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ON THE ANALYTICAL DETERMINATION OF SOLAR CELL FILL FACTOR AND EFFICIENCY

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Summary

A comparison of several analytical, approximate expressions proposed by various authors to calculate the fill factor and the efficiency of a solar cell is presented. The comparison is referred to the conventional single-exponential solar cell model and it has been made by using a normalized nomenclature. The accuracy of the expressions has been examined as a function of the concentration factor and it is shown that the usefulness of the approximations decreases as the concentration factor increases. However, the loss in accuracy is different for each expression and, consequently, some of them can still be used for high concentration factors.

1. Introduction

For theoretical purposes it is interesting to have some means of predicting the performance of a solar cell, supposing its internal parameters are known (theoretically or experimentally). The main application of such theoretical expressions is to compare the expected performance of different devices under several operating conditions, thus allowing design criteria and research trends to be obtained.

The mathematical calculation of the performance parameters (fill factor and efficiency) of a solar cell has been treated by a number of researchers, who have suggested different expressions for determining them. Most of these expressions are based on the assumption that the single-exponential Shockley equation describes the current–voltage characteristic of the device.

It is well known that the single-exponential model is approximate and does not fit exactly the real behaviour of any photovoltaic device. However, it is still accurate enough to describe the performance of many current solar cells (such as conventional low injection silicon solar cells and GaAlAs/GaAs solar cells), particularly when medium and high sunlight concentration factors are concerned.

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When using the above-mentioned model, the exact values of the fill factor and efficiency cannot be calculated by means of explicit expressions as functions of the device internal parameters (photogenerated and dark saturation currents, parasitic resistances and the diode ideality factor). Although these values can be easily and exactly determined using conventional numerical procedures [1 - 3], a number of approximate analytical expressions have been proposed to obtain them in an explicit way. The main purpose of such approximations is to simplify further the mathematical treatment and to show directly the influence of the internal parameters on the performance of the solar cell. However, these two objectives can be reached only when the analytical expressions have a reasonable degree of accuracy.

The aim of this paper is to review some approximate expressions that have been proposed by several researchers [4 - 9] as substitutes for the exact calculation of the fill factor and efficiency of single-exponential modelled solar cells, thus allowing a comparison between them to be obtained, particularly for concentrator devices.

2. Mathematical expressions to calculate the fill factor and efficiency of a solar cell

As stated in the previous section, the solar cell model considered in the present work is the conventional single-exponential model (called the absolute solar cell model in Table 1) summarized in Table 1. A discussion of the assumptions concerning the solar cell model can be found in ref. 1 and in the references cited therein.

For comparison purposes, a set of non-dimensional parameters was used in the present work, so that the absolute solar cell model can be converted directly to the normalized model shown in Table 1.

When using this normalized nomenclature, the expressions proposed by several researchers can be rewritten as indicated in Table 2. The exact expression has been derived from previous publications [1, 8] and it can be shown that it is in agreement with those obtained from the exact expressions suggested by Pulfrey [2] and Govil and Agarwal [3]. The remaining expressions in Table 2 have been obtained by applying the normalized treatment of Table 1 to the corresponding equations suggested by the above researchers without additional changes.

As can be seen in Table 2, the expressions allow the fill factor and, consequently, the efficiency to be determined in a manner that differs from one researcher to another. However, in all cases, the fill factor can be expressed as a function of the concentration factor (v_{oc} in the normalized treatment) and the non-dimensional parameter v_o. Thus, v_o can be considered as a quality factor of the solar cell under dark conditions. v_o is greater the higher the quality of the solar cell; practical values of v_o lie between 20 (for poor quality silicon solar cells) and 40 (for conventional GaAs concentrator solar

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TABLE 1

Absolute and normalized solar cell models used in this work

Absolute solar cell model

$$J = XJ_{\ell 1} - J_o\left(\exp\frac{V_j}{mV_t} - 1\right)$$

$$\approx XJ_{\ell 1} - J_o \exp\frac{V_j}{mV_t}$$

$$V_j = V + Jr_s$$

$$V_t = (kT)/q$$

$$V_{oc} = mV_t \ln\frac{XJ_{\ell 1}}{J_o}$$

$$\mathrm{FF} = \frac{J_m V_m}{J_{sc} V_{oc}} = \frac{J_m V_m}{XJ_{\ell 1} V_{oc}}$$

$$\eta = \frac{J_m V_m}{XP_1} = \frac{J_{\ell 1} V_{oc} \mathrm{FF}}{P_1}$$

Non-dimensional parameters

$$j = J/(XJ_{\ell 1})$$

$$v = V/(mV_t);\ \ v_j = V_j/(mV_t)$$

$$v_o = \ln\frac{mV_t}{r_s J_o}$$

$$v_{oc} = V_{oc}/(mV_t) = \ln\frac{XJ_{\ell 1}}{J_o}$$

$$\eta_j = \frac{mV_t J_{\ell 1}}{P_1}$$

$$v_r = \exp(v_{oc} - v_o) = \frac{r_s X J_{\ell 1}}{mV_t}$$

Normalized solar cell model

$$j = 1 - \exp(v_j - v_{oc})$$

$$v_j = v + jv_r$$

$$\mathrm{FF} = (j_m v_m)/v_{oc}$$

$$\eta = \eta_j v_{oc} \mathrm{FF} = \eta_j j_m v_m$$

TABLE 2

Mathematical expressions to calculate the fill factor of a solar cell

Author	*Expressions*
Exact	$v_{jm} = 2v_r - 1 - 2\exp(v_{jm} - v_o) + \exp(v_{oc} - v_{jm})$
	$j_m = 1 - \exp(v_{jm} - v_o) \qquad v_m = v_{jm} - v_r j_m$
	$FF = j_m v_m / v_{oc}$
Araújo	$v_{jm} = v_{oc} - b\ln(a) \qquad a = v_{oc} - 2v_r + 1 \qquad b = a/(a+1)$
	$j_m = 1 - \exp(v_{jm} - v_o) \qquad v_m = v_{jm} - v_r j_m$
	$FF = j_m v_m / v_{oc}$
Singal	$j_m = 1 - c - 2c^3 d \qquad c = (v_{oc} + 1 - \ln v_{oc})^{-1} \qquad d = v_r(v_{oc} - \ln v_{oc})$
	$v_m = v_{oc} + \ln(1 + j_m) - j_m cd/(1 - c)$
	$FF = (j_m v_m)/v_{oc}$
Warashina (1)	$v_m = v_{oc} - \dfrac{v_{oc}+1}{v_{oc}+2}\ln\{(1+v_r)(1+v_{oc})\}$
	$FF = \dfrac{v_m}{v_{oc}}\left\{\dfrac{1 - \exp(v_m - v_{oc})}{1 + \exp(v_m - v_o)}\right\}$
Warashina (2)	$v_m = v_{oc} - v_r - \ln(1 + v_{oc})$
	$FF = v_m\{1 - \exp(v_m - v_{oc})\}/v_{oc}$
Green	$(FF)_o = \dfrac{v_{oc} - \ln(0.72 + v_{oc})}{1 + v_{oc}}$
	$FF = (FF)_o\left(1 - 1.1\dfrac{v_r}{v_{oc}}\right) + \dfrac{1}{5.4}\left(\dfrac{v_r}{v_{oc}}\right)^2$
Ruiz	$(FF)_o = \dfrac{v_{oc} - \ln(1 + v_{oc})}{1 + v_{oc}}$
	$FF = (FF)_o\left(1 - \dfrac{v_r}{v_{oc} + v_r/v_{oc}}\right) = (FF)_o\left\{1 - \dfrac{v_r}{v_{oc}} + \dfrac{1}{v_{oc} + v_r/v_{oc}}\left(\dfrac{v_r}{v_{oc}}\right)^2\right\}$

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cells). Similarly, the non-dimensional parameter η_j, that directly converts the normalized into the absolute efficiency, is representative of the device quality under illumination.

The main purpose of using the normalized nomenclature is to simplify the mathematical treatment and to reduce the number of practical, particular cases to a small number of general cases. To do this, the following considerations must be taken into account.

(a) It should be noted that the normalized and the absolute fill factors are identical, so that the FF–v_{oc} characteristic can be interpreted as the FF–ln X characteristic of a particular solar cell at a fixed temperature.

(b) As shown in ref. 1, the concentration factor X_M at which the maximum efficiency of the solar cell occurs is given, in a normalized manner and with great accuracy, by the expression $v_{ocM} = v_o$. In absolute terms, this relationship implies that

$$X_M = \frac{mV_t}{r_s J_{\ell 1}}$$

(c) The concentration factor X_M at which the maximum efficiency of the solar cell occurs is not the same as the concentration factor at which the maximum value of the fill factor occurs.

(d) Slight increases in v_{oc} beyond the maximum efficiency point are equivalent to large increases in X, due to the logarithmic dependence of v_{oc} on X. For instance, for a solar cell with $J_{\ell 1}$ = 35 mA cm^{-2}, $J_o = 10^{-13}$ A cm^{-2}, r_s = 24.3 mΩ cm^2 and mV_t = 26 mV (*i.e.* $v_o = 30$), the v_{oc} values 27, 30 and 33 are equivalent to X values of 1.5, 30.5 and 613 respectively. This relationship between v_{oc} and X explains the limits of variation considered in this work.

3. Discussion

The comparison was made by calculating the exact value of the fill factor and then comparing it with the values obtained from each analytical approximation. Values of v_o between 20 and 40 and values of v_{oc} between $v_o - 3$ and $v_o + 3$ were considered. The main results are summarized in Fig. 1, in which the number of exact decimal digits in the value of the fill factor is depicted as a function of the concentration factor for each approximation. The exact value of the fill factor was obtained from the exact expressions in Table 2 and depends on the numerical procedure used to solve the transcendent equation that gives the value of v_{jm}. In this work the exact value was calculated with an error less than 10^{-10} (see Appendix A). The accuracy of the efficiency is obviously the same due to the direct relationship between the two performance parameters.

From this study, the following results were obtained.

(a) The accuracy of each approximation is greater the greater the value of v_o (for the sake of simplicity, only the worst of all the cases considered in this work, $v_o = 20$, is shown). In other words, the accuracy of each approximation is better for the best quality solar cells.

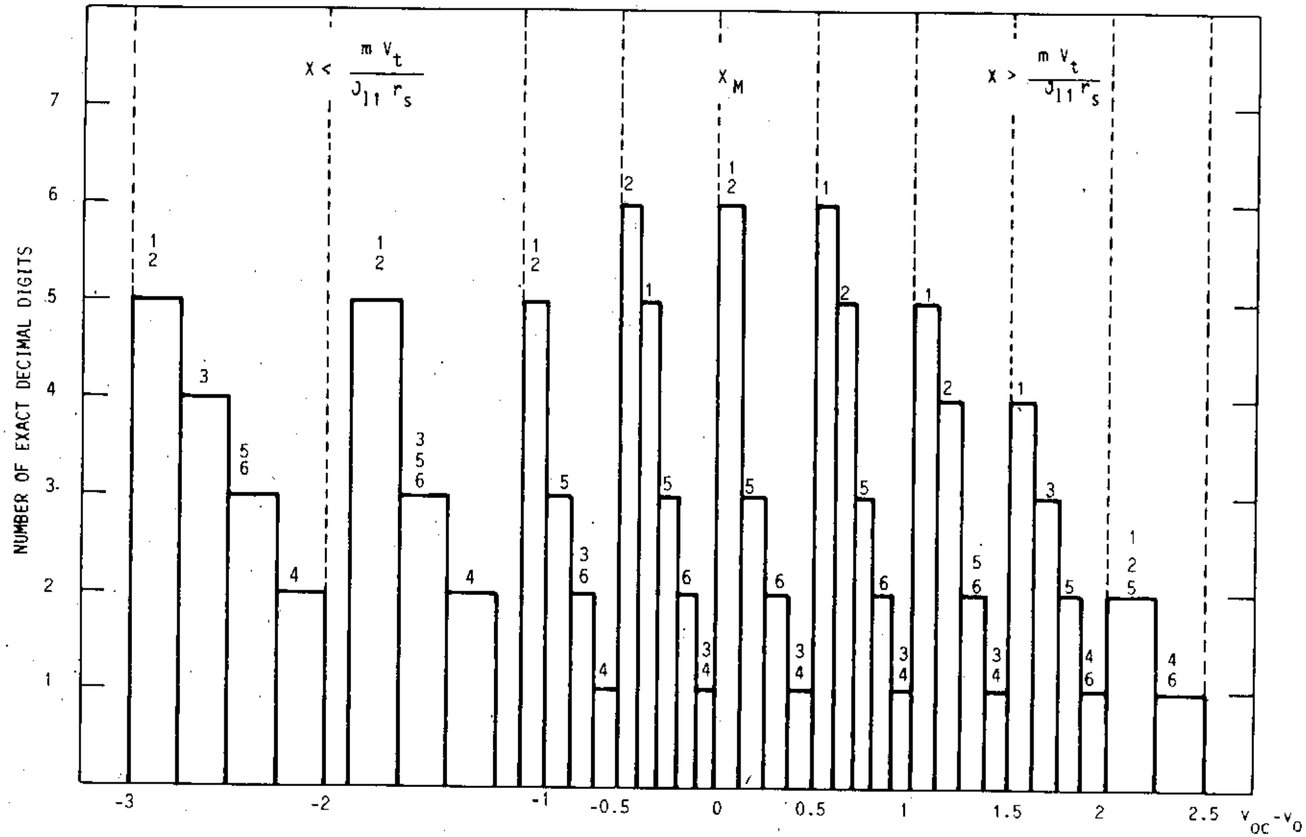

Fig. 1. Number of exact decimal digits in the fill factor as a function of the concentration factor for several analytical approximations: 1, Araújo and Sánchez; 2, Singal; 3, Warashina and Ushirokawa (1); 4, Warashina and Ushirokawa (2); 5, Green; 6, Ruiz.

(b) The dependence of the accuracy on the v_{oc} value is different for each approximation. In fact, every approximation is exact, *i.e.* the approximate value obtained by using it equals the exact value, for a particular set of values for v_o and v_{oc}. However, it may be observed that, in a general way, the accuracy of the expressions suggested by Green, Ruiz and Warashina and Ushirokawa decreases as the concentration factor increases whereas the expressions suggested by Singal and Araújo and Sánchez are more accurate around the maximum efficiency point. This can also be deduced from Fig. 1, where the fill factor decimal digit in which error occurs has been shown for each approximation in the worst case ($v_o = 20$) considered in this work.

(c) The accuracy of every approximation is poor for high values of v_{oc}. It can be deduced from Fig. 1 that, for X values lying in the very high concentration range, none of them is useful for predicting the fill factor and efficiency of a solar cell. However, it should be noted that the lack of usefulness occurs for v_{oc} values where the practical interest of the device, from an energy conversion viewpoint, is very small.

(d) The approximations suggested by Green and Ruiz have the advantage of showing explicitly the influence of the series resistance on the solar cell efficiency when the concentration factor increases, since they give the solar cell fill factor as a function of the fill factor that the device would exhibit if the parasitic resistance were zero. Green's approximation can be modified slightly to take into account the influence of the shunt resistance, but this possibility is not significant for concentrator solar cells, since the influence of the shunt resistance on the device performance decreases dramatically as the concentration factor increases. Furthermore, Green's approximation can be "reduced" to the approximation proposed by Ruiz when $v_r/v_{oc} \ll 1$, which is the case for many concentrator solar cells now used in most practical operation conditions.

4. Conclusions

From this work the following conclusions may be drawn.

(a) In the low concentration range (about 1 sun), all the approximations considered can be used for mathematical purposes. Although the approximations suggested by Green, Ruiz and Warashina and Ushirokawa are clearly less accurate than those proposed by Singal and Araújo and Sánchez, the practical difference between the two sets of expressions is not significant.

(b) In the medium and high concentration range, only the expressions suggested by Singal and Araújo and Sánchez are mathematically accurate (the accuracy of Green's approximation is comparable with that of the abovementioned researchers in the very high concentration range, but in this range none of them have an acceptable degree of accuracy). However, the second expression seems to be the most useful since (1) it maintains a large degree of accuracy over the whole range of practical v_{oc} values; (2) it is less accurate than that of Singal in the low concentration range, but the practical difference

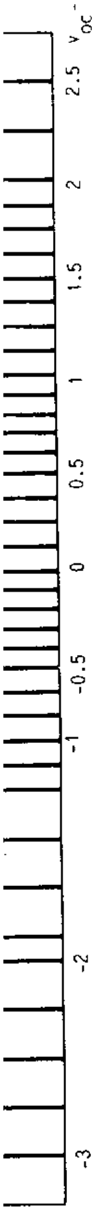

Fig. 1. Number of exact decimal digits in the fill factor as a function of the concentration factor for several analytical approximations: 1, Araújo and Sánchez; 2, Singal; 3, Warashina and Ushirokawa (1); 4, Warashina and Ushirokawa (2); 5, Green; 6, Ruiz.

between them is negligible; (3) it is more accurate than that of Singal by an order of magnitude for a concentration factor higher than that at maximum efficiency; (4) its mathematical expression is simpler than that of Singal.

(c) Green's and Ruiz's approximations are not interesting for mathematical purposes, except in the low concentration range, since they are clearly less accurate than those of Singal and Araújo and Sánchez. However, as has been discussed in the previous section, they are very useful for illustrating, in a qualitative way, the manner in which the series resistance of the solar cell affects the device performance as the concentration factor increases. From this viewpoint, Ruiz's approximation or Green's "reduced" approximation seem to be more useful than Green's expression included in Table 2.

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Appendix A: the determination of the exact value of the fill factor $(FF)_e$

The determination of $(FF)_e$ requires the previous calculation of v_{jm} by solving the transcendent equation

$$0 = v_{jm} - 2\exp(v_{oc} - v_o) + 1 + 2\exp(v_{jm} - v_o) - \exp(v_{oc} - v_{jm}) \qquad (1)$$

However, the exact value of v_{jm} cannot be determined. The numerical procedure allows a "quasi-exact" value v'_{jm} to be obtained, so that the following relationship is derived:

$$\Delta = v'_{jm} - 2\exp(v_{oc} - v_o) + 1 + 2\exp(v'_{jm} - v_o) - \exp(v_{oc} - v'_{jm}) \qquad (2)$$

If the exact value is expressed in the form

$$v_{jm} = v'_{jm} - \delta \qquad (3)$$

the following relationship between δ and Δ can be obtained from eqns. (1) and (2):

$$\delta = \frac{\Delta}{1 + 2\exp(v'_{jm} - v_o) + \exp(v_{oc} - v_{jm})} \qquad (4)$$

This is accurate if $|\delta|$ is small enough (less than 0.001, for instance), since in this case

$$\exp \delta \approx 1 + \delta \tag{5}$$

Consequently, the exact value

$$j_m = 1 - \exp(v_{jm} - v_{oc}) \tag{6}$$

is not determined, but a "quasi-exact" value is derived:

$$j'_m = j_m + \Delta_j = 1 - \exp(v_{oc} - v_o) \tag{7}$$

By introducing eqn. (3) in eqn. (6) and combining eqns. (6) and (7) the following equation is obtained:

$$\Delta_j = -\delta(1 - j'_m) \tag{8}$$

Similarly, the exact value

$$v_m = v_{jm} - j_m \exp(v_{oc} - v_o) \tag{9}$$

cannot be calculated, but a "quasi-exact" value

$$v'_m = v_m + \Delta_v = v'_{jm} - j'_m \exp(v_{oc} - v_o) \tag{10}$$

can be calculated, where

$$\Delta_v = \delta + \frac{\Delta_j(v'_m - v'_{jm})}{j'_m} \tag{11}$$

Thus, a "quasi-exact" value

$$(\mathrm{FF})'_e = \frac{v'_m j'_m}{v_{oc}} = (\mathrm{FF})_e + \Delta_f \tag{12}$$

of the fill factor is determined instead of the exact value

$$(\mathrm{FF})_e = \frac{v_m j_m}{v_{oc}} \tag{13}$$

and the intrinsic error in the numerical procedure is given by

$$\Delta_f = \frac{v'_m \Delta_j + j'_m \Delta_v - \Delta_j \Delta_v}{v_{oc}} \tag{14}$$

From eqns. (4), (8) and (11) the following conclusions may be drawn:

(a) $|\delta| < |\Delta|$;

(b) $|\Delta_j| < |\delta|$ and the sign of Δ_j is opposite to that of δ;

(c) Δ_v is of the same sign and order of magnitude as δ.

Consequently, there is a strong degree of compensation between Δ_v and Δ_j when determining Δ_f.

Therefore, the difference between the exact and a "quasi-exact" value of the fill factor is much lower than Δ.

In this work, the value of v'_{jm} was calculated using a pocket calculator program that generates different values of v'_{jm}. The procedure is stopped when the value of v'_{jm} is such that the corresponding value of Δ in eqn. (2) is less, in absolute value, than a previously fixed value (Δ').

The above-mentioned compensation in the value of Δ_f can be seen in Table 3, in which different values of Δ' have been considered. The differences in the "quasi-exact" values of the fill factor are observed to be smaller than the difference in v'_{jm} when Δ' is changed.

Thus, there is no appreciable difference, for the purpose of the present work, between the exact and "quasi-exact" values of the fill factor, thus confirming the conclusions about the accuracy of several analytical approximations used to calculate the fill factor. (In the calculations included in the main body of the paper, Δ was always less than 10^{-6}.)

TABLE 3

Influence of the numerical procedure on the determination of the exact value of the fill factor

v_o	v_{oc}	Δ'	Δ	δ	v'_{jm}	FF'_e
30	27	10^{-2}	-2.60×10^{-4}	-1.01×10^{-5}	23.79329814	0.843854523
30	27	10^{-4}	4.0×10^{-8}	2×10^{-9}	23.79328803	0.843854523
30	27	10^{-6}	4.0×10^{-8}	2×10^{-9}	23.79328803	0.843854523
30	30	10^{-2}	-5.38×10^{-4}	-2×10^{-5}	26.74863604	0.826294531
30	30	10^{-4}	2.20×10^{-7}	8×10^{-9}	26.74861604	0.826294530
30	30	10^{-6}	2.20×10^{-7}	8×10^{-9}	26.74861604	0.826294530
30	32	10^{-2}	-7.02×10^{-3}	-3.85×10^{-4}	29.20716986	0.653332807
30	32	10^{-4}	-3.18×10^{-5}	-1.74×10^{-6}	29.20678667	0.653332810
30	32	10^{-6}	1.10×10^{-7}	6×10^{-9}	29.20678492	0.653332810

Appendix B: nomenclature

J current density delivered by the solar cell
V voltage drop at solar cell terminals
$J_{\ell 1}$ photogenerated current density under 1 sun illumination
X concentration factor
J_{sc} short-circuit current density (in the present work, $J_{sc} \approx XJ_{\ell 1}$)
V_{oc} open-circuit voltage
P_1 incident power for 1 sun illumination
J_o dark saturation current density
m diode ideality factor
k Boltzmann's constant
q electron charge
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FF fill factor
η efficiency
r_s series area resistance

Subscript
m maximum power point of any illumination J–V curve