KINETICS OF INTERACTING QUASIPARTICLES IN STRONG EXTERNAL FIELDS

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Contents:

Abstract:

This report investigates the modern state of description of kinetic phenomena in solids exposed to intense external fields. It provides the results obtained so far in the studies of nonlinear and nonequilibrium effects determined by the field impact on the elementary acts of interactions of quasiparticles. The systems of electrons, phonons, magnons in semiconductors (magnetic semiconductors including) placed in strong alternating or constant electric fields, in ferro- and antiferrodielectrics under an alternating magnetic field, in tunnelling junctions under high voltages (oscillating voltages including) are described from the unified positions.

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1. Introduction

The problem of constructing a theory of kinetic phenomena in the systems under strong external fields has recently become increasingly urgent. It is attributed, to a large extent, to both great progress in creating the sources of intense fields (lasers, sources of SHF electromagnetic fields of radio frequency, intense sound pumping, constant electric field, etc.) and to the fact that under the influence of intense external actions the properties of the system may vary cardinally as compared with the usual thermodynamical equilibrium ones.

A great variety of nonlinear and nonequilibrium properties of the systems realized in strong external fields is determined not only by the characteristics of the external action, but also by the type of the system itself subjected to this action. The scope of the nonlinear and nonequilibrium phenomena studied by the present moment is exceedingly wide. Among them one can mention the study of elementary processes in quantum electrodynamics in strong electromagnetic and gravitational fields, nonlinear effects in gas plasma acted upon by the intense electromagnetic field, nonequilibrium phenomena in solids (metals, semiconductors, dielectrics (magnetic dielectrics including)), in strong high-frequency or constant (electric, magnetic) fields, etc. The majority of these questions mentioned above have already been considered in monographs and reports. The problems of quantum electrodynamics of the phenomena under intense field, for example, are set forth in refs. [1–6], the questions of interaction of high power irradiation with plasma are surveyed in refs. [7–11], the problems of propagation of strong electromagnetic waves in dielectrics (i.e., the problems of nonlinear optics) are described in refs. [12–16], the intense irradiation effect on metals is considered in ref. [17], some electronic properties of wide-band semiconductors under the action of a high-frequency electromagnetic field ($\Omega\tau \gg 1$, Ω is the frequency of the field, τ is the electron relaxation time) are discussed in [18]. We will refer to the articles of reviewing character and the original papers later on, and now we will touch upon the history of the question and the specificity of the problems treated in this report.

In the theoretical research into the intense field effect on the system of interacting particles it is not sufficient, in a number of cases, to take into account the external field according to the perturbation theory, but it is necessary to take into consideration that the field changes basically the states of interacting particles (as compared, for example, with the states described by plane waves) and, consequently, the very character of the interactions (see section 2). It turned out that in a large number of examples being of great interest for the study of kinetic phenomena in solids, while formulating the main relations of the kinetic theory, the external intense field can be allowed for accurately (not in terms of the perturbation theory), whereas the perturbation theory is sufficient to be developed only in respect to weak interaction between the particles (quasiparticles). This approach proved to be very fruitful and enabled the description of a whole class of new nonlinear and nonequilibrium effects in solids.

One of the most interesting and important, from the point of view of their practical application, are the problems of generation and amplification of waves (electromagnetic, spin and acoustic ones) under strong external fields, damping of intense irradiation by various systems, etc. These problems will be treated with due attention in the present report.

In contrast to the single-particle problems considered by quantum electrodynamics where strong external fields are taken into account, the problem of constructing the kinetics in solids (as well as in gas plasma) under a strong field is much more complicated as it is a many-body problem. The description of the kinetics of interacting quasiparticles (electrons, phonons, magnons) under an intense external field is not reduced only to the investigation of the changed probabilities of their interaction with each other due to the varied wave functions of the interacting quasiparticles caused by the external field. It is essential that the exact account of the external field in kinetic equations for quasiparticles tends to involve a basic change in the corresponding collision integrals (as compared, say, with the Boltzmann equation) and therefore a significant change in the quasiparticle distribution function. In this report we will treat only the problems related to the construction of kinetic equations for quasiparticles in solids exposed to strong homogeneous external fields when the programme for the construction of nonequilibrium thermodynamics can be fully realized. We will also consider the properties of the systems already studied and predicted by the present moment. They are determined by the fact that an intense external field alters essentially the state of quasiparticles for the duration of their collision.

Interest in the quantum phenomena in strong fields was aroused long ago. Swinger [19] considered the problem of the probability of creation of electron–positron pairs generated by the electric field from vacuum in quantum electrodynamics. Exact nonstationary solutions to the Dirac equations for an electron under a wave field found by Volkov [20] were used as a basis for the calculation of quantum transitions [19]. These investigations did not only serve as a powerful impetus for studying various problems in quantum electrodynamics (see refs. [1–6; 21]), but also in a sense initiated the development of the kinetic theory of solids in strong electromagnetic fields. Later on the processes of irradiation, photogeneration and those of annihilation and breakdown of pairs under an intense plane-wave field were examined in detail (see refs. [1–5] and the literature cited there). The mass and the polarization operators of an electron and a photon under the intense field were built and their analytical properties studied [3, 4]. A variety of quantum effects in intense external fields including also along with the electromagnetic field the strong gravitational one, was considered in a number of works surveyed in the monograph [5]. The problem concerning the forced bremsstrahlung and absorption (nonlinear ones including), the forced biphoton Compton scattering and that of electrons in the field of an intense standing wave were dealt with in the studies presented in ref. [21].

The typical field of the nonlinear quantum-electrodynamic effects under which they reach their optimum values, is m^2c^3/e (later on we will consider the Plank constant to be $\hbar = 1$) [3]. A field of such intensity performs the work mc^2 for the Compton length $1/mc$. In contrast to the conditions of nonlinearity in quantum-electrodynamic effects in a solid or plasma, the nonlinearities that occur due to the field impact on the interaction of the particles, take place under incomparably smaller fields. Thus, for example, the effect of the high-frequency electromagnetic field in metals on the electron-impurity scatterings resulting in the multiquantum absorption of the field is significant, as can be easily shown, even for the power introduced $W > 10^{-6}$ W/cm^2 provided that the field frequency $\Omega < 10^{11}$ sec^{-1}. In the case of a strong constant electric field (see sections 2 and 5) the phonon kinetics in wide-band semiconductors may have a number of basic peculiarities related to the field effect on the acts of electron–phonon interactions at comparatively weak intensities $E \lesssim 100$ V/cm. Thus, in a wide region of external fields under which the kinetic and nonequilibrium processes in solids are practically studied, it proved currently topical to consider those effects which are due to the field influence on the elementary acts of interactions of particles.

As has already been mentioned, the action of a strong electromagnetic wave on a solid or plasma involves a change in a very wide range of properties. For example, in gas plasma or plasma of solids, parametrical instability is possible under the action of the field due to the build up of the internal field of fluctuations with time. The research into the nonlinear properties of plasma determined by the parametrical action of the field on plasma, was initiated by the studies [22–24] (see also [8, 10]). As a consequence of the changed dispersion properties of plasma due to the strong field of pumping, the effective interaction of charged particles may significantly vary too, i.e. the field can influence the acts of particles interaction not only directly, but also by way of changing the collective properties. These problems are covered in detail in the monograph [9]. A number of items concerning the kinetics of plasma under strong fields are discussed also in the monographs [25, 26]. In the present report we will not touch upon parametric instabilities supposing that either due to the given conditions these effects are not realized or the time of the external field action is so small that the corresponding collective instabilities do not manage to develop.

In this report we will treat only those problems of the kinetics of quasiparticles in solids in which the intense external field determines to a large extent the character of the interaction between quasiparticles and thereby determines the explicit dependence of the corresponding collision integrals of quasiparticles in kinetic equations upon the field. By now quite a number of the results has been obtained for semiconductors (semimetals), ferro- and antiferrodielectrics, ferromagnetic semiconductors etc. pertaining to those physical situations in which one cannot be reduced to considering the impact of the field only on the states of quasiparticles between their collisions. In other words, there is a set of problems already studied by the present moment where conventional kinetic equations (for example, of the Boltzmann type) are inapplicable and it is necessary to go beyond the scope of the simplest plane-wave description of the states of interacting quasiparticles. The evolution of these problems has a somewhat shorter history as compared to that of the problems of quantum electrodynamics in strong fields.

One of the first studies dealing with the effect of the external electromagnetic field in solids on the processes of scattering of electrons on impurities or phonons were the works [27–29]. However, quantum kinetic equations in these studies were considered only in the linear field approximation. The papers [30, 31] treated the changed probability of interband transitions in semiconductors under a strong constant electric field. It was shown that the exact account of the electric field in the probabilities of the processes leads to a very interesting effect – the possibility of absorption of photons of the frequency Ω at interband transitions when $\Omega < \Delta$ (Δ is the width of the forbidden band). The paper [31], in particular, was a vivid example demonstrating that the kinetic approach, using as the basic state of interacting quasiparticles the exact wave functions of the particles in the field, may involve important and nontrivial effects. The papers [32–34] (see references cited in [35]) also belong to the studies that calculated only the section of the electronic scattering (on impurities or ions) in the presence of the strong high-frequency field.

The systematic research into the quantum kinetic phenomena in solids under strong electromagnetic fields was initiated in 1969. On the one hand, the quantum kinetic equations were constructed describing the nonequilibrium states of electrons (holes) in semiconductors with the allowance only for the interband transitions due to the intense high-frequency field of the frequency $\Omega > \Delta$ [36], on the other hand, assuming that the probabilities of interband transitions are small ($\Omega \ll \Delta$) in the case of an arbitrarily strong high-frequency electromagnetic field ($\Omega\tau \gg 1$, τ is the electron relaxation time), quantum kinetic equations were obtained for the electron distribution function f_p [37, 38] including intraband processes of electron–phonon scattering. In [39] electron-impurity scatterings were studied taking into account their changed probability in the electro-magnetic field. It is important to note the papers [40, 41] where kinetic equations for electrons in the strong electro-magnetic field were derived without allowing for the limitation on the value $\Omega\tau$.

In this report we will not dwell on the problems of kinetics of electrons under strong (quantizing) magnetic fields. Some of them are excellently described in the studies [42, 43].

Further development of the theory of kinetic phenomena in the systems of interacting quasiparticles in solids under strong external fields advanced both towards the growing number of the systems under consideration (semiconductors, semimetals, ferro- and antiferromagnets, magnetic semiconductors, etc.), the study of various properties (see sections 3 and 5) and the search for more consistent and universal ways of constructing kinetic equations proceeding from the most general principles of nonequilibrium thermodynamics. We should also note that while carrying out a programme like this various types of external fields were considered (alternating electromagnetic, constant electric field in semiconductors (semimetals, magnetic ones included), alternating magnetic field in ferro- and antiferrodielectrics). In [44] using the method of Zubarev's nonequilibrium statistical operator [45] in studying the kinetics of electrons due to the electron–electron collisions, the kinetic equations for the case of the strong and, generally speaking, inhomogeneous electromagnetic field were consistently derived. Another method of constructing kinetics was suggested by the authors of [46–48]. It allowed, in particular, to obtain kinetic equations in a rather simple and universal form both for electrons and phonons in the electron-impurity, electron–phonon systems under the strong alternating electromagnetic field, kinetic equations for tunnelling junctions [49] at high oscillating voltages as well as for a number of other systems of which we will speak later on. The investigation carried out in [46–48] made it possible also to formulate the kinetic equations within the limit $\Omega \to 0$, i.e. for the case of an arbitrarily strong constant electric field (naturally, less than Ziener's breakdown). We should place special emphasis on [50] where an interesting method was worked out enabling the derivation of a quantum kinetic equation for the spatially inhomogeneous Wigner function of the distribution of electrons interacting with chaotically arranged impurities under strong constant electromagnetic fields.

Interest in the discussion of nonlinear effects related to the impact of the constant electric field on the processes of interaction of electrons with other quasiparticles was accounted not only by the effects due to interband transitions [31] or intraband interactions [46–48, 50] in wide-band semiconductors (also in metals and semimetals), but by the extraordinarily interesting peculiarities of electron kinetics in narrow-band semiconductors [51, 52]. The theory of transport phenomena in semiconductors under strong constant electric field was elaborated in [51, 52] (see also the references cited in these works). A general relation for current was obtained valid for any width of the electron band and arbitrary coupling of electrons with phonons. Besides, in [51] a graphical technique was devised permitting to get specific results both for the weak electron–phonon interaction and for the case of a small radius polaron. Within the strong electric field limit, the authors of the papers mentioned above took into account the quantization of the longitudinal motion of electrons which permitted to arrive at some interesting and important conclusions, namely, to calculate the "jump" conductivity (between the levels of the Stark ladder) in the presence of resonance scattering on optical phonons (electro-phonon resonance [51]). The peculiarity of [51, 52] is the construction of stationary theories. The graphical technique, though, of [51] does not permit to build kinetic equations, i.e. to examine the evolution of the system with time under a strong constant electric field. Along with the mentioned studies, of great interest are the works [53–55] in which the kinetic equations for electrons in the wide-band case were constructed taking accurately into account the effect of the constant electric field [53, 54] and that of the arbitrarily varying electric field [55] on the intraband electron–phonon processes. The study [53a] seems to have been the first to draw attention to the fact that kinetics of electrons in semiconductors under high constant electric field can largely be determined by the durations of electron–phonon interactions being finite (the intracollision effect). We should also note that kinetics of phonons (as well as of other bosons) interacting with electrons taking into consideration the effect of the strong constant electric field has also a number of nontrivial peculiarities. For example, conditions are possible under which the non-Cherenkovian mechanisms of the amplification of phonons, spin waves and other effects are realizable [56, 57].

Concurrent with the development of quantum kinetic phenomena in solids under strong external fields some studies appeared recently devoted to the construction of quantum kinetic equations for plasma under strong electromagnetic field. These works originated from the one by Zeldovich and Raizer [60] where for the first time proceeding from the physical considerations, a quantum kinetic equation was built for the zeroth harmonic of the spherically symmetrical part of the electron distribution function including the effect of the high-frequency electromagnetic field on the elastical collisions of electrons with the atoms. The field in [60] was allowed for only in the lowest approximation (the single-quantum case). The article [58] presented a stationary kinetic equation (for the zeroth harmonic of the distribution function) as applied to the ion–electron scattering in the multiquantum case. Kas'yanov and Starostin [59] were the first to derive consistently a quantum kinetic equation for arbitrary harmonics of the distribution function. Though we will not dwell here on the problems of kinetics of plasma under strong fields, we have noted the mentioned papers for two reasons. Firstly, these works [58–60] serve as an example of studying the effect of the high-frequency fields on the acts of elementary interactions of particles which determines the kinetic effects in the system. Secondly, the approach to the construction of kinetic equations and the results of [58] in particular were widely used when considering the kinetic phenomena in ferromagnetic semiconductors [61–64] exposed to the strong high-frequency electromagnetic field.

Apart from the problems of studying kinetic conducting systems where strong electromagnetic field determines the character of interaction of the current carriers (electrons, holes) with other quasiparticles, various investigations have been undertaken lately dealing with the kinetics of quasiparticles in magnetic dielectrics under strong alternating magnetic field. The papers [47, 65–70] can serve as an example of the approach accurately including the external alternating magnetic field and elaborating the perturbation theory only in terms of weak interaction between the quasiparticles. These papers presented a theory of the so-called nonresonance parallel pumping in ferro- and antiferromagnets. These problems will also be discussed here.

All the problems described below are treated from the unified positions, viz. it is believed that a problem of a free particle (without interaction with other particles) in the external classical field admits of an accurate solution whereas for constructing nonequilibrium thermodynamics of the system an assumption of weakness of interactions between quasiparticles is used.

This report does not claim to cover all the problems of kinetics in solids under strong fields. Firstly, as has already been said, we will not dwell here on the collective effects originating from the changed dispersion properties of the system, nor will we consider here strong constant magnetic fields. Secondly, we will analyze only those kinetic phenomena which are mainly attributed to the effect of external fields on the elementary acts of interactions of quasiparticles. A number of questions concerning the electronic properties of semiconductors under the intense high-frequency ($\Omega\tau \gg 1$) electromagnetic field are discussed in detail in ref. [18], therefore in some cases we will only refer to the corresponding studies. Lastly, we will not touch upon the similar problems pertaining to the systems with pairing though considerable progress has been made in their studies (see, for example [71–73]).

Though this report will mainly embrace the results of the theoretical investigations, the author would like to hope that this report will be of interest not only for those engaged in theoretical physics, but for experimental researchers too.

Unfortunately, in this relatively new, but steadily developing trend of solid state physics theory is far ahead of the experiment. Nevertheless, even now one can claim that the theoretical study of nonequilibrium and nonlinear phenomena due to the effect of intense external fields on the interactions of quasiparticles is very promising and has resulted in the prediction of extremely interesting and important (viewed practically) effects. Some of these effects have already been proved experimentally, but the majority of them still await the experimental check. However, it should also be noted that the ideas arisen in recent years concerning the peculiarities of quasiparticle kinetics in high external fields due to the influence of external fields on the elementary acts of interactions between the quasiparticles turned out to be obtained while estimating some experimental results already available.

The problems related to the possibility of experimental realization of the phenomena discussed in this report, and the analysis of the experimental results available by the present moment will be touched upon as the material is described. However, we think it expedient to dwell in more detail on the problems related to the experiment in a separate place of the report, its Conclusion. Section 2 contains the methods of constructing kinetic equations for the system of weak-interacting quasiparticles under strong external fields and the derivation of equations for a number of concrete systems. Section 3 is devoted to the description of the properties of semiconductors (magnetic semiconductors including) under high-frequency electromagnetic field, both the electron and the boson (phonon, magnon) properties being discussed. Section 4 contains the results of studying the nonequilibrium states of magnetic dielectrics exposed to the strong alternating magnetic field. The properties of the electron–phonon, electron–magnon systems under strong constant electric field are considered in section 5. Section 6 covers the kinetic phenomena in tunnelling junctions of normal metals under high voltages (alternating voltages including). As has been mentioned above, the Conclusion deals with those experimental results which have a direct bearing on the problems examined in sections 2–6. Also included are the most essential, to our mind, ways of possible future experimental studies. Apart from this, the problems and the prospects of development of quasiparticle kinetics under strong fielda are outlined.

The author would consider his goal fully achieved if the interest in this quite peculiar trend in solid state physics aroused in the reader (both a specialist in theory and in experiment) after reading the report gave rise to new and intriguing results.

2. Kinetic equations for the systems of weak-interacting quasiparticles in a strong field

One of the assumptions that allow the description of the kinetics of particles in strong external fields is that of invariability of the aggregate state of the medium, i.e. the state of the latter does not change considerably during the time of exposure to the field. For example, in the case of a high-frequency electromagnetic field the condition that quick ionization of atoms should be absent, is expressed by the inequality below [21]

$$I \ll mc\Omega^2\Delta/4\pi e^2 \tag{2.1}$$

where I is the intensity of the acting irradiation, and Δ is the energy of the external electron level in the atom.

In this section we will first explain the procedure of deriving kinetic equations for weak-interacting quasiparticles in a strong homogeneous external field having generalized the well-known "golden rule" for constructing collision integrals in the case of accurate account taken of the field. We will show this using as an example the electron–phonon interactions under the high-frequency electromagnet field and the strong constant electric field (subsection 2.1.1). Then we will indicate one of the ways of consistent construction of nonequilibrium thermodynamics for the weak-interacting systems of quasiparticles in arbitrarily strong fields (subsection 2.1.2). And, finally, using the results from 2.1.2 we will derive some kinetic equations describing highly nonequilibrium states of various systems.

2.1. Probabilities of transitions

2.1.1

Let us dwell on the motion of an electron in the conduction band (having formally an infinite width) acted upon by the homogeneous electromagnetic field in the approximation of the isotropic effective mass. Choosing the gauge in which the scalar potential of the field φ is equal to zero, we will write the Schrödinger equation in the form

$$\mathrm{i}\frac{\partial\psi(\boldsymbol{r},t)}{\partial t}=\frac{1}{2m}\left|\hat{\boldsymbol{p}}-\frac{e}{c}\boldsymbol{A}(t)\right|^2\psi(\boldsymbol{r},t) \tag{2.2}$$

where $\boldsymbol{p}=-\mathrm{i}\nabla$ is the canonical operator of the momentum, $\boldsymbol{A}(t)$ is the vector potential of the electromagnetic field.

Since the canonical momentum operator commutes with the Hamiltonian and, therefore, the canonical momentum $\boldsymbol{p}$ is the motion integral, the equation will be solved in the following way

$$\psi(\boldsymbol{r},t)=C(t)\exp(\mathrm{i}\boldsymbol{pr}).$$

Substituting the wave function in this form to eq. (2.2) and solving the corresponding equation for $C(t)$, we obtain [74–76]

$$\psi(\boldsymbol{r},t)=\exp\left\{\mathrm{i}\left(\boldsymbol{pr}-\int_0^t \mathrm{d}t'\frac{1}{2m}\left(\boldsymbol{p}-\frac{e}{c}\boldsymbol{A}(t')\right)^2\right\}. \tag{2.3}$$

It is implied that the field is included in the moment of time $t=0$.

With the help of the obtained wave functions one can study now the probabilities of electron transitions from the state described by the canonical momentum $\boldsymbol{p}$ to the state $\boldsymbol{p}'$ exposed to the weak potential V.

We will first consider the case of the high-frequency electric field $\boldsymbol{E}(t)=\boldsymbol{E}_0\sin\Omega t$ when the function $\psi(\boldsymbol{r},t)$ acquires the form of

$$\psi(\boldsymbol{r},t)=\exp\left\{\mathrm{i}\left[\boldsymbol{pr}-E_{\boldsymbol{p}}t+\frac{e\boldsymbol{E}_0\boldsymbol{p}}{m\Omega^2}\sin\Omega t-\frac{e^2E_0^2}{8m\Omega^3}\sin 2\Omega t\right]\right\}$$

where $E_{\boldsymbol{p}}=\varepsilon_{\boldsymbol{p}}+e^2E_0^2/4m\Omega^2$ is called the main value of the quasi-energy [77], $\varepsilon_{\boldsymbol{p}}=p^2/2m$.

As an example we will calculate the probability of transition from the state $\boldsymbol{p}$ to the state $\boldsymbol{p}'$ with the creation of the phonon having the frequency $\omega_{\boldsymbol{q}}$ ($\boldsymbol{q}$ is the phonon wave vector). The probability of the transition $w_{\boldsymbol{p}\to\boldsymbol{p}'}$ at the moment of time t per unit time is determined [78] by the following relation

$$w_{\boldsymbol{p}\to\boldsymbol{p}'}=\frac{\mathrm{d}}{\mathrm{d}t}\left|\int_0^t \mathrm{d}\tau\,\langle\psi^*_{\boldsymbol{p}'}(\tau)|V|\psi_{\boldsymbol{p}}(\tau)\rangle\right|^2. \tag{2.4}$$

After substituting the wave function in the explicit form to (2.4), it is necessary to calculate the value

$$\frac{\mathrm{d}}{\mathrm{d}t}\left|\int_0^t \mathrm{d}\tau\exp\left\{\mathrm{i}\left[(\varepsilon_{\boldsymbol{p}'}-\varepsilon_{\boldsymbol{p}}+\omega_{\boldsymbol{q}})\tau-\frac{e\boldsymbol{E}_0(\boldsymbol{p}'-\boldsymbol{p})}{m\Omega^2}\sin\Omega\tau\right]\right\}\right|$$

which is equal to

$$2\int_0^t \mathrm{d}\tau \cos\left[(\varepsilon_{p'} - \varepsilon_p + \omega_q)(\tau - t) - \frac{eE_0}{m\Omega^2}(p' - p)(\sin \Omega\tau - \sin \Omega t)\right]. \tag{2.5}$$

To derive a kinetic equation for, say, the electron distribution function f_p in addition to the probability of the transition (2.4) one should allow, in a standard procedure, for the kinetic bracket (see, for example, [79]) corresponding to the processes in question. The "golden rule" for constructing a kinetic equation appropriate for the case under consideration may be written in the form of

$$\frac{\partial f_p}{\partial t} = 2\sum_{p',q} |M(q)|^2 \int_0^t \mathrm{d}\tau \{L^{(1)}_{pp'q}\tilde{\Gamma}^{(\mathrm{ep})}_{pp'q}\delta_{p'-p-q,0} + L^{(2)}_{pp'q}\tilde{\Gamma}^{(\mathrm{ep})}_{p'pq}\delta_{p'-p+q,0}\}\, \mathrm{d}\tau\, \mathrm{e}^{\eta\tau}, \qquad \eta \to -0 \tag{2.6}$$

$$L^{(1)}_{pp'q} = (1 - f_p)f_{p'}(1 + N_q) - f_p(1 - f_{p'})N_q$$

$$L^{(2)}_{pp'q} = (1 - f_p)f_{p'}N_q - f_p(1 - f_{p'})(1 + N_q)$$

where

$$\tilde{\Gamma}^{(\mathrm{ep})}_{pp'q} = \cos\left[(\varepsilon_p - \varepsilon_{p'} + \omega_q)(\tau - t) - \frac{eE_0}{m\Omega^2}(p - p')(\sin \Omega\tau - \sin \Omega t)\right]. \tag{2.7}$$

$M(q)$ is the matrix element of the electron–phonon interaction, N_q is the phonon distribution function, $\delta_{x,x'}$ is the Kroneker symbol, in accordance with the general rules for constructing kinetic equations [43, 45], it is assumed in (2.6) that $f_p = f_p(\tau)$, $N_q = N_q(\tau)$. The latter consideration is important (see section 3) since the probability of the transition (2.4) is in explicit dependence upon time.

For the convenience of using the kinetic equation (2.6) in specific problems, we will make a substitution $\tau \to t + \tau$ in it, the kernels $\tilde{\Gamma}$ turning into Γ being equal to

$$\Gamma^{(\mathrm{ep})}_{pp'q} = \cos\left[(\varepsilon_p - \varepsilon_{p'} + \omega_q)\tau - \frac{eE_0}{m\Omega^2}(p - p')(\sin \Omega(t + \tau) - \sin \Omega t)\right]. \tag{2.8}$$

In this case integration over τ is fulfilled within the limit from $-t$ to 0, and the dependence upon the distribution functions is realized by means of the arguments $t + \tau$, i.e. $f = f(t + \tau)$, $N = N(t + \tau)$. Let us bear in mind now that, in practice, it is expedient to consider the moments of time t which are much greater than some typical times τ_0 for which precisely the kinetic stage of the system evolution manages to establish, i.e. the stage at which, generally speaking, one may describe the state of the system with the help of the single-particle density matrices (of the distribution functions). In the order of magnitude the time τ_0 [43] is equal to

$$\tau_0 \sim \max\{1/\bar{\varepsilon},\, r_0/\bar{v},\, R/\bar{v}\}$$

where $\bar{\varepsilon}$ is the mean energy of the particles, $\bar{v}$ is the mean velocity, r_0 is the radius of interaction, R is the radius of initial correlations. At $t \gg \tau_0$ the lower limit of integration can be assumed as equal to $-\infty$, though, the dependence of the distribution functions arguments on time through $t+\tau$ must be, generally speaking, preserved.

We have finally

$$\frac{\partial f_p}{\partial t} = 2\sum_{p',q} |M(q)|^2 \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau}\{L^{(1)}_{pp'q}\Gamma^{(\mathrm{ep})}_{pp'q}\delta_{p'-q-q,0} + L^{(2)}_{pp'q}\Gamma^{(\mathrm{ep})}_{p'pq}\delta_{p'-p+q,0}\}\,, \qquad \eta \to +0\,. \tag{2.9}$$

As is evident, by neglecting the electric field in the collision integrals and the latter's time nonlocality, eq. (2.9) converts into a standard kinetic equation [79] describing the electron and phonon relaxation towards the thermodynamically equilibrium state when external fields are absent. It is clearly seen from the fact that at $\boldsymbol{E}_0 = 0$

$$\int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} \cos(\xi\tau) = \pi\,\delta(\xi)$$

where $\delta(\xi)$ is the Dirac delta-function.

Kinetic equation (2.9) is represented in the form including the canonical moments of electrons. The transformation from the equation containing the gradient-invariant distribution functions, i.e. the functions dependent of the kinetic momentum $\boldsymbol{P} = \boldsymbol{p} - (e/c)\boldsymbol{A}(t)$ ($\boldsymbol{p}$ is the canonical momentum) is attained quite easily and the corresponding kinetic equations will be given further on. Here we will only supply the result for the kinetic equation for the distribution functions f_p and N_q averaged over the period of the high-frequency field [37, 38] for the case of $\Omega\tau \gg 1$ (here τ is the electron relaxation time). It may be demonstrated (see the supplement in [66]) that an equation like this is obtained from eq. (2.9) by the substitution of $f_k(t+\tau)$, $N_q(t+\tau)$ included in the collision integral for the mean values of f_k, N_q which, accurate within $(\Omega\tau)^{-1}$, are actually the zeroth harmonics (with respect to oscillations) of the distribution functions. Even the averaged functions f_k, N_q describe the "slow" relaxation in the system and depend only on the "slow" time t. Thus, averaging is reduced to that of the integrals having the form of

$$\int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} \cos[E\tau + \beta(\sin\Omega(t+\tau) - \sin\Omega t)]$$

which are expressed by the Bessel function $I_n(x)$ as

$$\frac{\pi}{2}\sum_{m,n=-\infty}^{\infty} (-1)^n\, \mathrm{e}^{\mathrm{i}m\Omega t} I_n(\beta) I_{m-n}(\beta)\{\delta_+(E-n\Omega) + (-1)^m \delta_-(E+n\Omega)\} \tag{2.10}$$

where

$$\delta_-(\alpha) = \delta_+^*(\alpha) = \frac{1}{\pi(\eta + \mathrm{i}\alpha)}\,, \qquad \eta \to +0\,.$$

As a result of averaging we have instead of (2.10)

$$\pi \sum_{n=-\infty}^{\infty} I_n^2(\beta)\,\delta(E+n\Omega)$$

and, finally, the kinetic equation (2.9) takes on the form (see [37, 38])

$$\frac{\partial f_p}{\partial t} = 2\pi \sum_{n=-\infty}^{\infty} \sum_{p',q} |M(q)|^2 I_n^2\Big(\frac{eE_0(p-p')}{m\Omega^2}\Big) \Big\{[(1-f_p)f_{p'}(1+N_q) - f_p(1-f_{p'})N_q]\,\delta_{p'-p-q,0}\,\delta(\varepsilon_p-\varepsilon_{p'}+\omega_q+n\Omega) + [(1-f_p)f_{p'}N_q - f_p(1-f_{p'})(1+N_q)]\,\delta_{p'-p+q,0}\,\delta(\varepsilon_{p'}-\varepsilon_p+\omega_q+n\Omega)\Big\}\,. \tag{2.11}$$

Eq. (2.11) proved to be exceedingly helpful in the discussion of various electronic properties of semiconductors under strong high-frequency fields [18].

Let us consider now, using the same system, i.e. that of an electron interacting with phonons, the case of the strong homogeneous constant electric field of the value $\boldsymbol{E}$. According to the result (2.3) the wave function of electrons under such a field is given by

$$\psi_p(t) = \exp\Big\{\mathrm{i}\Big[\boldsymbol{pr} - \frac{1}{2m}(p^2t + e\boldsymbol{Ep}t^2 + \tfrac{1}{3}e^2E^2t^3)\Big]\Big\}\,. \tag{2.12}$$

The calculation of the probability of the transition (2.4), as is seen, is reduced to the calculation of the value

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big|\int_0^t \mathrm{d}\tau\,\exp\Big\{\mathrm{i}\Big[(\varepsilon_{p'}-\varepsilon_p+\omega_q)\tau + \frac{e\boldsymbol{E}(\boldsymbol{p}'-\boldsymbol{p})}{2m}\tau^2\Big]\Big\}\Big|^2$$

which is equal to

$$2\int_0^t \mathrm{d}\tau\,\cos\Big[(\varepsilon_{p'}-\varepsilon_p+\omega_q)(\tau-t) + \frac{e\boldsymbol{E}(\boldsymbol{p}'-\boldsymbol{p})}{2m}(\tau^2-t^2)\Big]\,.$$

Reasoning similarly to the case of the alternating electric field (see above) we arrive at the following kinetic equation (in the representation of the canonical momentum) [46, 48, 57]

$$\frac{\partial f_p}{\partial t} = 2\sum_{p',q}|M(q)|^2 \int_{-\infty}^{0} \mathrm{d}\tau\,\mathrm{e}^{\eta\tau}\{L^{(1)}_{pp'q}\tilde{R}_{pp'q}\delta_{p'-p-q,0} + L^{(2)}_{pp'q}\tilde{R}_{p'pq}\delta_{p'-p+q,0}\}\,, \qquad \eta\to+0 \tag{2.13}$$

where

$$\tilde{R}_{pp'q} = \cos\Big\{(\varepsilon_p-\varepsilon_{p'}+\omega_q)\tau + \frac{e\boldsymbol{E}(\boldsymbol{p}-\boldsymbol{p}')}{2m}\tau(\tau+2t)\Big\}\,.$$

The absence of the dynamic terms related to the field in the left-hand side of the kinetic equation is only the result of the chosen representation. In the absence of collisions, the electron canonical momentum (in contrast to the kinetic one) remains, therefore it is a good quantum number. Nevertheless, it is often more convenient to use the kinetic equation having a more "conventional" representation which includes the gradient-invariant distribution function. In order to pass over to the representation in which the functions depend on the kinetic momentum $\boldsymbol{P} = \boldsymbol{p} + e\boldsymbol{E}t$ one should adopt the following procedure. Designating by $F(\boldsymbol{P}, t)$ the gradient-invariant distribution function, we have

$$f_p(t) = F(\boldsymbol{P}_t, t) = F(\boldsymbol{p} + e\boldsymbol{E}t, t)$$
$$f_p(t+\tau) = F(\boldsymbol{p} + e\boldsymbol{E}(t+\tau), t+\tau) = F(\boldsymbol{P} + e\boldsymbol{E}\tau, t+\tau)\,.$$

Making also certain corresponding substitutions in the kernels $\tilde{R}$ of eq. (2.13) and using the same symbols, but regarding $f_p(t)$ as the gradient-invariant function and $\boldsymbol{p}$ as a kinetic momentum, we will present eq. (2.13) in the form [48]

$$\frac{\partial f_p}{\partial t} + e\boldsymbol{E}\frac{\partial f_p}{\partial \boldsymbol{p}} = 2\sum_q \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} |M(q)|^2$$
$$\times \{[(1 - f_{p+eE\tau}) f_{p+eE\tau+q}(1+N_q) - f_{p+eE\tau}(1 - f_{p+eE\tau+q})N_q]\, R_{p,p+q,q}$$
$$+ [(1 - f_{p+eE\tau}) f_{p+eE\tau-q} N_q - f_{p+eE\tau}(1 - f_{p+eE\tau-q})\,(1+N_q)] R_{p-q,p,q}\} \tag{2.14}$$

where

$$R_{p,p',q} = \cos\left[(\varepsilon_p - \varepsilon_{p'} + \omega_q)\tau + \frac{e\boldsymbol{E}(\boldsymbol{p} - \boldsymbol{q}')}{2m}\tau^2\right].$$

We will remind here that the arguments in the functions f_k, N_q in the collision integral have the value $t+\tau$.

Similarly to the case of the strong alternating field (2.9) the changeover in (2.14) to the standard kinetic equation [79] describing relaxation of the particles to the equilibrium state in the absence of the external field is performed assuming that the collision integral is local in time and neglecting the explicit dependence of the right-hand side of (2.14) on the field.

The considered elementary prescriptions for constructing kinetic equations allowing for the effect of external fields on the acts of the elementary processes of quasiparticles interactions, naturally, do not need substitution by the "first" principles. For this we have a number of consistent methods suggested, for example, in [43, 45, 80]. Later on we will demonstrate one of the possible approaches [48] permitting to obtain consistently and in a rather simple form general relations by means of which it is easy in every specific case to write the explicit form of the kinetic equation.

The examples of kinetic equations furnished here and the ones to be given in this section (subsections 2.2–2.5) indicate that they can bring about an extraordinarily wide range of nonlinear properties. Nonlinearities may be roughly divided into two types. The first one is related to the nonequilibrium form of the distribution functions that must serve as solutions to the kinetic equations. Such a class of nonlinearities is the outcome, as is clear, of the kinetic processes that do not involve the field effect on the acts of particles interaction (see [81–83]). The other type of nonlinearities is related mainly to the apparent field dependence of the collision integrals of quasiparticles. In fact, such an apparent dependence is the manifestation of the fact that for the time of quasiparticles interaction the field manages to change noticeably their state and, therefore, the character of interactions also varies. For instance, for electrons interacting with phonons under the strong constant electric field the changed nature of the interaction shown in the fact that for the time of the phonon emission by the electron (this time in its order of magnitude equals to ω_q^{-1}) due to its acceleration in the field the electron succeeds in gaining additional energy equal to $e\boldsymbol{E}\boldsymbol{q}/2m\omega_q$. It is quite clear that both types of nonlinearities exist, as a rule, simultaneously.

The appearance of the explicit dependence of the collision integrals on the field results in some important and nontrivial properties. For example, the field can allow those processes which earlier (i.e. without field) were prohibited by the energy and momentum conservation laws. Such examples will be discussed in sections 3–5.

2.1.2. *Equation for the density matrix and kinetic equations for quasiparticles*

Let us assume that due to the action of the external field, the Hamiltonian of the system is in explicit time dependence and can be divided into two terms

$$\mathcal{H} = \mathcal{H}_0(t) + V(t) \tag{2.15}$$

where $\mathcal{H}_0(t)$ is the Hamiltonian of free particles interacting with the external field, for instance

$$\mathcal{H}_0(t) = \sum_p (\varepsilon_p + h_p(t))\, a_p^+ a_p\,. \tag{2.16}$$

ε_p is the spectrum of quasiparticles, $h_p(t)$ is the external field, (a_p^+, a_p) are the operators of creation and annihilation of the particles, $V(t)$ is the Hamiltonian of the interaction which for applications will be regarded as weak.

We will assume also that at $t \to -\infty$ the external field is absent and the system is in the state of static equilibrium and afterwards the field is adiabatically included. In the case like this, as was shown in [43, 84], one can build correctly the kinetic equations describing the nonequilibrium states of the system at arbitrary times. Taking advantage of the method in [84], we will formulate the equations in such a form, starting from the assumption (2.15) that the external field could be included explicitly.

We will introduce the evolution operator determined by the relation

$$\begin{aligned} &\partial U(t)/\partial t = -\mathrm{i}\mathcal{H}_0(t)\, U(t)\,, \qquad U(0) = U^+(0) = 1 \\ &U^+(t)\, U(t) = U(t)\, U^+(t) = 1\,. \end{aligned} \tag{2.17}$$

Now we will formulate the ergodic relation for the density matrix ρ. If it satisfies the principle of spatial weakening of correlations [85], then at $\tau \to \pm\infty$ it has the following relation

$$U(\tau)\, \rho\, U^+(\tau) \xrightarrow[\tau\to\pm\infty]{} \rho^{(0)}(\gamma^{(0)}(\tau)) \tag{2.18}$$

i.e. the density matrix ρ is "mixed" to the density matrix of a special kind [43]

$$\rho^{(0)}(\gamma) = \exp\Big(\Omega(\gamma) - \sum_k X_k(\gamma)\hat{\gamma}_k\Big)$$
$$\mathrm{Sp}\,\rho^{(0)}(\gamma)\,\hat{\gamma}_k = \gamma_k\,, \qquad \mathrm{Sp}\,\rho^{(0)}(\gamma) = 1\,. \tag{2.19}$$

Here $\hat{\gamma}_k$ are the operators corresponding to the parameters γ_k. In (2.18) the parameters $\gamma_k^{(0)}(\tau)$ are determined by the density matrix ρ and the evolution operator $U(\tau)$:

$$\gamma_k^{(0)}(\tau) = \mathrm{Sp}\,\rho\,U^+(\tau)\,\hat{\gamma}_k\,U(\tau)\,.$$

Further on we will dwell only on those systems for which the following condition is satisfield:

$$[\mathscr{H}_0(t), \hat{\gamma}_k] = 0\,.$$

At the end of this section we will supply the results for a more general case too. Then the operators $\hat{\gamma}_k$ commute also with the evolution operator and, thus, in formula (2.19) we have

$$\gamma_k = \mathrm{Sp}\,\rho\hat{\gamma}_k\,.$$

Let $\gamma_k(t)$ be an arbitrary time function of t, then it is easily shown (see [84]) that there exists the following identity

$$\rho^{(0)}(\gamma(t)) - \rho^{(0)}(\gamma(0)) = \int_{-t}^{0} \mathrm{d}\tau\,U^+(t+\tau, t)\,\frac{\partial\rho^{(0)}(\gamma(t+\tau))}{\partial\gamma_k}\,U(t+\tau, t)\,\dot{\gamma}_k(t+\tau) \tag{2.20}$$

where the double-time evolution operator $U(t, t')$ is determined by the formula

$$U^+(t+\tau, t) = U(t)\,U^+(t+\tau)\,.$$

After these preliminary values of arbitrary density matrices that satisfy the principle of spatial weakening of correlations [85], we will consider the density matrix satisfying the Liouville equation

$$\partial\rho/\partial t = \mathrm{i}[\rho, \mathscr{H}_0(t) + V(t)]\,. \tag{2.21}$$

We multiply the equation at the left-hand side by $U^+(t)$ and the one at the right-hand side by $U(t)$ and integrate over time from $-t$ to 0; then we get

$$\rho(t) = U(t)\,\rho(0)\,U^+(t) + \mathrm{i}\int_{-t}^{0} \mathrm{d}\tau\,U^+(t+\tau, t)[\rho(t+\tau), V(t+\tau)]\,U(t+\tau, t)\,. \tag{2.22}$$

We will use (2.20) where the parameters $\gamma_k(t)$ are not defined by the density matrix of the system

$$\gamma_k(t) = \mathrm{Sp}\,\rho(t)\,\hat{\gamma}_k\,, \tag{2.23}$$

i.e. they are true values of the parameters γ_k at the moment of time t. Then we have

$$\rho(t)=\rho^{(0)}(\gamma(t))+U(t)\{\rho(0)-\rho^{(0)}(\gamma(0))\}\,U^{+}(t)$$
$$+\mathrm{i}\int_{-t}^{0}\mathrm{d}\tau\,U^{+}(t+\tau,t)\left\{[\rho(t+\tau),V(t+\tau)]+\mathrm{i}\frac{\partial\rho^{(0)}(\gamma(t+\tau))}{\partial\gamma_k}L_k(t+\tau)\right\}U(t+\tau,t) \tag{2.24}$$

where $L_k(t)$ is the "collision integral" that determines the evolution of the parameters $\gamma_k(t)$ with time

$$\mathrm{d}\gamma_k/\mathrm{d}t=\mathrm{i}\,\mathrm{Sp}\,\rho(t)\,[V(t),\hat{\gamma}_k]\equiv L_k(t)\,. \tag{2.25}$$

Let us study the limiting transition in (2.24) where t tends to $-\infty$. Since with $t=-\infty$, $\rho(-\infty)=w$ (w is the equilibrium density matrix) and $\gamma_k(-\infty)=\mathrm{Sp}\,w\hat{\gamma}_k$ then

$$U^{+}(t)\{\rho(t)-\rho^{(0)}(\gamma(t))\}\,U(t)\xrightarrow[t\to-\infty]{}0$$

and consequently

$$\rho(0)-\rho^{(0)}(\gamma(0))=\mathrm{i}\int_{-\infty}^{0}\mathrm{d}\tau\,U^{+}(\tau)\left\{[\rho(\tau),V(\tau)]+\mathrm{i}\frac{\partial\rho^{(0)}(\gamma(\tau))}{\partial\gamma_k}L_k(\tau)\right\}U(\tau)\,.$$

The final integral equation [48] for the density matrix ρ is given by

$$\rho(t)=\rho^{(0)}(\gamma(t))+\int_{-\infty}^{0}\mathrm{d}\tau\,U^{+}(t+\tau,t)\left\{\mathrm{i}[\rho(t+\tau),V(t+\tau)]\right.$$
$$\left.-L_k(t+\tau)\left(\frac{\partial\rho^{(0)}(\gamma(t+\tau))}{\partial\gamma_k}\right)\right\}U(t+\tau,t)\,. \tag{2.26}$$

We expand the density matrix $\rho(t)$ over the interaction $V(t)$. The zeroth approximation for $\rho(t)$ is the density matrix $\rho^{(0)}(\gamma(t))$. As together with the condition $[\mathscr{H}_0,\hat{\gamma}_k]=0$ the condition $[\hat{\gamma}_k,\hat{\gamma}_l]=0$ is also satisfied, then

$$L_k^{(1)}(t)=\mathrm{i}\,\mathrm{Sp}\,\rho^{(0)}(\gamma(t))\,[V(t),\hat{\gamma}_k]=0\,.$$

Therefore $\rho(t)=\rho^{(0)}+\rho^{(1)}+\cdots$ where

$$\rho^{(1)}(t)=\mathrm{i}\int_{-\infty}^{0}\mathrm{d}\tau\,U^{+}(t+\tau,t)\,[\rho^{(0)}(\gamma(t+\tau)),V(t+\tau)]\,U(t+\tau,t)$$

and the evolution of the parameters $\gamma_k(t)$ is defined by the equation

$$\mathrm{d}\gamma_k(t)/\mathrm{d}t = L_k^{(2)}(t) + \cdots \tag{2.27}$$

where the collision integral $L_k^{(2)}$ [44, 48] is given by

$$L_k^{(2)}(t) = -\int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{Sp}\, \rho^{(0)}(\gamma(t+\tau))\,[U^{+}(t+\tau, t)\, V(t+\tau)\, U(t+\tau, t), [V(t), \hat{\gamma}_k]]\,.$$

The equation for the parameters $\gamma_k(t)$ is a nonlocal equation since the collision integral $L_k(t)$ is determined by the values of $\gamma_k(\tau)$ at $\tau < t$. However, in a number of cases this nonlocality may be neglected. For example, if the interaction V between the particles is small (which precisely is assumed further on) and consequently $\mathrm{d}\gamma_k/\mathrm{d}t \sim V^2$, then

$$\gamma(t+\tau) = \gamma(t) + \tau\, \mathrm{d}\gamma/\mathrm{d}t + \cdots = \gamma(t) + \sim\tau V^2$$

and nonlocality of the collision integral is significant only in higher approximations in powers of interaction. If, for example, we consider the interaction of the particles with the high-frequency field, nonlocality can be neglected when the frequency of the field $\Omega \ll \bar{\varepsilon}$ ($\bar{\varepsilon}$ is the mean energy of the particles) (see section 4). Nonlocality, though, is significant when studying the high-frequency properties for $\Omega \sim \bar{\varepsilon}$. An example of this kind will be discussed in section 3.1.

We have already considered the case when the operators commute with the Hamiltonian $\mathscr{H}_0(t)$. However, the formulae are easily generalized also for the case when

$$[\mathscr{H}_0(t), \hat{\gamma}_k] = a_{kl}(t)\, \hat{\gamma}_l\,. \tag{2.28}$$

The integral equation for the density matrix $\rho(t)$ is here of the same form, but the parameters $\gamma_k(t)$ satisfy the equation

$$\mathrm{d}\gamma_k(t)/\mathrm{d}t = \mathscr{L}_k^{(0)}(t) + L_k(t)$$

where

$$\mathscr{L}_k^{(0)}(t) = \mathrm{i}a_{kl}(t)\, \gamma_l(t)\,.$$

The expansion of the value L_k begins with the first order over the interaction

$$L_k(t) = L_k^{(1)}(t) + L_k^{(2)}(t) + \cdots$$
$$L_k^{(1)}(t) = \mathrm{i}\, \mathrm{Sp}\, \rho^{(0)}(\gamma(t))\,[V(t), \hat{\gamma}_k] \tag{2.29}$$

and the collision integral [48, 44] is given by

$$L_k^{(2)}(t) = -\int_{-\infty}^{0} \mathrm{d}\tau \Big\{ \mathrm{Sp}\, \rho^{(0)}(\gamma(t+\tau))\,[U^{+}(t+\tau, t)\, V(t+\tau)\, U(t+\tau, t), [V(t), \hat{\gamma}_k]]$$
$$+ \mathrm{i}L_k^{(1)}(t+\tau) \frac{\partial}{\partial \gamma_l} \mathrm{Sp}\, \rho^{(0)}(\gamma(t+\tau))\, U(t+\tau, t)\,[V(t), \hat{\gamma}_l]\, U^{+}(t+\tau, t) \Big\}\,. \tag{2.30}$$

We have shown here the general relations that make it possible for any specific system, i.e. for any Hamiltonian, to derive kinetic equations with any degree of accuracy in powers of interaction between quasiparticles. Further on we will discuss some systems of a special kind from the point of view of applying the results of (2.27), (2.30) for concrete calculations.

2.2. *Kinetic equations for electron–phonon, electron–impurity systems in a strong alternating electromagnetic field*

We will now obtain kinetic equations describing the nonequilibrium states of the thin metal film (semiconductor or metal) exposed to the intense electromagnetic field impinging normally on the surface (for example, superhigh-frequency electromagnetic, laser irradiation). For simplicity it is supposed that the film thickness d is much smaller than the penetration depth of the field owing to which we will analyze below the homogeneous case. The main mechanism of electron relaxation is meant to be the electron–phonon and electron–impurity interactions, the impurities being chaotically arranged.

The Hamiltonian of the system in question takes on the form

$$\mathcal{H} = \sum_{p} \varepsilon_p a_p^+ a_p + \sum_{q} \omega_q b_q^+ b_q + \sum_{p,p'} F_{p'p} a_{p'}^+ a_p + \sum_{p,p'} U(p-p')\, a_p^+ a_{p'} + \sum_{q=p'-p} (M(p,p')\, a_p^+ a_{p'} b_q^+ + \text{h.c.}) \qquad (2.31)$$

where (a_p^+, a_p) are the electron creation and annihilation operators, $\varepsilon_p = p^2/2m$,

$$F_{p'p} = -\frac{e}{2mc}(p+p')\, A_{p'-p}(t) + \frac{1}{2m}\left(\frac{e}{c}\right)^2 A^2_{p'-p}(t)\,, \qquad e<0\,.$$

A_q is the Fourier-component of the vector potential (the gauge is chosen in such a way that its scalar potential $\varphi = 0$), $U(p-p')$ is the matrix element of the electron interaction with a random impurity, $M(p,p') = M(q)$ is the matrix element of the electron–phonon interaction, (b_q^+, b_q) are the phonon creation and annihilation operators, $\omega_q = sq$ is the spectrum of phonons, s is the sound velocity (we remind here that $\hbar = 1$ throughout the report), formula (2.31) is assumed to include the corresponding summation over the spins of the electron σ and the polarizations of the phonon λ.

Allowing for the space inhomogeneity of the problem and regarding the potential F as an oscillating one with the frequency Ω, the part of the Hamiltonian $\mathcal{H}_0(t)$ (see (2.15)) related to the "free electrons" and their interaction with the field can be included in the form

$$\mathcal{H}_0(t) = \sum_{p} (\varepsilon_p + F_p(t))\, a_p^+ a_p$$

where

$$F_p(t) = \lambda_p \cos(\Omega t + \varphi_0)\,, \qquad \lambda_p = (-e/mc)\,(pA_0)\,;$$

φ_0 is the initial phase, A_0 is the potential amplitude.

Thus, we obtain a Hamiltonian for which relation (2.27) holds true. We will restrict ourselves to the consideration of only the second order of the nonequilibrium theory of perturbations both for the electron–impurity and electron–phonon interactions.

The double-time evolution operator according to (2.17) is given by

$$U(t+\tau, t) = \exp\Big\{-\mathrm{i}\sum_p \Big[\varepsilon_p\tau + \frac{\lambda_p}{\Omega}(\sin(\Omega(t+\tau)+\varphi_0) - \sin(\Omega t+\varphi_0))\Big] a_p^+ a_p\Big\}\,.$$

So, the calculation in (2.27) is reduced to that of the mean of double commutators. For example, calculating the collision integral for the phonon distribution function $N_k = \langle b_k^+ b_k\rangle$ caused by the electron–phonon interactions, it is necessary to study the average value

$$\sum_{\substack{p,p',q\\ \bar p,\bar p',\bar q}} \langle[a_p^+ a_{p'} b_q^+, [a_{\bar p}^+ a_{\bar p'} b_{\bar q}, b_k^+ b_k]]\rangle$$

$$\times \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} \exp\Big\{\mathrm{i}\Big[(\varepsilon_p - \varepsilon_{p'} + \omega_q)\tau + \frac{\lambda_p - \lambda_{p'}}{\Omega}(\sin(\Omega(t+\tau)+\varphi_0) - \sin(\Omega t+\varphi_0))\Big]\Big\}\,;$$

here $\langle\cdots\rangle$, for brevity, stands for the averaging with the Gibbs nonequilibrium distribution.

Since averaging concerns the density matrix of a special kind (2.19) where the set of $\hat\gamma$-operators includes $a_p^+ a_p$ and $b_q^+ b_q$, we can make use of the conventional Wick-rules assuming that only the following pairings are other than zero

$$\underbrace{a_p^+ a_{p'}} \equiv f_{pp'}\,, \qquad \underbrace{b_q^+ b_{q'}} \equiv N_{qq'}\,.$$

The changeover to the Wigner space-homogeneous distribution functions is attained through the equalities

$$f_{pp'} = f_p \delta_{pp'}\,, \qquad N_{qq'} = N_q \delta_{qq'}\,.$$

The use of the above-mentioned rules culminates in the following kinetic equation for the phonon distribution function N_q

$$\frac{\partial N_q}{\partial t} = 2\sum_p \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau}\, |M(q)|^2 \{(1+N_q)(1-f_{p-q})f_p - N_q(1-f_p)f_{p-q}\}\, \tilde\Gamma_{p-q,p}\,, \qquad \eta \to +0 \tag{2.32}$$

where

$$N = N(t+\tau)\,, \qquad f = f(t+\tau)$$

$$\tilde\Gamma_{p',p} = \cos\Big\{(\varepsilon_{p'} - \varepsilon_p + \omega_q)\tau + \frac{\lambda_{p'} - \lambda_p}{\Omega}(\sin(\Omega(t+\tau)+\varphi_0) - \sin(\Omega t+\varphi_0))\Big\}\,. \tag{2.33}$$

Eq. (2.32) as follows from (2.31) is written in the representation of the canonical momentum for the distribution function of electrons. The changeover to the gradient-invariant distribution function f_p (where by $p = P - (e/c)A(t)$ we designate the kinetic momentum, P is the canonical momentum and to designate the gradient-invariant distribution function we preserve the originally used symbol f) is realized in the following way [48]

$$f_p(t+\tau) \to f_{p+l}(t+\tau), \qquad l = \frac{e}{c}(A(t) - A(t+\tau)) \tag{2.34}$$

$$\tilde{\Gamma}_{pp'} \to \Gamma_{pp'} = \cos\left\{\left[\varepsilon_p - \varepsilon_{p'} + \omega_q + \frac{eA(t)}{mc}(p - p')\right]\tau + \frac{\lambda_p - \lambda_{p'}}{\Omega}(\sin(\Omega(t+\tau) + \varphi_0) - \sin(\Omega t + \varphi_0))\right\}. \tag{2.35}$$

The equation for the electron distribution function f_p is derived similarly. The only difference is that one should average the random-arranged impurities. The result for the gradient-invariant distribution function (i.e. the function dependent only of the kinetic momentum p) acquires the form of

$$\frac{\partial f_p}{\partial t} + eE(t)\frac{\partial f_p}{\partial p} = \mathscr{L}_p^{(\mathrm{imp})}\{f, A\} + \mathscr{L}_p^{(\mathrm{ep})}\{f, A\} \tag{2.36}$$

where

$$\mathscr{L}_p^{(\mathrm{imp})}\{f, A\} = -2n \sum_{p'} \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau}\, |U(p-p')|^2 (f_{p+l} - f_{p'+l})\, \Gamma_{pp'}^{(\mathrm{imp})}$$

$$\Gamma_{pp'}^{(\mathrm{imp})} = \cos\left\{\left[\varepsilon_p - \varepsilon_{p'} + \frac{eA(t)}{mc}(p - p')\right]\tau + \frac{\lambda_p - \lambda_{p'}}{\Omega}(\sin(\Omega(t+\tau) + \varphi_0) - \sin(\Omega t + \varphi_0))\right\}$$

the collision integral $\mathscr{L}_p^{(\mathrm{ep})}\{f, A\}$ coincides with (2.9), the substitutions of (2.34), (2.35) must still be made.

The system of kinetic equations (2.32), (2.36) enables the solution of a wide range of problems concerning the highly nonequilibrium states of metals (semimetals), semiconductors under intense alternating electromagnetic fields. It is common knowledge [81–83] that even neglecting the explicit field dependence of the collision integrals, the joint solution for the kinetic equations for electrons and phonons leads to insurmountable difficulties. It is possible only to construct some model solutions, for example, for the approximation of the quasi-equilibrium function with drift velocity, the approximation of the effective temperature, etc. As for eqs. (2.32), (2.36), they are incomparably more complicated and there is not any perceptible progress in their solution as yet. Nevertheless, a number of reasonable physical assumptions and conditions is possible which permits to carry out the analysis of kinetic equations and to consider the properties of the system determined by the field effect on the processes of interaction of quasiparticles.

2.3. *Kinetic equations for electrons and magnons in ferromagnetic semiconductors in a strong alternating electromagnetic field*

Ferromagnetic semiconductors are a special type of semiconductors whose properties depend on the interaction of electrons with the quanta of magnetic subsystems – magnons. Usually as a model for describing the interaction of the conduction electrons with the localized spins the so-called s-d exchange model is used [86–89]. This model, evidently, can describe the real physical situation for wide-band semiconductors in a sufficiently adequate way. The effect of the s-d exchange interaction on the conduction electrons eliminates the degeneration of spins and the conduction band splits into two subbands having different spin orientations. The subbands are shifted in reference of each other for the energy

$$\Delta = I\langle S_z\rangle$$

where I is the exchange energy, axis z coincides with the direction of the mean magnetic moment, $\langle s_z\rangle$ is the mean value of the localized spin. So, the spectrum of electrons is given by

$$\varepsilon_{p\uparrow,\downarrow} = \varepsilon_p \mp \Delta/2\,. \tag{2.37}$$

The value Δ as a rule, is in order of magnitude of $10^2 \div 10^3$ K.

Another effect can be described rather consistently within the framework of the s-d exchange model. It consists in the interaction of electrons with the fluctuations of the magnetic moment. Such interaction may be regarded as the collision of the conduction electrons with magnons. The main contribution to the kinetic processes of electron–magnon interactions is made by single- and double-magnon processes whose Hamiltonian takes on the form [86]

$$\mathscr{H}_{\mathrm{em}} = \mathscr{H}^{(3)}_{\mathrm{em}} + \mathscr{H}^{(4)}_{\mathrm{em}} \tag{2.38}$$

where

$$\mathscr{H}^{(3)}_{\mathrm{em}} = \sum_{p,p',q} \psi_3(a^+_{p\uparrow}a_{p'\downarrow}b^+_q\delta_{p+q-p',0} + a^+_{p\downarrow}a_{p'\uparrow}b_q\delta_{p'-p+q,0}) \tag{2.39}$$

$$\mathscr{H}^{(4)}_{\mathrm{em}} = \sum_{p,p',q,q',\sigma} \psi_4(a^+_{p\sigma}a_{p'\sigma}b^+_q b_{q'}\delta_{p+q-p'-q',0} + \mathrm{h.c.})\,; \tag{2.40}$$

here $(a^+_{p\sigma}, a_{p\sigma})$ are the electron creation and annihilation operators with the momentum $\boldsymbol{p}$ and the spin projection $\sigma(\uparrow, \downarrow)$; (b^+_q, b_q) are the magnon creation and annihilation operators; ψ_3, ψ_4 are the amplitudes of the s-d exchange interaction ($\psi_3 \sim \Delta/\sqrt{N}$, $\psi_4 \sim \Delta/N$; N is the number of elementary cells) which are usually considered in the model as independent of the momenta of the interacting particles.

In addition to the interaction (2.38) in the system in question there are also other types of interaction, for example, those between electrons, electrons and phonons, electrons and impurities. For the magnetic subsystem magnon–magnon, magnon–phonon interactions are essential too. When constructing kinetic equations for a ferromagnetic semiconductor under a strong alternating electric field, the electron–impurity and electron–phonon processes are taken into account as in section 2.2. For a homogeneous external field the integral of electron–electron collisions is of the standard form which does not include the field explicitly. This fact, as can be easily understood, follows from the law of the kinetic momentum conservation for colliding particles. As for allowance made for other types of interactions, except those of magnons, the corresponding integrals will be discussed in section 2.4 since they are the same both for magnetic conductors and magnetic dielectrics.

The derivation of the kinetic equations for the electron distribution function in the subbands $f_{p\uparrow} = \langle a^+_{p\uparrow} a_{p\uparrow}\rangle$, $f_{p\downarrow} = \langle a^+_{p\downarrow} a_{p\downarrow}\rangle$ and the magnon distribution function $N_q = \langle b^+_q b_q\rangle$ is analogous to that in section 2.2. The only difference is that for deriving kinetic equations a larger set of γ-parameters is introduced (see section 2.1). They are $f_{p\uparrow}$, $f_{p\downarrow}$, N_q. The technique of calculations for the mean values in formula (2.27) is that the following pairings are considered to be equal to zero by the Wick rule,

$$\underbrace{a^{(+)}_{p\uparrow} a^{(+)}_{p\downarrow}}, \qquad \underbrace{a^{(+)}_{p\uparrow} a^{(+)}_{p\uparrow}}, \qquad \underbrace{a^{(+)}_{p\downarrow} a^{(+)}_{p\downarrow}}, \qquad \underbrace{b_q b_q}.$$

Let us supply now the kinetic equations only for the values of the electron and the magnon distribution functions averaged over the period of the field oscillations [61–63, 90]. The system of equations for the case of $\Omega\tau \gg 1$ (τ is the relaxation time of electrons and magnons) is given by

$$\begin{aligned}\frac{\partial N_k}{\partial t} = 2\pi \sum_n \sum_{p,p'} I_n^2\left(\frac{eE_0(p-p')}{m\Omega^2}\right) |\psi_3|^2 \{(1+N_q)(1-f_{p\uparrow}) f_{p'\downarrow} \\ - N_k(1-f_{p'\downarrow}) f_{p\uparrow}\}\, \delta(\varepsilon_{p\uparrow} - \varepsilon_{p'\downarrow} + \omega_k + n\Omega)\, \delta_{p'-p-k,0} + 2\pi \sum_n \sum_{p,p',k',\sigma} |\psi_4|^2 I_n^2\left(\frac{eE_0(p-p')}{m\Omega^2}\right) \\ \times \{(1+N_k) N_{k'} (1-f_{p\sigma}) f_{p'\sigma} - N_k (1+N_{k'})(1-f_{p'\sigma}) f_{p\sigma}\} \\ \times \delta(\varepsilon_{p\sigma} - \varepsilon_{p'\sigma} + \omega_k - \omega_{k'} + n\Omega)\, \delta_{k-k'+p-p',0}\end{aligned} \tag{2.41}$$

where ω_k is the spectrum of magnons (for more detail see [91]) which is equal to $\omega_0 + \theta_c(ak)^2$, in the isotropic model, ω_0 is the activation energy, θ_c is the Curie energy, a is the lattice constant,

$$\begin{aligned}\frac{\partial f_{p\uparrow}}{\partial t} = 2\pi \sum_n \sum_{p',k} I_n^2\left(\frac{eE_0(p-p')}{m\Omega^2}\right) |\psi_3|^2 \\ \times \{(1-f_{p\uparrow}) f_{p'\downarrow} (1+N_k) - (1-f_{p'\downarrow}) f_{p\uparrow} N_k\} \\ \times \delta(\varepsilon_{p\uparrow} - \varepsilon_{p'\downarrow} + \omega_k + n\Omega)\, \delta_{p-p'+k,0} + \mathscr{L}^{(4)}_{p\uparrow}\{f, N\},\end{aligned} \tag{2.42}$$

$$\begin{aligned}\frac{\partial f_{p\downarrow}}{\partial t} = 2\pi \sum_n \sum_{p',k} I_n^2\left(\frac{eE_0(p-p')}{m\Omega^2}\right) |\psi_3|^2 \\ \times \{(1-f_{p\downarrow}) f_{p'\uparrow} N_k - f_{p\downarrow}(1-f_{p'\uparrow})(1+N_k)\} \\ \times \delta(\varepsilon_{p\downarrow} - \varepsilon_{p'\uparrow} - \omega_k + n\Omega)\, \delta_{p-p'-k,0} + \mathscr{L}^{(4)}_{p\downarrow}\{f, N\}\end{aligned} \tag{2.43}$$

where

$$\begin{aligned}\mathscr{L}^{(4)}_{p\sigma}\{f, N\} = 2\pi \sum_n \sum_{p',k,k'} I_n^2\left(\frac{eE_0(p-p')}{m\Omega^2}\right) |\psi_4|^2 \\ \times \{(1-f_{p\sigma}) f_{p'\sigma}(1+N_k) N_{k'} - (1-f_{p'\sigma}) f_{p\sigma} (1+N_{k'}) N_k\} \\ \times \delta(\varepsilon_p - \varepsilon_{p'} + \omega_k - \omega_{k'} + n\Omega)\, \delta_{p-p'+k-k',0}\,.\end{aligned} \tag{2.44}$$

Eqs. (2.42) and (2.43) are different since according to the Hamiltonian (2.39) the transition of the electron from the "lower" subband to the "upper" one due to the single-magnon processes, may occur only if the spin wave is absorbed. In the common case (i.e. when the high-frequency field is not present) such a process is a threshold one and at the temperature $T < \Delta$ it is exponentially small. The reverse process, in accordance with (2.39), is not a threshold one. This point is one of the reasons enabling the opinion that both the kinetics of magnons and that of electrons in ferromagnetic semiconductors are largely determined by double-magnon (2.40) interactions [92] since they are not threshold ones. Under the conditions of the electron warming the effective electron temperature T_e may exceed or become in the order of Δ and in this case the kinetic phenomena are mainly determined by the three-particle electron–magnon interactions [88, 93]. It is different when the processes of electron–magnon collisions occur in the presence of a high-frequency field. The presence of the term $n\Omega$ in the laws of energy conservation of the interacting quasiparticles (see formulae (2.41)–(2.44)) even for the case of $T \ll \Delta$ can eliminate the threshold character of the three-particle processes (2.39) either at sufficiently high frequencies or at a sufficiently great intensity [63]. Even the kinetic equations given in this section permit us to study a number of interesting and basic questions such as the nonlinear absorption of the intense high-frequency electromagnetic field [94], the effect of "defreezing" of magnons [90], the amplification of spin waves [63], and also some other questions which will be touched upon in section 3.

2.4. Kinetic equations for magnons in ferro- and antiferrodielectrics in strong nonresonance alternating magnetic field

The character of the action of the alternating magnetic field on ferro- and antiferromagnets is, to a large extent, controlled both by the field polarization with respect to the direction of magnetization and the relation between the field frequency Ω and the typical frequencies of spin waves. As for the orientation of the alternating magnetic field, two cases, as a rule, are distinguished, namely, the so-called parallel pumping when the alternating field is oriented along the magnetization (of the ferromagnet, for example) [91] and the so-called cross-pumping [91, 95] when the external alternating field is perpendicular to the magnetic moment orientation. For the case of cross pumping the typical field frequency at which nonequilibrium effects are most prominent, is the value ε_0 – the activation in the spectrum of magnons*. If these frequencies coincide, the so-called ferro- or antiferromagnetic resonance takes place (for more detail see [91, 95]). The range of characteristic frequencies in parallel pumping is divided into two regions: $\Omega > 2\varepsilon_0$, $\Omega < 2\varepsilon_0$. In the first case the process of breakdown of the space homogeneous wave into two spin waves is believed conceivable

$$\Omega = \varepsilon_{k} + \varepsilon_{-k} \tag{2.45}$$

where ε_k is the magnon energy with the wave vector $\boldsymbol{k}$. The process (2.45) enables the parametrically resonance exitation of spin waves in ferro- and antiferromagnets and serves as the main channel of transmitting energy from the source of pumping to the magnon subsystem in the ideal (without magnetic inhomogeneities) magnets. The study of the dynamic theory of parametric resonance was the purpose of the works [96, 97]. The kinetic theory of parametric excitation of spin waves in ferromagnets under parallel pumping is propounded in [98].

For the magnetic field frequencies $\Omega > 2\varepsilon_0$ the processes of the (2.45)-type are realized, naturally, even in the absence of interactions between quasiparticles, therefore this case will not be tackled here. The alternating field of such a frequency influences the processes of interaction between the magnons, but usually this effect is comparatively small [66].

* Here in contrast to section 2.3, activation is designated by ε_0, not by ω_0. We will preserve the symbol ω_q for the spectrum of phonons.

The situation when $\Omega < 2\varepsilon_0$ and the pumping is parallel, will be of greatest interest to us. This is accounted by the following facts. The direct channel for the absorption of the field energy by magnons (2.45) which, in principle, could involve nonequilibrium states, under the given conditions is absent since it is prohibited by the laws of energy conservation (2.45). Therefore, the absorption of the field energy is caused only by the interaction between the quasiparticles (magnon–magnon, magnon–phonon processes, the action of magnons with magnetic inhomogeneities – impurities, dislocations). The action of the alternating magnetic field on the quantum processes of interaction in ferro- and antiferromagnets brings about the nonequilibrium states that may have some interesting and unexpected properties (section 4).

2.4.1. Kinetic equations for ferromagnets

Let us consider a ferromagnet (a single-domain sample, the static magnetic field $H > 4\pi M_0$, M_0 is the saturation magnetization) exposed to the alternating magnetic field $h(t) = h_0 \cos \Omega t$, parallel to the static field and to the direction of $\boldsymbol{M}_0$. The Hamiltonian of the system including only the magnon–magnon interactions* is given by [91]

$$\mathscr{H} = \sum_p \varepsilon_p c_p^+ c_p + \sum_p h_p(t)\, c_p^+ c_p + \sum_{123} (V_{1,23} c_1^+ c_2 c_3 + \text{h.c.})$$
$$+ \sum_{1234} W_{12;34} c_1^+ c_2^+ c_3 c_4 + \sum_{1234} (W'_{1,234} c_1^+ c_2 c_3 c_4 + \text{h.c.}) + \sum_{12} U_{12} c_1^+ c_2 + \sum_{12} (V_{12} c_1 c_2 + \text{h.c.}) , \quad (2.46)$$

where

$$\varepsilon_p = \sqrt{A_p^2 - |B_p|^2} , \qquad A_p = \theta_c (ap)^2 + \varepsilon_0 + m \sin^2 \theta_p ,$$
$$B_p = m \sin^2 \theta_p \exp(-2i\varphi_p) , \qquad \varepsilon_0 = \mu(H + \beta M_0) , \qquad h_p(t) = h_p^0 \cos \Omega t ,$$
$$h_p^0 = \mu h_0 \frac{\theta_c (ap)^2 + \varepsilon_0 + m \sin^2 \theta_p}{[(\varepsilon_0 + \theta_c (ap)^2)^2 + 2(\varepsilon_0 + \theta_c (ap)^2)\, m \sin^2 \theta_p]^{1/2}} ;$$

here h_0 is the amplitude of the alternating magnetic field; $\mu = g\mu_0$, μ_0 is the Bohr magneton; θ_k, φ_k are polar angles in the space of the wave vectors; (c^+, c) are the magnon operators; $V_{1,23}$, W, W' are the matrix elements of the magnon–magnon interactions, their notations are assumed to include the law of the momentum conservation, for example, $V_{1,23} = V_{1,23}\delta_{1-2-3,0}$; for the sake of brevity the digital indices stand for the corresponding momenta: $1 \equiv p_1$, $2 \equiv p_2, \ldots$, $m = 4\pi\mu M_0$; β is the anisotropy constant; U_{12}, V_{12} are the matrix elements of the magnon–impurity interactions.

In the Hamiltonian (2.46) we are reduced to the purely nonresonance terms of the interaction between the particles and the field leaving out a term of the following type

$$H_p(t) c_p^+ c_{-p}^+ + \text{h.c.} , \qquad H_p(t) = H_p^0 \, e^{-i\Omega t} . \quad (2.47)$$

It is associated with the fact (see [66, 68]) that if $\Omega < 2\varepsilon_0$ the processes of excitation of spin waves (2.47) in the second order of the perturbation theory are forbidden by the law of energy conservation. The

* Not to encumber the explanation, we will consider the magnon–phonon interactions in the next subsection devoted to antiferromagnets. It is also noteworthy that the influence of the magnon–phonon interactions on the kinetics of magnons in ferromagnets, as a rule, is not very significant [91]. For antiferromagnets, though, it is essential.

inclusion of these terms in the higher orders of the perturbation theory produces by far lesser effects than those already allowed for in (2.46) at least for the parameter [66]

$$\left(\frac{\Omega}{\varepsilon_0-\Omega/2}\right)^2\left(\frac{M_0}{H}\right)^2 \ll 1\,.$$

Thus, again we are confronted with an example of the system whose kinetic equations being constructed can include explicitly the strong alternating field. Since the Hamiltonian of the magnon interaction with the field commutes with the operator of the particles number, it is sufficient to provide only the normal averages $f_p=\langle c_p^+c_p\rangle$ for consideration ($\langle\cdots\rangle$, as before, stands for the averaging over the Gibbs nonequilibrium distribution).

From (2.27) it is inferred that one should calculate only the following averages included in the kinetic equation

$$\begin{aligned}\frac{\partial f_p}{\partial t} = &\sum_{1231'2'3'}\int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} I^{(V)1,23}_{1',2'3'}(t,t+\tau)\langle[c_1^+c_2c_3,[c_{3'}^+c_{2'}^+c_{1'},c_p^+c_p]]\rangle\\
&+\sum_{12341'2'3'4'}\int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} I^{(W)12,34}_{1'2',3'4'}(t,t+\tau)\langle[c_1^+c_2^+c_3c_4,[c_{3'}^+c_{4'}^+c_{1'}c_{2'},c_p^+c_p]]\rangle\\
&+\sum_{12341'2'3'4'}\int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} I^{(W')1,234}_{1'2'3'4'}(t,t+\tau)\langle[c_1^+c_2c_3c_4,[c_{2'}^+c_{3'}^+c_{4'}^+c_{1'},c_p^+c_p]]\rangle\\
&+\sum_{121'2'}\int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} I^{(U)12}_{1'2'}(t,t+\tau)\langle[c_1^+c_2,[c_{2'}^+c_{1'},c_p^+c_p]]\rangle\\
&+\sum_{121'2'}\int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} I^{(V)12}_{1'2'}(t,t+\tau)\langle[c_1c_2,[c_{2'}^+c_{1'}^+,c_p^+c_p]]\rangle + \text{h.c.}\,,\end{aligned} \tag{2.48}$$

where

$$I^{(V)1,23}_{1',2'3'} = -V_{1,23}V^*_{1',2'3'}\exp\left\{\mathrm{i}\left[(\varepsilon_1-\varepsilon_2-\varepsilon_3)\tau+\frac{h_1^0-h_2^0-h_3^0}{\Omega}\,\theta(t,\tau)\right]\right\}$$

$$I^{(W)12,34}_{1'2',3'4'} = -W_{12,34}W^*_{1'2',3'4'}\exp\left\{\mathrm{i}\left[(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4)\tau+\frac{h_1^0+h_2^0-h_3^0-h_4^0}{\Omega}\,\theta(t,\tau)\right]\right\}$$

$$I^{(W')1,234}_{1',2'3'4'} = -W'_{1,234}W'^*_{1',2'3'4'}\exp\left\{\mathrm{i}\left[(\varepsilon_1-\varepsilon_2-\varepsilon_3-\varepsilon_4)\tau+\frac{h_1^0-h_2^0-h_4^0}{\Omega}\,\theta(t,\tau)\right]\right\}$$

$$I^{(U)12}_{1'2'} = -\overline{U_{12}U^*_{1'2'}}\exp\left\{\mathrm{i}\left[(\varepsilon_1-\varepsilon_2)\tau+\frac{h_1^0-h_2^0}{\Omega}\,\theta(t,\tau)\right]\right\}$$

$$I^{(V)12}_{1'2'} = -\overline{V_{12}V^*_{1'2'}}\exp\left\{\mathrm{i}\left[(\varepsilon_1+\varepsilon_2)\tau+\frac{h_1^0+h_2^0}{\Omega}\,\theta(t,\tau)\right]\right\};$$

here

$$\theta(t,\tau) \equiv \sin \Omega(t+\tau) - \sin \Omega t$$

$$\langle\cdots\rangle \equiv \mathrm{Sp}\, \rho_0\, (f(t+\tau)) \ldots$$

$$\rho_0\{f\} = \exp\Big\{-\sum_p \Big[\ln(1+f_p) - c_p^+ c_p \ln\Big(\frac{1+f_p}{f_p}\Big)\Big]\Big\}$$

the horizontal bar stands for the averaging over the random arrangement of impurities [43].

Calculating the spurs in (2.48) for the case of the quadriparticle interactions is quite a cumbersome procedure. It may, however, be simplified if the pairings are placed immediately below the commutator symbol. For instance,

$$\mathrm{Sp}\, \rho_0\, [a_1^+ a_2^+ a_3 a_4,\, a_{1'}^+ a_{2'}^+ a_{3'} a_{4'}]$$
$$= [f_1 f_2 (1+f_3)(1+f_4) - (1+f_1)(1+f_2) f_3 f_4]\, \delta_{1,4'} \delta_{2,3'} \delta_{3,2'} \delta_{4,1'}\,.$$

Here we have made use of the Bose statistics and the space homogeneous approximation. Acting similarly in the case of the W'-interaction too, upon averaging over the random arrangements of impurities we obtain [65–68]

$$\frac{\partial f_p}{\partial t} = 4\pi \sum_{123} |V_{1,23}|^2 \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} K_{1,23}(h,t,\tau)\, (\delta_{p3} + \delta_{p2} - \delta_{p1}) \{(1+f_2)(1+f_3) f_1 - (1+f_1) f_2 f_3\}$$
$$+ 16\pi \sum_{1234} |W_{12,34}|^2 \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} K_{12,34}(h,t,\tau)\, \delta_{p1}\, \{(1+f_1)(1+f_2) f_3 f_4 - f_1 f_2 (1+f_3)(1+f_4)\}$$
$$+ 12\pi \sum_{1234} |W'_{1,234}|^2 \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} K_{1,234}(h,t,\tau)\, (\delta_{p4} + \delta_{p2} + \delta_{p3} - \delta_{p1})$$
$$\times \{f_1 (1+f_2)(1+f_3)(1+f_4) - f_2 f_3 f_4 (1+f_1)\}$$
$$+ 2n\pi \sum_{p'} |U_{pp'}|^2 \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} K_{pp'}(h,t,\tau)\, (f_{p'} - f_p)$$
$$+ 2n\pi \sum_{p'} |V_{pp'}|^2 \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} \tilde{K}_{pp'}(h,t,\tau)\, (1 + f_p + f_{p'}) \qquad (2.49)$$

where

$$K_i(h,t,\tau)=\frac{1}{2}\sum_m\sum_n(-1)^n\,e^{im\Omega t}I_n\left(\frac{\lambda_i}{\Omega}\right)I_{m-n}\left(\frac{\lambda_i}{\Omega}\right)\{\delta_+(E_i-n\Omega)+(-1)^m\delta_-(E_i+n\Omega)\}$$

$$i=\{(1,23),(12,34),(1,234),(12)\}\,,\qquad f_p=f_p(t+\tau)$$

$$E_{1,23}=\varepsilon_1-\varepsilon_2-\varepsilon_3\,,\qquad E_{12,34}=\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4\,,$$

$$E_{1,234}=\varepsilon_1-\varepsilon_2-\varepsilon_3-\varepsilon_4\,,\qquad E_{12}=\varepsilon_1-\varepsilon_2\,,\qquad \lambda_{1,23}=h_1^0-h_2^0-h_3^0\,,$$

$$\lambda_{12,34}=h_1^0+h_2^0-h_3^0-h_4^0\,,\qquad \lambda_{1,234}=h_1^0-h_2^0-h_3^0-h_4^0\,,\qquad \lambda_{12}=h_1^0-h_2^0\,;$$

for the last term in eq. (2.49) $E_{12}=\varepsilon_1+\varepsilon_2$, $\lambda_{12}=h_1^0+h_2^0$, n is the concentration of impurities (or the concentration of dislocations if the defects of the dislocation type are analyzed).

In the case of a high-frequency field ($\Omega\tau_s\gg 1$, τ_s is the magnon relaxation time), as in the previous sections, it is not difficult to change over to the kinetic equation for the distribution function "smoothed out" at the times Δt satisfying the inequality

$$\tau_s\gg\Delta t\gg\Omega^{-1}\,.$$

Viewed formally the procedure of this changeover consists in the substitution $f(t+\tau)\to f(t)$ where t is the "slow" time and

$$\int_{-\infty}^{0}d\tau\,e^{\eta\tau}K_i(h,t,\tau)\ldots\to\sum_n\delta(E_i+n\Omega)\,I_n^2\left(\frac{\lambda_i}{\Omega}\right)\ldots$$

It is in this form that equation (2.49) was used in [65–70] when studying various properties.

The obtained kinetic equation describes both the change in the overall number of magnons and their redistribution over frequencies under the action of the external pumping. Equation (2.49) also allows for the fact that the nonresonance generation of two magnons by the field quantum Ω ("through the impurity") is perceivable too. A process like this is, certainly, quite possible when $\Omega>2\varepsilon_0$. But, as has already been stated in the introduction to section 2.4, for the case of $\Omega>2\varepsilon_0$ there is a direct resonance channel of pumping spin waves. Therefore, it would appear senseless to take into account the nonresonance processes of magnon pumping and at the same time not to consider the nonresonance processes. In section 4 we will demonstrate that it is not so. If the concentrations of magnetic inhomogeneities are not too small, the nonresonance processes regarding the effectiveness of particles pumping can be much higher than that of the resonance processes.

Let us pass over now to the discussion of the specificity of antiferromagnets with magnetic anisotropy of the "light-plane"-type.

2.4.2. Kinetic equations for antiferromagnets

We will dwell now on the antiferromagnet with magnetic anisotropy of the "light plane"-type exposed to the homogeneous alternating magnetic field parallel to the constant external field lying in the basis plane of the crystal [69]. We will concern ourselves, as in section 2.4.1, with the parametrically nonresonance case when $\Omega<2\Delta_1$ (Δ_1 is the activation of the low-frequency branch of the magnon spectrum in antiferromagnets). Here, similarly to the cases discussed so far, it is not sufficient to be confined to the second order of the perturbation theory in powers of each of the interactions in the system (with the field, between the particles), but it is necessary to analyze consistently the kinetic equations including the "interference" of the particles interactions with the field and other particles. To put it differently, one should consistently take into account the impact of the alternating magnetic field on the processes of interaction of magnons.

To describe the processes of interaction in the system of spin waves and the interaction of spin waves with the alternating magnetic field, the usual procedure is to start from the following expression for the antiferromagnetic Hamiltonian with the magnetic anisotropy of the "light plane"-type (see, for example, [99–101])

$$\mathcal{H} = \mathcal{H}_2 + \mathcal{H}_{\mathrm{f}} + \mathcal{H}_3 + \mathcal{H}_4 + \mathcal{H}_{\mathrm{sp}} + \mathcal{H}_{\mathrm{imp}} + \mathcal{H}_{\mathrm{sd}} \tag{2.50}$$

where

$$\mathcal{H}_2 = \sum_k \{\varepsilon_k c_k^+ c_k + E_k d_k^+ d_k\} + \sum_q \omega_q b_q^+ b_q$$

(c_k^+, c_k), (d_k^+, d_k) are the magnon creation and annihilation operators of the low- and high-frequency branches, respectively,

$$\varepsilon_k = \sqrt{\Delta_1^2 + (sk)^2}\,, \qquad E_k = \sqrt{\Delta_2^2 + (sk)^2}$$

$$\Delta_1^2 = \mu^2 H(H + H_{\mathrm{D}}) + \Delta_0^2\,, \qquad \Delta_2^2 = 2\mu^2 H_{\mathrm{E}} H_{\mathrm{A}}\,, \qquad s = \theta_{\mathrm{N}} a\,;$$

H is the constant external field, H_{D} is the Dzyaloshinski field, H_{A} is the anisotropy field, H_{E} is the exchange field, θ_{N} is the order value of Néel's temperature, Δ_0 is the activation due to the superfine and magneto-elastic interaction, ω_q is the phonon energy, (b_q^+, b_q) are the phonon operators,

$$\mathcal{H}_{\mathrm{f}} = \mathcal{H}_{\mathrm{f}}^{(1)} + \mathcal{H}_{\mathrm{f}}^{(2)}$$

$\mathcal{H}_{\mathrm{f}}^{(1)}$ describes the effect of pumping on the magnons of the lower branch, and $\mathcal{H}_{\mathrm{f}}^{(2)}$ on those of the upper branch

$$\begin{aligned} \mathcal{H}_{\mathrm{f}}^{(1)} &= \sum_k \{v_k(t)\, c_k^+ c_k + \tfrac{1}{2} w_k(t)\,(c_k c_{-k} + c_k^+ c_{-k}^+)\}\,, \\ v_k(t) &= v_{0k} \cos \Omega t\,, \qquad v_{0k} = \frac{\mu h \omega_H}{\varepsilon_k}\,, \qquad \omega_H = \mu(2H + H_{\mathrm{D}})\,; \end{aligned} \tag{2.51}$$

h is the amplitude of the alternating field.

The Hamiltonians $\mathcal{H}_3$, $\mathcal{H}_4$, $\mathcal{H}_{\mathrm{sp}}$, $\mathcal{H}_{\mathrm{imp}}$, $\mathcal{H}_{\mathrm{sd}}$ describe three- and four-magnon, magnon–phonon, magnon–impurity, magnon–dislocation interactions, respectively. The amplitudes of these processes are well-known and can be found in [99–101]. Therefore, for the sake of brevity, we will not give them here.

Since, as a rule, the inequality $\Delta_1 \ll \Delta_2$ is observed, the effect of the nonresonance field on the magnons of the high-frequency branch can be neglected at least with respect to the parameter $(\Delta_1/\Delta_2)^2$ [69]. Next it should be taken into consideration that in the case $\Omega < 2\Delta_1 (\ll \Delta_2)$, we are interested in, nonequilibrium states will mainly occur only in the system of low-frequency magnons. That is why the Hamiltonians $\mathcal{H}_{sp}$, $\mathcal{H}_{imp}$, $\mathcal{H}_{sd}$ should involve only the parts associated with the processes in which the magnons of the low-frequency branch of the spectrum take part.

The second term in (2.51) describes the parametrically resonance action of the field on the magnons. In the case of $\Omega < 2\Delta_1$ the direct processes described by $w_k(t)\, c_k^+ c_{-k}^+$ are prohibited by the law of energy conservation. These processes, however, as well as the nonresonance ones have influence on the processes of interaction of magnons. In [69] it was shown that one may neglect this influence at least for $\Omega \ll \Delta_1$ and consider only the interaction $v_k(t)\, c_k^+ c_k$.

Under the given situation for constructing the kinetic equations directly one can resort to the method given in formula (2.57). The kinetic equation for the magnons of the spectrum lower branch has the form ($\Omega\tau_s \gg 1$)

$$\begin{aligned}\frac{\partial f_p}{\partial t} &= 4\pi \sum_{123} |\psi_{1,23}|^2 \sum_n I_n^2\left(\frac{\lambda_{1,23}^{(1)}}{\Omega}\right) L_{1,23}^{(1)}\{f, F\}\,(\delta_{3p} + \delta_{2p}) \\ &+ 8\pi \sum_{1234} |\Phi_{12,34}|^2 \sum_n I_n^2\left(\frac{\lambda_{12,34}^{(2)}}{\Omega}\right) (\delta_{1p} + \delta_{2p})\, L_{12,34}^{(2)}\{f\} \\ &+ 4\pi \sum_{123} |\varphi_{1,23}|^2 \sum_n I_n^2\left(\frac{\lambda_{1,23}^{(3)}}{\Omega}\right) (\delta_{2p} + \delta_{3p})\, L_{1,23}^{(3)}\{f, N\} \\ &+ 2\pi \sum_{p'} \overline{|\chi_{pp'}|^2} \sum_n I_n^2\left(\frac{\lambda_{pp'}^{(4)}}{\Omega}\right) L_{pp'}^{(4)}\{f\} \\ &+ 2\pi \sum_{p'} \overline{|\zeta_{pp'}|^2} \sum_n I_n^2\left(\frac{\lambda_{pp'}^{(4)}}{\Omega}\right) L_{pp'}^{(4)}\{f\}\,,\end{aligned} \tag{2.52}$$

where the bar stands for averaging with respect to the random arrangement of defects, $\psi_{1,23}$, $\Phi_{12,34}$ are the amplitudes of three- and four-magnon interactions, $\varphi_{1,23}$ is the amplitude of the magnon–phonon interactions; χ, ζ are the matrix elements of the magnon interactions with impurities and dislocations, respectively,

$$\begin{aligned} L_{1,23}^{(1)}\{f, F\} &= \{(1+f_2)(1+f_3)F_1 - (1+F_1)f_2 f_3\}\, \delta(\varepsilon_2 + \varepsilon_3 - E_1 + n\Omega)\,, \\ L_{12,34}^{(2)}\{f\} &= \{(1+f_1)(1+f_2)f_3 f_4 - (1+f_3)(1+f_4)f_1 f_2\}\delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_3 - \varepsilon_4 + n\Omega)\,, \\ L_{1,23}^{(3)}\{f, N\} &= \{(1+f_2)(1+f_3)N_1 - f_2 f_3(1+N_1)\}\, \delta(\varepsilon_2 + \varepsilon_3 - \omega_1 + n\Omega)\,, \\ L_{pp'}^{(4)}\{f\} &= \{f_{p'} - f_p\}\, \delta(\varepsilon_p - \varepsilon_{p'} + n\Omega)\,, \end{aligned} \tag{2.53}$$

$f_p \equiv \langle c_p^+ c_p \rangle$, $F_p \equiv \langle d_p^+ d_p \rangle$ is the distribution function of the high-frequency magnons, $N_q \equiv \langle b_q^+ b_q \rangle$ is the phonon distribution function

$$\lambda_{1,23}^{(1)} = \lambda_{1,23}^{(3)} = v_{02} + v_{03}\,, \qquad \lambda_{12,34}^{(2)} = v_{01} + v_{02} - v_{03} - v_{04}\,, \qquad \lambda_{pp'}^{(4)} = v_{0p} - v_{0p'}\,.$$

Equations (2.52), (2.53), in principle, can be supplemented with the similar kinetic equations for phonons and for the upper branch magnons. Since, however, kinetic phenomena in antiferromagnets are largely determined by the low-frequency magnons, it is usually sufficient to be confined to the equations for the values of f, N assuming that the function F is an equilibrium one.

2.5. Kinetic equations for electrons and bosons interacting with them in a strong constant electric field

We will discuss now the rules of constructing kinetic equations for the electron–phonon, electron–magnon systems exposed to the homogeneous constant electric field of intensity $\boldsymbol{E}$.

The kinetic equations both for the distribution function f_p and the distribution function of bosons N_q (phonons, magnons, plasmons) for the case of the constant electric field are derived similarly to those of the alternating electric field (see sections 2.2 and 2.3). Thus, we will not repeat the whole procedure here, but do the following: we will make the limiting transition $\Omega \to 0$ [48] in the formulae of sections 2.2 and 2.3. In the assumption that the electron distribution function is dependent on the kinetic momentum [46–48, 56, 57] we obtain

$$\frac{\partial N_q}{\partial t} = 2\sum_p \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau}\, |\psi_i|^2 \{(1+N_q)(1-f_{p+eE\tau-q})f_{p+eE\tau} - N_q(1-f_{p+eE\tau})f_{p+eE\tau-q}\}$$
$$\times \cos\left[(\varepsilon_{p-q} - \varepsilon_p + \omega_q)\tau - \frac{eEq}{2m}\tau^2\right] \tag{2.54}$$

where ψ_i is the amplitude of the electron–boson interaction, ω_q is the spectrum of bosons, $\varepsilon_p = p^2/2m$ is the electron energy, $f_p = f_p(t+\tau)$.

The kinetic equation for electrons involving only the nonelastic interaction of electrons acquires the form of (2.14). When electrons are scattered on the chaotically arranged impurities the kinetic equation for the space inhomogeneous Wigner distribution function $f_p(r, t)$ [50, 48] is given by

$$\frac{\partial f_p}{\partial t} + \frac{p}{m}\frac{\partial f_p}{\partial r} + eE\frac{\partial f_p}{\partial p} = -\frac{n}{(2\pi)^3}\int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau}\, |U(q)|^2 \left\{f\left(r - \frac{q\tau}{2m}, p + eE\tau\right) - f\left(r - \frac{q\tau}{2m}, p + eE\tau - q\right)\right\}$$
$$\times \cos\left\{(\varepsilon_p - \varepsilon_{p-q})\tau + \frac{eEq}{2m}\tau^2\right\}. \tag{2.55}$$

Kinetic equation (2.54) is substantially simplified if one considers sufficiently large times when a stable regime is possible in the system, i.e. one may assume the right-hand side of the equation having $f_p = f_p(t)$. In this case after the substitution of the variable $p + eE\tau \to p$ the kernel of the collision integral is represented by means of the Sine and Cosine Fresnel integrals:

$$\int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} \cos(E\tau + \beta\tau^2) = \sqrt{\frac{\pi}{2|\beta|}}\{[\tfrac{1}{2} + S(|\xi|)]\sin\xi^2 + [\tfrac{1}{2} + C(|\xi|)\,\mathrm{sign}(\xi\beta)]\cos\xi^2\} \tag{2.56}$$

where $\xi = E/2\sqrt{|\beta|}$, $C(x)$, $S(x)$ are the Sine and Cosine Fresnel integrals, respectively.

Though, as before, the equations remain to be sufficiently complicated, there appears some possibility of considering the limiting cases of the big and small ξ.

2.6. Kinetic equations for stochastic pumping

A large number of problems in physics and their applications in engineering lead to the study of the systems of many particles in stochastic fields [102]. Though a case like this, as it may seem, is not directly connected with the problems dealt with in the given report, we will dwell on them in brief for the following reason. In reality, the problems of the stochastic field impact on the kinetics of quasiparticles and that of the coherent monochromatic field (see sections 2.2–2.4) on the states of quasiparticles in the presence of the stochastic processes of quasiparticles and scattering are largely analogous. In both cases the absorption of the homogeneous field energy by the system is caused by the stochastic processes. Here we supply the results of the theory [103–104] where the kinetic equations for the systems described by the following Hamiltonian, were formulated

$$\mathcal{H} = \mathcal{H}_0 + \varphi(\boldsymbol{x}, t) \tag{2.57}$$

where $\mathcal{H}_0$ is the Hamiltonian of the system of free particles, $\varphi(\boldsymbol{x}, t)$ is the operator of the field, stochastic with time and in space, generated by stochastic sources. It is assumed [103] that the external field is a steady stochastic process.

The kinetic equations for the single-particle operator f characterizing the state of the system exposed to the field is naturally written in the following form

$$\overline{\partial f/\partial t} = \mathrm{i}\overline{[f, \mathcal{H}_0]} + \mathrm{i}\overline{[f, \varphi(\boldsymbol{x}, t)]} \tag{2.58}$$

where the bars stand for averaging with respect to the stochastic field.

The studies [103, 104] formulated a method for constructing the perturbation theory in the stochastic field φ enabling the calculation in any necessary orders of the second term in the right-hand side of equation (2.58).

In the space nonhomogeneous case the following kinetic equation takes place in the representation of the momentum for the distribution function [103, 104]

$$\frac{\partial f_{12}}{\partial t} = L_{12}^{(0)}\{f\} + L_{12}^{(1)}\{f\} + L_{12}^{(2)}\{f\} \tag{2.59}$$

where $L_{12}^{(0)} = \mathrm{i}(\varepsilon_2 - \varepsilon_1) f_{12}$, $\varepsilon_{\boldsymbol{p}}$ is the energy of the free particle with the momentum $\boldsymbol{p}$

$$L_{12}^{(1)} = \mathrm{i}(\bar{\varphi}_2 - \bar{\varphi}_1) f_{12}$$

$$L_{12}^{(2)} = A_{12} + A_{12}^*$$

$$A_{12} = -\Big(\pi \sum_{1'2'} \int \mathrm{d}\omega\, \{f_{11'}\, I(2'-1', 2-2', -\omega)\, \delta_-(\omega_{1'2'} - \omega) - f_{1'2'}\, I(1-1', 2-2', \omega)\, \delta_-(\omega_{11'} - \omega)\} + \text{h.c.}\Big)$$

where $\boldsymbol{p}_i \overline{\equiv} \mathrm{i}$, $\omega_{12} = \varepsilon_1 - \varepsilon_2$; $I(1-2, 3-4, \omega)$ is the Fourier component of the correlation function $I(1-2, 3-4, \tau)$ which is determined as

$$I(1-2, 3-4, \tau) = g(1-2, 3-4, \tau) - \bar{\varphi}(1-2)\, \bar{\varphi}(3-4)\,,$$

$$g(1-2, 3-4, \tau) = \frac{(2\pi)^6}{V^2} \int \mathrm{d}^3x_1\, \mathrm{d}^3x_2\, \varphi(x_1, \tau)\, \varphi(x_2, 0) \exp(\mathrm{i}(1-2)x_1) \exp(\mathrm{i}(3-4)x_2)\,,$$

$$\bar{\varphi}(1-2) = \frac{(2\pi)^3}{V} \int \mathrm{d}^3x_1\, \overline{\varphi(x_1)} \exp(\mathrm{i}(1-2)x_1)\,.$$

The terms, quadratic in φ in eq. (2.59), correspond to the collision integral.

In the space homogeneous case when $f_{12} = f_1\delta_{12}$, $\bar{\varphi} = 0$, the kinetic equation is considerably simplified

$$\frac{\partial f_p}{\partial t} = -\frac{\pi}{2}\sum_{12}\int d\omega\,(\delta_{1p} - \delta_{2p})(f_1 - f_2)\{R(2-1, -\omega) + R(2-1, \omega)\}\,\delta(\omega_{12} - \omega)\,. \tag{2.60}$$

Here it is taken into account that

$$I(1-2, 3-4, \omega) = R(1-2, \omega)\,\delta_{1-2,4-3}$$

and that the correlation function R has the inversion centre, i.e.

$$R(2-1, \pm\omega) = R(1-2, \pm\omega)\,.$$

If the interaction between the particles is allowed for, the kinetic equation (2.60) can be supplemented with the corresponding collision integrals in its right-hand side. Due to this it becomes applicable to the description of the nonlinear evolution of the system of interacting quasiparticles under stochastic pumping [103].

3. Nonequilibrium properties of electrons and bosons interacting with them in a high-frequency electromagnetic field

Let us consider now the specific kinetic properties of the systems which stem from the equations given in sections 2.2 and 2.3. As has already been underlined in the Introduction, we will touch upon only those properties and effects which are due to the field impact on the quasistatic interactions. The choice of the material for this section is largely dictated by the fact that there is a report [18] devoted to the problems of nonlinear electronic properties of semiconductors under strong electromagnetic pumping. Therefore, drawing the reader's attention to ref. [18] we will discuss predominantly the results obtained recently in this sphere and go into the properties of ferromagnetic semiconductors in highly nonequilibrium states.

3.1. High-frequency conductivity of electron gas of semiconductors. "Linear" effects

It is common knowledge that if the external field frequency Ω is comparable with the mean energy $\bar{\varepsilon}$ of the particle, the quantum approach is fundamentally essential for describing kinetic phenomena. Such an approach, as has been shown in sections 2.2, 2.3, is based on the use of the quantum kinetic equations of the type (2.9), (2.33)–(2.36), (2.41)–(2.44). In order to see how essential even under a weak field the impact of the alternating electromagnetic field on the acts of the electron interactions is, we will concern ourselves with the simplest system where the electrons are scattered on the impurities in an elastic way. We will calculate the high-frequency conductivity of the electron gas in the lowest linear field approximation. When analyzing this problem, we will, firstly, allow for the non-Markovian nature of the scattering processes, i.e. the time nonlocality of the electron-impurity collision integral, and, secondly, the dependence of the collision integral on the alternating electric field. A procedure of this kind was made for the first time in [48].

The kinetic equation for the distribution function of electrons interacting with the impurities in the field

$$E(t) = E \sin \Omega t \tag{3.1}$$

according to eq. (2.36) in the space homogeneous case is given by

$$\frac{\partial f_p}{\partial t} + eE \sin \Omega t \frac{\partial f_p}{\partial p} = \frac{2n}{V} \sum_q \int \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} |U(p-q)|^2 (f_{q+\Delta}(t+\tau) - f_{p+\Delta}(t+\tau))\, \Gamma_{pq} \tag{3.2}$$

where

$$\Delta = \frac{eE}{\Omega}(\cos \Omega t - \cos \Omega(t+\tau))$$

$$\Gamma_{pq} = \cos\{\mathscr{E}\tau - Z[\Omega\tau \cos \Omega t + \sin \Omega t - \sin(\Omega t + \Omega\tau)]\}$$

$$\mathscr{E} = \varepsilon_p - \epsilon_q\,, \qquad Z = eE(p-q)/m\Omega^2\,.$$

In order to be restricted only to the linear approximation, one should expand both the kinetic bracket in the collision integral and the kernel Γ_{pq}. The collision integral is finally transformed to

$$\begin{aligned} \mathscr{L}_p^{(\mathrm{imp})}\{f\} = \frac{2n}{V} \sum_q |U(p-q)|^2 \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} \Big\{ & (f_q(t+\tau) - f_p(t+\tau)) \cos \mathscr{E}\tau \\ & + \frac{eE}{\Omega}(\cos \Omega t - \cos \Omega(t+\tau)) \cos \mathscr{E}\tau \left(\frac{\partial f_q(t+\tau)}{\partial q} - \frac{\partial f_p(t+\tau)}{\partial p} \right) \\ & - Z \sin \mathscr{E}\tau\, (\Omega\tau \cos \Omega t + \sin \Omega t - \sin \Omega(t+\tau))\, (f_q(t+\tau) - f_p(t+\tau)) \Big\}\,. \end{aligned} \tag{3.3}$$

In obtaining (3.3) the following condition has been used

$$\left\{ \frac{|e|E}{\Omega \bar{p}}, \frac{|e|E\bar{p}}{m\Omega^2} \right\} \ll 1 \tag{3.4}$$

where $\bar{p}$ is the mean electronic momentum.

Let us calculate the current j of the system

$$j = \frac{e}{m} \frac{V}{(2\pi)^3} \int \mathrm{d}p\, p f_p\,. \tag{3.5}$$

The conductivity of the system σ and the phase shift between the field and the current is defined by the relation

$$j = \sigma E \sin(\Omega t - \varphi)\,. \tag{3.6}$$

Thus, the procedure is not reduced only to the harmonics expansion of the values included in (3.3) and to the expansion of the distribution functions in Legendre's polynomials:

$$f_p = f_p^0 + f_p^1 \cos\theta, \qquad \theta = (\widehat{p, E}).$$

The equation for the magnitude f_p^1 [48] has the form of ($f_p^1 \equiv f_1, f_p^0 \equiv f_0$)

$$\frac{\partial f_1}{\partial t} + eE \sin\Omega t \frac{\partial f_0}{\partial p} = -\left(\nu_1 + \frac{\nu_2}{\Omega}\frac{\partial}{\partial t}\right) f_1 + \frac{eE}{\Omega}\frac{\partial f_0}{\partial p}[(\nu_1 - \nu)\cos\Omega t - \nu_2 \sin\Omega t]$$

$$- \nu eE\Omega^{-2}\sqrt{\frac{2}{m}}\int_0^\infty \mathrm{d}\varepsilon' \sqrt{\varepsilon'}\,[f_0(\varepsilon) - f_0(\varepsilon')]\, G(\varepsilon, \varepsilon') \tag{3.7}$$

where

$$G(\varepsilon, \varepsilon') = \cos\Omega t \left[\Omega \frac{\partial}{\partial\varepsilon}\delta(\varepsilon - \varepsilon') + \tfrac{1}{2}(\delta(\varepsilon - \varepsilon' - \Omega) - \delta(\varepsilon - \varepsilon' + \Omega))\right]$$

$$+ \frac{1}{2\pi}\sin\Omega t \left[\frac{2}{\varepsilon - \varepsilon'} - \frac{1}{\varepsilon - \varepsilon' + \Omega} - \frac{1}{\varepsilon - \varepsilon' - \Omega}\right]$$

$\varepsilon \equiv \varepsilon_p$, $\varepsilon' \equiv \varepsilon_{p'}$, ν is the frequency of the electron-collisions with impurities,

$$\nu_1 = \frac{\nu}{\pi m p}\frac{\pi}{2}\int_0^\infty q^2\,\mathrm{d}q\,[\delta(\varepsilon_p - \varepsilon_q + \Omega) + \delta(\varepsilon_p - \varepsilon_q - \Omega)]$$

$$\nu_2 = \frac{\nu}{\pi m p}\frac{1}{2}\int_0^\infty q^2\,\mathrm{d}q\left(\frac{1}{\varepsilon_p - \varepsilon_q - \Omega} - \frac{1}{\varepsilon_p - \varepsilon_q + \Omega}\right).$$

If the time nonlocality of the collision integral is neglected and only the Γ dependence of the kernel on the electric field is taken into account, the result (3.7), as for its form, remains the same, but $\nu_1 \equiv \nu$, $\nu_2 \equiv 0$. The magnitude $f_0(\varepsilon)$ in eq. (3.7) must be assumed to be an equilibrium one as the change in $f_0(\varepsilon)$ with respect to its equilibrium type is allowed for only in studying the field nonlinear effects. We will examine the nondegenerate case under the conditions that

$$\Omega \ll T, \qquad \nu \ll T.$$

The case of $\Omega \sim T$ is of great interest, though, the analytical solution of eq. (3.7) in a situation like this is impossible. The calculation of current (3.6) yields the following results [48]

$$\sigma \simeq \sigma_0\left[1 + \frac{2}{3\pi}\frac{\nu}{T}\left(1 + \frac{4}{15\sqrt{\pi}}\sqrt{\frac{\Omega}{T}}\right)\right], \qquad \sigma_0 = \frac{e^2 n_e}{3m\sqrt{\nu^2 + \Omega^2}} \tag{3.8}$$

$$\operatorname{tr}\varphi \simeq \frac{\Omega}{\nu}\left[1+\frac{1}{6}\frac{\Omega^2}{T^2}\left(1+\frac{1}{2}\frac{\nu^2}{\Omega^2+\nu^2}\right)\right] \tag{3.9}$$

where n_e is the concentration of electrons.

The simplest analysis of the above-mentioned results shows that in the case of $\Omega \sim T$ it can be expected that the relative corrections for the high-frequency conductivity become of the unit order. Thus, even in the field linear approximation the enumerated effects can vary noticeably the well-known form of standard kinetic coefficients.

3.2. Nonlinear high-frequency effects in semiconductors

We will dwell here on the problem of the intraband energy absorption of the high-frequency ($\Omega\tau \gg 1$) electromagnetic field in semiconductors. The absorption of the homogeneous field by the electrons of conduction is caused by their stochastization due to various scattering processes: on impurities, acoustic, optical phonons, etc. The absorption can be calculated using a kinetic equation of the (2.36)-type.

The energy absorbed by the system per unit time by the unit volume is found from the relation

$$\dot{Q} = \frac{1}{V}\sum_{p,i} \varepsilon_p \mathscr{L}_p^{(i)}\{f\} \tag{3.10}$$

where $\mathscr{L}_p^{(i)}\{f\}$ are the phonon collision integrals, the index i indicates the association with the definite type of electronic scattering (on impurities, dislocations, phonons, magnons, etc.), V is the volume of the system.

The function f in (3.10), in principle, must be chosen as a solution to the corresponding kinetic equation which is the case with equation (2.36), for example. A problem like this, however, has not yet been solved in view of its exceeding complexity. Consideration was given only to the problems of relative effectiveness of various channels of the field absorption. Here we mean the channels connected with the electron–impurity, electron–phonon and other interactions. For calculating the rate of absorption of the field energy (3.10), one may resort to the following simplifying consideration. Not to deal with the problem of the type of the nonequilibrium distribution function, we will assume the field to be a pulsed one having such a pulse duration τ_p that for the time of the field action on the system, the field may be considered, on the one hand, quasistationary ($\tau_p \gg \Omega^{-1}$) and, on the other hand, the quasistationary state of the system is believed to be essentially invariable ($\tau_p \ll \tau$, τ is the electron relaxation time). In this case, evidently, the electron distribution function must be substituted in (3.10). One more important fact must be also mentioned. A strong electromagnetic wave can influence significantly the scattering properties of the medium [10], change, in particular, the Coulomb screening of, say, the charged ionized impurities, in semiconductors [105]. However, [106] if the varied electronic quasimomentum in the scattering induced by the field greatly exceeds the reciprocal value of the Debye screening radius, the effect of the charged screening may be neglected.

The multiquantum field absorption by the electrons of conduction in semiconductors was examined in [106–109, 111] (see also the references cited in [18]). The work [106] contains the calculations of the field absorption coefficient in semiconductors when the main mechanism of the electronic scattering is the scattering on the ionized atoms. The authors succeeded in representing the final expression only as the expansion in the small parameter $\beta = e^2E^2/m\Omega^3$. The expression obtained in [106] for laser irradiation ($\Omega \sim 10^{14}$–$10^{15}\,\mathrm{sec}^{-1}$) actually holds true for sufficiently great powers introduced as the value

$\beta = 1$ has the corresponding density of the energy flux of the order of 10^9 W/cm^2. The coefficient of the ν-photon absorption of intense light by the current carriers in [107] was calculated for three cases: the electronic scattering on acoustic, optical phonons and ionized impurities. It was shown that the ν-photon absorption coefficient depends on the number ν of the absorbed quanta quite differently which enables comparison of the effectiveness of various channels of the field absorption within the limit of strong fields (a multiquantum case). The calculation of the field absorption coefficient (by means of numerical methods) for the case of acoustic phonons was fulfilled in [108]. The authors of [109] studied the energy absorption of the high-frequency electromagnetic field in a polar semiconductor at low temperature when the scattering of the electrons by optical phonons predominates and the phonon energy is great as compared to the mean energy of electrons. In addition, wide use was made of those regions in the vicinity of the single-photon absorption threshold when $\Omega < \omega_0$ (ω_0 is the phonon frequency) the field absorption is caused completely by the nonlinear effects. In [111] the rate of the energy accumulation by an electron (3.10) was calculated for the case of electron–phonon interactions.

We consider now in more detail the problem of the intense electromagnetic field using as an example the electron–impurity interactions when the situation is essentially a multiquantum one [110].

In accordance with the kinetic equation for the electrons interacting with the impurities under the high-frequency field ($\Omega\tau \gg 1$) the absorbed energy (3.10) can be represented by

$$\dot{Q} = 2\pi n_i \sum_{p,p'} \sum_n \varepsilon_p I_n^2\left(\frac{eE(p-p')}{m\Omega^2}\right) |U(p-p')|^2 (f_{p'} - f_p)\, \delta(n\Omega - \varepsilon_{p'} + \varepsilon_p)\,. \tag{3.11}$$

When analyzing the multiquantum case it is convenient to make use of the following approximate equality [58]

$$\sum_n \int \mathrm{d}E\, I_n^2\left(\frac{\lambda}{\Omega}\right) \varphi(E)\, \delta(E - n\Omega) \simeq \tfrac{1}{2}[\varphi(\lambda) + \varphi(-\lambda)]\,, \qquad \lambda \gg \Omega\,. \tag{3.12}$$

The transformation of expression (3.11) including (3.12) yields [110]

$$\dot{Q} = n_i \frac{V}{(2\pi)^3} ma \int_0^\infty \mathrm{d}p\, p f_p \int_0^\infty \mathrm{d}r\, r^2 |U(r)|^2 \int_{-1}^{+1} \mathrm{d}y\, y\, \theta(2p - |r - 2may|)\,, \tag{3.13}$$

where $a = eA_0/mc$.

First we supply the results for the charged impurities: $|U(r)|^2 = U_0^2/r^2$ and for the nondegenerate case

$$\dot{Q} \simeq n_i n_e V U_0^2 \begin{cases} \dfrac{\sqrt{2}}{24\pi^{9/2}} (mT)^{-1/2} (ma)^2\,, & ma^2 \ll T \\ \dfrac{2}{(2\pi)^4 \sqrt{\pi}}\, ma\, \Phi\left(a\sqrt{\dfrac{m}{2T}}\right), & ma^2 \gg T \end{cases} \tag{3.14}$$

where n_e is the concentration of electrons and

$$\Phi(x_0) = \int_0^{x_0} \mathrm{d}x\, \mathrm{e}^{-x^2}\,.$$

In the case of neutral impurities $|U(r)|^2 = U_0^2$ the field dependence varies. As before, for the nondegenerate case we have

$$\dot{Q} \simeq n_i n_e V U_0^2 \begin{cases} \dfrac{\sqrt{2}}{3\pi^{9/2}} (mT)^{1/2} (ma)^2 , & ma^2 \ll T \\ \dfrac{4}{(2\pi)^4 \sqrt{\pi}} (ma)^3 \, \Phi\Big(a\sqrt{\dfrac{m}{2T}}\Big) , & ma^2 \gg T . \end{cases} \tag{3.15}$$

The difference in dependence of the absorbed energy on the field value within the limit of strong fields in formulae (3.14) and (3.15) permits, in principle, to determine the character of the electron interactions with impurities. Analogous calculations for the case of gas plasma were done in [58].

In this section we should also note the studies [112, 113] which dealt with the generation of higher harmonics by free carriers in semiconductors; [112] contained the calculations of the nonlinear susceptibility that dictates the generation of the third harmonic. An important result was obtained in [113] where it was demonstrated that when the main mechanism of nonlinearity is the electromagnetic field impact on the electron–phonon interactions and the photon energy is by far larger than the mean electron energy, the main contribution to the linear susceptibility is made by the virtual processes.

Of great interest will be to set the problem concerning the propagation of the weak electromagnetic wave in the plasma of the semiconductor exposed to the strong homogeneous high-frequency electric field of a different frequency. In the study like this the possibility of investigating extraordinarily important nonlinear effects appears. The essence of formulating the problem is quite simple. The strong field changes substantially the probabilities of electronic interactions with other particles (quasiparticles) and against this background the "linear" absorption of the weak wave which is determined by the same interactions also acted upon by the strong field, varies radically as compared to the quasi-equilibrium case. Under such a situation one can expect not only the change in the value itself of the "linear" absorption, but also the change in sign of the weak wave damping. This consideration is essential for studying the possibilities of amplification (generation) of electromagnetic waves. The propagation of the weak electromagnetic wave as against the strong one of a different frequency was analyzed in [114–116]. For the case of gas plasma such a problem was surveyed in [117].

For solving the problem of propagation of the weak electromagnetic wave with the frequency Ω_2 against the background of the strong wave with the frequency Ω_1, the kinetic equations described in section 2 must be generalized. This task was fulfilled in [114]. For the case of plane polar waves when ($\Omega_1\tau, \Omega_2\tau \ll 1$), the kinetic equation [114] for the high-frequency part of $\tilde{n}_p$, the distribution function of the electrons interacting with the acoustic phonons, was represented as

$$\frac{\partial \tilde{n}_p}{\partial t} = \sum_q |M(q)|^2 \sum_{f,l,n,s} I_l(a_1 q)\, I_s(a_1 q)\, I_n(a_2 q)\, I_f(a_2 q) \exp\{\mathrm{i}[(s-l)\Omega_1 + (f-n)\Omega_2] + \mathrm{i}(f-n)\varphi\}$$

$$\times \int_{-\infty}^{t} \mathrm{d}t' \, \{[\bar{n}_{p+q}(1+N_q) - \bar{n}_p N_q] \exp[\mathrm{i}(\varepsilon_{p+q} - \varepsilon_p - \omega_q - s\Omega_1 - f\Omega_2 + \mathrm{i}\eta)(t-t')]$$

$$+ [\bar{n}_{p+q} N_q - \bar{n}_p(1+N_q)] \exp[\mathrm{i}(\varepsilon_{p+q} - \varepsilon_p + \omega_q - s\Omega_1 - f\Omega_2 + \mathrm{i}\eta)(t-t')]\} + \{p \to p - q\} , \tag{3.16}$$

where $\bar{n}_p$ is the stationary part of the electron distribution function with respect to the canonical momenta $\boldsymbol{p}$

$$\boldsymbol{a}_{1,2} = e\boldsymbol{E}_{1,2}/m\Omega_{1,2}^2 .$$

$\boldsymbol{E}_1$, $\boldsymbol{E}_2$ are the electric field amplitudes of the strong and weak waves, respectively, $\{\boldsymbol{p} \to \boldsymbol{p} - \boldsymbol{q}\}$ is the expression obtained by means of substitution from (3.16), φ is the phase shift between the waves, $\eta \to +0$.

The absorption coefficient α of the weak wave is obtained from (3.16) in the form

$$\alpha = \frac{8\pi}{c\sqrt{\varepsilon_0}E_2^2}\langle j(t)\,\boldsymbol{E}_2 \sin \Omega_2 t\rangle$$
$$= \frac{32\pi^2\Omega_2}{c\sqrt{\varepsilon_0}\,E_2^2}\sum_{k,l}\sum_{p,q}|M(q)|^2 N_q\bar{n}_p\; lI_k^2(\alpha_1 \boldsymbol{q})\, I_l^2(\boldsymbol{a}_2\boldsymbol{q})\,\delta(\varepsilon_{p+q} - \varepsilon_p - k\Omega_1 - l\Omega_2) \tag{3.17}$$

where $\langle\ldots\rangle$ stands for averaging over the time, $j(t)$ is the high-frequency current, ε_0 is the dielectric permeability, c is the light velocity.

In the lowest approximation to E_2 [114] we arrive at

$$\alpha = \alpha_0 F(a_1, \nu, \kappa, \gamma) \tag{3.18}$$

where α_0 is the coefficient of the single-particle absorption

$$F(a, \nu, \kappa, \gamma) = 3\int_0^1 [y^2\cos^2\gamma + \tfrac{1}{2}(1-y^2)\sin^2\gamma]\Big\{\sum_{n=0}^{\infty}(1+n\nu)^{3/2} I_n^2(a\kappa y\sqrt{1+n\nu})$$
$$+\sum_{n=1}^{\infty}|1-n\nu|^{3/2} I_n^2(a\kappa y\sqrt{|1-n\nu|})\,\mathrm{sign}(1-n\nu)\Big\}\,\mathrm{d}y \tag{3.19}$$

here $\nu = \Omega_1/\Omega_2$, $\kappa = \sqrt{2m\Omega_2}$, γ is the angle between the vectors $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$. Within the limit $a_1\kappa \ll 1$ and $\nu \ll 1$

$$F \simeq 1 + \tfrac{3}{16}(a_1\kappa)^2(1+2\cos^2\gamma)\,\nu^2 .$$

For $\nu \gg 1$

$$F \simeq 1 + \tfrac{1}{4}(a_1\kappa)^2(1+2\cos^2\gamma)\nu^{3/2} .$$

These results demonstrate that the effect of the strong electromagnetic wave involves the increase in the weak wave absorption. The above-mentioned formulae pertain to the case of scattering on acoustic phonons. Intriguing results were obtained in [114] for the case of electronic scattering on the ionized impurities. As before, within the limit $a_1\kappa \ll 1$ and for $\nu \ll 1$ we have

$$F \simeq 1 - \tfrac{1}{80}(a_1 \kappa)^2 (1 + 2\cos^2 \gamma)\nu^2 .$$

For $\nu \gg 1$

$$F \simeq 1 - \tfrac{1}{20}(a_1 \kappa)^2 (1 + 2\cos^2 \gamma)\, \nu^{-1/2} ,$$

i.e. the absorption is decreased. It should be noted that the study of the limit $(a_1\kappa) > 1$ could lead to a change in the sign of the absorption, i.e. to the possibility of the enhancement of the weak wave. A problem of this kind has not yet been considered.

Interesting peculiarities in the weak wave propagation as against the strong one were examined in [116]. The authors analyzed the range of field frequencies higher than that of the electronic collisions with the scattering particles. The external field was assumed to be inhomogeneous and has the form of the longitudinal electromagnetic wave. The main result [116] may be summarized as follows: the nonlinear interaction of the magnetic field of the weak transverse electromagnetic wave with the inhomogeneous current generated by the external field leads to the significant dependence of the weak wave length on the external field intensity at the frequencies lower than those of plasma. Of greatest interest is the fact [116] that the increased intensity of the external field may give rise to the increased length of the weak wave and after some threshold value – to the direction of the wave propagation changed to the opposite. With the usual n-Ge parameters, but with the concentration of electrons $n \sim 10^{16}\,\mathrm{cm}^{-3}$, $T \sim 10^{-15}$ erg, the external field frequency $\Omega \sim 10^{11}\,\mathrm{sec}^{-1}$, the weak field frequency $\omega \sim 10^{12}\,\mathrm{sec}^{-1}$, the electric field threshold value is several tens of V/cm.

3.3. Phonons in a strong electromagnetic wave

In addition to the peculiar features of the electromagnetic waves absorption when it is accompanied with the action of the strong high-frequency wave of a different frequency on the system of interacting electrons, the problem of the ultrasound attenuation and the generation of the acoustic phonons are interesting. Since this problem is currently central for those of acousto-electronics, we will dwell on it in more detail. First we go into the effect of "gigantic" oscillations of the ultrasound attenuation coefficient [118] ($ql \gg 1$, l is the free electron path, $\boldsymbol{q}$ is the sound wave vector). From the kinetic equation for phonons (2.32) it is obvious that the attenuation coefficient in the presence of the high-frequency field ($\Omega\tau \gg 1$) is equal to

$$\gamma_{\boldsymbol{q}} = \pi |M(q)|^2 \sum_n I_n^2\left(\frac{e\boldsymbol{E}_0\boldsymbol{q}}{m\Omega^2}\right) \sum_{\boldsymbol{p}} (f_{\boldsymbol{p}+\boldsymbol{q}} - f_{\boldsymbol{p}})\, \delta(\varepsilon_{\boldsymbol{p}+\boldsymbol{q}} - \varepsilon_{\boldsymbol{p}} - \omega_{\boldsymbol{q}} + n\Omega) . \tag{3.20}$$

If $\Omega \gg \omega_{\boldsymbol{q}}$ and $\Omega \gg q\bar{v}$ ($\bar{v}$ is the mean electron velocity), then

$$\gamma_{\boldsymbol{q}} \simeq \gamma_{\boldsymbol{q}}^{(0)}\, I_0^2\left(\frac{e\boldsymbol{E}_0\boldsymbol{q}}{m\Omega^2}\right) \tag{3.21}$$

where $\gamma_{\boldsymbol{q}}^{(0)}$ is the attenuation coefficient in the absence of the field.

Thus, with a change in the sound frequency and the field intensity the attenuation coefficient $\gamma_{\boldsymbol{q}}$ may experience "gigantic" oscillations. The study [18] may provide some considerations in favour of the fact that there is a certain analogy to this phenomenon in geometric resonance observed when sound propagating through the sample is placed in the magnetic field.

A very important example was analyzed in [119] where the electromagnetic pumping did not only alter the value of the electronic attenuation of ultrasound, but was the condition of interaction itself between the phonons and the electrons. Such a way stating the problem is clear from the following simple considerations. For the elementary act of the phonon absorption by the electron, the law of energy and momentum conservation must be observed simultaneously. If, for example, $q > 2m\bar{v}$, the process of interactions in the second order of the perturbation theory is under such conditions forbidden in the absence of the field. If the terms $n\Omega$ are included in the energy conservation law and the pumping is involved, these processes can be permitted and the sound attenuation itself can be determined. For the degenerate electron gas at $q \gg p_0$ (p_0 is the Fermi momentum), $\Omega \gg \varepsilon_0$ (ε_0 is the Fermi energy) it was shown [119] in the linear field intensity approximation that electrons can interact with sound in the region of the wave numbers,

$$\sqrt{2m\Omega + p_0^2} - p_0 \leq q \leq \sqrt{2m\Omega + p_0^2} + p_0 \,. \tag{3.22}$$

Note that for $q = \sqrt{2m\Omega}$ the attenuation coefficient (3.20) changes its sign and for

$$\sqrt{2m\Omega + p_0^2} - p_0 \leq q \leq \sqrt{2m\Omega}$$

it becomes negative. Thus, the effect of ultrasound amplification can occur. In fact, the region of wave vectors enabling the effect belongs rather to that of hypersound.

In [120] much attention was paid to the possibility of amplification of the optic phonons flux when the electromagnetic field is present. The parametric resonance of acoustic and optic phonons perceivable in the presence of electromagnetic pumping, was investigated in [121]. It was shown that during the interaction of both types of phonons with the conduction electrons, parametrical interaction occurs which is followed, for example, by the additional damping of some modes and by the enhancement of others on condition that the increment exceeds the corresponding damping decrement of electrons. Another aspect of the effect produced by the intense electromagnetic field on the phonons interacting with the conducting electrons is its impact on the processes of generation (emission) of phonons. Such a viewpoint is topical for the research into the spectral composition of phonon emission, the determination of optimal parameters for phonon generations, etc. Since we have already tackled the problem of phonon absorption with the given wave vector, we have all the rights to raise the question about the phonon peculiarities displayed during the forced irradiation when the electronic subsystem of the semiconductor is excited by electromagnetic irradiation. This problem for the case of optical phonons was given consideration in [122] and for acoustic phonons in [123].

The subject of ref. [122] was the single-phonon processes under the assumption that the electron–phonon interaction is weak. The field intensity, though, was assumed to be arbitrary and, therefore, the suggested theory was a nonlinear one with respect to the field. Calculations were also made of the rate of phonon generation for the case when the electron gas is nondegenerate and the electron distribution function is of the Boltzmann type. This paper also attached great importance to the possibility of establishing instability in the system of optical phonons. The instability is manifested in the changed sign of their linear (with respect to the electron–phonon interaction) absorption coefficient.

The rate of the phonon generation in compliance with the kinetic equation for phonons in the case of $\Omega\tau \gg 1$ is calculated from the expression like

$$G_q = 2\pi|M(q)|^2 \, V \sum_n I_n^2(aq) \int \frac{d^3p}{(2\pi)^3} f_{p+q}(1-f_p)\,\delta(\varepsilon_p - \varepsilon_{p+q} + \omega_q + n\Omega)\,. \tag{3.23}$$

We now study, using the acoustic phonons as an example, the results of [123] in the calculations of the spectral dependence of the generation rate (3.23). We will adopt the following approximations. Let us assume that the rate of the electronic relaxation due to the electron–electron collisions exceeds the rate of other types of collisions which allows us to consider the function f_p in expression (3.23) to be a quasi-equilibrium one with the effective temperature T [122, 123]. Since we care for the fulfilment of the given condition for electrons at the energies $\varepsilon_p \sim \varepsilon_0 + \Omega$ (ε_0 is the Fermi level), then, as can be demonstrated, the requirement appears to mean the inequality $\Omega \gg \varepsilon_0$. This inequality, on the one hand, does not need sufficiently high frequencies (for example, laser frequencies) and, on the other hand (at lower frequencies, for instance), picks out for consideration only semimetals or semiconductors whose ε_0 is relatively small. In the case of a pulsed field there is not such a restriction. Here we supply the results for the case when the external pumping is relatively weak (or the frequency is sufficiently high). The part of (3.23), proportional to the field intensity which is the very parameter to determine the forced generation of phonons [123], is equal to

$$\tilde{G}_q = \frac{|M(q)|^2 \, V}{4\pi q}\left(\frac{e}{mc\Omega} A_0 q\right)^2 I(\omega)\,. \tag{3.24}$$

The expression for $I(\omega)$ in the general case is rather cumbersome. Studying the condition $\Omega > \omega_D$ (the Debye frequency), $\Omega \gg T$, we have

$$I(\omega) \sim T \ln \frac{\exp((E(\omega) - \varepsilon_0)/T) + \exp(\Omega/T)}{\exp((E(\omega) - \varepsilon_0)/T) + 1} \tag{3.25}$$

where

$$E(\omega) = \tfrac{1}{2} m s^2 (\Omega/\omega)^2\,.$$

The results (3.24), (3.25) given above indicate that the generation of acoustic phonons occurs predominantly with the frequencies

$$\omega \gtrsim \Omega \sqrt{ms^2/2(\Omega + \varepsilon_0)}$$

which means that, if, for instance, $\Omega \sim 10^{15}\,\mathrm{sec}^{-1}$, then $\omega \gtrsim 10^{13}\,\mathrm{sec}^{-1}$. We can obtain in this way the sources of superhigh-frequency phonons.

3.4. Renormalized electronic spectrum

The previous sections were concerned with the impact of intense irradiation on the kinetics of quasiparticles, i.e. to put it differently, as applied to the damping, on the imaginary parts of quasiparticles mass operators. There is another aspect of the problem. In what way does the electromagnetic field determine directly the real parts of mass operators, i.e. the renormalization effects in the quasiparticles spectrum? This problem was discussed in [124, 126] (see also references cited in

[125]). It was demonstrated in [124] that a change in the electron effective mass caused by the high-frequency electromagnetic field effect on the electron–phonon interactions, may give rise to the effect of self-induced transparency of the semiconductor. The essence of this phenomenon is that, if in the weak field when the mass renormalization effects can be neglected and so can be the changes in the plasma frequency ω_p, the relation $\Omega < \omega_p$, for example, is fulfilled, then under the impact of the field due to the increasing effective mass the intensity may vary: $\Omega > \omega_p$. Thus, the semiconductor becomes "transparent" for the external field. This point needs a more detailed investigation. To determine the nonequilibrium spectrum renormalizations, it is necessary, similarly to section 2, to construct kinetic equations for anomalous averages of the type $g_p = \langle a_p a_{-p} \rangle$, then the true spectrum of quasiparticles ε_p can be defined (for more detail see [73, 127]) as the oscillation frequency of the anomalous average g_p. For the nondegenerate case and the temperature dependence neglected (i.e. the effect of "real" phonons being neglected) the electron mass operator for $\Omega\tau \gg 1$ takes on the form

$$\Sigma_p(a_0) = \sum_n \sum_q |M(q)|^2 \frac{I_n^2(a_0 q)}{\varepsilon_p - \varepsilon_{p-q} - \omega_q + n\Omega} . \tag{3.26}$$

We embark now on the study of the results for the case of dispersionless (optical) phonons $\omega_q = \omega$, the weak field of $a_0 q_{\max} \ll 1$ using the model $G/q\sqrt{V}$ for $M(q)$. The weak field limit permits to confine eq. (3.26) only to the three terms with $n = 0, \pm 1$.

The calculation of (3.26) yields the following correction $\Delta\varepsilon_p$ for the electron spectrum [124]

$$\Delta\varepsilon_p \simeq \frac{mG^2 a_0^2}{4\pi} \begin{cases} \frac{1}{3}\left(\frac{q_1+q_2}{2} - q_0\right) + \frac{1}{5}\left(\cos^2\theta + \frac{1}{2}\right)\left[\frac{1}{q_0} - \frac{1}{2}\left(\frac{1}{q_1} + \frac{1}{q_2}\right)\right] p^2 & \text{with } \Omega < \omega \\ \frac{1}{3}\left(\frac{q_2}{2} - q_0\right) + \frac{1}{5}\left(\cos^2\theta + \frac{1}{2}\right)\left(\frac{1}{q_0} - \frac{1}{2q_2}\right) p^2 & \text{with } \Omega > \omega \end{cases}$$

where $q_0^2 = 2m/\omega$, $q_1^2 = 2m/(\omega - \Omega)$, $q_2^2 = 2m/(\omega + \Omega)$, and θ is the angle between the direction of the electric field and the electron momentum.

The anisotropic effective mass is determined by

$$m^* = m(1 - \alpha/6)^{-1}$$

where

$$\alpha = \frac{m^{1/2} G^2}{2^{3/2} \pi \omega^{3/2}} \begin{cases} 1 - \frac{3}{5}(a_0 q_0)^2 (2\cos^2\theta + 1)\left[1 - \frac{q_0}{2}\left(\frac{1}{q_1} + \frac{1}{q_2}\right)\right], & \Omega < \omega \\ 1 - \frac{3}{5}(a_0 q_0)^2 (2\cos^2\theta + 1)\left(1 - \frac{q_0}{2q_2}\right), & \Omega > \omega . \end{cases}$$

As is evident from the expressions given above, the external field action, in fact, culminates in the increased effective electron mass.

In [125] the author treated the problem of the electron spectrum using the interband and intraband transitions of electrons.

Peculiar features are displayed by the electron spectrum if the effect of the strong constant electric field on the electronic excitations is taken into account. Though the specific problems associated with the constant electric field will be discussed later in section 5, we will nevertheless, furnish the results obtained in [126] since they deal with the renormalized electronic spectrum.

The expression for the real part of the electron mass operator in the considered case may be represented by

$$\Sigma_p(\boldsymbol{E}) = -\sum_q |M(q)|^2 \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau} \sin\left\{(\varepsilon_p - \varepsilon_{p-q} - \omega_q)\tau + \frac{e\boldsymbol{E}\boldsymbol{q}}{2m}\tau^2\right\}, \qquad \eta \to +0\,. \tag{3.27}$$

Let us examine the model of deformation interaction ($|M(q)|^2 = M_0^2 q/V$) and the acoustic phonons. For calculating (3.27) one can make the weak field expansion when

$$\frac{|e|Eq_{\mathrm{D}}}{m}\tau_0^2 \ll 1\,, \qquad \tau_0 = \max\left\{\frac{m}{q_{\mathrm{D}}^2}, \frac{m}{q_{\mathrm{D}}p}\right\}$$

where q_{D} is the Debye momentum.

The part of (3.27), linear with respect to the field, is given by [126]

$$\Sigma_p^{(1)} = \frac{M_0^2(e\boldsymbol{E}\boldsymbol{p})m^2 s}{2\pi p^3}\,\theta\left(\frac{p}{m} - s\right) \tag{3.28}$$

where $\theta(x)$ is the step function.

It should be emphasized here that the electron spectrum in the electric field varies not due to the electron drift, but due to the effect of the electron-acceleration by the field for the time of its interaction with the phonon.

3.5. Nonequilibrium magnons under intense electromagnetic irradiation

In section 2.3 we have derived the kinetic equations for electrons and magnons in a ferromagnetic semiconductor in case such a system is exposed to intense high-frequency electromagnetic pumping. Here we will study some of the effects which result from equations (2.41)–(2.43). It should be specified that for the present moment only a beginning is made for substantial research into the nonequilibrium properties of ferromagnetic semiconductors in those cases when the quantum kinetic equations allowing for the field impact on the processes of electron–magnon interactions must be used. Here we will discuss only the questions connected with the problem of the amplification of spin waves under the high-frequency field and with the pumping effect on the nonequilibrium value of the magnetization of the ferromagnetic semiconductor. In other words, we will dwell on those properties that are related to the spin subsystem.

Let us consider the damping of magnons γ_k associated with the triple-particle electron–magnon interactions in the high-frequency field ($\Omega\tau \gg 1$) [63]. In accordance with the kinetic equation (2.41) the expression for the magnon damping may be represented by

$$\gamma_k = \pi|\psi_3|^2 \sum_n \sum_p I_n^2\left(\frac{\lambda}{\Omega}\right)(f_{p\uparrow} - f_{p+k\downarrow})\,\delta(\varepsilon_{p+k,\downarrow} - \varepsilon_{p\uparrow} - \omega_k + n\Omega) \tag{3.29}$$

where $\lambda = e\boldsymbol{E}_0\boldsymbol{k}/m\Omega$, E_0 is the amplitude of the high-frequency electric field, and ω_k is the magnon spectrum.

Our task is to investigate whether the conditions are possible under which the value γ_k due to the varied propabilities of transitions into the field, can become negative, i.e. instability may be set in the system of magnons. Similarly to the previous examples considered in this section, we assume that the falue f_p in (3.29) is a quasi-equilibrium one (the field is pulsed and the distribution function does not manage to change noticeably). It is not difficult to check that within the weak field limit ($\lambda \ll \Omega$) one can obtain only a small correction for the usual linear damping. Therefore, we will pass over directly to the multiquantum limit ($\lambda \gg \Omega$) [63]. In this case expression (3.29) is simplified and reduced to

$$\gamma_k = \pi|\psi_3|^2 \sum_p (f_{p\uparrow} - f_{p+k\downarrow})[\delta(\varepsilon_{p+k\downarrow} - \varepsilon_{p\uparrow} - \omega_k - \lambda) + \delta(\varepsilon_{p+k\downarrow} - \varepsilon_{p\uparrow} - \omega_k + \lambda)]\,. \tag{3.30}$$

Of special interest is the fact that at $\lambda \gg \varepsilon_0$ (the degenerate case being considered) one may neglect the processes in which photons are emitted and thus

$$\gamma_k = \pi|\psi_3|^2 \sum_p (f(\varepsilon_{p\uparrow}) - f(\varepsilon_{p\uparrow} + \omega_k - \lambda))\, \delta(\varepsilon_{p+k\downarrow} - \varepsilon_{p\uparrow} - \omega_k + \lambda)\,.$$

In the approximation of the step function for the magnitude f, we arrive at the following expression [63]

$$\gamma_k = -\frac{m^2 V|\psi_3|^2}{4\pi k}\left(\frac{ekE_0}{m\Omega} - \omega_k\right) \tag{3.31}$$

and therefore, the condition $\gamma_k < 0$ in this case resembles the Cherenkov instability. If $|\gamma_k| > \gamma_{\mathrm{mm}}$ (γ_{mm} is the magnon damping caused by other types of interactions), the given effect may serve as a way of amplification of spin waves. It might be well to point out, however, that with so high intensities ($\lambda \gg \varepsilon_0$, Ω) studied the realization of parametrical instability of spin waves is perceivable, but the possibility of experimental observation of the phenomenon in question is, to our mind, doubtful.

Let us proceed to the discussion of the peculiarities of the quasistationary nonequilibrium states of magnons that can be obtained in the ferromagnetic semiconductor affected by the high-frequency electromagnetic field. The fact that under the action of the field the probabilities of transition included into the kinetic equation for magnons (2.41) are modified, is of basic importance for studying the nonequilibrium states of the magnon subsystem as the magnon nonequilibraty occurs not only due to the electron energy transferred to magnons, but also due to the direct absorption of the external field quanta by magnons via electrons. It should be noted that usually nonequilibrium states of magnons were regarded as those formed as a result of the electron energy transferred to magnons [92].

Let us dwell on the problem where of primary importance for us will be the change in the overall number of magnons brought about by the high-frequency electromagnetic field. Therefore, we will confine ourselves only to the analysis of the electron–magnon interactions that do not preserve their number of spin waves [90].

Equation (2.41) indicates that the magnon nonequilibraty is dictated both by the changed distribution function in the high-frequency field and by the direct participation of quanta of the external field changing the magnon equilibrium distribution function (by means of processes of the type $\varepsilon_{p'\uparrow} \rightleftarrows \varepsilon_{p\downarrow} - \omega \pm \Omega$). If the time of the external field action on the electron–magnon system τ_{p} exceeds the time τ_{me} of the magnon–electron relaxation, the electrons heated by the external field manage to transfer their energy to magnons. If, however, $\tau_p < \tau_{me}$, i.e. the field is a pulsed one, the only channel of nonequilibraty of the magnon system are the processes of direct transfer of the field energy to magnons, we will assume also that $\tau_p \gg \tau_{mm}$ (τ_{mm} is the magnon–magnon relaxation time), i.e. the quasistationary nonequilibrium state has enough time to be inserted in the magnon subsystem. Prior to giving the final results, the conclusion may be drawn about the properties inherent in the state.

The electronic absorption of quanta Ω of the external field appears to induce the electronic transition from the lower spin subband to the upper one (the electronic energy must increase due to the external field absorption), whereas the electronic transition from the lower subband to the upper one gives rise to a kind of extraction of magnons from the sample, i.e. tends to decrease the overall number of magnons.

Let us consider the nondegenerate case [90] and the single-quantum limit. We assume that the state is a weakly nonequilibrium one, so that the integral of magnon–magnon interactions can be approached in the τ-approximation. Defining the correction for the equilibrium distribution function from (2.41), we can write the expression for the changed magnetization relative to the saturation magnetization in the form

$$\frac{\delta M}{(\mu/v_0)} \simeq \frac{1}{2}\left(\frac{2\pi n_e}{mT}\right)^{3/2} \frac{\tau_{mm} v_0^2 (eE_0)^2 I_{sd}^2}{\Omega^2} \frac{4mT}{\theta_c a^2} I(\Delta, T) \tag{3.32}$$

where

$$I(\Delta, T) = \int_{\Delta/2T}^{\infty} \mathrm{d}x\, \mathrm{e}^{-x} \sqrt{\frac{x}{x+\Delta/T}} \left(x + \frac{\Delta}{T} - 1\right),$$

$v_0 = a^3$, a is the lattice constant, n_e is the concentration of electrons, and I_{sd} is the exchange interaction constant. Formula (3.32) is obtained under the condition that Ω, $\omega_0 \ll T$. Here are some numerical estimates of the effect. At $\theta_c = 150$ K, $\Delta \sim T \sim 100$ K, $n_e \sim 10^{16}$, $\Omega \sim 10^9\ \mathrm{sec}^{-1}$, $E_0 = 0.5$ V/cm, $\tau_{mm} \sim 10^{-7}$ sec, $I_{sd} \sim 10^{13}\ \mathrm{sec}^{-1}$ the relative value of the increase of magnetization is 10 per cent. The condition $\tau_p < \tau_{me}$ is easily satisfied since τ_{me}, as a rule [92], is much greater than 10^{-6} sec. As for taking into account the quadriparticle processes preserving the magnon number, it is reduced only to the additional channel of changing the magnetization (not nearly so effective) associated with the effect of the stimulation of magnetization [67].

The example discussed here is of interest in the sense that we can clearly see here to what, at first sight, unusual effects the exact allowance for the field may lead in the collision integrals of quasiparticles. A similar example will be analyzed also in section 4.1.

4. Nonequilibrium states of ferro- and antiferrodielectrics caused by a strong high-frequency magnetic field

This section deals with various mechanisms of excitation of spin waves in ferro- and antiferromagnets under nonresonance parallel pumping (NPP). We will also furnish here the already known results obtained in the studies of the properties of magnets in nonequilibrium states under NPPP. All the questions are discussed below on the basis of the use of the kinetic equations given in section 2.4.

4.1. Nonequilibrium magnetization of ferromagnets

We would like to remind that we will deal here only with the field frequency Ω less than the double activation energy of magnons $2\varepsilon_0$ so that parametrical resonance under parallel pumping is impossible.

A high-frequency magnetic field ($\Omega\tau_s \gg 1$, τ_s is the magnon relaxation time) affects the ferromagnet and gives rise to the nonequilibrium state in it. Besides, due to the fact that the magnon relaxation time is final such a nonequilibrium state can be stationary. The degree of nonequilibraty is dependent both on the power introduced in the system (to be more precise, of the absorbed field energy) and on the value of the relaxation times. In the nonequilibrium state that is characterized by a magnon distribution function f_p different from the Bose (equilibrium) function, the properties of the ferromagnet vary and one may observe some very interesting effects.

When analyzing the nonequilibrium properties of ferromagnets one has to solve an extraordinarily complicated problem – to find a nonequilibrium distribution function satisfying the nonlinear integral equation in the finite differences which is the case with the kinetic equation (2.49). It is clear that such a problem defies analytical solution in the general case, moreover, there is not any significant progress made in solving kinetic equations of the type (2.49). Similarly to the views expressed in [66, 68], we will consider the weak field limit where the analysis can be confined only to the single-quantum processes in the field square approximation.

$$\mu h_0/\Omega \ll 1 . \tag{4.1}$$

We will first examine the case of an ideal (without magnetic inhomogeneities) ferromagnet [65, 66].

The ferromagnet magnetization in the thermodynamically equilibrium state [91] is, as is well-known, determined by the magnon occupation numbers and its temperature dependence is attained via the dependence of the magnon distribution function $f_p(T)$. Being in the nonequilibrium state, the magnetization is determined both by the temperature and the alternating magnetic field amplitude and frequency since the distribution function starts to depend on the external pumping:

$$M(h, T) = M_0 - \mu \sum_p (|u_p|^2 + |v_p|^2) f_p \equiv M(T) - \delta M \tag{4.2}$$

where M_0 is the saturation magnetization, u_p, v_p are the coefficients of the so-called Bogolubov u-v transformations [91], $M(T)$ is the equilibrium magnetization at temperature T, and δM is the addition to the magnetization caused by the nonequilibrium state.

In the case of $M_0 \ll H$ (we remind that H is the external constant field directed along the magnetization), $|u_p| \sim 1$ and $|v_p| \ll 1$, so the nonequilibrium addition δM to the magnetization is determined mainly by the processes induced by the external field which do not preserve their magnon number [65, 66]. In the case under study the kinetic equation can be written as

$$\partial f_p/\partial t = \mathcal{L}^s_p\{f, h\} + \mathcal{L}^r_p\{f\} \tag{4.3}$$

where

$$\mathscr{L}^{\mathrm{s}}_{p}\{f, h\} = 4\pi\left(\frac{\mu h_0}{2\Omega}\right)^2 \left\{\sum_{12} |V_{p,12}|^2 [(1+f_p)f_1 f_2 - f_p(1+f_1)(1+f_2)] \right.$$
$$\times [\delta(\varepsilon_p - \varepsilon_1 - \varepsilon_2 - \Omega) + \delta(\varepsilon_p - \varepsilon_1 - \varepsilon_2 + \Omega)]$$
$$+ 2\sum_{12} |V_{1,p2}|^2 [(1+f_p)(1+f_2)f_1 - f_p f_2(1+f_1)]$$
$$\left. \times [\delta(\varepsilon_1 - \varepsilon_p - \varepsilon_2 - \Omega) + \delta(\varepsilon_1 - \varepsilon_p - \varepsilon_2 + \Omega)] \right\};$$

$\mathscr{L}^{\mathrm{r}}_{p}$ is the usual (see [91]) collision integral for magnons that differs only in a factor of the order of $1 - \frac{1}{2}(\mu h_0/\Omega)^2$ which appears by expanding the Bessel function of the zeroth order,

$$I_0^2(x) \sim 1 - \tfrac{1}{2}x^2 .$$

Equation (4.3) can be solved only by numerical methods. Here we examine the states when the nonequilibrium addition δf_p to the equilibrium distribution function $f_0(\varepsilon_p)$ is small. In this case the τ-approximation to the collision integral $\mathscr{L}^{\mathrm{r}}_{p}$ can be used [66] for determining δf_p. Allowing for the magnon spectrum in the simplest form $\varepsilon_p = \varepsilon_0 + \theta_{\mathrm{c}}(ap)^2$, we may represent the structure of the distribution function as:

$$\delta f_p \simeq \pi^{-1}\left(\frac{\mu h_0}{2\Omega}\right)^2 \frac{(\mu M_0)^2}{4\theta_{\mathrm{c}}^{3/2}} \frac{\pi}{\sqrt{\varepsilon - \varepsilon_0}} \left\{ \theta(\varepsilon - 3\varepsilon_0 - 2\Omega) \int_{\varepsilon_{01}^-}^{\varepsilon_{01}^+} \mathrm{d}\varepsilon_1 \, [(1+f_\varepsilon) \right.$$
$$\times f_{\varepsilon 1} f_{\varepsilon - \varepsilon 1 - \Omega} - f_\varepsilon (1+f_{\varepsilon 1})(1+f_{\varepsilon - \varepsilon 1 - \Omega})] + \theta(\varepsilon - 3\varepsilon_0 + 2\Omega)$$
$$\times \int_{\varepsilon_{02}^-}^{\varepsilon_{02}^+} \mathrm{d}\varepsilon_1 [(1+f_\varepsilon) f_{\varepsilon 1} f_{\varepsilon - \varepsilon 1 + \Omega} - f_\varepsilon (1+f_{\varepsilon 1})(1+f_{\varepsilon - \varepsilon 1 + \Omega})]$$
$$+ 2\left[\int_{\varepsilon_{03}^+}^{\infty} \mathrm{d}\varepsilon_1 \, ((1+f_\varepsilon)(1+f_{\varepsilon 1 - \varepsilon - \Omega}) f_{\varepsilon 1} - f_\varepsilon (1+f_{\varepsilon 1}) f_{\varepsilon 1 - \varepsilon - \Omega}) \right.$$
$$\left.\left. + \int_{\varepsilon_{03}^-}^{\infty} \mathrm{d}\varepsilon' ((1+f_\varepsilon)(1+f_{\varepsilon 1 - \varepsilon + \Omega}) f_{\varepsilon 1} - f_\varepsilon (1+f_{\varepsilon 1}) f_{\varepsilon 1 - \varepsilon + \Omega}) \right] \right\}$$

where

$$f_\varepsilon \equiv f_0(\varepsilon_p), \qquad \varepsilon_{01}^{\pm} = \tfrac{1}{2}(\varepsilon - \Omega) \pm \tfrac{1}{2}\sqrt{(\varepsilon - \varepsilon_0)(\varepsilon - 3\varepsilon_0 - 2\Omega)},$$
$$\varepsilon_{02}^{\pm} = \tfrac{1}{2}(\varepsilon + \Omega) \pm \tfrac{1}{2}\sqrt{(\varepsilon - \varepsilon_0)(\varepsilon - 3\varepsilon_0 + 2\Omega)}, \qquad \varepsilon_{03}^{\pm} = \varepsilon_0 + \tfrac{1}{4}(2\varepsilon - \varepsilon_0 \pm \Omega)^2/(\varepsilon - \varepsilon_0),$$

and $\theta(x)$ is the step function.

This distribution function has a number of peculiar features which can be associated with the nonequilibrium properties of ferromagnets. Calculating the integrals in the explicit form, we can find that in different energy regions the high-frequency field affects the state of magnons in essentially different ways. In the low-frequency region the field can induce processes of the type $\varepsilon_1 + \varepsilon_2 \pm \Omega \rightarrow \varepsilon_3$ decreasing the magnon occupation numbers. In the region $\varepsilon_p \gtrsim T$ the number of magnons grows, as a rule. This concerns the energy local change in the states. The integral contribution of the change in the distribution function to the magnetization (4.2) is given by [66]

$$\frac{\delta M(h, T)}{M_0} \simeq \frac{\mu}{VM_0} \sum_p \delta f_p = \frac{\mu^3 \tau}{16\pi^3} \left(\frac{\mu h_0}{2\Omega}\right)^2 \overline{\frac{M_0 T^2}{\theta_c^3 v_0}} I(\omega, x_0) \tag{4.4}$$

where v_0 is the elementary cell volume, $\omega = \Omega/T$, $x_0 = \varepsilon_0/T$

$$I(\omega, x_0) = I(\omega, x_0) + I(-\omega, x_0)$$

$$I(\omega, x_0) = \int_{3x_0+\omega}^{\infty} \mathrm{d}x \int_{(x-\omega)/2-\sqrt{(x-x_0)(x-3x_0-2\omega)/2}}^{(x-\omega)/2+\sqrt{(x-x_0)(x-3x_0-2\omega)/2}} \mathrm{d}x' \frac{\mathrm{e}^x(1-\mathrm{e}^{-\omega})}{(\mathrm{e}^x - 1)(\mathrm{e}^{x'} - 1)(\mathrm{e}^{x-x'-\omega} - 1)}.$$

A marked effect occurs when the inequality $T \gg \varepsilon_0$ is fulfilled, i.e. under the conditions in which ferromagnets are usually investigated experimentally. The asymptotic (4.4) at $T \gg \varepsilon_0 \gg \Omega$ yields the following expression [65]

$$\frac{\delta M(h, T)}{M_0} \simeq \frac{\sigma}{2^6 \pi^3} \frac{\mu}{M_0 v_0} \left(\frac{\mu h_0}{\varepsilon_0}\right)^2 \left(\frac{\mu M_0}{\theta_c}\right)^2 \frac{T}{\theta_c} (T\tau), \qquad \sigma \sim 1 .$$

In order to observe the effect of the nonequilibrium suppression of magnetization as well as the related phenomena, it is preferable to deal with magnets having, firstly, a small exchange energy (less or in the order of some tens of absolute degrees) and, secondly, small activation energies ε_0 (for example, films magnetized in the plane). Besides, very pure samples are preferable whose eigenvalue of the spin relaxation time is relatively small ($\tau^{-1} \sim 10^6$–$10^7\ \mathrm{sec}^{-1}$). Equally intriguing, to our mind, is the situation when the ferromagnet is in the vicinity of the phase transition of the second kind of the spin reorientation type. In this case a drastic decrease of the magnon mode $\varepsilon_0 \sim \mu\sqrt{|H_c - H|H + \Delta_0^2}$ takes place (H_c is the field of the spin reorientation, Δ_0 is the gap due to the magnetostriction interaction). However, not less perceivable is the interval $\tau^{-1} \ll \Omega \ll \varepsilon_0$ in which the effect [55] leads to the nonequilibrium renormalization of the transition fields.

We now turn our attention to the description of another limiting situation when magnon collisions with impurities [67] and dislocations [68] are the most frequent, and the nonequilibrium state of the magnons is therefore dictated by the elastic collisions of magnons with magnetic impurities.

The kinetic equation including the effect of the alternating magnetic field on the magnon–impurity collisions has been given in section 2.4 (formula (2.49)). We confine ourselves, as before, to the case of the weak field (4.1) and the high-frequency approximation $\Omega\tau_s \gg 1$. In contrast to the case (4.3) the source of nonequilibraty associated with the magnon–impurity interactions preserves the number of magnons changing only their distribution over the momenta. For the momentum isotropic part of the distribution function, the term related to pumping in the kinetic equation can be expressed in the nonintegral form. Expanding the function in terms of the Legendre polynomial, the equation for the isotropic function f_ε [68] ($M_0 \ll H$) is written in the form

$$\frac{\partial f_\varepsilon}{\partial t} = -\frac{n_i v_0 U^2 m}{16\pi\theta_c^{3/2}}\left(\frac{\mu h_0}{\Omega}\right)^2 \left\{\sqrt{\varepsilon+\Omega-\varepsilon_0}\left[\frac{3}{2}\left(\frac{1}{\varepsilon}+\frac{1}{\varepsilon+\Omega}\right)-\frac{2}{\sqrt{\varepsilon(\varepsilon+\Omega)}}\right](f_\varepsilon - f_{\varepsilon+\Omega})\right.$$

$$\left. + \sqrt{\varepsilon-\Omega-\varepsilon_0}\left[\frac{3}{2}\left(\frac{1}{\varepsilon}+\frac{1}{\varepsilon-\Omega}\right)-\frac{2}{\sqrt{\varepsilon(\varepsilon-\Omega)}}\right](f_\varepsilon - f_{\varepsilon-\Omega})\,\theta(\varepsilon-\Omega-\varepsilon_0)\right\} + \mathscr{L}^{\tau}_{\varepsilon}\{f\} \tag{4.5}$$

where $\mathscr{L}^{\tau}_{\varepsilon}(f)$ designates, similarly to (4.3), the collision integrals due to the nonelastic processes of relaxation; $m \equiv 4\pi\mu M_0$, n_i is the concentration of impurities, U is the constant of the magnon interaction with the impurity (in the exchange mechanism of interaction $V \sim \theta_c$).

With not too small concentrations of impurities [67] the mechanism of spin waves excitation controlled by the source of nonequilibraty (4.5) is far more effective than the mechanism involving the magnon–magnon interactions induced by the field. However, in contrast to the latter, we would like to emphasize again, that the magnon–impurity mechanism only redistributes the magnons, over the energies. It is not difficult to notice that according to (4.5) the absorption of the external field quanta by the magnon system gives rise to the increase in the mean magnon energy accompanied by the decreased number of magnons in the low-frequency part of the spectrum at the expense of their increase in the high-frequency region. Speaking in terms of the effective temperature, local with respect to the energy, this means that the general "warming" of the system is associated with the "cooling" of magnons in the low-frequency part of the spectrum.

In spite of the preserved overall number of magnons in the case under consideration, the ferromagnetic magnetization in compliance with formula (4.2) must vary. It remains now to be ascertained what direction such a variation may take. The structure of the coefficient $(|u_p|^2 + |v_p|^2)$ [91] is such that it decreases with the increasing wave vector $\boldsymbol{p}$ which indicates that the magnetization responds, mainly, to the change in the low-frequency states of magnons.

But, as has already been noted, it is in the low-frequency region that magnons are cooled by pumping. Consequently, the ferromagnetic magnetization may, when the main mechanism of nonresonance parallel pumping is the scattering of the spin waves on the impurities, exceed the thermodynamically equilibrium value determined by the temperature T. This phenomenon by analogy with the corresponding phenomenon in the theory of superconductivity (see [71]) received the name of the stimulation of magnetization [67].

If we study the kinetic equation (4.5) in the τ-approximation, then in the case of $\Omega \ll 2\varepsilon_0 \ll T$ the relative increase in the magnetization [67] is equal to

$$\frac{|\delta M|}{M_0} \sim 10^{-1}(n_i v_0)(\Omega\tau)\left(\frac{U}{\theta_c}\right)^2\left(\frac{\mu h_0}{\Omega}\right)^2\left(\frac{\Omega}{\theta_c}\right)\left(\frac{T}{\varepsilon_0}\right)\left(\frac{\mu M_0}{\varepsilon_0}\right)^3 \tag{4.6}$$

where τ is the nonelastic relaxation time of magnons.

The effect of the stimulation of magnetization begins to dominate the effect of suppression [68] with the concentrations of impurities satisfying the inequality

$$10^2(n_i v_0)\left(\frac{\mu M_0}{T}\right)\left(\frac{U}{\theta_c}\right)^2\left(\frac{\theta_c}{\varepsilon_0}\right)^2 > 1\,. \tag{4.7}$$

In view of the great value of the matrix element for the magnon–impurity interactions ($U \sim \theta_c$ in the exchange model), the condition (4.7) is actually fulfilled even with the sufficiently small concentrations n_i. For example, at $T \sim 300$ K, $\theta_c \sim 500$ K, $\mu M_0 \sim 0.02$ K, $\varepsilon_0 \sim 0.3$ K, the condition (4.7) consists in the inequality $(n_i v_0) > 10^{-4}$.

A more pronounced effect may be expected in the study of the multiquantum case [70]. Modern experimental facilities make it possible to obtain the fields of the amplitude $h_0 \gtrsim 10^2$ Oe within the pulsed regime. So, for the field frequencies $\Omega \lesssim 10^9\,\mathrm{sec}^{-1}$ the situation is realized when the field is absorbed in the multiquantum way ($\mu h_0 > \Omega$). To examine the change in the ferromagnetic magnetization in the multiquantum case is a rather complicated task. Still, its solution is possible with the sufficiently small concentration of impurities ($(n_i v_0) \ll 1$) when, irrespective of the big value of the field ($\mu h_0 \gg \Omega$), the effective constant of the field coupling with magnons is small [70]. In this case the following result is obtained for the relative increase in the magnetization ($\Omega \ll \mu h_0 \ll \varepsilon_0 \ll T$)

$$\frac{|\delta M|}{M_0} \simeq \gamma_0 (n_i v_0) \left(\frac{\mu M_0}{\theta_c}\right) \left(\frac{\mu M_0}{\tau^{-1}}\right) \left(\frac{\mu h_0}{\varepsilon_0}\right)^2 \left(\frac{\mu M_0}{\varepsilon_0}\right)^4 \frac{T}{\varepsilon_0}$$

where $\gamma_0 \sim 10$.

Linear magnetic inhomogeneities – dislocations – as contrasted to point magnetic inhomogeneities – impurities – usually influence the processes of spin waves scattering much more effectively [128, 129]. Also more effective is the nonresonance parallel pumping due to the magnon-dislocation interactions. It is very convenient to consider the model of ring dislocations arranged chaotically [128, 129]. For a model like this under the condition of $\Omega \ll 2\varepsilon_0 \ll T$, the effect of increasing magnetization in the single-quantum case [68] is equal to

$$\frac{|\delta M|}{M_0} \sim 10^{-2} (n_d b^2) \left(\frac{R^3}{v_0}\right) \left(\frac{\mu M_0}{\theta_c}\right)^3 \left(\frac{\mu h_0}{\varepsilon_0}\right)^2 (T\tau) \tag{4.8}$$

where n_d is the concentration of dislocations (the ratio of the full length of dislocations to the crystal volume), b is the mean Burgers vector, R is the mean radius of the dislocation ring. For $h_0 \sim 1$ Oe, $n_d \sim 10^8\,\mathrm{cm}^{-2}$, $b \sim 10^2\,a$, $R \sim 10^{-3}$ cm, $m/\theta_c \sim 10^{-3}$, $\Omega/2\varepsilon_0 \sim 10^{-1}$, $T \sim 300$ K, $\tau \sim 10^{-7}$–10^{-8} sec, the relative increase of magnetization may be 10^{-3}–10^{-2}.

The work [130] mentioned the fact that similar effects may show up also in the case of intense sound pumping in the ferromagnet if the sound frequency Ω is less than $2\varepsilon_0$. In contrast to the longitudinal alternating magnetic field (having the frequency $\Omega < 2\varepsilon_0$) which, as has been stated, is absorbed only due to the processes of magnon interactions, the sound field quanta are absorbed directly due to the magnetostriction interactions.

The processes like $\varepsilon_k + \Omega \to \varepsilon_{k'}$ lead to the increased mean energy of magnons with their overall number being preserved ($\Omega < 2\varepsilon_0$). The kinetic theory of the intense sound pumping effect on magnons is developed in [131]. We will not go into these problems here since they are beyond the scope of the present report. For the sake of comparison we will supply the result [130] for the increased magnetization. For the case of longitudinal sound ($ql \gg 1$) whose amplitude is u_0, and directed parallel to the constant magnetic field, we have

$$\frac{|\delta M|}{M_0} \simeq \frac{1}{8\pi} \left(\frac{u_0}{a}\right)^2 (\Omega\tau) \left(\frac{\Omega}{\theta_c}\right)^2 \left(\frac{T}{\theta_c}\right) I\left(\frac{v_0}{T}\right) \tag{4.9}$$