Low-frequency technique for the underwater calibration of individual elements of a line hydrophone array

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A calibrator for use with long line hydrophones (seismic streamers) has been designed, constructed, and tested. Comparison calibrations may be obtained for individual elements or small groups of interconnected elements of a line hydrophone array in the frequency range of 20 to 3000 Hz. Absolute calibrations may be obtained in the frequency range of 20 to 100 Hz. The calibrator may be operated in either of two configurations. The first configuration has the calibrator submerged in a water trough. In the second configuration the calibrator, with the hydrophone inserted, forms a closed chamber through which water is continuously circulated during operation. A comparison between results obtained by using the calibrator, and independent measurements is presented. The theory of operation of the device, and of the sonic resistors used to reduce standing waves within the calibrator, is presented in the appendixes.

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INTRODUCTION

Conventional hydrophones are calibrated by free-field reciprocity or comparison-calibration techniques or by one of the closed-chamber, low-frequency methods that simulate an acoustic wave by a sinusoidal variation of the chamber pressure.[1–3] None of these methods is convenient to use for calibrating the elements of a multielement line-hydrophone array (seismic streamer) that may be tens or even hundreds of meters long. A comparison calibration can be obtained in a tank or lake by suspending the array vertically and sequentially positioning each element at the calibration depth. The low-frequency limit for this type of calibration depends on the size of the tank or lake.

The typical line-hydrophone array, considered here, consists of a number of piezoelectric elements individually cabled through preamplifiers to the receiving system. Since the array is used at frequencies well below the element resonant frequency, where the element is acoustically stiff and equivalent to a point receiver, it can be calibrated in a simulated acoustic field like the sinusoidal pressure variation in a closed chamber. The sound pressure incident on an acoustically stiff point receiver is equal to the free-field acoustic pressure, so any calibration method that measures the open circuit voltage for a measured incident sound pressure yields the free-field voltage sensitivity of the hydrophone.

The calibrator described here is designed primarily to give comparison calibrations in the frequency range of 20–3000 Hz. At low frequencies (below 100 Hz) absolute calibrations may also be obtained. Mechanical design, associated electronic instrumentation, modification of basic calibrator to eliminate the need for a water trough, and theory of operation are presented.

I. MECHANICAL DESIGN

The calibrator shown on Fig. 1 contains three centrally located, coaxial, internally radiating, piezoceramic transducers. At each end of the transducer assembly is a 0.9144-m (36 in.)-long stainless-steel tube manufactured out of standard weight 0.1143-m (4.5 in.)-o.d. pipe. Thus the calibrator can accept a 0.0889-m (3.5 in.)-o.d. line-hydrophone array. The ten sonic resistors inserted into the wall of each tube were made out of 0.0191-m (0.75 in.) stainless-steel pipe plugs. The sonic resistors reduce the frequency-dependent pressure fluctuations within the calibrator. Each sonic resistor is designed to have a resistance of 1.0×10^9 mks Ω. Theory and design of the sonic resistors are presented in Appendix B. The entire calibrator assembly, when submerged in water, forms an acoustic transmission line.

Each transducer, shown on Fig. 2, consists of piezoceramic tube inside a stainless-steel housing. The piezoceramic tube is potted in polyurethane. The dimensions of the piezoceramic elements are 0.1143-m (4.5 in.) i.d., 0.1270-m (5 in.) o.d., and 0.0889-m (3.5 in.) long.

Two of the three calibrator transducers produce the sound pressure within the acoustic transmission line. The central or monitor transducer measures this sound pressure in the region of the line-array element being calibrated. There must be adequate margin between acoustic coupling and structural (mechanical) coupling of the two sound-producing transducers to the monitor transducer. That is, in order that the output of the monitor transducer truly represent the sound pressure

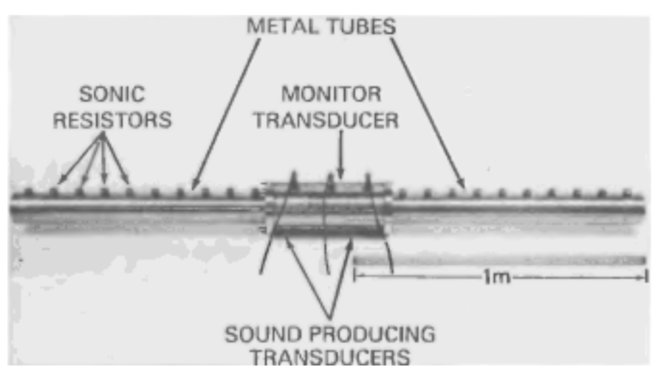


FIG. 1. Calibrator.

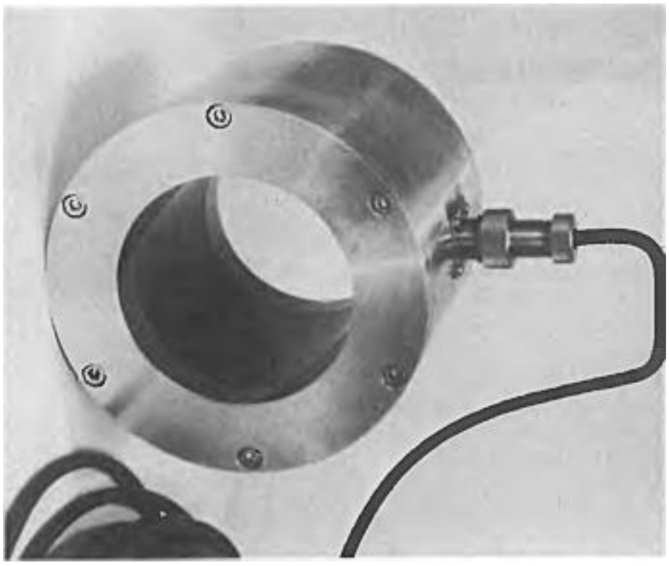


FIG. 2. Picture of one transducer.

level at the center of the calibrator, this output must be much larger when the calibrator is operated in water (acoustic coupling) than when it is operated in air (mechanical coupling). Mechanical coupling is through a path from the piezoceramic driver to its housing to the receiver housing and finally to the piezoceramic receiver. A compliant support of the ceramic, symmetric with respect to its center of gravity reduces the excitation of the housing. A massive housing relative to the mass of the ceramic driver further reduces mechanical coupling. Since the calibrator must operate over a wide frequency range, the compliant support must be highly damped so that mechanical coupling is not amplified at some frequencies by high Q, low-frequency structural resonances.

A cut-away view of the calibrator is shown in Fig. 3. The piezoceramic tube within the transducer is lead zirconate-lead titanate (Channelite 5400). An aluminum ring is cemented to each end of the tube and contains an O-ring seal. Three small strips of Corprene pressure-release material cemented to each aluminum ring keep the piezoceramic tube-ring assembly centered in the housing. End flanges and tie bolts clamp the three transducers together. Terminating stainless-steel tubes, approximately one meter long, screw into the end flanges. Each transducer was assembled, mounted in a lathe, and rotated at about 0.1 rad/sec (1 rpm), while air-free polyurethane liquid slowly filled the slits up to the O-ring seal. The speed of rotation was then increased to 125 rad/sec (1200 rpm), and polyurethane liquid was added up to the inside diameter (0.1016 m) of the transducer housing. Rotation was continued until the polyurethane cured. In this way the piezoceramic tube is mounted in its housing with its outer surface and each end free to move (air backed). The ceramic tube is supported on each end by the O-ring, symmetric with respect to its center of gravity. The polyurethane in the slit (0.4×4 mm) has a high enough stiffness to retain the sound pressure but its shear compliance is much greater than the length or radial compliance of the ceramic tube. The polyurethane in shear produces enough damping so that mechanical vibration of the transducer housing is not excited. The ratio of the housing mass to the piezoceramic mass is about 5. This mounting technique has been previously used by one of the authors[3] in a portable hydrophone calibrator. The object of this carefully configured mechanical design is to reduce the mechanical coupling between the sound-producing transducers and the monitor transducer so that the output of the monitor transducer is a true measure of the sound pressure within the calibrator, and not a measure of the vibration of the monitor transducer housing assembly. A way to determine the effectiveness of this mechanical design is to measure the output of the monitor transducer with the calibrator operated in air and with the calibrator submerged in water. If the "in air" measurement is significantly less than the submerged measurement then the decoupling achieved by the mechanical design is adequate. The measurement of the "in air" output of the monitor transducer is at least 40 dB below the submerged output over the entire frequency range of the calibrator.

II. ELECTRONIC INSTRUMENTATION AND CALIBRATION PROCEDURE

Figure 4 is a block diagram of the electronic circuitry. The oscillator, amplifier, and transformer pro-

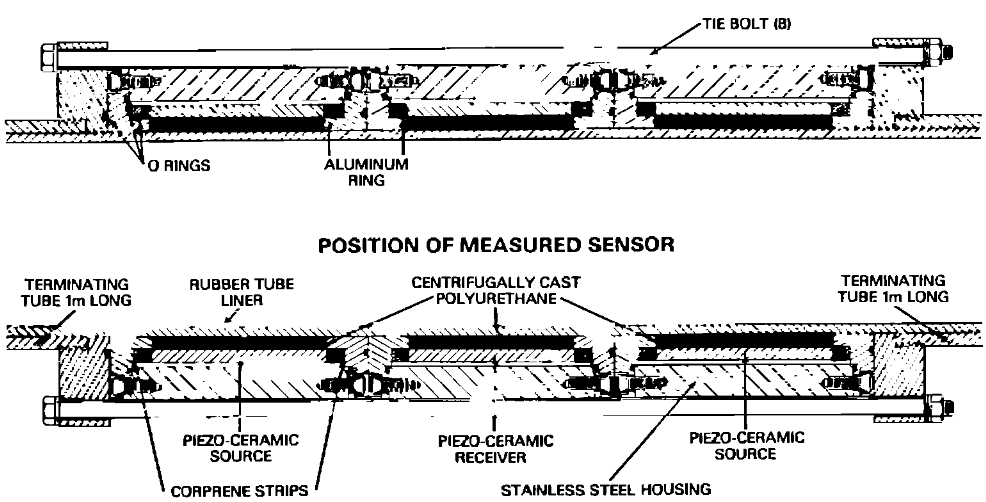
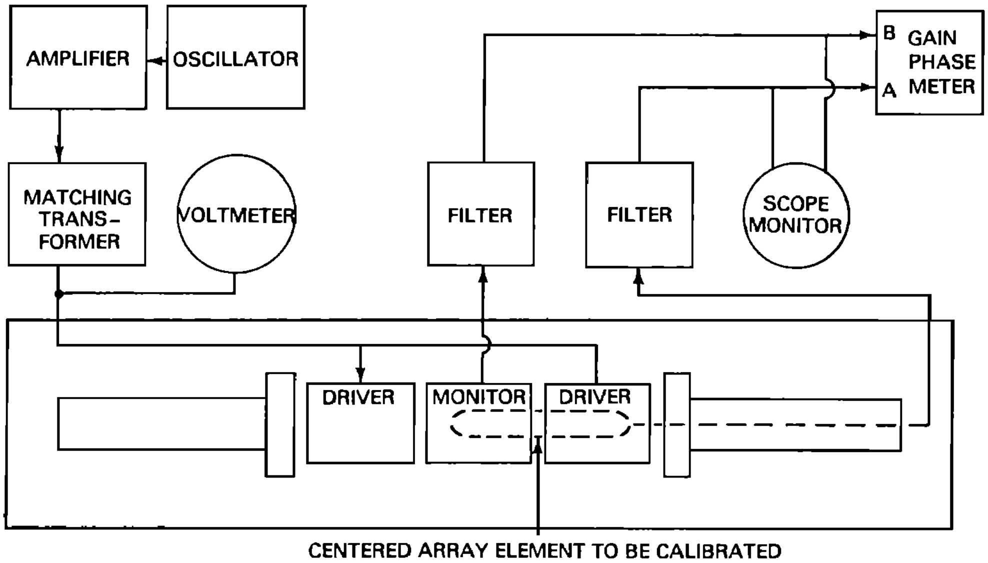


FIG. 3. Cut-away view of calibrator.



EQUIPMENT USED:
KRON-HITE FILTERS MODEL 3103
KRON-HITE 50 WATT AMPLIFIER MODEL DCA-50
KRON-HITE MATCHING TRANSFORMER 20 CPS-20KC 50 WATT MODEL MT-55
HEWLETT PACKARD OSCILLATOR MODEL 200CD
HEWLETT PACKARD GAIN/PHASE METER MODEL 3575A
HEWLETT PACKARD VTVM MODEL 400H

FIG. 4. Block diagram of electronic circuitry.

duce a voltage up to 500 V over the frequency range of the calibrator. The output voltages from the centered element of the line array and the calibrator receiving transducer are connected through identical filters to the A and B terminals of the gain/phase meter. A voltmeter measured the driving voltage and the oscilloscope measures the waveform of the received signals. The gain/phase meter is used to measure the magnitude of the ratio of the line-array element voltage to the calibrator output voltage for comparison calibrations.

The uniformity of the acoustic field within the calibrator was checked using a small probe. The acoustic pressure 0.1524 m (6 inches) from the center was within 0.5 dB of the acoustic pressure at the center from 20 to 350 Hz and within 3 dB from 350 to 1500 Hz. The acoustic pressure 0.3048 m (12 in.) from the center was approximately 1 dB lower than the pressure at the center from 20 to 240 Hz and from 240 to 1300 Hz differed by from 1 to 5 dB.

In order to calibrate an element of a line-hydrophone array, the calibrator is first submerged in a water-filled trough as shown on Fig. 5. The line-hydrophone array is then threaded through the calibrator until the array element to be calibrated is located at the center of the calibrator. The array must be threaded through the calibrator with special care to insure that the array surface is thoroughly wetted and that no air bubbles are carried into the calibrator. In general air bubbles within the calibrator will lower the sound pressure level which can be obtained within the calibrator, and will give rise to spurious resonances. Air bubbles near the center of the calibrator will cause erroneous calibrations because of the large pressure gradient near an air bubble. After the array has been inserted into the calibrator, a calibration frequency is chosen and the sound-producing or driver transducers are electrically driven at a power level such that adequate signals are obtained from both the monitor transducer and the line-hydrophone array element. After both filters are set to pass the calibration frequency the relative sensitivity (in decibels) of the line-hydrophone array element with respect to the monitor transducer is read directly on the gain/phase meter. This amount is then added to the sensitivity of the monitor transducer (approximately -183 dB *re* 1 V/μPa for the transducers described in this paper) to obtain the calibration of the line-hydrophone array. A comparison of the calibrations obtained by using the calibrator with those obtained by independent measurements is shown in Fig. 6. Of particular interest is the calibration of the type F36 hydrophone at the higher frequencies. Line-hydrophone arrays frequently employ clusters of interconnected elements. Each cluster would then be equivalent to a short-line hydrophone. The type F36 hydrophone is equivalent to a line hydrophone eight inches in length. The sensitivity of this hydrophone is constant throughout the range of 20–3000 Hz for sound incident in the radial direction. However, at 3 kHz, the axial sensitivity of an 8-in. line hydrophone is 2.6 dB less than the radial sensitivity. Since the type F36 hydrophone was oriented within the calibrator so that sound was incident in the axial direction (any other orientation being impossible since the

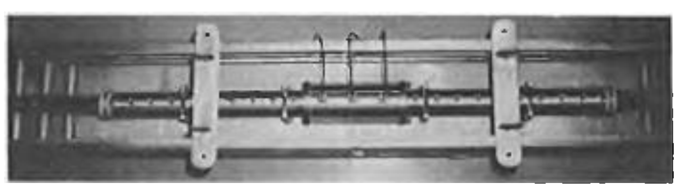


FIG. 5. Complete calibrator in water-filled trough.

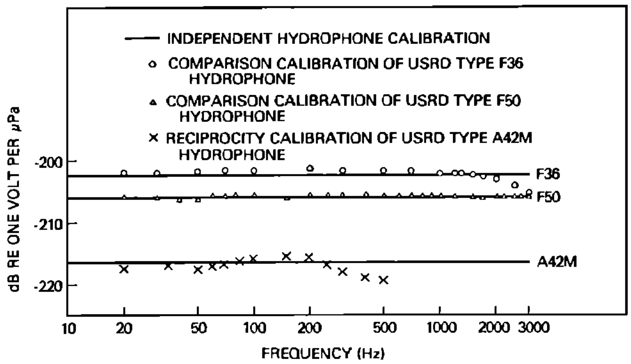


FIG. 6. Comparison of calibrations obtained using the calibrator with independent calibrations.

calibrator has a 4 in. i.d.), a 2.6-dB decrease in sensitivity should be observed at 3 kHz. From Fig. 6 it can be seen that the sensitivity of the F36 in the axial direction does decrease by approximately the correct amount as the frequency approaches 3 kHz. Thus, in order to determine the sensitivity of a cluster of elements within a line-hydrophone array at the higher frequencies, it is necessary to know the effective length of the cluster so that the appropriate correction factor may be applied.

At low frequencies (below 100 Hz) it is possible to obtain absolute calibrations from the calibrator provided that some means of measuring the current delivered by the power-amplifier–matching-transformer combination may be determined. This can be achieved by using a current toroid. Also, absolute voltages rather than voltage ratios are needed. The gain/phase meter will read voltages in dB *re* 1 V. If the gain/phase meter is connected to the current toroid then the current delivered by the impedance-matching transformer can be measured in dB *re* 1 amp. Also needed is the low-frequency open tube reciprocity parameter J derived in Appendix A. For the calibrator in this paper its value is given by $2.362\times10^{-6}/f$ mks inverse acoustic ohms, where f is the frequency of calibration. All measurements are to be made in dB *re* 1 V and dB *re* 1 amp. First drive the sound-producing transducers and measure the output of the line-hydrophone array element E_{1H} and the output of the monitor transducer E_{12}. Then drive the monitor transducer and measure the output of the line-hydrophone array element E_{2H} and the current into the monitor transducer I_1. The sensitivity of the line-hydrophone array element $(M_h)_H$ is then

$$(M_h)_H=\tfrac{1}{2}(20\log_{10}J+E_{2H}+E_{1H}-E_{12}-I_2)-120\,, \qquad (1)$$

where the 120 dB is the conversion from Pa to μPa. This sensitivity is in units of dB *re* 1 V/μPa. An absolute calibration of a type A42-M hydrophone is shown on Fig. 6 along with an independent calibration for comparison. The deviation of the absolute calibration from the actual calibration for frequencies above 100 Hz is caused by the failure of the expression for J to yield the actual value of the reciprocity parameter for frequencies above 100 Hz. The data obtained so far is sufficient to determine the sensitivity of the monitor transducer. As shown in Appendix A, the sensitivity of the monitor transducer $(M_h)_2$ is given by

$$(M_h)_2=\tfrac{1}{2}(20\log_{10}J+E_{2H}+E_{12}-E_{1H}-I_2)-120\,, \qquad (2)$$

where this sensitivity is again in units of dB *re* 1 V/μPa. If one of the signals to be measured contains considerable distortion and it becomes necessary to set the high- and low-pass sections of a filter near the operating frequency, then all signals should be passed through that same filter. The loss through the filter will then cancel in the computations as can be seen by examining Eqs. (1) and (2). This same procedure should also be adopted for comparison calibrations if one of the signals is distorted, provided the distortion is not caused by the overload in the receive section of the electronic circuitry. An overload in the receive section of the electronic circuitry (including hydrophone preamplifiers) can only be corrected by reducing the power from the power amplifier.

III. MODIFICATIONS TO THE PHYSICAL CONFIGURATION OF THE CALIBRATOR

In order to insert a long line hydrophone into the calibrator it is necessary to bend the array at the points where it enters the trough or the calibrator itself. This is impossible to achieve if the array contains long telemetry cans, or if a long array is to be calibrated as it is being deployed. The metal trough was eliminated and the ends of the long metal calibrator tubes were fitted with rubber diaphragms as shown on Fig. 7. Holes were cut into the rubber diaphragm somewhat smaller than the cross section of the array to be calibrated. When the array is inserted through the calibrator a closed chamber is formed. Water is continuously recirculated through the calibrator to remove any air which has entered the calibrator and to compensate for the water lost at the points where the array passes through the rubber diaphragms. A bladder arrangement is inserted into the pumped water supply. The outer side of the bladder applies air pressure to the sonic resistors to pressure compensate them. In this manner the air is eliminated and long line hydrophones

FIG. 7. Calibrator modified to eliminate the need for the water-filled trough.

are easily inserted into the calibrator. The bladder also prevents pump noise from entering the calibrator. The electroacoustical performance of the calibrator was not degraded by this modification.

IV. RECOMMENDATIONS FOR FUTURE STUDY AND DEVELOPMENT

There is an increasing tendency for line-hydrophone arrays to employ clusters of interconnected elements instead of single elements. These clusters are sometimes quite long. A study needs to be done to determine the frequency range over which valid calibrations can be made on clusters of elements. Some questions concerning the calibration of line-hydrophone arrays which contain clusters are: Can the valid calibration frequency range be extended by the use of some correction factor? What useful information can be obtained from the calibrator for frequencies outside its valid calibration frequency range? Can any useful information be obtained from the calibrator if the cluster is longer than the calibrator?

There are two other factors which can affect the performance of the calibrator when measuring line-hydrophone array elements. First, line-hydrophone arrays are inside some sort of hose. This hose has some stiffness so that the acoustic pressure within the hose may not be the same as the acoustic pressure outside the hose. Also the hose has some finite length and hence standing waves may be set up within the hose since it can act as an acoustic transmission line. These resonances affect the calibration of the line-hydrophone array element as determined by the calibrator. The second factor which can affect the performance of the calibrator is the fact that the hose of a line-hydrophone array is typically filled with a fluid whose density and sound speed are considerably different than those of water. This last factor may or may not be important.

It is difficult to thread a long line-hydrophone array through the calibrator by hand. If the calibrator were split in half lengthwise and fitted with an appropriate hinge and clamp, then a section of a line hydrophone could easily be placed in the open calibrator. Air bubbles could easily be removed with the calibrator open. The calibrator would merely have to be closed and clamped shut in order to make measurements. It is recommended that this approach be considered in future designs for the calibrator.

ACKNOWLEDGMENT

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APPENDIX A: THEORY OF OPERATION OF THE LINE-ARRAY ELEMENT CALIBRATOR

The theory of operation of the line-array element calibrator is divided into two sections, the general theory and the low-frequency approximations. The low-frequency theory applies at frequencies well below the first acoustic resonance within the calibrator.

A. General theory

The calibrator shown on Fig. 1, when submerged in a water-filled trough, is essentially an open-ended, water-filled acoustic transmission line symmetrically driven near its center. Since the calibrator is symmetric about its center, only one-half of it need be considered in setting up the analysis. The mathematically important sections of the calibrator are the active piezoceramic regions of the transducers themselves, the junction regions where the various elements are interconnected (short transmission lines), and the long metal tubes (long transmission lines) including the end correction. Variables associated with each of these regions will be identified by the subscripts p, j, and t, respectively.

The active region of any of the three identified transducers consists of a radially polarized piezoceramic tube as shown on Fig. A-1. In this figure r_p is the mean radius, a_p is the inside radius, t_p is the wall thickness, and l_p is the length of the piezoceramic tube. The equivalent circuit (the relationship between the voltage, the current, the force applied at the mean radius, and the mean radial velocity) is easily obtained by standard techniques.[A1] The equivalent circuit with acoustic and mechanical circuit elements represented in the impedance analogy is shown on Fig. A-2(a) and the corresponding mathematical expressions are

$$F=(Z_m^{sc}+\phi^2 Z_e^b)V+\phi Z_e^b I \tag{A1}$$

and

$$E=\phi Z_e^b V+Z_e^b I\ , \tag{A2}$$

where F is the total radially inward force exerted on the mean radius of the piezoceramic tube, V is the inward mean radial velocity, E is the voltage applied across the thickness of the tube, I is the electrical current into the tube, Z_m^{sc} is the short circuit mechanical impedance of the tube, Z_e^b is the mechanically blocked electrical impedance of the tube, and ϕ is the turns ratio of the equivalent electromechanical transformer. The impedance and the turns ratio above are defined by

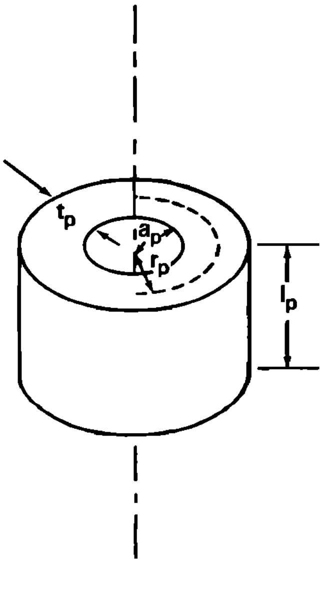

FIG. A-1. Piezoceramic ring geometry nomenclature.

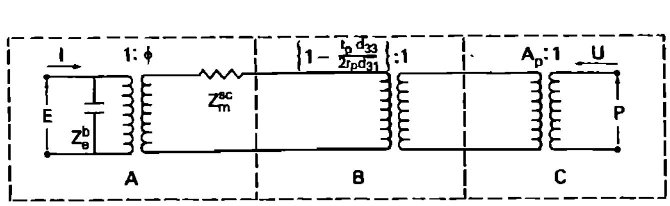

FIG. A-2. Electroacoustical equivalent circuit of calibrator transducers.

$$Z_e^b=[j\omega(2\pi r_p l_p \epsilon_{33}^T/t_p)(1-d_{31}^2 Y_{11}^E/\epsilon_{33}^T)]^{-1}\ , \tag{A3}$$

$$Z_m^{sc}=j\omega\rho_p 2\pi r_p t_p l_p+2\pi l_p t_p Y_{11}^E/j\omega r_p\ , \tag{A4}$$

and

$$\phi=2\pi l_p d_{31} Y_{11}^E\ , \tag{A5}$$

where ω is the angular frequency of operation, i.e., $\omega=2\pi f$ where f is the frequency of operation, ϵ_{33}^T is the permittivity of the piezoceramic material parallel to the direction of polarization, Y_{11}^E is the Young's modulus of the piezoceramic material at constant electric field perpendicular to the direction of polarization, d_{31} is the piezoelectric coefficient relating the strain produced normal to the direction of polarization of the electric field parallel to the direction of polarization of the piezoceramic material, ρ_p is the density of the piezoceramic material, and $j=\sqrt{-1}$. Some other useful quantities are defined as follows: The electromechanical coupling coefficient k_{31} is defined by

$$k_{31}^2=d_{31}^2 Y_{11}^E/\epsilon_{33}^T\ . \tag{A6}$$

The free capacitance of the piezoceramic tube C^f is given by

$$C^f=2\pi r_p l_p \epsilon_{33}^T/t_p\ . \tag{A7}$$

The blocked capacitance C^b is given by

$$C^b=C^f(1-k_{31}^2)\ . \tag{A8}$$

Thus, the expression for Z_e^b may be written as

$$Z_e^b=1/j\omega C^b=1/j\omega C^f(1-k_{31}^2)\ . \tag{A9}$$

Thus far, the electrical quantities of voltage and current have been related to the mechanical quantities of force and velocity at the mean radius of the piezoceramic tube, but what is needed is the relationship of electrical quantities to mechanical quantities on the inside surface of the piezoceramic tube. The relationship between the mean radial velocity V and the inside surface velocity V_i has been previously determined[A2] for a piezoceramic tube at low frequencies. This relationship is applicable to the present situation and is given by

$$V_i=(1-t_p d_{33}/2r_p d_{31})V\ . \tag{A10}$$

Since velocity is the mechanical analogue of current, a voltage transformer whose turns ratio is $(1-t_p d_{33}/2r_p d_{31}):1$ is needed in the equivalent circuit to relate mechanical quantities at the mean radius of the piezoceramic tube to mechanical quantities at the inside surface of the piezoceramic tube. This additional transformer is shown in Fig. A-2(b). Ultimately the calibrator will be used to determine the pressure sensitivity of a hydrophone, so a voltage transformer whose turns ratio is $A_p:1$, where A_p is the inside surface area of the piezoceramic tube, has been included in the equivalent circuit as shown in Fig. A-2(c). The three transformers can be combined into a single $\psi:1$ transformer as shown in Fig. A-3 where ψ is given by

$$\psi=(A_p/k_{31}^2)(g_{31}-g_{33}t_p/2r_p)\ . \tag{A11}$$

In this expression

$$g_{31}=d_{31}/\epsilon_{33}^T \tag{A12}$$

and

$$g_{33}=d_{33}/\epsilon_{33}^T\ . \tag{A13}$$

At this point the low-frequency open-circuit voltage output of the piezoceramic tube due to a unit internal pressure M_h can easily be determined and is given by

$$M_h=-k_{31}^2\psi=a_p(g_{33}t_p/2r_p-g_{31}) \tag{A14}$$

which is the same as the expression one would obtain from the theory of Langevan[A3] by making the appropriate modifications to his theory. If an electrical cable is connected to the active element the cable capacitance should be added in parallel with Z_e^b of Fig. A-3. Thus cable effects may be taken into account.

In order to include the acoustic transmission line within the piezoceramic tube it is necessary to assume that the volume velocity is injected at the center of the piezoceramic tube and enters two parallel transmission lines whose lengths are each one-half the length of a piezoceramic tube. It is also necessary to assume plane-wave propagation within the transmission lines and pressure uniformity over the entire inside surface of the piezoceramic radiator. The complete equivalent circuit is assembled by adding appropriate transmission lines and boundary conditions. All the transmissions lines are derived in the same manner so only the transmission line corresponding to the water-filled metal tubes will be presented. The characteristic acoustic impedance Z_{t0} of the acoustic transmission line within the long metal tubes is given by[A4]

$$Z_{t0}=\rho_0 c_t/S_t\ , \tag{A15}$$

where ρ_0 is the density of the water within the tube, S_t is the inside cross-sectional area of the tube, and c_t is the velocity of propagation for acoustic plane waves within the metal tube. The speed of sound c_t for a water filled tube with a vacuum outside the tube can be calculated from the Korteweg equation[A5]

$$c_0/c_t=[1+2\rho_0 c_0^2 a_t/Yt_t(1-5t_t/6a_t)]^{1/2}\ , \tag{A16}$$

where c_0 is the free-field sound speed in water, a_t is the inside radius of the metal tube, t_t is the wall thickness of the metal tube, and Y is the Young's modulus of

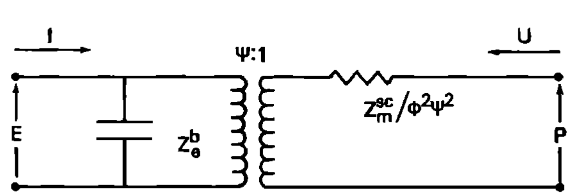

FIG. A-3. Simplified equivalent circuit of calibrator transducers.

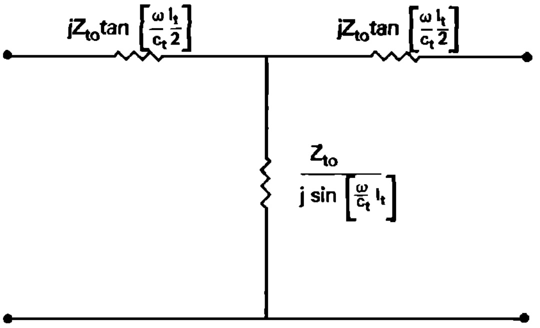

FIG. A-4. Equivalent circuit for acoustic transmission lines.

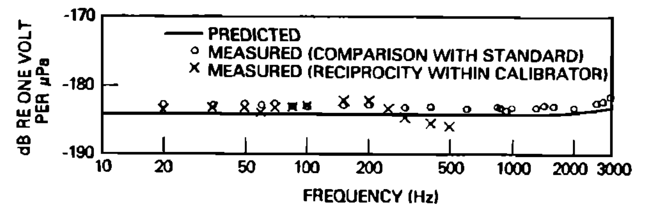

FIG. A-5. Sensitivity of monitor transducer (open circuit output voltage for a unit internal acoustic pressure) as measured within the calibrator.

the tube material. Junger[A6] has shown that for $\omega a_t/c_t < 1$ and $t_t/a_t \geq \frac{1}{16}$, the speed of sound is the same for a tube submerged in water or surrounded by vacuum. The calibrator is designed so that these conditions hold. The equivalent T circuit for the lossless propagation of acoustic waves in the water-filled metal tubes is shown on Fig. A-4 where l_t is the length of the tubes. The boundary condition at the center of the calibrator is that the velocity is zero. The boundary condition at the open ends of the long metal tubes is that there is an additional cylindrical mass of water at the ends. The effective cylinder has the same diameter as the inside diameter of the long metal tubes, and a length of 0.6 times the inside radius of the long metal tubes.[A4] This is taken into account in the analysis by increasing the length of the long metal tubes by $0.6 r_t$.

The equivalent circuit describing the operation of the calibrator is too complicated to permit hand computations. However, it is a simple matter to evaluate the expressions contained in the equivalent circuit with a small computer. The physical dimensions and the Young's modulus for the various elements within the calibrator are listed in Table A-1. The inside surfaces of the piezoceramic surfaces were coated with polyurethane. However, the acoustic properties of the thin layer of polyurethane are sufficiently similar to that of water, so that the piezoceramic tubes may be considered to be completely water filled. The material parameters of the piezoceramic material are $\rho = 7600$ kg/m^3, $d_{31} = -1.25\times10^{-10}$ m/V, $g_{31} = -1.06\times10^{-2}$ V m/N, $g_{33} = 2.53\times10^{-2}$ V m/N, and $\epsilon_{33}^T = -1.15\times10^{-8}$ F/m. The cable capacitance for each cable is 6500 pF. The fluid within the acoustic transmission line is water and its properties are well known. The calculated and measured open-circuit voltages of the central or monitor transducer for a unit pressure are shown on Fig. A-5. In operation the two outer or driving transducers produce the sound within the calibrator and these two transducers are electrically connected in parallel. The calculated and measured pressures at the center of the calibrator for a 1-amp input to the driving transducers are shown on Fig. A-6. The calculated and measured pressures at the center of the calibrator for a 1-V input to the driving transducers are shown on Fig. A-7. On Figs. A-6 and A-7 one can see the rapid and extreme pressure fluctuations as the frequency of operation of the calibrator is varied. These pressure fluctuations are undesirable for two reasons. First, the peaks in pressure amplify any harmonics present within the system whose frequency lies near a peak, thus making calibration difficult. Second, calibration at a frequency near a null in pressure response is difficult because there is insufficient pressure within the calibrator at that frequency. For these reasons it was decided to insert sonic resistors into the walls of the metal tubes to reduce the standing waves within the calibrator. If the acoustic resistance per unit length added to the metal tubes is equal to the characteristic acoustic impedance of the metal tubes then the pressure fluctuations will be reduced to an acceptable level, a useable level of sound pressure will be sustained within the calibrator throughout the operating frequency range of the calibrator, and the low-frequency characteristics of the calibrator will be unchanged. The sonic resistors should be evenly spaced along the metal tubes and should be spaced by only a small fraction of a wavelength in order to appear to be continuous to the acoustic wave. Ten sonic resistors were inserted into the wall of each metal tube. The value of the resistance for the sonic resistors was determined by calculating the response of the calibrator for various values of resistance, and choosing the value of resistance which yields the most desirable response. In this manner a value of 1.0×10^9 mks Ω was chosen

TABLE A-I. The physical dimensions and Young's modulus for the various components of the calibrator.

	Ceramic elements	Junctions between ceramic elements	Junctions between ceramic elements and metal tubes	Metal tubes including end connection
Inside radius (m)	0.0572	0.0508	0.0508	0.0511
Wall thickness (m)	0.0064	0.0203	0.0203	0.0060
Length (m)	0.0889	0.0413	0.0206	0.9562
Young's modulus (N/m^2)	8.20×10^{10}	1.93×10^{11}	1.93×10^{11}	1.93×10^{11}

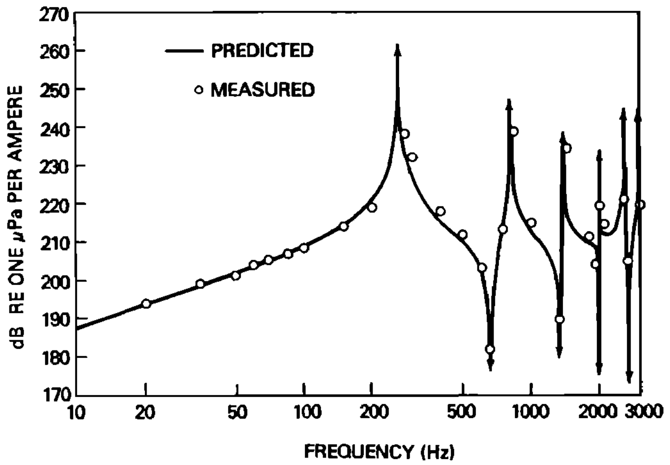

FIG. A-6. Acoustic pressure at the center of the calibrator with 1 amp into the sound-producing transducers.

for the sonic resistors. The calculated and measured pressures at the center of the calibrator with the sonic resistors in place for a one ampere input to the driving transducers are shown on Fig. A-8. The calculated and measured pressures at the center of the calibrator with the sonic resistors in place for a 1-V input to the driving transducers are shown in Fig. A-9. These last two figures show a significant reduction in standing waves within the calibrator while retaining a useful level of sound pressure throughout the operating frequency of the calibrator.

B. Low-frequency characteristics of the calibrator

For this analysis it will be convenient to electrically drive either the monitor transducer or the sound-producing transducers. The low-frequency equivalent circuit for one-half the calibrator is shown on Fig. A-10. When driving the monitor transducer, C^f and C^b should be replaced by $\frac{1}{2}C^f$ and $\frac{1}{2}C^b$ to account for the fact that only one-half of this transducer operates on each side of the calibrator. The quantity m_i is the effective mass of the water moved by the transducer under consideration, where a subscript 1 refers to a sound-producing transducer and a subscript 2 refers to the monitor transducer. If all sections of the calibrator had the same inside diameter the effective mass would be the mass of the water actually moved by the transducer. By examining the low-frequency equivalent circuit of the various transmission lines in question, one can show that the recipe for determining the effective mass is to divide the mass of the water within each section of the calibrator by the square of the inside cross-sectional area of that section, sum these quantities, and multiply the result by the square of the inside cross-sectional area of the piezoceramic tubes. For a sound-producing transducer the effective mass m_i is the effective mass of the water from the center of the sound-producing transducer all the way out to and including the end correction of the long metal tube. For a monitor transducer the effective mass m_2 includes one-quarter the mass of water within the monitor transducer plus all the water out to and including the end correction of the long metal tube. For the calibrator in this paper $m_2 = 14.24$ kg, whereas the actual mass of the water is only 9.47 kg. The sound-producing transducers are connected in parallel. The hypothetical halves of the monitor are physically in parallel. The sound pressure per volt produced within the calibrator is equal to that predicted by the equivalent circuit for one-half the calibrator. The sound pressure per ampere produced within the calibrator is equal to one-half that predicted

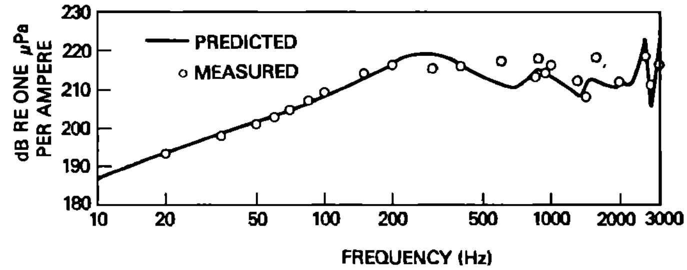

FIG. A-8. Acoustic pressure at the center of the calibrator with 1 amp into the sound-producing transducers and with sonic resistors inserted in the long metal tubes.

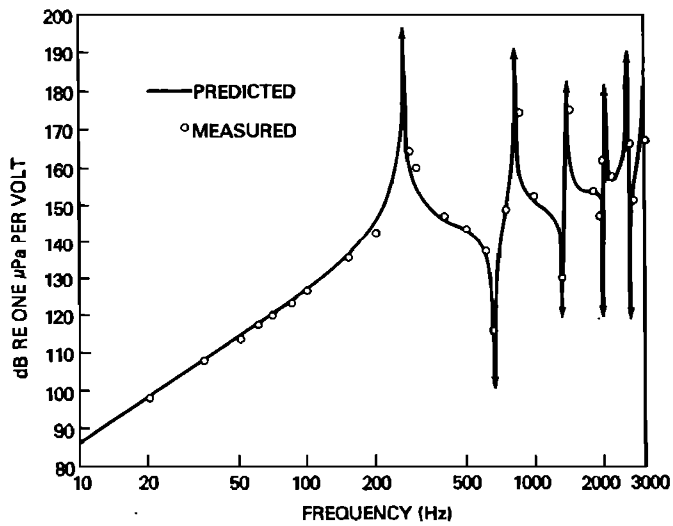

FIG. A-7. Acoustic pressure at the center of the calibrator with 1V applied to the sound-producing transducers.

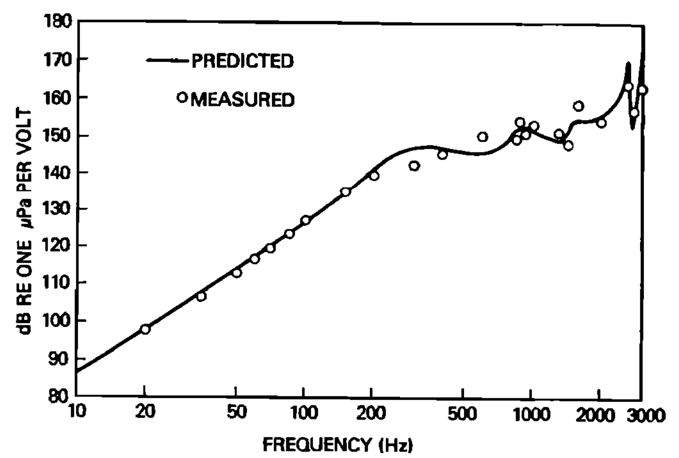

FIG. A-9. Acoustic pressure at the center of the calibrator with 1 V applied to the sound producing transducers and with sonic resistors inserted in the long metal tubes.

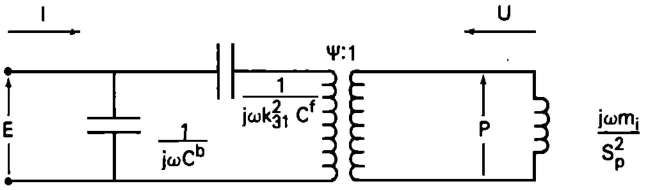

FIG. A-10. Low-frequency equivalent circuit for one-half the calibrator.

by the equivalent circuit for one-half the calibrator. The quantity P on the equivalent circuit is the pressure produced at the center of the transducer which is actually driven. When driving the sound-producing transducer the sound pressure at the monitor transducer is equal to the pressure at the sound-producing transducer. When driving the monitor transducer the sound pressure at the center of the other transducers is reduced by the ratio m_1/m_2. As in the definitions of the masses let the subscript 1 refer to the location at the center of either of the sound producing transducer and let the subscript 2 refer to the location at the center of the monitor transducer. For doubly subscripted variables let the first subscript refer to the location of the electrical drive and the second subscript refer to the location at which acoustic pressure is to be determined. Consider the two sound-producing transducers connected in parallel to be a single transducer identified by the subscript 1. Then the voltage sensitivity $(M_h)_i$ (the magnitude of the open-circuit voltage produced by a unit pressure) of the transducers is given by

$$(M_h)_1 = (M_h)_2 = |k_{31}^2 \psi| \ . \tag{A17}$$

This follows directly from Fig. A-10. Also from Fig. A-10 and the considerations above, the current responses $(S_I)_{ij}$ (the magnitude of the sound pressure produced by a unit current) are given by

$$(S_I)_{11} = (S_I)_{12} = (M_h)_1 \omega m_1/2S_p^2 \ , \tag{A18}$$

$$(S_I)_{22} = (M_h)_2 \omega m_2/2S_p^2 \ , \tag{A19}$$

and

$$(S_I)_{21} = (M_h)_2 \omega m_1/2S_p^2 \ . \tag{A20}$$

Similarly, the voltage responses $(S_E)_{ij}$ (the magnitude of the sound pressure produced by a unit voltage) are given by

$$(S_E)_{11} = (S_E)_{12} = (M_h)_1 \omega^2 m_1 C^f/S_p^2 \ , \tag{A21}$$

$$(S_E)_{22} = (M_h)_2 \omega^2 m_2 C^f/2S_p^2 \ , \tag{A22}$$

and

$$(S_E)_{21} = (M_h)_2 \omega^2 m_1 C^f/2S_p^2 \ . \tag{A23}$$

The reason for retaining the identities of $(M_h)_1$ and $(M_h)_2$ is that $(M_h)_2$ may change in time. It is possible to do absolute hydrophone calibrations within the calibrator. Define the reciprocity parameter J by

$$J = (M_h)_2/(S_I)_{22} = 2S_p^2/\omega m_2 \tag{A24}$$

then J depends only on geometrical parameters, the density of water, and the frequency of operation. This reciprocity parameter is then an open-tube reciprocity parameter and is the magnitude of the acoustic transfer admittance between the source at location 2 and the centrally located hydrophone. Note that reciprocity parameters defined by $(S_I)_{12}/(M_h)_1$ and $(S_I)_{21}/(M_h)_2$ are identical, as they must be if the systems are reciprocal.[A7] For comparison, the reciprocity parameter for a small closed chamber is ωC, where C is the volume compliance of the chamber–medium–hydrophone combination.[A7] For the calibrator in this paper the reciprocity parameter is $2.362\times 10^{-6}/f$ where f is the frequency of operation. Let H be any hydrophone located at position 2 (the center of the calibrator). Let E_{ij} be the open-circuit voltage produced by hydrophone i due to the action of sound pressure generated at location j. Drive the transducers at location 1 and obtain immediately

$$E_{12}/E_{1H} = (M_h)_2/(M_h)_H \tag{A25}$$

because the monitor hydrophone and the unknown hydrophone are sensing the same pressure. This equation is the basis for comparison calibrations within the calibrator. Combining Eq. (A25) and (A24) yields

$$(M_h)_H = (M_h)_2(E_{1H}/E_{12}) = (S_I)_{22} J (E_{1H}/E_{12}) \ . \tag{A26}$$

If the monitor transducer is driven we have

$$E_{2H} = (M_h)_H (S_I)_{22} I_2 \tag{A27}$$

because the pressure sensed by hydrophone H is $(S_I)_{22}I_2$. Combining Eqs. (A27), (A26), and (A24) yields

$$(M_h)_H = (JE_{2H}E_{1H}/E_{12}I_2)^{1/2} \tag{A28}$$

and

$$(M_h)_2 = (JE_{2H}E_{12}/E_{1H}I_2)^{1/2} \ , \tag{A29}$$

where I_2 is the current into the monitor transducer.

In this manner it is possible to perform low-frequency absolute calibrations of any pressure hydrophone within the calibrator. In particular it is possible to calibrate the monitor hydrophone. The addition of sonic resistors increases the range over which the formula for J is approximately valid. For the calibrator constructed as set forth in this paper absolute calibrations can be performed up to 100 Hz. Reciprocity calibrations for the monitor hydrophone and for a probe hydrophone are shown on Figs. A-5 and A-6, respectively.

APPENDIX B: SONIC RESISTORS

The configuration for the sonic resistors to be used with the calibrator is shown on Fig. B-1. Acoustic pressure within the calibrator acts through the rubber diaphragm and the silicone fluid, and causes the magnesium cylinder to move. The small gap between the magnesium cylinder and the sonic-resistor housing contains a viscous silicone fluid. The shearing action between the magnesium cylinder and the housing causes energy to be dissipated in the silicone fluid. The sonic resistor is effective as long as its resistance is greater than the total mass reactance of the fluid within the gap, the magnesium cylinder, and the effective mass of any fluid carried along by the axial motion of the magnesium cylinder. The gap between the magnesium cylinder and the housing is small so that the problem can be set up in rectangular coordinates as shown in

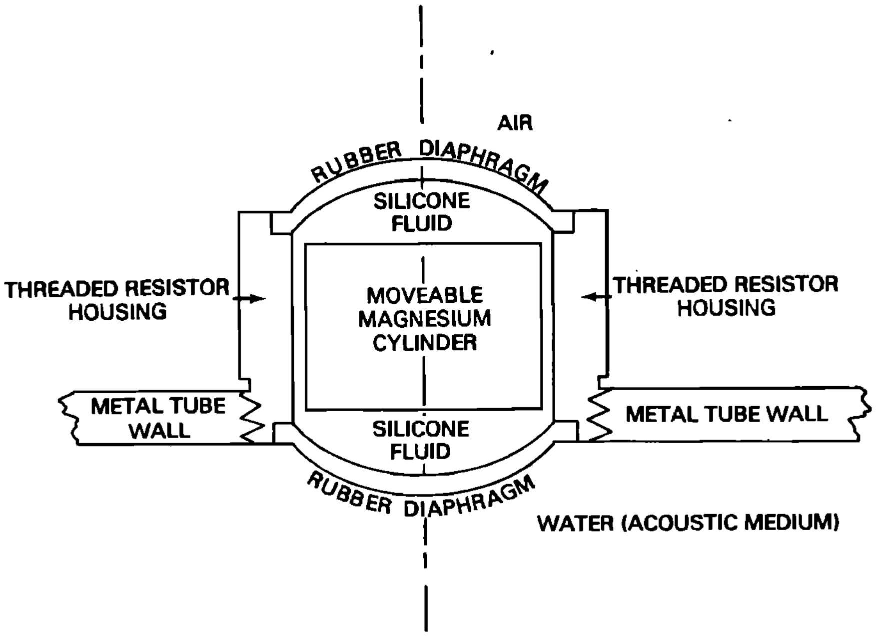

FIG. B-1. Cross section of a sonic resistor.

Fig. B-2. At $x=0$ there is a flat plate parallel to the $Y-Z$ plane moving with velocity $\dot{\xi}(0)=\dot{\xi}_0 e^{j\omega t}$ in the z direction, and at $x=\Delta r$ the velocity in the z direction is $\dot{\xi}(\Delta r)=0$, where Δr is the gap between the magnesium cylinder and the resistor housing. The plates have a height h and a width $2\pi r$ where h is the height of the magnesium cylinder and r is the radius of the magnesium cylinder. The gap between the two plates is filled with silicone fluid and the velocity in the z direction of any plane of fluid parallel to the $Y-Z$ plane is the quantity $\dot{\xi}(x)$. The differential equation describing the motion of the fluid is

$$\frac{\partial \dot{\xi}}{\partial t}=\frac{\mu}{\rho}\frac{\partial^2 \dot{\xi}}{\partial x^2}, \qquad \text{(B1)}$$

where μ is the absolute viscosity of the silicone fluid, and ρ is the density of the silicone fluid. Assume a solution of the form

$$\dot{\xi}(x)=C\,e^{j(\omega t-kx)}. \qquad \text{(B2)}$$

Substitution of Eq. (B2) into Eq. (B1) yields two allowed values for k, namely

$$k=\pm(\omega\rho/2\mu)^{1/2}(1-j)\equiv\pm\beta(1-j) \qquad \text{(B3)}$$

which defines the quantity β. Application of the boundary conditions at $x=0$ and $x=\Delta r$ yields

$$\dot{\xi}(x)=\frac{\dot{\xi}_0 e^{j\omega t}\sinh[\beta(1+j)(\Delta r-x)]}{\sinh[\beta(1+j)\Delta r]}. \qquad \text{(B4)}$$

The definition of the mechanical impedance of the moving plate is

$$F=Z_m\dot{\xi}\big|_{x=0}, \qquad \text{(B5)}$$

where F is the force on the plate and Z_m is the mechanical impedance of the plate. The force on the plate is

$$F=-\mu S\frac{\partial\dot{\xi}}{\partial x}\bigg|_{x=0}, \qquad \text{(B6)}$$

where S is the surface area of the plate, i.e.,

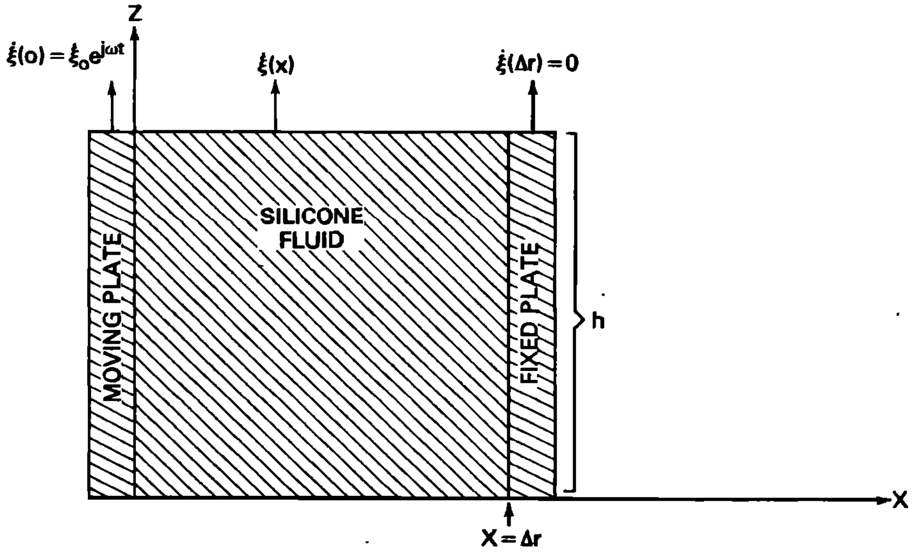

FIG. B-2. Coordinate system for determining acoustic properties of sonic resistors.

$$S=2\pi rh\ . \tag{B7}$$

The mechanical impedance of the sonic resistor is

$$Z_m=2\pi rh\mu\beta(1+j)\coth[\beta\Delta r(1+j)]\ . \tag{B8}$$

To obtain the acoustical impedance divide Z_m by the square of the cross-sectional area A of the magnesium cylinder, i.e., divide by $(\pi r^2)^2$. For $\beta\Delta r$ small, the approximate expression for the acoustical impedance Z_a of the sonic resistors is

$$Z_a\simeq 2\mu h/\pi r^3\Delta r+j(2\omega\rho h\Delta r/3\pi r^3)\ , \tag{B9}$$

or even more simply

$$Z_a\simeq \mu S/A^2\Delta r+j\omega(\rho V/3A^2)\ , \tag{B10}$$

where V is the volume of the silicone fluid within the gap and A is the area of the magnesium cylinder exposed to the acoustic waves. For $r=0.0080$ m (0.316 in.), $\Delta r=0.0001$ m (0.004 in.), $h=0.0075$ m (0.297 in.), and $\mu=12.0$ N sec/m^2, the resistive portion of the mechanical impedance is 1.06×10^9 mks Ω. A number of sonic resistors were constructed and the measured reduction in the standing wave pattern within the calibrator indicated that the loss was too great. A silicone fluid with a viscosity of 4.8 N sec/m^2 was used to replace the 12 N sec/m^2 fluid within the sonic resistors. The performance of the calibrator with this less viscous silicone fluid was much closer to that predicted for a sonic resistor with an acoustic resistance of 1.06×10^9 mks Ω. The roughness of the surfaces containing the silicone fluid is the probable cause of the excessive loss.[B1]

[a)]Present address: Underwater Sound Reference Division, Orlando, FL 32806.

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