Spin relaxation of two-dimensional electrons in noncentrosymmetric semiconductors

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An effective mechanism of spin relaxation of electrons associated with the spin–orbit splitting of the conduction band is active in centrosymmetric semiconductors.[1] In crystals with the zinc-blende structure the splitting is proportional to the cube of the electron momentum. In the case of III-V semiconductors, for which spin relaxation has been investigated in detail experimentally,[2] this mechanism is frequently the dominant one.

We shall report a study of spin relaxation of two-dimensional electrons in GaAs-type semiconductors due to this mechanism. This is of interest in view of experimental investigations of the magnetoresistance of thin films[3,4] and optical orientation of electrons in quantum-well heterostructures.[5]

The specific feature of the two-dimensional case is that the projection q of the electron momentum along the normal to the film (layer) plane is much greater than the two-dimensional momentum k in this plane: $q \sim \hbar/a \gg k$, where a is the film thickness. Averaging of the spin–orbit splitting along the motion of an electron in the direction perpendicular to the film has the effect that the splitting of a two-dimensional size-quantization band is proportional to q^2k. Therefore, in contrast to the three-dimensional case, the splitting is a linear function of the momentum k. For a given electron energy ε the splitting in the two-dimensional case is greater than in three dimensions and the difference is a factor of $\hbar^2/(ma^2\varepsilon \gg 1$, where m is the effective mass (the energy of two-dimensional electrons is measured from the lowest size-quantization level).

This splitting of the conduction band can be described as the result of action on the electron spin of an effective magnetic field of intensity and direction governed by the magnitude and direction of the momentum. Spin precession around this field in the intervals between collisions gives rise to spin relaxation. In the case of frequent collisons the rate of relaxation is proportional to the square of the effective field. It follows from the above that in the two-dimensional case the effective magnetic field is considerably greater than in the three-dimensional case and the spin relaxation rate increases by a factor $(\hbar^2/ma\varepsilon)^2$. Here $\varepsilon \sim T$ or $\varepsilon \sim \varepsilon_F$ applies to nondegenerate or degenerate electrons, respectively (T is the temperature in energy units and ε_F is the Fermi energy).

We shall also show that in the two-dimensional case the various components of the electron spin relax at different rates and the nature of such relaxation depends on the orientation of the film relative to the crystallographic axes. This spin relaxation anisotropy appears also in the three-dimensional case when a crystal is subjected to uniaxial deformation.[6]

The Hamiltonian describing an electron in the conduction band of GaAs-type semiconductors is

$$H = \frac{p^2}{2m} + \frac{\hbar}{2}(\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}) \tag{1}$$

where

$$\Omega = \frac{\alpha\varkappa}{\hbar m^{3/2}(2E_g)^{1/2}}, \tag{2}$$

$$\varkappa_x = p_x(p_y^2 - p_z^2), \quad \varkappa_y = p_y(p_z^2 - p_x^2), \quad \varkappa_z = p_z(p_x^2 - p_y^2). \tag{3}$$

Here, m is the effective mass; **p** is the electron momentum; $\boldsymbol{\sigma}$ are the Pauli matrices; E_g is the band gap; α is a numerical coefficient governing the spin splitting of the conduction band (in the case of GaAs, we have $\alpha = 0.07$).[2] In Eq. (3), the quantities p_x, p_y, and p_z are the projections of the momentum along the principal axes of a crystal.

Going over to the two-dimensional case, we shall write down the Hamiltonian in the form

$$H = \frac{\mathbf{k}^2}{2m} + \frac{\hbar}{2}(\boldsymbol{\sigma}\cdot\boldsymbol{\omega}) \tag{4}$$

where **k** is a two-dimensional momentum in the film plane. The smallness of the spin–orbit term in the Hamiltonian (1) makes it possible to obtain an expression for $\boldsymbol{\omega}$ if we average the value of $\boldsymbol{\Omega}$ along the motion of an electron in a direction perpendicular to the film: $\boldsymbol{\omega} = \langle\boldsymbol{\Omega}\rangle$.. We shall do this by writing down $\mathbf{p} = \mathbf{q} + \mathbf{k}$, where $\mathbf{q} = \mathbf{n}\times(\mathbf{p}\cdot\mathbf{n})$, $\mathbf{k}\cdot\mathbf{n} = 0$, and **n** is a unit vector along the normal to the film plane. Obviously, we have

$$\langle q_i\rangle = 0, \quad \langle q_i q_j\rangle = \langle q^2\rangle n_i n_j, \quad \langle q_i q_j q_l\rangle = 0. \tag{5}$$

The quantity $\langle q^2\rangle$ is governed by the shape of the potential well which confines the motion of an electron to a direction perpendicular to the film. In the case of a deep rectangular well of width a, we have $\langle q^2\rangle = (\pi\hbar/a)^2$. It follows from Eqs. (3) and (5) that

$$\langle\varkappa_x\rangle = \langle q^2\rangle\left[2n_x(n_y k_y - n_z k_z) + (n_y^2 - n_z^2)k_x\right]. \tag{6}$$

The quantities $\langle\varkappa_y\rangle$ and $\langle\varkappa_z\rangle$ are obtained from Eq. (6) by cyclic transposition of the indices. Equation (6) is simplified by dropping small terms of the order of k^3, since in the case of two-dimensional electrons we have $\langle q^2\rangle \gg k^2$.

It therefore follows that the effective magnetic field acting on an electron spin (proportional in the two-dimensional case to the vector $\boldsymbol{\omega} \sim \langle\boldsymbol{\varkappa}\rangle$) is linear in respect of the two-dimensional momentum **k** and depends strongly on the orientation of the film relative to the axes of a crystal.

An equation describing spin relaxation can be derived in a manner fully analogous to that used in the three-dimensional case.[1] This equation is

$$dS_i/dt = -\sum_j \Gamma_{ij}S_j, \tag{7}$$

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where S is the average spin of an electron and the relaxation tensor is described by the formula

$$\Gamma_{ij}=\frac{\int\limits_0^\infty d\varepsilon\,(\partial F/\partial\varepsilon)\,\tau(\varepsilon)\left(\overline{\omega^2}\delta_{ij}-\overline{\omega_i\omega_j}\right)}{\int\limits_0^\infty d\varepsilon\,(\partial F/\partial\varepsilon)}. \tag{8}$$

Here $F(\varepsilon)$ is the distribution function of the electron energies ε; $\tau(\varepsilon)$ is the momentum relaxation time; the bar in the above formula denotes averaging over the direction of the two-dimensional momentum **k**. Such averaging can be carried out using the relationship

$$\overline{k_ik_j}=(\delta_{ij}-n_in_j)\,k^2/2. \tag{9}$$

In the derivation of Eqs. (7) and (8) it is assumed (as in the three-dimensional case) that $\omega\tau \ll 1$ and the energy relaxation time is much longer than the momentum relaxation time, but much shorter than the spin relaxation time. Moreover, in the case when the electrons are degenerate, their average spin is assumed to be small.

The formula (8) differs from the corresponding formula for the three-dimensional case because of the replacement of $\boldsymbol{\Omega}$ and $\boldsymbol{\omega}$. It is important to note that the tensor Γ_{ij} no longer reduces to a scalar, in contrast to the three-dimensional case, where $\overline{\Omega_i\Omega_j}=\delta_{ij}\Omega/3$.

Using Eqs. (6), (8), and (9), we obtain

$$\Gamma_{ij}=\tau_s^{-1}(\varepsilon_{ij}\operatorname{Tr}\hat{B}-B_{ij}), \tag{10}$$

where

$$\frac{1}{\tau_s}=\frac{a^2\langle q^2\rangle^2}{2\hbar^2m^2E_g}\,\frac{\int\limits_0^\infty d\varepsilon\,\varepsilon\tau(\varepsilon)\,\partial F/\partial\varepsilon}{\int\limits_0^\infty d\varepsilon\,\partial F/\partial\varepsilon}. \tag{11}$$

A symmetric tensor $\hat{B}$ is governed by the orientation of the two-dimensional film or layer relative to the principal axes of a crystal. In a coordinate system linked to these axes, the components of the tensor $\hat{B}$ become

$$B_{xx}=(n_y^2-n_z^2)^2+4n_x^2(n_y^2+n_z^2)-9n_x^2(n_y^2-n_z^2), \tag{12a}$$

$$B_{xy}=-2n_xn_y(n_x^2+n_y^2)-9n_xn_y(n_y^2-n_z^2)(n_z^2-n_x^2). \tag{12b}$$

The remaining components of $\hat{B}$ can be obtained by cyclic transposition of the indices x, y, and z.

In the case of nondegenerate electrons, Eq. (11) becomes

$$\frac{1}{\tau_s}=\frac{a^2\langle q^2\rangle^2\tau T}{2\hbar^2m^2E_g}, \tag{13}$$

where τ is the average relaxation time governing the mobility $\mu=e\tau/m$. In the degenerate case, Eq. (13) should be modified by replacing the temperature T with the Fermi energy ε_F.

Equations (7), (10), (12), and (13) represent the solution of the problem formulated above for an arbitrary orientation of a two-dimensional film or layer. We shall now consider the most interesting special cases.

a) $\langle 001\rangle$ orientation. The projections of the vector of the normal along the crystal axes are $n_x=n_y=0$ and $n_z=1$. It then follows from Eqs. (10) and (12) that the nonzero components of the tensor $\hat{\Gamma}$ are $\Gamma_{xx}=\Gamma_{yy}=\Gamma_{zz}/2=\tau_s^{-1}$. Therefore, the relaxation rate of the spin components parallel to the film or layer is governed by the time $\tau_\parallel=\tau_s$. The rate of relaxation perpendicular to the film is twice as fast: $\tau_\perp=\tau_s/2$. This difference between the relaxation rates is due to the fact that, for a given orientation of the film, we can see from Eq. (6) that $\langle\varkappa_z\rangle=0$, i.e., the effective magnetic field varying randomly with time remains parallel to the film.

b) $\langle 111\rangle$ orientation. In this case we have $n_x=n_y=n_z=3^{-1/2}$, $\Gamma_{xx}=\Gamma_{yy}=\Gamma_{zz}=16/(9\tau_s)$, and $\Gamma_{xy}=\Gamma_{xz}=\Gamma_{yz}=4/(9\tau_s)$. Reducing the tensor $\hat{\Gamma}$ to the principal axes, we obtain $\tau_\parallel=2\tau_\perp=3\tau_s/4$. As in the preceding case, the effective field is parallel to the film.

c) $\langle 110\rangle$ orientation. We now have $n_x=n_y=2^{-1/2}$ $n_z=0$, $\Gamma_{xx}=\Gamma_{yy}=\Gamma_{zz}/2=-\Gamma_{xy}=(8\tau_s)^{-1}$, and $\Gamma_{xz}=\Gamma_{yz}=0$. Hence, we find that $\tau_\parallel=4\tau_s$ and $\tau_\perp=\infty$. Therefore, the spin component perpendicular to the film does not relax. This, at first sight, surprising result is explained by the fact that – as can be seen from Eq. (6) – the effective magnetic field is always parallel to the vector of the normal to the two-dimensional film. When the electron momentum changes because of collisions, only the intensity and sign of this field are affected. In fact, the slow relaxation of the perpendicular component of the spin does occur. We can obtain the finite value of the time $\tau_\perp$ if we include the small terms of the order of k^3 dropped from Eq. (6). When such terms are included, it is found that $\tau_\perp/\tau_\parallel\sim(\hbar^2/ma^2\varepsilon)^2\gg 1$.

If the potential well in which size quantization takes place is not symmetric, then the Hamiltonian (4) should be supplemented by an additional term[7] of the type $V=\beta\boldsymbol{\sigma}[\mathbf{k}\times\mathbf{n}]$. Splitting of the conduction band, which is due to this term, also results in spin relaxation. If the dominant role is played by the interaction V, the tensor $\hat{\Gamma}$ should be of the same form as in the case of the $\langle 001\rangle$ orientation considered above, but Eqs. (11) and (13) should be modified by the substitution

$$\frac{a^2\langle q^2\rangle^2}{2\hbar^2m^2E_g}\rightarrow\frac{4m\beta^2}{\hbar^2}. \tag{14}$$

The spin splitting for two-dimensional electrons in a GaAs–GaAlAs heterostructure was investigated experimentally by Stein, von Klitzing, and Weimann.[8] An estimate obtained in Ref. 9 indicates that the spin–orbit interaction described by the Hamiltonian (4), but without the correction V, predicts splitting of the order of that found experimentally. Therefore, it is possible that in the case of n-type GaAs the role of the interaction V is small even in the case of a strongly asymmetric potential well of the kind found in the experiments of Stein et al.[8]

We shall conclude by noting that a distinguishing feature of the mechanism discussed here is a strong dependence of the nature of spin relaxation on the orientation of a two-dimensional film or layer relative to the crystal axes.

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Influence of trapping levels on the recombination of nonequilibrium carriers in silicon

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Trapping levels, which in contrast to recombination centers, trap for a time carriers of just one sign, can have a considerable influence on photoconductivity relaxation.[1] As a rule, the presence of trapping levels is manifested by a strong dependence of the kinetics of recombination processes on continuous illumination which partly fills these levels and, therefore, reduces their influence. A nondestructive contactless microwave method,[2] based on determination of the form of relaxation curves from the change in the microwave conductivity, makes it possible to study the kinetics of recombination processes in which trapping levels are manifested most fully. In the interpretation of the microwave relaxometry data we need to calculate the change in the density of nonequilibrium carriers due to the presence of traps. We shall report a numerical solution to the relevant equations and compare the results with the experimental curves obtained for test samples. Our samples were silicon wafers annealed in an argon atmosphere at 1150°C. This treatment was selected because, firstly, annealing most probably alters only the bulk characteristics and, secondly, the nature of the recombination curves now differs greatly from the characteristic curves of the original silicon.

The original samples were polished wafers of silicon of the KDB-10 grade; they were 380 μ thick and their surfaces were subjected to the usual treatments employed in microelectronic technology (after these treatments of the surface recombination velocity was of the order of $3 \cdot 10^3$ cm/sec on the working side and $9 \cdot 10^3$ cm/sec on the reverse side[2]). A control sample 1 was not subjected to any additional treatment, whereas samples 2 and 3 were annealed for 10 and 30 min, respectively. Preliminary measurements indicated that the surface recombination velocities and the characteristics of the bulk recombination centers were not affected significantly by annealing for these durations. Generation of nonequilibrium carriers occurred approximately homogeneously across the thickness of a wafer under the influence of an array of infrared light-emitting diodes generating pulses of 0.5 μsec duration (the wavelength was of the order of 0.9 μ); an analog signal representing the photoconductivity decay was stored and recorded with a boxcar integrator.

The photoconductivity relaxation curve of sample 1 was exponential with a characteristic decay time of 3.5 μsec, which corresponded – after allowance for the diffusion and recombination processes on the surface[2] – to the bulk lifetime of ~20 μsec. The experimental kinetic curves for samples 2 and 3 are shown in Fig. 1. Each of them was close to a sum of two exponential functions with very different characteristic times and weights, and the weight of an exponential function with the longer decay time decreased under the influence of continuous illumination (Fig. 1b).

The system of levels was proposed in Ref. 1 for the transition to and thermal release from trapping levels located close to the top of the band gap and also the dropping of an electron to the valence band via a deep re-

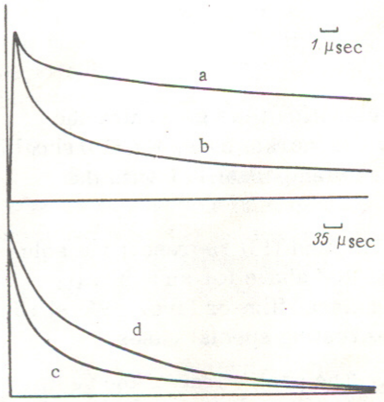

FIG. 1. Relaxation curves of the nonequilibrium conductivity of sample 2 (a, c) and 3 (d), and also of sample 2 in the presence of continuous illumination (b).