Optimizing waveguide array mode-locking for high-power fiber lasers

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ABSTRACT

A comprehensive theoretical treatment is given of the mode-locking dynamics produced by the intensity discrimination (saturable absorption) generated by the nonlinear mode-coupling in a waveguide array. Emphasis is placed on the mode-locking stability as a function of the critical physical parameters in the laser cavity. The theoretical characterization of the laser cavity's stability and dynamics allows for a comprehensive optimization of the laser cavity parameters towards achieving high peak-power, high-energy pulses in both the anomalous and normal dispersion regimes.

Keywords: Mode-locked lasers, waveguide arrays

1. INTRODUCTION

High-power pulsed lasers are an increasingly important technological innovation as their conjectured and envisioned applications have grown significantly over the past decade. Indeed, this promising photonic technology has a wide number of applications ranging from military devices and precision medical surgery to optical interconnection networks.[1–4] Such technologies have placed a premium on the engineering and optimization of mode-locked laser cavities that produce stable and robust high-power pulses. Thus the technological demand for novel techniques for producing and stabilizing high-power pulses has pushed mode-locked lasers to the forefront of commercially viable, nonlinear photonic devices. In this manuscript, we consider theoretically the mode-locking stability and dynamics of a laser cavity mode-locked by the nonlinear mode-coupling effect in a waveguide array.[5–9] The performance of the waveguide array mode-locking model developed is optimized so as to produce high-power pulses in both the anomalous and normal dispersion regimes. Additionally, spectral filtering techniques are presented and shown to further enhance the output peak powers and energies of the laser cavity. The stability of the mode-locked solutions are completely characterized as a function of the cavity energy and the waveguide array parameters.

In the past decade, the developments and understanding of mode-locked lasers have rendered them a standard, commercially available photonic technology. Indeed, it is well understood that some form of cavity saturable absorption or intensity discrimination is fundamental to producing stable mode-locked pulses in a passive laser cavity.[1,2,10] Such intensity discrimination can be produced by a number of methods ranging from placing a linear polarizer in a fiber ring laser,[11–14] using a coupler in a figure-eight laser to produce nonlinear interferometry,[15–18] placing a semiconductor saturable absorber in a linear-cavity configuration,[19–21] or using a combination of spectral filtering with polarization filters in a dispersion controlled cavity.[22–25] Alternatively, active mode-locking can be used to produce mode-locked pulses by directly modulating the output electromagnetic field or using an acousto-optic modulator.[27,28] In all these cases, an effective intensity discrimination is generated to stabilize and control the mode-locked pulses. A relatively new method for generating intensity discrimination in a laser cavity is due to the nonlinear mode-coupling generated in a waveguide array.[5–9] Although nonlinear mode-coupling has been proposed previously as a theoretical method for producing stable mode-locking,[29–31] the waveguide array is the only nonlinear mode-coupling device that has been experimentally verified to produce the requisite pulse shaping required for mode-locking.[32] Although theoretical models have been developed towards

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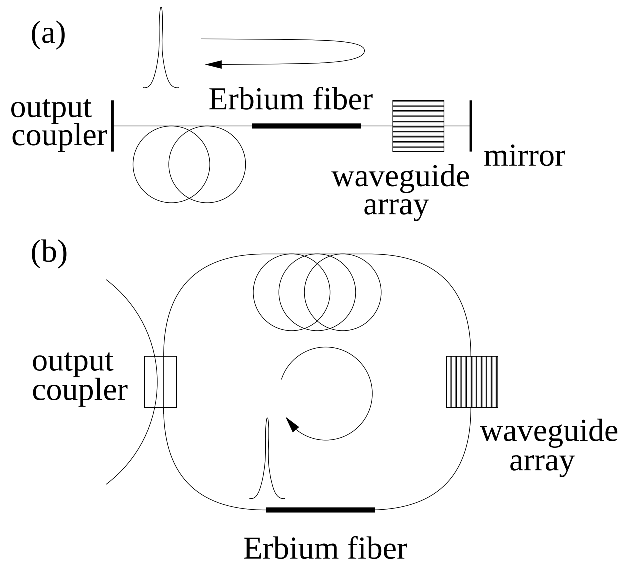


Figure 1. Two possible laser cavity configurations which include nonlinear mode-coupling from the waveguide array as the mode-locking element. The fiber coupling in and out of the waveguide array occurs at the central waveguide as illustrated. Any electromagnetic field which is propagated into the neighboring waveguides is ejected (attenuated) from the laser cavity. In addition to the basic setup, polarization controllers, isolators, and other stabilization mechanisms may be useful or required for successful operation.

understanding the mode-locking dynamics and stability of waveguide array based lasers,[5–9] a characterization of its optimal performance and ability to generate high peak-power and high-energy pulses has not previously been performed. The aim of this paper is to optimize the performance of the laser cavity and engineer the physical parameter specifications so as to produce the high peak-power, high-energy pulses that are robust and stable.

Figure 1 illustrates two possible mode-locking configurations in which the waveguide array provides the critical effect of intensity discrimination (saturable absorption). In Fig. 1(a) a linear cavity configuration is considered whereas in Fig. 1(b) a ring cavity geometry is considered. In either case, the waveguide array provides an intensity dependent pulse shaping by coupling out low intensity wings to the neighboring waveguides. This low intensity field is then ejected from the laser cavity. In contrast, high intensity portions of the pulse are retained in the central waveguide due to self-focusing. Thus high intensities are only minimally attenuated. This intensity selection mechanism generates the necessary pulse shaping for producing stable mode-locked pulse trains. Understanding the output pulse power and energy are the focus of this manuscript.

2. THE WAVEGUIDE ARRAY

Coupled mode theory for characterizing the evanescent electromagnetic field interactions between neighboring waveguides has a long and well-established history of successfully quantifying the dynamics observed in many coupled mode devices.[33,34] With the advent of nonlinear materials and high intensity laser sources, a broader range of nonlinear photonic devices has been envisioned, including those based upon waveguide arrays. The waveguide array has been proposed as a technologically feasible and promising technology[35] that has applications ranging from optical switching[35–39] to mode-locking.[5–9] In such applications, the interaction of the Kerr nonlinearity with the linear waveguide coupling allows for the manifestation of fundamentally new physical phenomena and applications. Indeed, it has been well known that nonlinear coupling leads to a nontrivial modification of the linear dynamics.[40]

The leading-order equations governing the nearest-neighbor coupling of electromagnetic energy in the waveguide array is given by[35–39]

$$i\frac{dA_n}{d\xi} + C(A_{n-1} + A_{n+1}) + \beta |A_n|^2 A_n = 0\,, \tag{1}$$

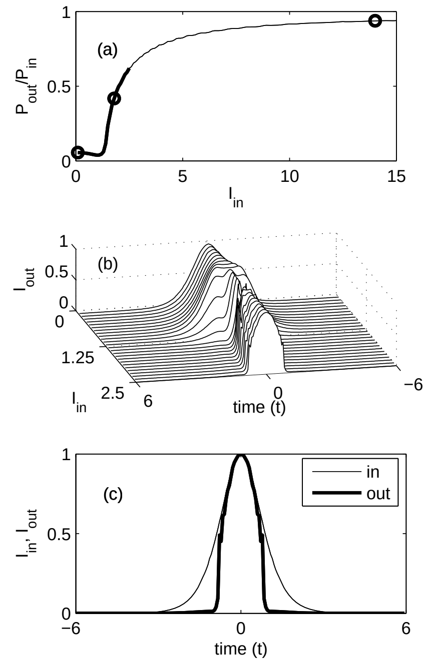


Figure 2. Performance characteristics of the waveguide array as a function of intensity. (a) Transmission (saturation fluency) curve of the waveguide array showing the dependence of the ratio of the output to input integrated power or energy (P_{out}/P_{in} where $P = \int_{-\infty}^{\infty} |A_0|dt$) in the center waveguide A_0 as a function of the input intensity I_{in}. At $I_{in} \approx 2$, a rapid change occurs in the transmission performance. (b) Pulse shaping as a function of input intensity that occurs over the bolded region of (a). As the intensity is increased, the pulse is shortened by the self-focusing effects of the waveguide array.[32] (c) Input and output pulse profiles for the final input intensity $I_{in} = 2.5$ depicted in (b). The pulse shortening is clearly demonstrated.

where A_n represents the normalized electric field amplitude in the n^{th} waveguide ($n=-N, \cdots, -1, 0, 1, \cdots, N$ and there are $2N+1$ waveguides). Using the normalizations presented previously[5,6] gives $C = 4.92$ and $\beta = 15.1$ for the physically measured values of a 6 mm long waveguide array with the linear coupling coefficient $c = 0.82$ mm^{-1} and the nonlinear self-phase modulation parameter $\gamma^* = 3.6$ m^{-1}W^{-1}.[38,39] In this waveguide array, the core-to-core spacing between the 41 waveguides is 8 μm so that the nearest neighbor coupling dominates the waveguide interactions. To more accurately capture the pulse shaping dynamics in the waveguide array, the normal chromatic dispersion should be included.[32] However, it will suffice here to simulate the leading order governing behavior with the coupled equations (1).

To demonstrate the basic pulse shaping effect in the waveguide array, numerical simulations are performed of the governing equation (1) with 41 ($N = 20$) waveguides[39] with pulses of different launch peak powers (energies). In the simulations, we fix the pulse width and investigate the input/output relationship between the launched pulse and the output of the waveguide array. Although the parameters assumed are based upon an experimentally realizable waveguide,[32,38,39] the behavior demonstrated is generic and holds for any waveguide array. Thus the results presented are nondimensional to demonstrate the universality of the phenomena.

Figure 2 demonstrates the hallmark features of the waveguide array and the ideal saturable absorption it produces for use in mode-locking. For these simulations, light was launched in the center waveguide with initial amplitude

$$A(0, t) = \sqrt{I_{in}}\operatorname{sech} t \tag{2}$$

where I_{in} measures the initial launch peak power. Figure 2(a) shows the saturation fluency curve, or transmission curve, associated with the waveguide array and its central waveguide. This clearly demonstrates the impact of low and high intensities on propagation. Specifically, lower intensities are discretely diffracted via nearest-neighbor coupling whereas the higher intensities remain spatially localized due to self-focusing. The spatial self-focusing can be understood as a consequence of Eq. (1) being a second-order accurate, finite-difference discretization of the focusing nonlinear Schrödinger equation.[35] This fundamental behavior has been extensively verified experimentally.[36–39] Thus low intensity pulses are almost completely attenuated upon propagation through the waveguide array, whereas high intensities are temporally reshaped (shortened) upon transmission.[32]

Figure 2(a) shows that, as the intensity is increased, a rapid transition happens in the transmission. Figure 2(b) demonstrates the transition from linear pulse shaping to nonlinear pulse shaping that occurs through the nonlinear transmission threshold which begins at $I_{in} \approx 2$. Specifically, the output pulse goes from simple linear attenuation at low intensities to strong pulse shaping and pulse shortening at high intensities. The bolded region of the transmission curve in Fig. 2(a) is the region of intensity depicted in Fig. 2(b). Figure 2(c) demonstrates the pulse shaping for a high peak-power pulse ($I_{in} = 2.5$). This pulse shaping process has been verified and characterized experimentally.[32] Indeed, it is the basis for using waveguide arrays for modelocking[5–9] since the low peak power tails of the higher peak power pulses are strongly attenuated due to coupling to neighboring waveguides.

3. MODE-LOCKING DYNAMICS

The evolution of electromagnetic energy in a laser cavity is composed of two different parts: the optical fiber cavity and the intensity discrimination element that is responsible for initiating mode-locking from initial white-noise. The pulse propagation in a laser cavity is governed by the interaction of chromatic dispersion, self-phase modulation, linear attenuation, and bandwidth limited gain. These effects are modelled by the equation

$$i\frac{\partial u}{\partial z}+\frac{D}{2}\frac{\partial^2 u}{\partial t^2}+\beta|u|^2u+i\gamma u-ig(z)\left(1+\tau\frac{\partial^2}{\partial t^2}\right)u=0\,, \tag{3}$$

where the saturated gain behavior[1] is given by

$$g(z)=\frac{2g_0}{1+\|u\|^2/e_0}\,. \tag{4}$$

Here $\|u\|=\int_{-\infty}^{\infty}|u|^2dt$, u represents the electric field envelope normalized by the peak field power $|E_0|^2$ and D indicates the average laser cavity dispersion. The variable t represents the physical time in the rest frame of the pulse normalized by t_0 where t_0=200 fs is the typical full-width at half-maximum of the pulse. The variable z is scaled on the dispersion length $z_0=(2\pi c)/(\lambda_0^2\bar{D})(t_0)^2$ corresponding to the average cavity dispersion. This gives the peak field power scaling to be $|E_0|^2=\lambda_0 A_{\text{eff}}/(4\pi n_2 z_0)$. Here $n_2=2.6\times10^{-16}$ cm^2/W is the nonlinear coefficient in the fiber, $A_{\text{eff}}=60$ μm^2 is the effective cross-sectional area, $\lambda_0=1.55$ μm is the free-space wavelength, c is the speed of light, and $\gamma=\Gamma z_0$ ($\Gamma=0.2$ dB/km) is the fiber loss. The bandwidth limited gain in the fiber is incorporated through the dimensionless parameters g and $\tau=(1/(t_0\Omega)^2)$. For a gain bandwidth which can vary from $\Delta\lambda$=20–40 nm, $\Omega=(2\pi c/\lambda_0^2)\Delta\lambda$ so that typically $\tau\approx0.08-0.32$. The parameter τ controls the spectral gain bandwidth of the mode-locking process, limiting the pulse width.

The intensity discrimination in the specific laser considered here is provided by the nonlinear mode-coupling in the waveguides as described in the previous section. When placed within an optical fiber cavity, the pulse shaping mechanism of the waveguide array leads to stable and robust mode-locking.[5–7] In its most simple form, the nonlinear mode-coupling is averaged into the laser cavity dynamics.[8] Numerical simulations have shown that the fundamental behavior in the laser cavity does not change when considering more than five waveguides.[8] It is interesting to note that if a three waveguide system is considered (one central and a neighboring waveguide on each side), mode-locking is not achieved. This can be explained due to the large attenuation required in the neighboring waveguides. This attenuation effectively reduces the coupling to the central waveguide which is critical for stable and robust mode-locking. Although it is not possible to consider the three waveguide model, further simplifications to the five waveguide model can be achieved by making use of the symmetric nature of the coupling and lower intensities in the neighboring waveguides.[9] The resulting approximate evolution dynamics

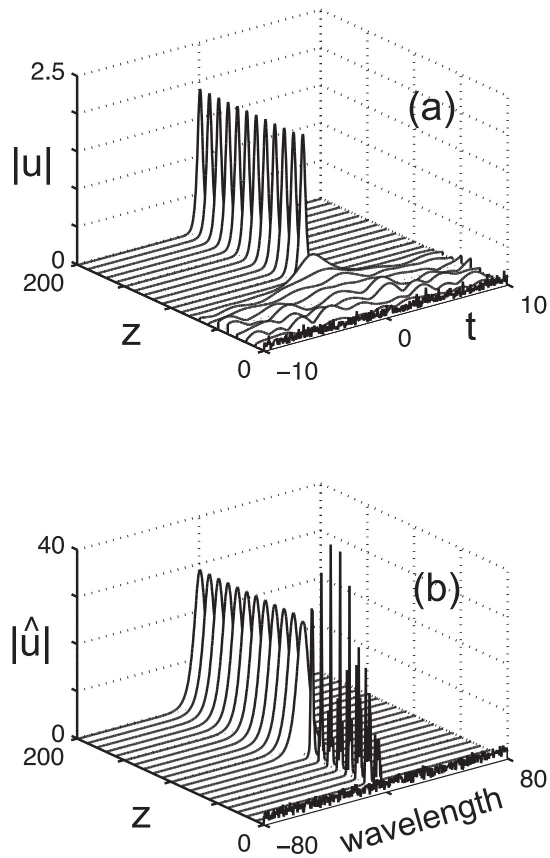


Figure 3. Typical (a) time and (b) spectral mode-locking dynamics of the waveguide array mode-locking model Eq. (5) in the anomalous dispersion regime from initial white-noise. The steady state solution is a short, nearly transform-limited pulse which acts as an attractor to the mode-locked system.

describing the waveguide array mode-locking is given by[9]

$$i\frac{\partial u}{\partial z}+\frac{D}{2}\frac{\partial^2 u}{\partial t^2}+\beta|u|^2u+Cv+i\gamma_0 u-ig(z)\left(1+\tau\frac{\partial^2}{\partial t^2}\right)u=0 \tag{5a}$$

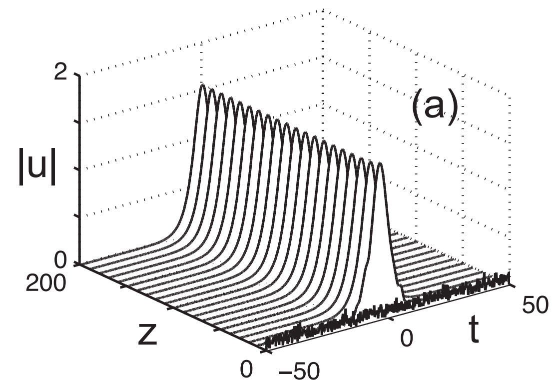
$$i\frac{\partial v}{\partial z}+C(w+u)+i\gamma_1 v=0 \tag{5b}$$

$$i\frac{\partial w}{\partial z}+Cv+i\gamma_2 w=0\,, \tag{5c}$$

where the $v(z,t)$ and $w(z,t)$ fields model the electromagnetic energy in the neighboring channels of the waveguide array. Note that the equations governing these neighboring fields are ordinary differential equations. All fiber propagation and gain effects occur in the central waveguide. It is this approximate system which will be the basis for our analytic findings. In fact, Eq. (5) provides a great deal of analytic insight due to its hyperbolic secant solutions

$$u(z,t)=\eta\,\mathrm{sech}\,\omega t^{1+iA}e^{i\theta z}, \tag{6}$$

where the solution amplitude η, width ω, chirp parameter A, and phase θ satisfy a set of nonlinear equations.[9] Further, this solution forms from any arbitrary initial condition, thus acting as a global attractor to the system. This is in contrast to the master mode-locked equation[1] for which initial conditions must be carefully prepared to observe stable mode-locking.



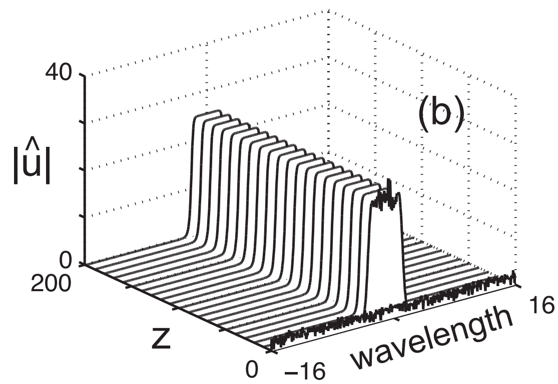


Figure 4. Typical (a) time and (b) spectral mode-locking dynamics of Eq. (5) in the normal dispersion regime from initial white-noise. The steady state solution is a broad, highly-chirped pulse which acts as an attractor to the mode-locked system.

In the anomalous dispersion regime ($D = 1 > 0$), solitonlike pulses can be formed as a result of the balance of anomalous dispersion and positive (i.e. self-focusing) nonlinearity. Typically mode-locked fiber lasers operating in the anomalous dispersion regime are limited in pulse energy by restrictions among the soliton parameters which is often referred to as the soliton area theorem.[25] However, ultra-short, nearly transform-limited output pulses are desired for many applications. This encourages exploration of possible laser cavity configurations that could potentially maximize pulse energy in the anomalous dispersion regime. Figure 3 shows the typical time- and spectral-domain mode-locking dynamics of the waveguide array model (5) in the anomalous dispersion regime. Here the equation parameters are $\beta = 8$, $C = 5$, $\gamma_0 = \gamma_1 = 0$, $\gamma_2 = 10$, $g_0 = 1.5$, and $e_0 = 1$. Stable and robust mode-locking is achieved from initial white-noise after $z \sim 100$ units. The steady state pulse solution has a short pulse duration and is nearly transform-limited, which is in agreement with experiments performed in the anomalous dispersion regime.[1,2]

Mode-locking in the normal dispersion regime ($D = -1 < 0$) relies on non-soliton processes and has been shown experimentally to have stable high-chirped, high-energy pulse solutions.[22,23] Figure 4 shows the typical time and spectral mode-locking dynamics of the waveguide array model (5) in the normal dispersion regime. Here the equation parameters are $\beta = 1$, $C = 3$, $\gamma_0 = 0$, $\gamma_1 = 1$, $\gamma_2 = 10$, $g_0 = 10$, and $e_0 = 1$. In contrast to mode-locking in the anomalous dispersion regime, the mode-locked solution is quickly formed from initial white-noise after $z \sim 10$ units. The mode-locked pulse is broad in the time domain and has the squared-off spectral profile characteristic of a highly chirped pulse ($A \gg 1$). These characteristics are in agreement with observed experimental pulse solutions in the normal dispersion regime.[20,22,23] Although these properties

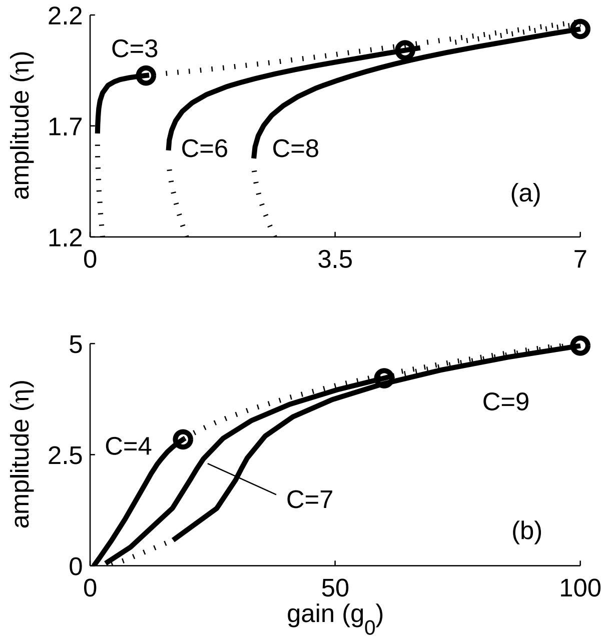


Figure 5. Bifurcation structure of the mode-locked solution (6) in the (a) anomalous and (b) normal dispersion regimes as a function of the coupling parameter C. The solid lines indicate stable solutions while the dotted lines represent the unstable solutions. For both anomalous and normal dispersion, an increase in the coupling constant leads to higher peak power pulses. The increase is $\approx$ 15% for anomalous dispersion and $\approx$ 100% for normal dispersion. The circles approximately represent the highest peak power pulses possible for a given coupling constant. The gain parameter is $g_0 = 0.8, 4.5$ and 7 for anomalous dispersion and $g_0 = 19, 60$ and 100 for normal dispersion.

make the pulse solutions impractical for photonic applications, the potential for high-energy pulses from normal dispersion mode-locked lasers has generated a great deal of interest.[25, 26, 43, 44]

4. OPTIMIZING FOR HIGH POWER

As demonstrated in the previous two sections, the waveguide array provides an ideal intensity discrimination effect that generates stable and robust mode-locking in the anomalous and normal dispersion regimes. The aim here is to try to optimize (maximize) the energy and peak power output of the laser cavity. Intuitively, one can think of simply increasing the pump energy supplied to the erbium amplifier in the laser cavity in order to increase the output peak power and energy. However, the mode-locked laser then simply undergoes a bifurcation to multi-pulse operation.[9] Thus, for high-energy pulses, it becomes imperative to understand how to pump more energy into the cavity without inducing a multi-pulsing instability.

In the subsections that follow the stability of single pulse per round trip operation in the laser cavity is investigated as a function of the physically relevant control parameters. Two specific parameters that can be easily engineered are the coupling coefficient C and the loss parameter γ_1. Varying these two parameters demonstrates how the output peak power and energy can be greatly enhanced in both the normal and anomalous cavity dispersion regimes.

In order to assess the laser performance, the stability of the mode-locked solutions must be calculated. A standard way for determining stability is to calculate the spectrum of the linearization of the governing equations (5) about the exact mode-locked solution (6).[9, 45] The spectrum is composed of two components: the radiation modes and eigenvalues. The radiation modes are determined by the asymptotic background state where $(u, v, w) = (0, 0, 0)$, whereas the eigenvalues are associated with the shape of the mode-locked solution (6). Details of the linear stability calculation and its associated spectrum are given in,[9] while an explicit representation of the associated eigenvalue problem and its spectral content is given in.[45] As in,[9] a numerical continuation method is used here in conjunction with a spectral method for determining the spectrum of the linearized operator to produce both the solution curves and their associated stability. Our interest is in simultaneously finding stable solution curves and maximizing their associated output peak power and energy as a function of the parameters C and γ_1. In the normal and anomalous dispersion regimes, stable high peak power curves can be generated by increasing the input peak power via g_0. For the anomalous dispersion regime, while the peak power increase is a marginal $\approx$20%, the energy output can be doubled. For normal dispersion, the peak power increase is four fold with an order of magnitude increase in the output energy. These solutions then undergo a Hopf bifurcation before producing multi-pulse lasing.[9]

To explore the laser cavity performance as a function of the coupling constant C, we consider the solution curves and their stability for a number of values of the coupling constant. Figure 5 shows the solution curves (η versus g_0) for both the anomalous and normal dispersion regimes as a function of the increasing coupling constant C. This figure demonstrates that an increased coupling constant allows for the possibility of increased peak power from the laser cavity. In the case of anomalous mode-locking, the peak power increase is only $\approx$15%, while for normal mode-locking the peak power is nearly doubled by increasing the linear coupling. The steady-state solution profiles verify the increased peak power associated with the increase in coupling constant C. Although the peak power is increased for the output pulse, it comes at the expense of requiring to pump the laser cavity with more gain. Although this makes intuitive sense, it should be recalled that the peak power and pulse energy levels are being increased without the transition to multi-pulse instabilities in the laser cavity.

5. CONCLUSION

In conclusion, we have provided an extensive study of the robust and stable mode-locking that can be achieved by using the nonlinear mode-coupling in a waveguide array as the intensity discrimination (saturable absorption) element in a laser cavity. Indeed, the spatial self-focusing behavior which arises from the nonlinear mode-coupling of this mode-locking element gives the ideal intensity discrimination (or saturable absorption) required for temporal pulse shaping and mode-locking. Extensive numerical simulations of the laser cavity with a waveguide array show a remarkably robust mode-locking behavior. Specifically, the cavity parameters can be modified significantly, the coupling losses can be increased, and the gain model altered, and yet the mode-locking persists for a sufficiently high value of g_0. This demonstrates, in theory, the promising technological implementation of this device in an experiment. Here, the robust behavior as a function of the physical parameters is specifically investigated towards producing high peak-power and high-energy mode-locked pulses in both the normal and anomalous dispersion regimes.

In practice, the technology and components to construct a mode-locked laser based upon a waveguide array are available.[39] An advantage of this technology is the short, nonlinear interaction region and robust intensity-discrimination (saturable absorption) provided by the waveguide array. The results here also suggest how the waveguide array spacing and waveguide array losses should be engineered so as to maximize the output intensity (peak power) and energy. Indeed, the theoretical framework established here provides solution curves and their stability as a function of the key parameters C and γ_1. These curves can be directly related to the design and optimization of laser cavities. A clear trend in anomalous mode-locking, normal mode-locking and spectrally filtered normal mode-locking is the increase in peak power and energy for the coupling coefficient C. For anomalous mode-locking this increase is only $\approx$25%, due to the soliton area theorem and the onset of multi-pulse instabilities. However, the pulse energy can be nearly doubled. For normal cavity fibers with or without filtering, the peak power increase can be four-fold with appropriate engineering, and the energy increase can be an order of magnitude. Thus the theory presented provides a critical design component for a physically realizable laser cavity based upon the waveguide array.

In the laser cavities proposed, index-matching materials, tapered couplers, polarization controllers and isolators may be useful and necessary to help further stabilize the theoretically idealized dynamics in the waveguide array model (5) presented here. Further, fiber tapering or free-space optics may be helpful to circumvent the losses incurred from coupling. Regardless, the theoretical results demonstrate that a mode-locked laser cavity operating by the nonlinear mode-coupling generated in a waveguide array is an excellent candidate for a compact, robust, cheap, and reliable high-energy pulse source based upon the union of the emerging technology of waveguide arrays with traditional fiber optical engineering.

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