastic deformation is suppressed and the metal surface can yield in compression, because of the high compressive stress caused by bending (Fig. 5(*d*)).

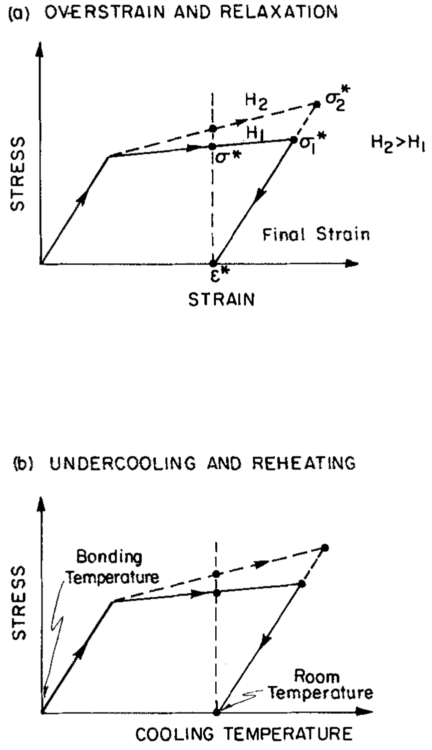

Fig. 6. Schematic showing reduction of stresses by (*a*) overstraining and relaxing and (*b*) undercooling and reheating; effects of work-hardening rate are also shown.

III. Elimination of Residual Stresses

The procedure used for elimination of the residual stress by undercooling and reheating can be exemplified and simulated by considering a standard elastic/plastic material subject to a sequence of overstraining and relaxation (Fig. 6). Upon straining, the stress increases linearly until yield, whereupon the stress increases at a reduced rate (dictated by the work-hardening rate) to a final stress σ^* (Fig. 6(*a*)). However, overstraining followed by relaxation can

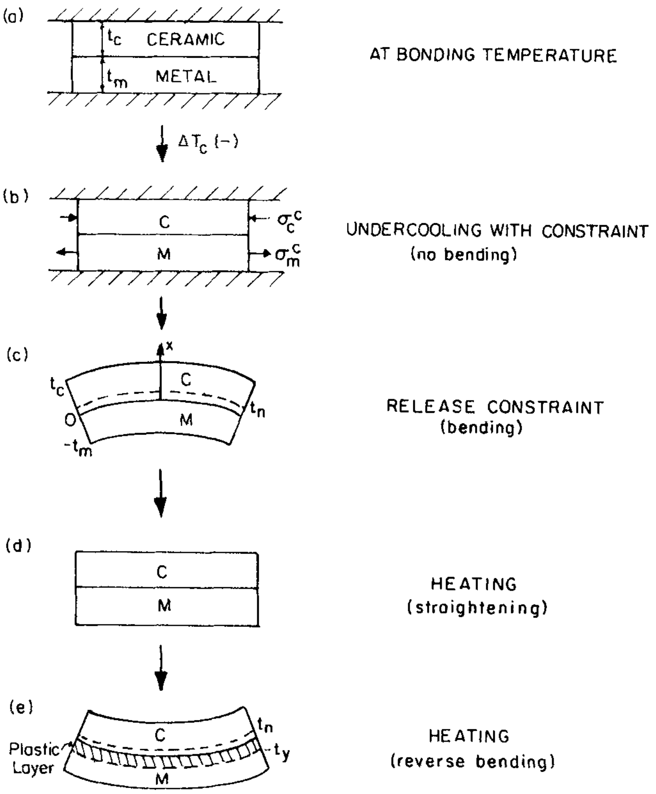

Fig. 7. Schematic showing (*a*) metal and ceramic bonded at high temperature; (*b*) constraint imposed on cooling to prevent bending (cooling temperature ΔT_c); (*c*) release of constraint, which induces bending; (*d*) strip straightened by reheating; and (*e*) further heating, which results in reverse bending.

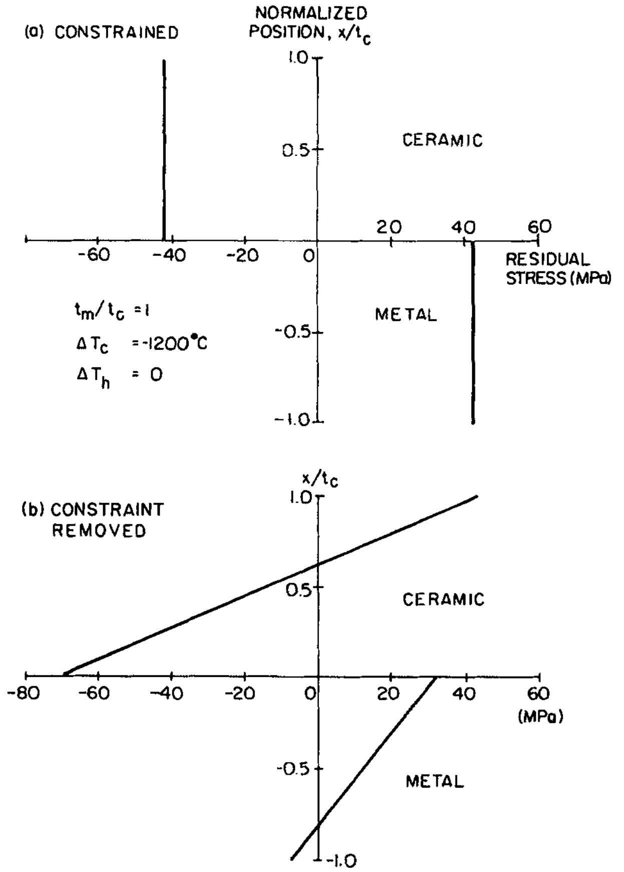

Fig. 8. Stress distributions within Cu/Al_2O_3 strip for $t_m/t_c = 1$ and $\Delta T_c = -1200°C$, showing (*a*) uniform tension and compression in metal and ceramic, respectively, after constrained cooling and (*b*) residual stresses modified by bending after removal of constraint.

appreciably reduce the final stress, because the stress relaxes elastically. By suitable choice of the overstraining stress σ^*, the final stress can be reduced to zero (Fig. 6(*a*)).

A similar sequence obtains for the undercooling and reheating process (Fig. 6(*b*)). The metal/ceramic strip is stress free at the bonding temperature. Elastic stress develops during initial cooling. Yielding of the metal then occurs and the stress increases at a diminished rate. Furthermore, by undercooling and reheating to room temperature, the residual stress can be eliminated (Fig. 6(*b*)).

(1) *Constrained Cooling*

The metal/ceramic strip is bonded at elevated temperature and then undercooled, with an external constraint imposed, over a temperature range ΔT_c (ΔT_c is negative), as depicted in Figs. 7(*a*) and (*b*). During this step, bending is prohibited and the metal is amenable to plastic deformation. Uniform tensile, σ_m^c, and compressive, σ_c^c, stresses develop in the metal and the ceramic, respectively (for a metal with a larger thermal expansion coefficient than the ceramic). The cooling stresses, σ_m^c and σ_c^c, are determined, subject to strain uniformity within the strip, such that

$$\frac{\sigma_c^c}{E_c} + \alpha_c \Delta T_c = \frac{\sigma_m^c}{E_m} + \frac{\sigma_m^c - \sigma_y}{H} + \alpha_m \Delta T_c \qquad (10)$$

Then, by noting that the total force on the system is zero, such that

$$\sigma_c^c t_c + \sigma_m^c t_m = 0 \qquad (11)$$

the stresses become

$$\sigma_m^c = \left[(\alpha_c - \alpha_m)\Delta T_c + \left(\frac{\sigma_y}{H}\right)\right]\left(\frac{1}{E_m} + \frac{t_m}{t_c}\frac{1}{E_c} + \frac{1}{H}\right)^{-1} \qquad (12a)$$

$$\sigma_c^c = -\sigma_m^c t_m / t_c \qquad (12b)$$

(2) *Relaxation and Heating*

Removal of the constraint after undercooling induces bending, with the metal on the concave side of the strip (Fig. 7(*c*)). Subsequent heating relaxes the residual stresses, and reduces the curvature of the strip until the strip is straightened (Fig. 7(*d*)). Further heating then causes reverse bending and raises the residual stresses, such that the metal eventually yields in compression (Fig. 7(*e*)). Consequently, during heating the metal can either behave elastically or be partially or fully plastic, depending on the material properties and the reheat temperature.

To evaluate the stresses that develop due to loss of constraint and reheating, the bending strains are regarded as being proportional to the distance from the neutral axis and inversely proportional to the radius of curvature.[5] Then the strain, ε^h, becomes

$$\varepsilon^h = c + (x - t_n)/r \qquad (-t_m \le x \le t_c) \qquad (13)$$

where c is a constant (to be determined), t_n is the position of the neutral axis, and r is the radius of curvature (r is positive when the metal is on the concave side).

The stresses σ_m^h and σ_c^h can be derived directly from the strains. In the linear elastic *ceramic*, the stress, σ_c^h, is directly related to the strains by

$$\sigma_c^h = E_c\left(c + \frac{x - t_n}{r} - \alpha_c \Delta T_h\right) \qquad (0 \le x \le t_c) \qquad (14)$$

In the *metal*, plastic strain may also be involved. Specifically, the metal may yield in compression during reverse bending. Furthermore, since the metal near the metal/ceramic interface is subject to the largest compressive stress, yield initiates at the interface ($x = 0$). The relations between the stress and the strain that develop in the metal are thus formulated for three cases, depending

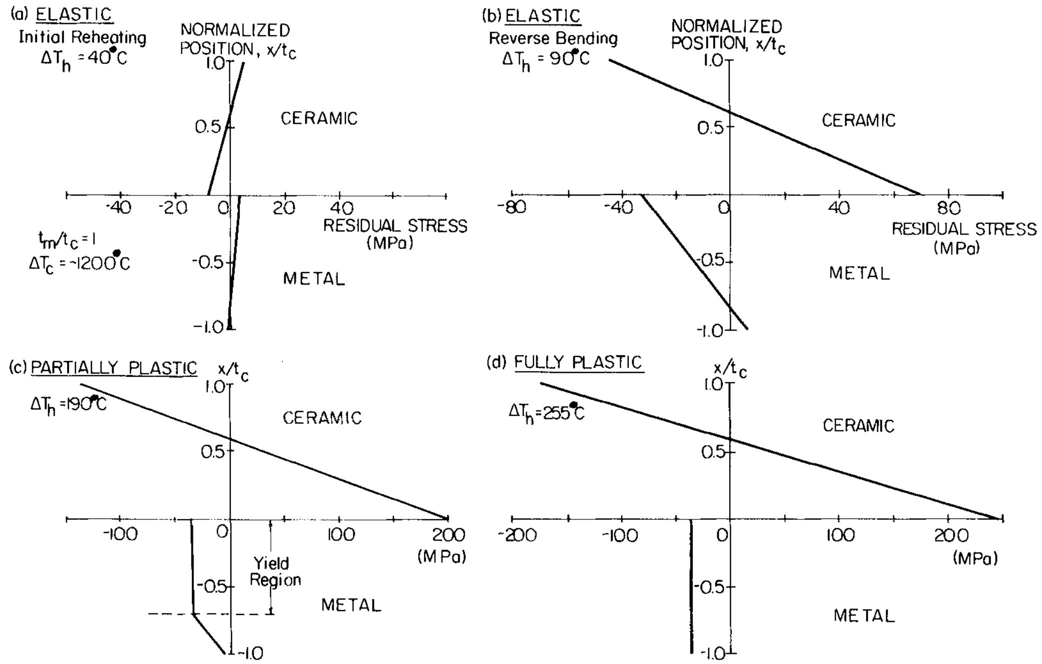

Fig. 9. Stress distributions within Cu/Al_2O_3 strip for $t_m/t_c = 1$ and $\Delta T_c = -1200°C$, showing (*a*) reduction of residual stresses by heating to $\Delta T_h = 40°C$, (*b*) reverse yielding and increase in residual stresses at $\Delta T_h = 90°C$, (*c*) partial yielding of metal at $\Delta T_h = 180°C$, and (*d*) full plasticity of metal at $\Delta T_h = 225°C$.

on the material properties and the reheat temperature: elastic, partially plastic, and fully plastic.

The elastic stresses are simply

$$\sigma_m^h = E_m\left(c + \frac{x - t_n}{r} - \alpha_m \Delta T_h\right) \qquad (-t_m \le x \le 0) \qquad (15)$$

In the partially plastic case, the stresses may be deduced as (Appendix B)

$$\sigma_m^h = E_m\left(c + \frac{x - t_n}{r} - \alpha_m \Delta T_h\right) \qquad (-t_m \le x \le -t_y) \qquad (16a)$$

$$\sigma_m^h = \left(c + \frac{x - t_n}{r} - \alpha_m \Delta T_h - \frac{\sigma_m^c + \sigma_y}{H}\right)\left(\frac{1}{E_m} + \frac{1}{H}\right)^{-1} \qquad (-t_y \le x \le 0) \qquad (16b)$$

where $-t_y$ is the position of the elastic/plastic boundary. Finally, for a fully plastic metal ($t_y = t_m$), Eq. (16) reduces to

$$\sigma_m^h = \left(c + \frac{x - t_n}{r} - \alpha_m \Delta T_h - \frac{\sigma_m^c + \sigma_y}{H}\right)\left(\frac{1}{E_m} + \frac{1}{H}\right)^{-1} \qquad (-t_m \le x \le 0) \qquad (17)$$

The total residual stresses are the sum of the cooling stresses and subsequent stresses due to loss of constraint and reheating, such that

$$\sigma_m = \sigma_m^c + \sigma_m^h \qquad (18a)$$

$$\sigma_c = \sigma_c^c + \sigma_c^h \qquad (18b)$$

The stresses are obtained subject to solutions for the constants c, t_n, and r (and t_y for partial yield). These constants can be solved (Appendix C) by imposing the following boundary conditions (Section II). The sum of the bending stresses (terms involving $(x - t_n)/r$) is zero. The sum of the total stresses is zero. The stress at the elastic/plastic boundary ($x = -t_y$) equals $-\sigma_y$ (for partial yield only). Finally, the sum of the bending moments, with respect to the neutral axis ($x = t_n$), is zero. Residual stresses computed for the system Cu/Al_2O_3 are summarized in Figs. 8 and 9 for the case $t_m/t_c = 1$ and $\Delta T_c = -1200°C$. After constrained cooling, uniform tension and compression develop in the metal and ceramic, respectively (Fig. 8(*a*)). As the constraint is removed, bending occurs and the stress distributions are modified (Fig. 8(*b*)). Then, upon reheating, the residual stresses are initially reduced (Fig. 9(*a*)). However, further heating results in reverse bending and an increase in the residual stresses (Fig. 9(*b*)). Eventually, compressive yielding of the metal initiates from the metal/ceramic interface, such that partially (Fig. 9(*c*)) or fully (Fig. 9(*d*)) plastic conditions may develop.

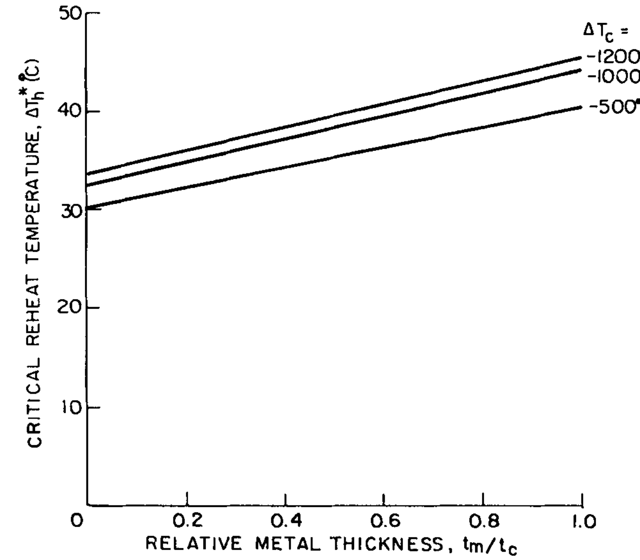

Fig. 10. Critical heating temperature to straighten strip as a function of relative metal thickness for $\Delta T_c = -500$, -1000, and $-1200°C$.

The reheat temperature needed to straighten the strip and eliminate the residual stresses (Eq. (C–3)) is evaluated as a function of the relative metal thickness, t_m/t_c, and plotted for several cooling temperatures in Fig. 10 (the essentially linear variation is attributed to the relative magnitude of the deformation parameters for Cu, for which $H \ll E_m$).

The variation of the curvature with the reheat temperature (for $t_m/t_c = 1$ and $\Delta T_c = -1200°C$) is plotted in Fig. 11. The regions of elastic, partially plastic, and fully plastic response of the Cu are indicated. It is noted that the curvature and residual stress exhibit relatively large changes while the response is elastic.

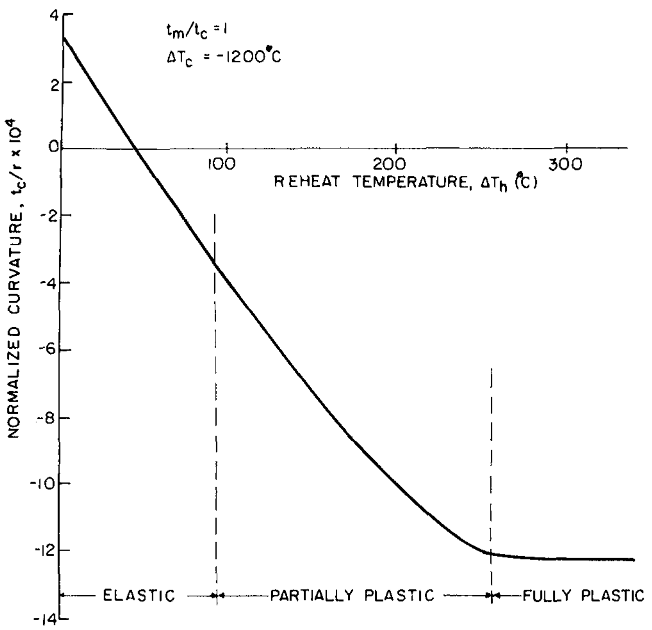

Fig. 11. Normalized curvature as a function of reheating temperature for $t_m/t_c = 1$ and $\Delta T_c = -1200°C$. Also shown are regions of elastic, partially plastic, and fully plastic deformation response of metal.

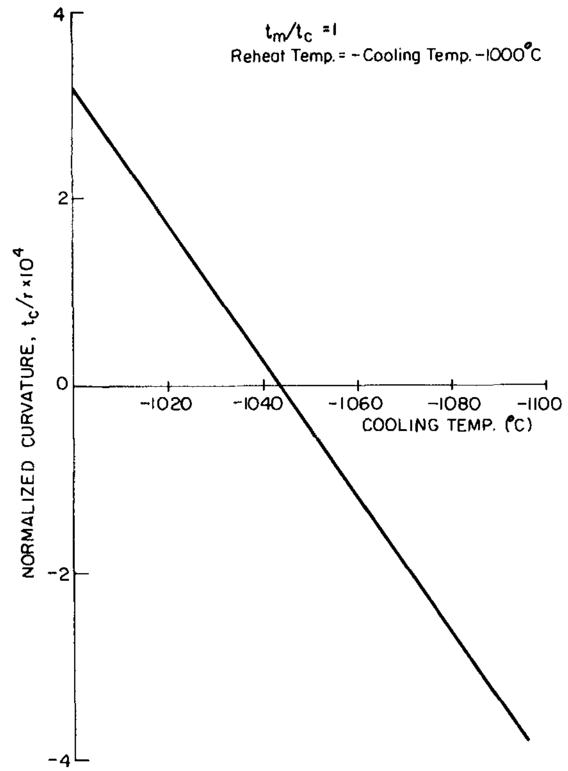

Fig. 12. Normalized curvature as a function of undercooling temperature for $t_m/t_c = 1$. Temperature difference before cooling and after heating is 1000°C.

Conversely, quite small changes occur when the Cu becomes fully plastic.

An example of optimum undercooling is illustrated in Fig. 12, using a temperature difference between bonding and room temperature of 1000°C. It can be determined from the figure that undercooling of the strip by $\Delta T_c = -1044°C$ and subsequent heating by $\Delta T_h = 44°C$ would straighten the strip and eliminate the residual stresses. The critical cooling and reheating temperature needed to straighten the strip (Eqs. (C–4) and (C–5)) are plotted in Fig. 13 as functions of the relative metal thickness and work-hardening rate. It is noted that the higher the work-hardening rate, the lower the critical undercooling temperature, and hence, the higher the critical reheat temperature (Fig. 13).

It is also interesting to compare the residual stresses that develop in the strip with and without a cooling constraint (Section II). The comparison evaluated for a nonhardening metal ($H = 0$) is plotted in Fig. 14 for $t_m/t_c = 1$. It is noted that both the curvature and the residual stresses at a specific undercooling are significantly lower when constraint is imposed. The imposition of constraint is thus of general desirability for the minimization of residual stress during undercooling and hence, inhibits the development of cracks or other modes of damage.

IV. Conclusions

A stress analysis has been conducted for a linear work-hardening metal in a metal/ceramic strip. Stresses develop during cooling to room temperature, due to the different mechanical and thermal properties of the materials. This asymmetry results in bending and stress redistribution. The calculations illustrate trends in the stresses, the locations of the neutral axis and the elastic/plastic boundary, and the curvature of the strip, with the mismatch in thermal expansion, the mechanical properties, and the thickness of the two constituents as variables. It is specifically demonstrated that, for a metal with a high work-hardening rate, fully plastic deformation is suppressed, because of the stress redistribution caused by bending. It is also noted that, for a perfectly plastic metal, once the fully plastic condition has been achieved, the curvature and the stresses do not change with further cooling. Brittle fracture tendencies of the ceramic are thus reduced by using a nonhardening metal strip.

The calculations presented above refer to the stresses within the beam, away from the free ends, where the interface is stress free (i.e., no normal or shear stresses at the interface). At the ends, the requirement that the surfaces be stress free perturbs the stress field to a distance about 3 times the strip thickness.[7] Within this region, shear stresses develop at the interface and exhibit a peak about one strip thickness away from the end. Large normal stresses also develop at the interface, which may be singular at the end. These end effects often result in debonding at the interface.

A constrained cooling procedure for the elimination of residual stress has also been analyzed. During constrained cooling, the strip is prevented from bending and uniform tensile and compressive stresses develop in the metal and the ceramic, respectively. As the constraint is removed, bending occurs due to the asymmetric stress. Upon reheating the strip, the curvature and the residual stresses initially reduce, as it straightens. However, further heating causes reverse bending and the metal starts to yield in compression, initiating from the metal/ceramic interface. The optimum undercooling needed to straighten the strip and eliminate the residual stresses upon subsequent heating is predicted from the present study. Finally, a comparison of unconstrained with constrained cooling indicates that the curvature and the residual stresses that develop during the latter are significantly lower. The use of constraint is thus of general desirability with regard to the minimization of residual stresses in metal/ceramic strips.

APPENDIX A

Stresses and Curvatures in a Ceramic/Metal Strip

The various bending conditions described in Section II may be used to evaluate the bending constants t_n, ε_0, $1/r$, and t_y and, hence, to determine the stresses. The essential results are summarized in the text.

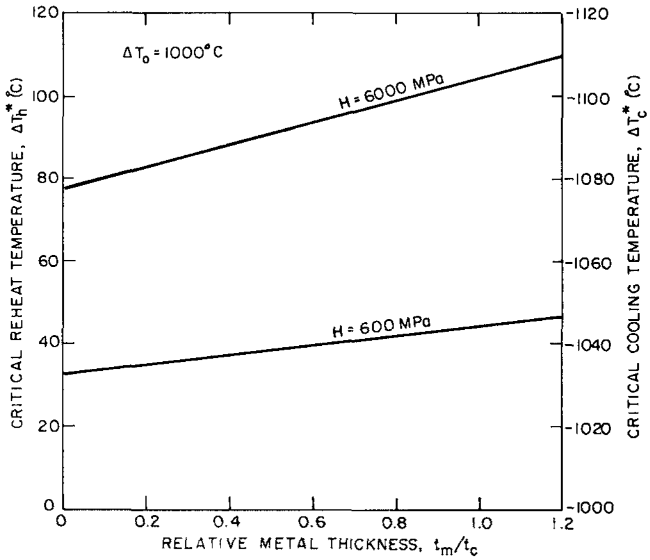

Fig. 13. Critical cooling and heating temperature to straighten strip as a function of metal thickness for temperature difference between bonding and room temperatures of 1000°C; H = 600 and 6000 MPa.

For the *elastic case,* the bending constants subject to the requirement, $\sigma_m(x = 0) \le \sigma_y$, are

$$t_n = \frac{E_c t_c^2 - E_m t_m^2}{2(E_m t_m + E_c t_c)} \qquad \text{(A-1)}$$

$$\varepsilon_0 = \frac{E_c(\alpha_c - \alpha_m)\Delta T t_c}{E_m t_m + E_c t_c}$$

$$\frac{1}{r} = \frac{6E_m E_c t_m t_c (t_m + t_c)(\alpha_c - \alpha_m)\Delta T}{E_m^2 t_m^4 + E_c^2 t_c^4 + 2E_m E_c t_m t_c (2t_m^2 + 2t_c^2 + 3t_m t_c)}$$

Partial plasticity requires that $\sigma_m(x = 0) > \sigma_y$ and $\sigma_m(x = -t_m) < \sigma_y$; whereupon, for a metal with a large work-hardening rate ($H \approx \infty$)

$$t_n = \frac{E_c t_c^2 - E_m t_m^2}{2(E_m t_m + E_c t_c)}$$

$$\varepsilon_0 = \frac{1}{2r}\frac{E_c t_c(t_c + 2t_y) - E_m t_m(t_m - 2t_y)}{E_m t_m + E_c t_c} + \frac{\sigma_y}{E_m}$$

$$\frac{1}{r} = \frac{6E_m E_c t_m t_c (t_m + t_c)(\alpha_c - \alpha_m)\Delta T}{E_m^2 t_m^4 + E_c^2 t_c^4 + 2E_m E_c t_m t_c (2t_m^2 + 2t_c^2 + 3t_m t_c)}$$

$$t_y = \frac{\{[E_m^2 t_m^4 + E_c^2 t_c^4 + 2E_m E_c t_m t_c (2t_m^2 + 2t_c^2 + 3t_m t_c)]\sigma_y + E_m E_c[E_c t_c^4 + E_m t_m^2 t_c (4t_m + 3t_c)](\alpha_m - \alpha_c)\Delta T\}}{6E_m^2 E_c t_m t_c (t_m + t_c)(\alpha_m - \alpha_c)\Delta T} \qquad \text{(A-2)}$$

and for *perfect metal plasticity* ($H = 0$)

$$t_n = \frac{1}{2}\frac{E_c t_c^2 - E_m(t_m^2 - t_y^2)}{E_m(t_m - t_y) + E_c t_c}$$

$$\varepsilon_0 = \frac{1}{2r}\frac{E_c t_c(t_c + 2t_y) - E_m(t_m - t_y)^2}{E_m(t_m - t_y) + E_c t_c}$$

$$\frac{1}{r} = 2\frac{(E_m t_m + E_c t_c)\sigma_y + E_m E_c t_c(\alpha_m - \alpha_c)\Delta T}{E_m^2(t_m - t_y)^2 - E_m E_c t_c(t_c + 2t_y)}$$

$$\begin{aligned}&[E_m^2 t_m^4 + E_c^2 t_c^4 + 2E_m E_c t_m t_c(2t_m^2 + 2t_c^2 + 3t_m t_c)]\sigma_y \\ &+ [2t_y(E_m t_m + E_c t_c) - 3(E_m t_m^2 - E_c t_c^2)]E_m t_y^2 \sigma_y \\ &= E_m E_c(\alpha_m - \alpha_c)\Delta T[-t_c(E_c t_c^3 + 3E_m t_m^2 t_c + 4E_m t_m^3) \\ &+ 6E_m t_m t_c t_y(t_m + t_c) - E_m t_c t_y^2(2t_y + 3t_c)]\end{aligned} \qquad \text{(A-3)}$$

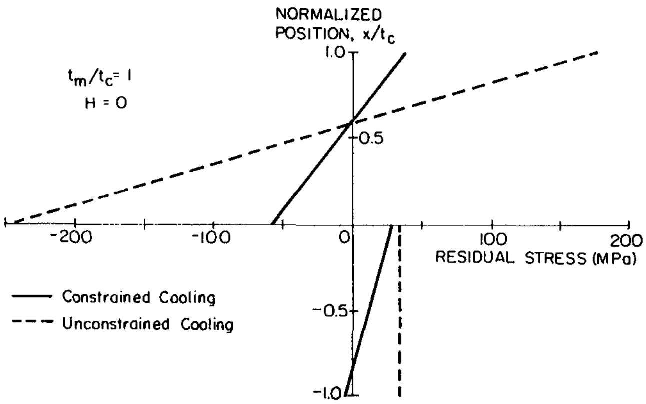

Fig. 14. Stress distribution within Cu/Al_2O_3 strip for $t_m/t_c = 1$ and $H = 0$ for constrained and unconstrained cooling.

Finally for the *fully plastic case* ($\sigma_m(x = -t_m) > \sigma_y$)

$$t_n = \frac{1}{2}\frac{E_c t_c^2 - \left(\frac{1}{E_m} + \frac{1}{H}\right)^{-1} t_m^2}{\left(\frac{1}{E_m} + \frac{1}{H}\right)^{-1} t_m + E_c t_c}$$

$$\varepsilon_0 = -\frac{E_c t_c(\alpha_m - \alpha_c)\Delta T + \sigma_y\left(1 + \frac{H}{E_m}\right)^{-1} t_m}{\left(\frac{1}{E_m} + \frac{1}{H}\right)^{-1} t_m + E_c t_c}$$

$$\frac{1}{r} = \frac{6E_m E_c t_m t_c(t_m + t_c)\left(1 + \frac{E_m}{H}\right)^{-1}\left[(\alpha_c - \alpha_m)\Delta T + \frac{\sigma_y}{H}\right]}{\left\{\left(\frac{1}{E_m} + \frac{1}{H}\right)^{-2} t_m^4 + E_c^2 t_c^4 + 2E_m E_c t_m t_c\left(1 + \frac{E_m}{H}\right)^{-1}(2t_m^2 + 2t_c^2 + 3t_m t_c)\right\}} \qquad \text{(A-4)}$$

Note that in the absence of work hardening ($H = 0$), the curvature reduces to

$$\frac{1}{r} = \frac{6t_m(t_m + t_c)\sigma_y}{E_c t_c^3}$$

and the stresses become

$$\sigma_c = -\sigma_y\frac{t_m}{t_c} + \left(x - \frac{t_c}{2}\right)\frac{1}{r}$$

$$\sigma_m = \sigma_y \qquad \text{(A-5)}$$

The fully plastic solutions for a nonhardening material have previously been derived by Wittmer *et al.*[4]

APPENDIX B

Relations between Stress and Strain for Reverse Yielding in the Metal

When the metal yields in compression upon reheating, the plastic strain, ε_p, is

$$\varepsilon_p = (\sigma_m + \sigma_y)/H \qquad \text{(B-1)}$$

where σ_m is the total stress in the metal. The total strain that develops during this step is $c + (x - t_n)/r$ (Eq. (13)), and the thermal strain is $\alpha_m \Delta T_h$. The reheat residual stress in the metal, σ_m^h, is thus

$$\sigma_m^h = E_m\left(c + \frac{x - t_n}{r} - \alpha_m \Delta T_h - \frac{\sigma_m + \sigma_y}{H}\right) \tag{B–2}$$

The total stress is the sum of the cooling and reheating stresses

$$\sigma_m = \sigma_m^c + \sigma_m^h \tag{B–3}$$

Hence, combination of Eqs. (B–2) and (B–3) gives

$$\sigma_m = \left(c + \frac{x - t_n}{r} - \alpha_m \Delta T_h + \frac{\sigma_m^c}{E_m} - \frac{\sigma_y}{H}\right)\left(\frac{1}{E_m} + \frac{1}{H}\right)^{-1} \tag{B–4}$$

Substitution of Eq. (B–4) into Eq. (B–2) then gives

$$\sigma_m^h = \left(c + \frac{x - t_n}{r} - \alpha_m \Delta T_h - \frac{\sigma_m^c + \sigma_y}{H}\right)\left(\frac{1}{E_m} + \frac{1}{H}\right)^{-1} \tag{B–5}$$

APPENDIX C

Stresses and Curvatures in a Constrained Metal/Ceramic Strip

The boundary conditions described in Section III may be used to evaluate the bending and curvature terms that determine the stresses in a strip after constrained cooling. For *elastic bending* of the metal, upon loss of constraint and reheating, the constants can be explicitly determined as

$$t_n = (E_c t_c^2 - E_m t_m^2)/2(E_m t_m + E_c t_c)$$

$$c = (E_m \alpha_m t_m + E_c \alpha_c t_c)\Delta T_h/(E_m t_m + E_c t_c) \tag{C–1}$$

$$\frac{1}{r} = \left[\sigma_m^c + \frac{E_m E_c t_c(\alpha_c - \alpha_m)\Delta T_h}{E_m t_m + E_c t_c}\right] \times \frac{6t_m(t_m + t_c)(E_m t_m + E_c t_c)}{E_m^2 t_m^4 + E_c^2 t_c^4 + 2E_m E_c t_m t_c(2t_m^2 + 2t_c^2 + 3t_m t_c)}$$

subject to the condition

$$\sigma_m(x = 0) \geq -\sigma_y \tag{C–2}$$

Note that the relation between the critical undercooling ΔT_c^* and reheating ΔT_h^* needed to produce a flat strip ($1/r = 0$), when the metal behaves elastically upon reheating, can be derived from Eq. (C–1) as

$$\Delta T_h^* = -\left[\Delta T_c^* + \frac{\sigma_y}{(\alpha_c - \alpha_m)H}\right]\left(1 - \frac{1}{1 + \dfrac{H}{E_m} + \dfrac{t_m}{t_c}\dfrac{H}{E_c}}\right) \tag{C–3}$$

At this critical temperature, the strip is free from residual stress. It is noted that, since the stress relaxation is elastic, the reheating ΔT_h^* (Eq. (C–3)) is essentially the same as can be derived from Section III(1) (without removing the constraint). It is also possible to derive the critical undercooling ΔT_c^* and reheating ΔT_h^* associated with a given temperature difference, ΔT_0, between the bonding temperature and room temperature:

$$\Delta T_c^* = -(\Delta T_0 + \Delta T_h^*) \tag{C–4}$$

where

$$\Delta T_h^* = \left[\Delta T_0 + \frac{\sigma_y}{(\alpha_m - \alpha_c)H}\right]\left(\frac{H}{E_m} + \frac{t_m}{t_c}\frac{H}{E_c}\right) \tag{C–5}$$

For *partial plasticity* in the metal, the solutions are complex and can be determined from the following four simultaneous equations:

$$\int_{-t_m}^{-t_y} E_m(x - t_n)/r\, dx + \int_{-t_y}^{0} (x - t_n)\left[\left(\frac{1}{E_m} + \frac{1}{H}\right)r\right]^{-1} dx + \int_0^{t_c} E_c(x - t_n)/r\, dx = 0$$

$$\int_{-t_m}^{-t_y} E_m(c - \alpha_m \Delta T_h)\, dx + \int_{-t_y}^{0}\left(c - \alpha_m \Delta T_h - \frac{\sigma_m^c + \sigma_y}{H}\right) \times \left(\frac{1}{E_m} + \frac{1}{H}\right)^{-1} dx + \int_0^{t_c} E_c(c - \alpha_c \Delta T_h)\, dx = 0$$

$$\sigma_m^c + E_m\left(c - \frac{t_y + t_n}{r} - \alpha_m \Delta T_h\right) = -\sigma_y \tag{C–6}$$

$$\int_{-t_m}^{-t_y}\left[\sigma_m^c + E_m\left(c + \frac{x - t_n}{r} - \alpha_m \Delta T_h\right)\right](x - t_n)\, dx + \int_{-t_y}^{0}\left(c + \frac{x - t_n}{r} - \alpha_m \Delta T_h + \frac{\sigma_m^c}{E_m} - \frac{\sigma_y}{H}\right)(x - t_n) \times \left(\frac{1}{E_m} + \frac{1}{H}\right)^{-1} dx + \int_0^{t_c}\left[\sigma_c^c + E_c\left(c + \frac{x - t_n}{r} - \alpha_c \Delta T_h\right)\right] \times (x - t_n)\, dx = 0$$

subject to the conditions

$$\sigma_m(x = 0) \leq -\sigma_y \tag{C–7a}$$

$$\sigma_m(x = -t_m) \geq -\sigma_y \tag{C–7b}$$

For *complete plasticity* in the metal, $\sigma_m(x = -t_m) \leq -\sigma_y$, specific solutions can be derived, as given by the following:

$$t_n = \frac{1}{2}\,\frac{E_c t_c^2 - \left(\dfrac{1}{E_m} + \dfrac{1}{H}\right)^{-1} t_m^2}{\left(\dfrac{1}{E_m} + \dfrac{1}{H}\right)^{-1} t_m + E_c t_c}$$

$$c = \frac{\left\{\left[\left(\dfrac{1}{E_m} + \dfrac{1}{H}\right)^{-1} \alpha_m t_m + E_c \alpha_c t_c\right]\Delta T_h + \left(\dfrac{1}{E_m} + \dfrac{1}{H}\right)^{-1} t_m \dfrac{\sigma_m^c + \sigma_y}{H}\right\}}{\left(\dfrac{1}{E_m} + \dfrac{1}{H}\right)^{-1} t_m + E_c t_c} \tag{C–8}$$

$$\frac{1}{r} = \frac{\left\{\left(\dfrac{1}{E_m} + \dfrac{1}{H}\right)^{-1} t_m\left(t_n + \dfrac{t_m}{2}\right)\left(c - \alpha_m \Delta T_h + \dfrac{\sigma_m^c}{E_m} - \dfrac{\sigma_y}{H}\right) + t_c\left(t_n - \dfrac{t_c}{2}\right)\left[\sigma_c^c + E_c(c - \alpha_c \Delta T_h)\right]\right\}}{\left(\dfrac{1}{E_m} + \dfrac{1}{H}\right)^{-1} t_m\left(\dfrac{t_m^2}{3} + t_n t_m + t_n^2\right) + E_c t_c\left(\dfrac{t_c^2}{3} - t_n t_c + t_n^2\right)}$$

The residual stresses are obtained by substitution of the constants t_n, c, t_y, and r into Eqs. (14) to (18).

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