

## DETERMINING CAMERA PARAMETERS FROM THE PERSPECTIVE PROJECTION OF A RECTANGLE

ROBERT M. HARALICK

Intelligent Systems Laboratory, Department of Electrical Engineering, FT-10, University of Washington, Seattle, WA 98195, U.S.A.

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**Abstract**—In this note we show how to use the 2D perspective projection of a rectangle of unknown size and position in 3D space to determine the camera look angle parameters relative to the plans of the rectangle. All equations are simple. In addition, if the size of the rectangle is known, it is possible to compute the exact 3D coordinates of the rectangle.

Perspective projection   camera calibration   Machine vision   Computer vision   Scene analysis

### 1. INTRODUCTION

Determination of surface orientation is one of the important tasks of a computer vision system. In this note we show that there is sufficient information in the 2D perspective projection of a rectangle of unknown size in 3D space to determine the camera look angle parameters. This in essence gives the relationship of surface normal of rectangle to camera viewing direction. We also show that if the size of the rectangle is given, then its exact 3D coordinates can easily be computed.

In photogrammetry it is widely known that given the coordinates of three 3D points and the corresponding positions of their perspective projection, then it is possible to compute the position of the camera as well as its look direction. A complete set of such relationships for a triangle of 3D points is given in Fischler and Bolles.<sup>(2)</sup> Certainly, the corresponding computation is possible for four points. However, if it is only known that the four points are in a rectangular configuration in a plane with unknown size for length and width of rectangle, then it is not immediately clear that the look angle is computable. The existence of the relationships derived in this note undoubtedly play a strong role in why people are able to accurately perceive the surface orientation of rectangular planar surfaces from man made objects.

The algebra used in the derivation is not particularly noteworthy. However, the resulting formulas are simple, of general use, and interesting since they seem not to appear in any known or convenient place in the literature.

### 2. THE PERSPECTIVE PROJECTION

We assume that the camera lens is the origin and that the lens views down the  $y$  axis. The image plane

is a known distance  $f$  in front of the lens and is orthogonal to the optical lens axis. The abscissa axis of the image plane is parallel to the  $x$  axis and the ordinate axis of the image plane is parallel to the  $z$  axis.

To permit the camera to be viewing into the 3D world in an arbitrary direction, we rotate the coordinate system so that in the rotated coordinate system the optic axis of the lens is the rotated  $y$  axis, the abscissa axis of the image plane is the rotated  $x$  axis and the ordinate axis of the image plane is the rotated  $z$  axis. Thus, we first counter clockwise rotate around the  $z$  axis by the pan angle  $\theta$ , then counter clockwise rotate around the  $x$  axis by the tilt angle  $\phi$ , and finally counter clockwise rotate about the  $y$  axis by the swing angle  $\xi$ . This convention as well as some of the other relationships we use here can be found in Haralick<sup>(1)</sup> and for reference purposes is shown in Fig. 1.

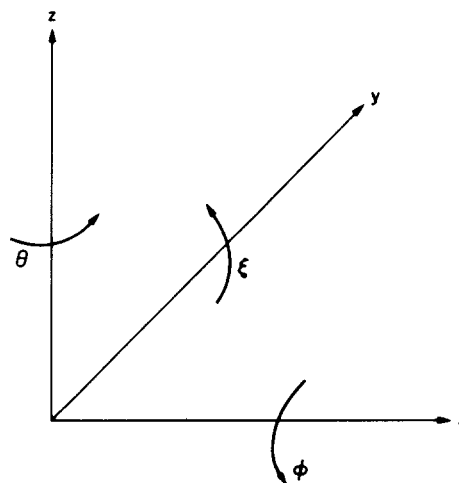


Fig. 1. Illustration of the convention for positive or counter-clockwise rotation of axes.

The perspective projection  $(x^*, z^*)$  of a 3D point  $(x, y, z)$  is given by

$$\begin{pmatrix} x^* \\ z^* \end{pmatrix} = \begin{pmatrix} \cos \xi & \sin \xi \\ -\sin \xi & \cos \xi \end{pmatrix} \begin{pmatrix} x' \\ z' \end{pmatrix} \quad (1)$$

where

$$x' = f \frac{x \cos \theta + y \sin \theta}{-x \cos \phi \sin \theta + y \cos \phi \cos \theta + z \sin \phi}$$

$$z' = f \frac{x \sin \phi \sin \theta - y \sin \phi \cos \theta + z \cos \phi}{-x \cos \phi \sin \theta + y \cos \phi \cos \theta + z \sin \phi}$$

If the perspective projection  $(x^*, z^*)$  is known then the ray of 3D points having  $(x^*, z^*)$  for its perspective projection can be determined. The ray is given by

$$\left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \middle| \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \begin{pmatrix} x' \cos \theta - f \sin \theta \cos \phi + z' \sin \theta \sin \phi \\ x' \sin \theta + f \cos \theta \cos \phi - z' \cos \theta \sin \phi \\ f \sin \phi + z' \cos \phi \end{pmatrix} \right\} \quad (2)$$

for some  $\lambda$  and where

$$\begin{pmatrix} x' \\ z' \end{pmatrix} = \begin{pmatrix} \cos \xi & -\sin \xi \\ \sin \xi & \cos \xi \end{pmatrix} \begin{pmatrix} x^* \\ z^* \end{pmatrix} \left. \right\}.$$

### 3. THE CAMERA PARAMETERS FROM THE PERSPECTIVE PROJECTION OF RECTANGLE

Suppose that a rectangle lies in the  $z = z_1$  plane and has unknown width  $W$  and length  $L$ . We assume that the corners of the rectangle are given by

$$p_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \quad p_2 = \begin{pmatrix} x_1 + W \\ y_1 \\ z_1 \end{pmatrix}$$

$$p_3 = \begin{pmatrix} x_1 \\ y_1 + L \\ z_1 \end{pmatrix} \quad p_4 = \begin{pmatrix} x_1 + W \\ y_1 + L \\ z_1 \end{pmatrix}$$

where  $y_1 > f$ , and that the corresponding perspective projection of these corners are

$$p_1^* = \begin{pmatrix} x_1^* \\ z_1^* \end{pmatrix} \quad p_2^* = \begin{pmatrix} x_2^* \\ z_2^* \end{pmatrix} \quad p_3^* = \begin{pmatrix} x_3^* \\ z_3^* \end{pmatrix} \quad p_4^* = \begin{pmatrix} x_4^* \\ z_4^* \end{pmatrix}.$$

Note that this knowledge implies that it is known that the line segment from  $(x_1^*, z_1^*)$  to  $(x_2^*, z_2^*)$  is caused by the side of the rectangle of length  $W$ . If this information is not known, the calculations must be repeated once under the assumption that the length of the side of the rectangle is  $W$  and once under the assumption that the length of the side of the rectangle is  $L$ . Our assumption about the position of the rectangle implies that the  $x$  and  $y$  axes of the coordinate system are parallel to the sides of the rectangle.

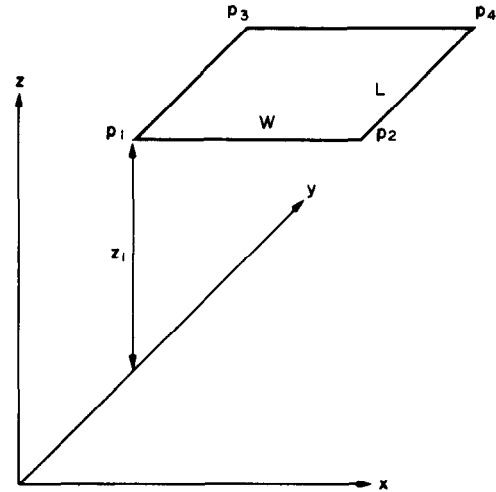


Fig. 2. Illustration of the position of the rectangle in 3D space.

Figure 2 illustrates the rectangle as it lies in the 3D world. Figure 3 illustrates the perspective projection of the rectangle.

As in equation (2) we let

$$\begin{pmatrix} x'_i \\ z'_i \end{pmatrix} = \begin{pmatrix} \cos \xi & -\sin \xi \\ \sin \xi & \cos \xi \end{pmatrix} \begin{pmatrix} x_i^* \\ z_i^* \end{pmatrix}, \quad i = 1, 2, 3, 4$$

where we understand that  $\xi$  is an unknown.

By equation (2) we must therefore have that for some  $\lambda_1, \lambda_2, \lambda_3$  and  $\lambda_4$

$$\begin{pmatrix} x'_1 \cos \theta - f \sin \theta \cos \phi + z'_1 \sin \theta \sin \phi \\ x'_1 \sin \theta + f \cos \theta \cos \phi - z'_1 \cos \theta \sin \phi \\ f \sin \phi + z'_1 \cos \phi \end{pmatrix} \lambda_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} \quad (3)$$

$$\begin{pmatrix} x'_2 \cos \theta - f \sin \theta \cos \phi + z'_2 \sin \theta \sin \phi \\ x'_2 \sin \theta + f \cos \theta \cos \phi - z'_2 \cos \theta \sin \phi \\ f \sin \phi + z'_2 \cos \phi \end{pmatrix} \lambda_2 = \begin{pmatrix} x_1 + W \\ y_1 \\ z_1 \end{pmatrix} \quad (4)$$

$$\begin{pmatrix} x'_3 \cos \theta - f \sin \theta \cos \phi + z'_3 \sin \theta \sin \phi \\ x'_3 \sin \theta + f \cos \theta \cos \phi - z'_3 \cos \theta \sin \phi \\ f \sin \phi + z'_3 \cos \phi \end{pmatrix} \lambda_3 = \begin{pmatrix} x_1 \\ y_1 + L \\ z_1 \end{pmatrix} \quad (5)$$

$$\begin{pmatrix} x'_4 \cos \theta - f \sin \theta \cos \phi + z'_4 \sin \theta \sin \phi \\ x'_4 \sin \theta + f \cos \theta \cos \phi - z'_4 \cos \theta \sin \phi \\ f \sin \phi + z'_4 \cos \phi \end{pmatrix} \lambda_4 = \begin{pmatrix} x_1 + W \\ y_1 + L \\ z_1 \end{pmatrix} \quad (6)$$

Equations (3), (4), (5) and (6) are sufficient to solve for  $\theta, \phi$  and  $\xi$  with  $\lambda_1, \lambda_2, \lambda_3, \lambda_4, x_1, y_1, z_1, W$  and

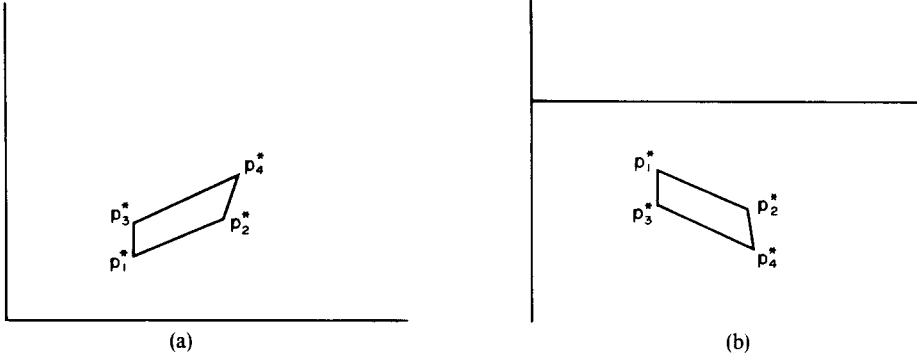


Fig. 3. (a) Illustration of the perspective projection of the rectangle when viewed from a position above the rectangle.  $\theta = 30^\circ$ ,  $\phi = 40^\circ$ ,  $\xi = 0^\circ$ . (b) Illustration of the perspective projection of the rectangle when viewed from a position below the rectangle  $\theta = 30^\circ$ ,  $\phi = 40^\circ$ ,  $\xi = 0^\circ$ .

$L$  all unknown. Notice that equations (3) and (5) and equations (4) and (6) have identical first and third components on the right-hand side. The first and third components of equations (3) and (5) can be used to establish that

$$\cos\theta\sin\phi f(x'_1 - x'_3) + \sin\theta f(z'_1 - z'_3) + \cos\theta\cos\phi(x'_1 z'_3 - x'_3 z'_1) = 0 \quad (7)$$

$$\theta = \tan^{-1} \frac{\cos\phi[(x'_1 z'_3 - x'_3 z'_1)(x'_2 - x'_4) - (x'_2 z'_4 - x'_4 z'_2)(x'_1 - x'_3)]}{f[(x'_1 - x'_3)(z'_2 - z'_4) - (x'_2 - x'_4)(z'_1 - z'_3)]} \quad (12)$$

In an identical manner, the first and third components of equations (4) and (6) can be used to establish that

$$\cos\theta\sin\phi f(x'_2 - x'_4) + \sin\theta f(z'_2 - z'_4) + \cos\theta\cos\phi(x'_2 z'_4 - x'_4 z'_2) = 0. \quad (8)$$

Multiplying (7) by  $(z'_2 - z'_4)$  and (8) by  $(z'_1 - z'_3)$  and subtracting yields

$$\cos\theta\sin\phi f[(x'_1 - x'_3)(z'_2 - z'_4) - (x'_2 - x'_4)(z'_1 - z'_3)] + \cos\theta\cos\phi[(x'_1 z'_3 - x'_3 z'_1)(z'_2 - z'_4) - (x'_2 z'_4 - x'_4 z'_2)(z'_1 - z'_3)] = 0. \quad (9)$$

Dividing out the  $\cos\theta$  from (9) and solving for  $\phi$  yields

$$\theta = \tan^{-1} \frac{(x'_2 z'_4 - x'_4 z'_2)(z'_1 - z'_3) - (x'_1 z'_3 - x'_3 z'_1)(z'_2 - z'_4)}{f[(x'_1 - x'_3)(z'_2 - z'_4) - (x'_2 - x'_4)(z'_1 - z'_3)]} \quad (10)$$

$$\phi = \tan^{-1} \frac{(x'_1 z'_2 - x'_2 z'_1)(z'_3 - z'_4) - (x'_3 z'_4 - x'_4 z'_3)(z'_1 - z'_2)}{f[(x'_3 - x'_4)(z'_1 - z'_2) - (x'_1 - x'_2)(z'_3 - z'_4)]} \quad (13)$$

The solution for  $\phi$  in equation (10) has an inherent ambiguity of  $180^\circ$ . The correct and unique value for

$$\frac{(x'_2 z'_4 - x'_4 z'_2)(z'_1 - z'_3) - (x'_1 z'_3 - x'_3 z'_1)(z'_2 - z'_4)}{f[(x'_1 - x'_3)(z'_2 - z'_4) - (x'_2 - x'_4)(z'_1 - z'_3)]} = \frac{(x'_1 z'_2 - x'_2 z'_1)(z'_3 - z'_4) - (x'_3 z'_4 - x'_4 z'_3)(z'_1 - z'_2)}{f[(x'_3 - x'_4)(z'_1 - z'_2) - (x'_1 - x'_2)(z'_3 - z'_4)]} \quad (14)$$

$\phi$  can be determined by selecting that value between  $-90^\circ$  and  $+90^\circ$  since angles outside the range from  $-90^\circ$  to  $+90^\circ$  make the camera tilt so much that the camera is looking in the hemisphere behind itself.

Multiplying (7) by  $(x'_2 - x'_4)$  and (8) by  $(x'_1 - x'_3)$  and subtracting yields

$$\sin\theta f[(z'_1 - z'_3)(x'_2 - x'_4) - (z'_2 - z'_4)(x'_1 - x'_3)] + \cos\theta\cos\phi[x'_1 z'_3 - x'_3 z'_1](x'_2 - x'_4) - (x'_2 z'_4 - x'_4 z'_2)(x'_1 - x'_3) = 0. \quad (11)$$

From (11) there results

The solution for  $\theta$  in equation (12) has an inherent ambiguity of  $180^\circ$ . The correct and unique value for  $\theta$  can be determined by selecting that value between  $-90^\circ$  and  $+90^\circ$  since angles outside the range from  $-90^\circ$  to  $+90^\circ$  make the camera pan so much that the camera is looking in the hemisphere behind itself.

From equations (10) and (12) it is apparent that once  $\xi$  is known, then  $\phi$  and  $\theta$  can be solved for. And it is the case that  $\xi$  can be solved for. Notice that equations (3) and (4) have second and third components equal and equations (5) and (6) have their second and third components equal. Thus, using similar manipulations to the ones just discussed we can determine an alternate and independent expression for  $\phi$

Equations (10) and (13) imply

Everything in equation (14) is known except  $\xi$  and we may now solve for  $\xi$ . This is an easier problem than at first sight because many of the terms are rotationally invariant. Specifically,

$$\begin{aligned}
x'_2 z'_4 - x'_4 z'_2 &= (x_2^* \cos \xi - z_2^* \sin \xi)(x_4^* \sin \xi + z_4^* \cos \xi) \\
&\quad - (x_4^* \cos \xi - z_4^* \sin \xi)(x_2^* \sin \xi + z_2^* \cos \xi) \\
&= (x_2^* z_4^* - x_4^* z_2^*) \cos^2 \xi + (x_2^* z_4^* - x_4^* z_2^*) \sin^2 \xi \\
&= x_2^* z_4^* - x_4^* z_2^*.
\end{aligned}$$

In a likewise manner the denominators are rotationally invariant. Equation (14) thus becomes

Once  $z_1$  is determined  $x_1$  and  $y_1$  can be determined from equation (2)

$$x_1 = z_1 \frac{x'_1 \cos \theta - f \sin \theta \cos \phi + z'_1 \sin \theta \sin \phi}{f \sin \phi + z'_1 \cos \phi} \quad (17)$$

$$y_1 = z_1 \frac{x'_1 \sin \theta + f \cos \theta \cos \phi - z'_1 \cos \theta \sin \phi}{f \sin \phi + z'_1 \cos \phi} \quad (18)$$

$$\begin{aligned}
&\frac{(x_2^* z_4^* - x_4^* z_2^*)[\sin \xi (x_1^* - x_3^*) + \cos \xi (z_1^* - z_3^*)] - (x_1^* z_3^* - x_3^* z_1^*)[\sin \xi (x_2^* - x_4^*) + \cos \xi (z_2^* - z_4^*)]}{f[(x_1^* - x_3^*)(z_2^* - z_4^*) - (x_2^* - x_4^*)(z_1^* - z_3^*)]} \\
&= \frac{(x_1^* z_2^* - x_2^* z_1^*)[\sin \xi (x_3^* - x_4^*) + \cos \xi (z_3^* - z_4^*)] - (x_3^* z_4^* - x_4^* z_3^*)[\sin \xi (x_1^* - x_2^*) + \cos \xi (z_1^* - z_2^*)]}{f[(x_3^* - x_4^*)(z_1^* - z_2^*) - (x_1^* - x_2^*)(z_3^* - z_4^*)]}
\end{aligned}$$

In essence, this states that a linear combination of the sine and cosine of  $\xi$  equals 0. Therefore, the swing angle  $\xi$  is given by

$$\xi = \tan^{-1} \frac{[A(z_1^* - z_3^*) - B(z_2^* - z_4^*) - C(z_1^* - z_2^*) + D(z_3^* - z_4^*)]}{[A(x_1^* - x_3^*) - B(x_2^* - x_4^*) - C(x_1^* - x_2^*) + D(x_3^* - x_4^*)]} \quad (15)$$

where

$$A = (x_2^* z_4^* - x_4^* z_2^*)/E$$

$$B = (x_1^* z_3^* - x_3^* z_1^*)/E$$

$$C = (x_3^* z_4^* - x_4^* z_3^*)/F$$

$$D = (x_1^* z_2^* - x_2^* z_1^*)/F$$

$$E = f[(x_1^* - x_3^*)(z_2^* - z_4^*) - (x_2^* - x_4^*)(z_1^* - z_3^*)]$$

$$F = f[(x_1^* - x_2^*)(z_3^* - z_4^*) - (x_3^* - x_4^*)(z_1^* - z_2^*)].$$

The solution for  $\xi$  in equation (15) has an inherent ambiguity of  $180^\circ$ . Thus if the arctangent function is taken to yield the principal value, then the second possible value for  $\xi$  is  $\xi + 180^\circ$ . Both values for  $\xi$  are legal and corresponding to each value of  $\xi$  is a value for  $\theta$  and  $\phi$  given by equations (10) and (12).

#### 4. DETERMINATION OF THE POSITION OF THE RECTANGLE GIVEN ITS SIZE

In this section, we suppose that the sides  $W$  and  $L$  of the rectangle are given. On the basis of the calculations of the previous section the camera viewing parameters  $\theta$ ,  $\phi$  and  $\xi$  can be determined and are now assumed known. Once  $\xi$  is known,  $(x', z')$  can be computed from the observed  $(x^*, z^*)$  by (2). From the second and third components of equations (3) and (5), we can solve for  $z_1$ :

#### 5. RECTANGLE IN A $y = y_1$ OR $x = x_1$ PLANE

If the rectangle is considered to lie in a  $y = y_1$  plane instead of a  $z = z_1$  plane, the same technique can be used with a minor modification. Solve the problem as if the rectangle is in the  $z = z_1$  plane. In effect, this sets up a coordinate system for the problem solution in which there is a prior rotation of  $+90^\circ$  or  $-90^\circ$  about the  $x$  axis. The choice of  $+90^\circ$  or  $-90^\circ$  taken depends on keeping the corners of the rectangle in front of the image plane. The solution yields angles  $\theta'$ ,  $\phi'$  and  $\xi'$ . From,  $\theta'$ ,  $\phi'$ , and  $\xi'$ , we can determine new angles  $\theta$ ,  $\phi$ , and  $\xi$  so that a rotation around the  $z$  axis of  $\theta$ , followed by a rotation around the  $x$  axis of  $\phi$ , followed by a rotation around the  $y$  axis of  $\xi$  gives the same effective rotation as a rotation around the  $x$  axis of  $\pm 90^\circ$ , followed by a rotation around the  $z$  axis of  $\theta'$ , followed by a rotation around the  $x$  axis of  $\phi'$ , followed by a rotation around the  $y$  axis of  $\xi'$ .

This determination of  $\theta$ ,  $\phi$ , and  $\xi$  from  $\theta'$ ,  $\phi'$ , and  $\xi'$  is simple. Choose  $\theta$ ,  $\phi$ , and  $\xi$  to make the entries of the corresponding composite rotation matrices equal. For the case where the prior  $x$  axis rotation is  $+90^\circ$ , we must have

$$z_1 = \frac{(f \sin \phi + z'_3 \cos \phi)(f \sin \phi + z'_1 \cos \phi)L}{f(x'_3 - x'_1) \sin \theta \sin \phi + (z'_1 - z'_3) f \cos \theta + (x'_3 z'_1 - x'_1 z'_3) \sin \theta \cos \phi} \quad (16)$$

$$\begin{pmatrix} \cos\xi'\cos\theta' + \sin\xi'\sin\phi'\sin\theta' & -\sin\xi'\cos\phi' & \cos\xi'\sin\phi' - \sin\xi'\sin\phi'\cos\theta' \\ -\cos\phi'\sin\theta' & -\sin\phi' & \cos\phi'\cos\theta' \\ -\sin\xi'\cos\theta' + \cos\xi'\sin\phi'\sin\theta' & -\cos\xi'\cos\phi' & -\sin\xi'\sin\theta' - \cos\xi'\sin\phi'\cos\theta' \end{pmatrix} \\
= \begin{pmatrix} \cos\xi\cos\theta + \sin\xi\sin\phi\sin\theta & \cos\xi\sin\theta - \sin\xi\sin\phi\cos\theta & \sin\xi\cos\phi \\ -\cos\phi\sin\theta & \cos\phi\cos\theta & \sin\phi \\ -\sin\xi\cos\theta + \cos\xi\sin\phi\sin\theta & -\sin\xi\sin\theta - \cos\xi\sin\phi\cos\theta & \cos\xi\cos\phi \end{pmatrix}.$$

From this there results

$$\phi = \sin^{-1}(\cos\phi'\cos\theta')$$

$$\theta = \tan^{-1}(\cos\phi'\sin\theta' / -\sin\phi')$$

$$\xi = \tan^{-1}((\cos\xi'\sin\phi' - \sin\xi'\sin\phi'\cos\theta') / (-\sin\xi'\sin\theta' - \cos\xi'\sin\phi'\cos\theta')).$$

To determine  $y_1$  we use

$$y_1 = \frac{W(x_2'\sin\theta + f\cos\theta\cos\phi - z_2'\cos\theta\sin\phi) \cdot (x_1'\sin\theta + f\cos\theta\cos\phi - z_1'\cos\theta\sin\phi)}{(x_2' - x_1')f\cos\phi + (x_1'z_2' - x_2'z_1')\sin\phi}.$$

Then from  $y_1$  we can determine  $x_1$  and  $z_1$  by

$$x_1 = y_1 \frac{x_1'\cos\theta - f\sin\theta\cos\phi + z_1'\sin\theta\sin\phi}{x_1'\sin\theta + f\cos\theta\cos\phi - z_1'\cos\theta\sin\phi}$$

$$z_1 = y_1 \frac{f\sin\phi + z_1'\cos\phi}{x_1'\sin\theta + f\cos\theta\cos\phi - z_1'\cos\theta\sin\phi}.$$

In a similar manner, if the rectangle is assumed to lie in a  $x = x_1$  plane, then by a 90° clockwise rotation around the  $y$  axis we can make it lie in the  $z = x_1$  plane in the rotated coordinate system. Determination of the viewing angles  $\theta'$ ,  $\phi'$ , and  $\xi'$  in the rotated coordinate system proceeds as given in Section 3. Equating the combined rotation matrix having the prior 90° rotation around the  $y$  axis with the rotation matrix for the desired  $\theta$ ,  $\phi$ , and  $\xi$  yields

$$\phi = \sin^{-1}(\cos\phi'\sin\theta')$$

$$\theta = \tan^{-1}(-\sin\phi' / \cos\phi'\cos\theta')$$

$$\xi = \tan^{-1}((\cos\xi'\cos\theta' - \sin\xi'\sin\phi'\sin\theta') / (\sin\xi'\cos\theta' - \cos\xi'\sin\phi'\sin\theta')).$$

To determine  $x_1$  we use

$$x_1 = \frac{W(x_1^*\cos\theta - f\sin\theta\cos\phi + z_1^*\sin\theta\sin\phi) \cdot (x_2^*\cos\theta - f\sin\theta\cos\phi + z_2^*\sin\theta\sin\phi)}{(x_1^* - x_2^*)f\cos\phi + (z_1^* - z_2^*)f\sin\theta + (x_1^*z_2^* - x_2^*z_1^*)\sin\phi}.$$

Then on the basis of equation (2) we can determine  $y_1$  and  $z_1$  from

$$y_1 = x_1 \frac{x_1^*\sin\theta + f\cos\theta\cos\phi - z_1^*\cos\theta\sin\phi}{x_1^*\cos\theta - f\sin\theta\cos\phi + z_1^*\sin\theta\sin\phi}$$

$$z_1 = x_1 \frac{f\sin\phi + z_1^*\cos\phi}{x_1^*\cos\theta - f\sin\theta\cos\phi + z_1^*\sin\theta\sin\phi}.$$

## 6. CONCLUSION

We have shown that if an observed quadrilateral is a 2D perspective projection and is known to have arisen from a rectangle situated on an unknown plane in the 3D world then the camera viewing parameters  $\theta$ ,  $\phi$  and  $\xi$  can be simply determined in closed form. If additional information about the size of the rectangle is known, then the exact position of the rectangle in the 3D world can be determined. The significance of these results is that from the observation of one rectangle, the surface orientation of the rectangle can be determined relative to the camera viewing direction. The only ambiguity in the determination is whether the camera is looking up and seeing the rectangle from below on one side or whether the camera is looking down and seeing the rectangle from above on the other side. This ambiguity is similar to the ambiguity in the perception of the Necker cube.

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**About the Author**—ROBERT M. HARALICK was born in Brooklyn, New York, on 30 September 1943. He received a B.A. degree in Mathematics from the University of Kansas in 1964, a B.S. degree in Electrical Engineering in 1966 and a M.S. degree in Electrical Engineering in 1967. In 1969, after completing his Ph.D. at the University of Kansas, he joined the faculty of the Electrical Engineering Department there where he last served as Professor from 1975 to 1978. In 1979 Dr Haralick joined the Electrical Engineering Department at Virginia Polytechnic Institute and State University where he was a Professor and Director of the Spatial Data Analysis Laboratory. From 1984 to 1986 Dr Haralick served as Vice President of Research at Machine Vision International, Ann Arbor, MI. Dr Haralick now occupies the Boeing Clairmont Egtvedt Professorship in the Department of Electrical Engineering at the University of Washington.

Professor Haralick has made a series of contributions in computer vision. In the high-level vision area, he has identified a variety of vision problems which are special cases of the consistent labeling problem. His papers on consistent labeling, arrangements, relation homomorphism, matching, and tree search translate some specific computer vision problems to the more general combinational consistent labeling problem and then discuss the theory of the look-ahead operators that speed up the tree search. This gives a framework for the control structure required in high-level vision problems. Recently, he has extended the forward-checking tree search technique to propositional logic.

In the low-level and mid-level areas, Professor Haralick has worked in image texture analysis using spatial gray tone co-occurrence texture features. These features have been used with success on biological cell images, X-ray images, satellite images, aerial images and many other kinds of images taken at small and large scales. In the feature detection area, Professor Haralick has developed the facet model for image processing. The facet model states that many low-level image processing operations can be interpreted relative to what the processing does to the estimated underlying gray tone intensity surface of which the given image is a sampled noisy version. The facet papers develop techniques for edge detection, line detection, noise removal, peak and pit detection, as well as a variety of other topographic gray tone surface features.

Professor Haralick's recent work is in shape analysis and extraction using the techniques of mathematical morphology. He has developed the morphological sampling theorem which establishes a sound shape/size basis for the focus of attention mechanisms which can process image data in a multiresolution mode, thereby making some of the image feature extraction processes execute more efficiently.

Professor Haralick is a Fellow of IEEE for his contributions in computer vision and image processing. He serves on the Editorial Board of *IEEE Transactions on Pattern Analysis and Machine Intelligence*. He is the computer vision area editor for *Communications of the ACM*. He also serves as associate editor for *Computer Vision, Graphics, and Image Processing* and *Pattern Recognition*.