where $\nu_0 = \varepsilon_0 + \theta_D^2/4\theta_c$,

$$I\left(\frac{\nu_0}{T}\right) = \int_{\nu_0/T}^{\infty} \frac{dx\,(x-x_0)\,e^x}{x\,(e^x-1)^2} \sqrt{x^2 - a^2 \frac{4(x-x_0)b - (\Omega/T)^2}{4(x-x_0)b}}\,,$$

$$b = \Omega^2/T^2\,, \qquad x_0 = \varepsilon_0/T\,, \qquad a = m/T\,.$$

For

$$\varepsilon_0/T \ll \theta_D^2/4\theta_c T \ll 1$$

$$I(\nu_0/T) \simeq \ln(\nu_0/T)\,.$$

4.2. Field absorption in ferromagnets

We will pass over to the discussion of the problems related to the employment of the kinetic equations (2.49) for the calculation of the imaginary part of the high-frequency magnetic susceptibility χ''. As before we will give only the results concerning the nonresonance parallel pumping for $\Omega\tau \gg 1$.

The field energy absorbed per unit time is calculated as $T\dot{S}$ where $\dot{S}$ is the derivative with respect to the time from the entropy S of the nonequilibrium magnon gas

$$S = \sum_p \{(1+f_p)\ln(1+f_p) - f_p \ln f_p\}\,.$$

For the magnitude χ'' there is a relation

$$\dot{Q} = \frac{h_0^2}{8\pi}\,\Omega\chi''\,. \tag{4.10}$$

Thus, for calculating χ'', it is necessary, firstly, to find the nonequilibrium distribution function of magnons satisfying the corresponding kinetic equation and, secondly, to calculate the integral entering the expression (4.10). A problem like this has not yet been solved completely, though for the single-quantum or even multiquantum case [70] when it is sufficient to be confined only to the linear approximation with respect to the introduced power, calculations of this kind were carried out in [66, 68] for ferromagnets, and for antiferromagnets in [69].

Let us start from the single-quantum case (4.1). In a pure (without impurities) ferromagnet, as has been stated above, the main contribution to the field absorption is made by triple magnon–magnon interactions [66]. Their corresponding imaginary part of the high-frequency magnetic susceptibility is equal to

$$\chi'' \simeq \frac{\nu}{2^3\pi^2}\frac{\mu^2}{v_0\Omega}\left(\frac{\mu M_0}{\theta_c}\right)^2\left(\frac{T}{\varepsilon_0}\right)\left(\frac{T}{\theta_c}\right), \qquad \nu \sim 0.03 \tag{4.11}$$

for $M_0 \ll H$, $T \gg \varepsilon_0 \gg \Omega$. The numerical estimates [66] of this magnitude indicate that for $\mu M_0 \sim 0.1$ K, $\varepsilon_0 \sim 2$ K, $T \sim 300$ K, $\theta_c \sim 500$ K, $\Omega \sim 10^7\,\text{sec}^{-1}$, $\chi'' \sim 10^{-3}$. Such an absorption can, evidently, be easily observed experimentally, though, we do not know as yet any experimental works devoted to this problem.

We will now pass over to the discussion of the magnon–impurity collisions. Since the interaction constant U can be in its order of magnitude of the exchange energy, we can expect in this case a greater value of absorption. In the single-quantum limit in the area of applicability of the result (4.11) the magnitude χ'' due to the magnon–impurity scatterings is equal to [68]

$$\chi'' \simeq \frac{1}{16\pi^2}\left(\frac{\mu^2 n_i}{\Omega}\right)\left(\frac{4\pi\mu M_0}{\theta_c}\right)\left(\frac{U}{\theta_c}\right)^2\left(\frac{T}{\varepsilon_0}\right). \tag{4.12}$$

Still greater absorption takes place in the multiquantum limit [70]. The "rate of energy accumulation by the magnon", i.e. the value of energy absorbed per unit time by the unit volume $\dot{Q}$ is calculable only in the multiquantum case under the condition that $\Omega \ll \mu h_0 \ll \varepsilon_0$ [70]. At sufficiently high temperature, $T \gg \varepsilon_0$, we have

$$\dot{Q} \sim (n_i v_0)\left(\frac{T}{\theta_c}\right)\left(\frac{\mu M_0}{\varepsilon_0}\right)^4 (\mu h_0)^2 . \tag{4.13}$$

4.3. Nonresonance parallel pumping in the conditions of parametric resonance

We will discuss here the results pertained to the case of $\Omega > 2\varepsilon_0$, i.e. to the situation where an external field quantum can create two magnons. However, we will dwell on the situation under which precisely the main channel of magnon creation is provided by the nonresonance processes that are guaranteed by the alternating magnetic field effect on the elementary acts of magnon interactions.

Let us assume that a ferromagnet is available whose magnon–impurity scatterings are essential. We will estimate then the probability of creation of two magnons "via impurity", for example, in the case of single-quantum processes. The estimations made below are rather crude and necessary only for the explanation why during parametric resonance, in a number of cases, one may neglect the purely resonance excitation of spin waves and consider only the nonresonance processes. As is evident from the Hamiltonian (2.46), the probability of creation of two magnons via the impurity is proportional to

$$(\mu h_0/\Omega)^2\, U^2 N_i$$

where N_i is the number of impurities per elementary cell. The probability of creation of two magnons due to the direct resonance action of the field is proportional to $(\mu h_0)^2$. Thus, if, for example, $\Omega \sim 10^{11}\,\mathrm{sec}^{-1}$ and $U \sim 10^{13}\,\mathrm{sec}^{-1}$ [132], it may be expected that with $N_i > 10^{-4}$ the nonresonance processes of creation of magnons are the principal ones. Indeed, as was shown in [68], the condition in question is more rigorous and takes on the form of

$$(n_i v_0)\sqrt{\frac{\theta_c}{\varepsilon_0}}\left(\frac{\Omega - 2\varepsilon_0}{2\varepsilon_0}\right)^{3/2} > 20\,,$$

i.e. the condition is defined not only by the concentration of impurities n_i, but also by the width of the source of nonequilibraty by the magnitude $(\Omega - 2\varepsilon_0)$.

In the case when the main mechanism of spin waves scattering is governed by the chaotically arranged dislocations, the magnon pumping takes, in the main, the nonresonance way [68] when

$$(n_d b^2)\frac{R^3}{v_0}\frac{\Omega^2}{\theta_c^{3/2}\varepsilon_0^{1/2}}\left(\frac{\Omega-2\varepsilon_0}{2\varepsilon_0}\right)^{7/2} > 10^2 .$$

If the relative width of the source of nonequilibraty $(\Omega - 2\varepsilon_0)/2\varepsilon_0 > 0.1$, then the specified inequality is attained beginning with $n_d \sim 10^8\,\mathrm{cm}^{-2}$, $R \sim 10^{-3}\,\mathrm{cm}$, $b \sim 10^2\, a$, $\theta_c \sim 500\,\mathrm{K}$. It should also be stressed that the resonance mechanism [96] of the spin waves pumping is of threshold character, which, too, distinguishes it from the nonresonance one. And, lastly, the character of the arising nonequilibrium states in both cases is different. The parametric resonance leads to the magnon pumping in the region with energy width in the order of τ_s^{-1} near $\Omega/2$, whereas the nonresonance creation of magnons develops in the interval $(\varepsilon_0, \Omega - \varepsilon_0)$ which exactly is determined by the notion of "width of the source" adopted above.

We will furnish now the results of the calculation of the imaginary part of the high-frequency magnetic susceptibility. In contrast to the example of $\Omega < 2\varepsilon_0$ studied in section 4.2, it is of practical value to survey only the single-quantum processes. This is connected with the fact that ordinarily when the value of the constant magnetic field H is of the order of or more than 1 kOe and, consequently, $\varepsilon_0 \gtrsim 10^{10}\,\mathrm{sec}^{-1}$, for obtaining the inequality $\mu h_0 > \Omega$ $(\mu h_0 > 2\varepsilon_0)$ experimentally, the following amplitudes of the alternating magnetic field are needed: $h_0 \gtrsim 10^3$ Oe. High-frequency magnetic fields of this amplitude, at least currently, are rather exotic.

The value of χ'' stemming from the processes of type $\Omega \to \varepsilon_k + \varepsilon_{k'}$ which develop in the impurities, is equal to [68]

$$\chi'' \simeq \frac{2\zeta_1}{\pi}(n_i v_0)\left(\frac{\mu^2}{v_0\Omega}\right)\frac{U^2 m^2}{\theta_c^3\Omega} I_1(x_0, \omega) \tag{4.14}$$

where $x_0 = \varepsilon_0/T$, $\omega = \Omega/T$, $\zeta_1 \sim 10^{-1}$,

$$I_1(x_0, \omega) = \int_{x_0}^{\omega - x_0} \mathrm{d}x \frac{x\sqrt{(x-x_0)(\omega - x_0 - x)}}{(\omega - x)^2(\mathrm{e}^x - 1)(\mathrm{e}^{\omega - x} - 1)} .$$

At room temperatures when $\Omega \ll T$ we have

$$\chi'' = \frac{\zeta_1}{4}(n_i v_0)\left(\frac{\mu^2}{\Omega v_0}\right)\left(\frac{U}{\theta_c}\right)^2\frac{(4\pi\mu M_0)^2}{\Omega\theta_c}\frac{T}{\varepsilon_0}\frac{(\Omega - 2\varepsilon_0)^2}{\varepsilon_0^{1/2}(\Omega - \varepsilon_0)^{3/2}} . \tag{4.15}$$

For the case of chaotically arranged dislocation rings we obtain the following result

$$\chi'' = \frac{16\zeta_1}{35} n_d \frac{b^2 R^3}{v_0}\left(\frac{\mu^2}{v_0\Omega}\right)\frac{(4\pi\mu M_0)^2 T^2}{\Omega\theta_c^3} I_2(x_0, \omega) \tag{4.16}$$

where

$$I_2(x_0, \omega) = \int_{x_0}^{\omega - x_0} \mathrm{d}x \frac{x(x - x_0)^{3/2}(\omega - x_0 - x)^{3/2}}{(\omega - x)^2(\mathrm{e}^x - 1)(\mathrm{e}^{\omega - x} - 1)} .$$

The magnitude (4.16) can be estimated at $T \gg \Omega$. Then

$$\chi'' = \frac{24\pi^3\zeta_1}{35}(n_d b^2)\left(\frac{R^3}{v_0}\right)\left(\frac{\mu^2}{v_0\Omega}\right)\left(\frac{\mu M_0}{\theta_c}\right)^2 \frac{T}{\theta_c}\frac{1}{\Omega}\left[\frac{\Omega^2 + 4\Omega\varepsilon_0 - 4\varepsilon_0^2}{4\sqrt{\varepsilon_0(\Omega - \varepsilon_0)}} - \Omega\right]. \tag{4.17}$$

Even with not too strong concentrations of dislocations n_d the value of χ'' due to the nonresonance pumping may be rather big. For example, for $n_d \sim 10^8\,\mathrm{cm}^{-2}$, $R \sim 10^{-3}\,\mathrm{cm}$, $b \sim 10^2 a$, $\theta_c \sim 500\,\mathrm{K}$, $\Omega - 2\varepsilon_0 \sim 2\varepsilon_0$ the value of $\chi'' \sim 10^{-1}$–10^{-2}.

The energy absorption of the alternating field produces nonequilibrium states in the system of magnons which are characterized by the excessive number of quasiparticles (magnons) as compared to the equilibrium state. This, in its turn, determines kinetic and thermodynamic parameters of the nonequilibrium ferromagnet. In particular, the nonequilibrium increase in the number of spin waves, as follows from formula (4.2), decreases the magnetization of the sample [68] under nonresonance pumping of spin waves. Here are some results concerning the change in the magnetization δM.

The kinetic equation for the zeroth harmonic of the isotropic part of the magnon distribution function f_p for $M_0 \ll H$ including the nonresonance creation of magnons ($\Omega > 2\varepsilon_0$) under the parallel pumping proceeding from (2.46) acquires the form

$$\frac{\partial f_\varepsilon}{\partial t} = \zeta_1(n_i v_0)\frac{U^2 m^2}{\theta_c^{3/2}}\left(\frac{\mu h_0}{\Omega}\right)^2 \frac{\sqrt{\Omega - \varepsilon_0 - \varepsilon}}{(\Omega - \varepsilon)^2}(1 + f_\varepsilon + f_{\Omega-\varepsilon})\,\theta(\Omega - \varepsilon_0 - \varepsilon) + I_\varepsilon\{f\} + \mathscr{L}_\varepsilon^{\mathrm{r}}\{f\}, \tag{4.18}$$

where $\zeta_1 \sim 10^{-1}$, $I_\varepsilon\{f\}$ is the source of nonequilibraty describing the processes of redistribution of magnons under the action of the field and coinciding in its form with the right-hand side of equation (4.5), $\mathscr{L}_\varepsilon^{\mathrm{r}}\{f\}$ stands, as before, for the usual collision integrals for magnons.

In [68] eq. (4.18) for the stationary case was studied in the τ-approximation for the collision integral $\mathscr{L}_\varepsilon^{\mathrm{r}}\{f\}$ and as a result the following expression was obtained for δM for small departures from equilibrium (Ω, $\varepsilon_0 < T$)

$$\frac{\delta M}{M_0} = \frac{\zeta_1}{4\pi}(n_i v_0)\frac{U^2 m^2}{\tau^{-1}\theta_c^3}\left(\frac{\mu h_0}{\Omega}\right)^2\left(\frac{T}{\Omega}\right)\frac{(\Omega - 2\varepsilon_0)^2}{\Omega\sqrt{\varepsilon_0(\Omega - \varepsilon_0)}}\left\{\frac{\Omega^2}{4\varepsilon_0(\Omega - \varepsilon_0)} + 1\right\}. \tag{4.19}$$

The suppression of magnetization in a ferromagnet having dislocations is described by the formula ($T \gg \Omega > 2\varepsilon_0$)

$$\frac{\delta M}{M_0} = \frac{8\pi^2\zeta_1}{70}(n_d b^2)\left(\frac{R^3}{v_0}\right)\left(\frac{\mu M_0}{\theta_c}\right)^2\left(\frac{\mu h_0}{\Omega}\right)^2(\Omega\tau)\left(\frac{T}{\varepsilon_0}\right)\frac{(\Omega - 2\varepsilon_0)^2}{\varepsilon_0^{1/2}(\Omega - \varepsilon_0)^{3/2}\theta_c}$$
$$\times\left\{\frac{(\Omega - 2\varepsilon_0)^2\,[\Omega^2 + 2(\Omega - \varepsilon_0)\varepsilon_0]}{\Omega^3} - \frac{[\sqrt{\Omega - \varepsilon_0} - \sqrt{\varepsilon_0}]^2}{[\sqrt{\Omega - \varepsilon_0} + \sqrt{\varepsilon_0}]^2}[\Omega + 4\sqrt{\varepsilon_0(\Omega - \varepsilon_0)}]\right\}. \tag{4.20}$$

Similar phenomena may also be developed in antiferromagnets. However, for this case the situation of $\Omega > 2\Delta_1$ (we remind that Δ_1 is the activation energy of low-frequency magnons) has been analyzed so far only with allowance for the parametrically resonance processes. We will furnish below the results concerning antiferromagnets with magnetic anisotropy of the "light plane"-type for which the equations given in section 2.4 are valid.

4.4. Mechanisms of absorption of a nonresonance field in antiferromagnets

Similarly to the previous sections, we will restrict our discussion to single-quantum processes changing over in eq. (2.49) to the limit of $\mu h_0 \ll \Omega$.

The basic processes that control kinetic and relaxation phenomena in an antiferromagnet with magnetic anisotropy of the "light-plane" type, have been studied in section 2.4. Here we will supply the results [69] enabling comparison of the effectiveness of various elementary processes for magnons during the absorption of the nonresonance alternating magnetic field ($\Omega < 2\Delta_1$). Let us remember that the energy absorption of the field is characterized by the imaginary part of the high-frequency magnetic susceptibility χ''. Taking into account the structure of the kinetic equation (2.49), the expression for χ'' can be represented as a sum

$$\chi'' = \chi''_{2s} + \chi''_{4s} + \chi''_{sp} + \chi''_{i} + \chi''_{d}$$

where each of the terms describes the contributions due to triple, quadruple magnon–magnon, magnon–phonon, magnon–impurity interactions, magnon scattering on dislocations.

For triple magnon–magnon interactions we obtain the result [69]

$$\chi''_{3s} = \frac{1}{2^4\pi^2}\left(\frac{\mu^2}{v_0\Omega}\right)\left(\frac{J_0}{\theta_N}\right)\left(\frac{\omega_H}{\theta_N}\right)^2\left(\frac{\mu H}{\theta_N}\right)^2\left(\frac{T}{\theta_N}\right)\left(\frac{\Delta_2}{\Delta_1}\right)^6\left(\frac{T}{\Delta_1}\right)^2 \varphi(\delta_1, \beta) \tag{4.21}$$

where J_0 is the constant of the homogeneous exchange interaction,

$$\delta_1 = \Delta_1/T,\ B = \Delta_2^2/2\Delta_1^2,\ \omega_H = \mu(2H + H_D),$$

$$\varphi(\delta_1, \beta) = \int_{\delta_1}^{\infty} \frac{dx \exp(-\beta x)}{x\,(e^x - 1)}\{(x + \sqrt{x^2 - \delta_1^2})^4 \exp(\beta\sqrt{x^2 - \delta_1^2}) - (x - \sqrt{x^2 - \delta_1^2})^4 \exp(-\beta\sqrt{x^2 - \delta_1^2})\}\,.$$

Expression (4.21) is obtained in the approximation $T \sim \Delta_1 \ll \Delta_2$ which is attainable experimentally. Under these conditions, as can easily be seen, the function φ is exponentially small.

The effect of nonresonance parallel pumping on the quadruple magnon–magnon interactions yields the following value in the order of magnitude ($T \sim \Delta_1$)

$$\chi''_{4s} \sim \left(\frac{\mu^2}{v_0\Omega}\right)\left(\frac{J_0}{\theta_N}\right)^2\left(\frac{\Delta_1}{\theta_N}\right)^4\left(\frac{\omega_H}{\theta_N}\right)^2\left(\frac{T}{\theta_N}\right). \tag{4.22}$$

Magnon–magnon processes determine the single-quantum absorption of the field which can be described by the imaginary part of the high-frequency susceptibility of the type

$$\chi''_{sp} = \frac{208}{3\pi^2}\left(\frac{\mu^2}{v_0\Omega}\right)\left(\frac{J_0}{\theta_N}\right)^2\left(\frac{B}{\theta_N}\right)^2\left(\frac{\omega_H}{\theta_N}\right)\left(\frac{\omega_H}{\theta_D}\right)\left(\frac{T}{\theta_F}\right)A(\delta_1) \tag{4.23}$$

where B is the magnetostriction constant, $\theta_F = \rho v_0 c^2$, ρ is the density of the matter, c is the sound velocity,

$$A(\delta_1) = \int_{\delta_1}^{\infty} \frac{dx\, e^x \sqrt{x^2 - \delta_1^2}}{(e^x - 1)\, \mathrm{sh}\, x} \{x\, \mathrm{cth}\, x - 2\}\,.$$

When analyzing the spin waves scattering on the impurities in antiferromagnets, two mechanisms are normally distinguished. The first one, which can be termed as an exchanged one, is due to the fact that exchange interaction in a sample with impurity may be represented as [133]

$$\Big\{I + I' v_0 \sum_{\alpha} \delta(\boldsymbol{r} - \boldsymbol{r}_\alpha)\Big\} \boldsymbol{M}_1 \boldsymbol{M}_2$$

where $\boldsymbol{r}_\alpha$ are impurity coordinates, I is the dimensionless constant of homogeneous exchange, and $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$ are magnetizations of sublattices. The other mechanism, which may be termed as a striction one, consists in magnon scattering on the field of deformations produced by the impurity in the lattice.

For the contribution to χ'' from the magnon scattering on impurities we obtain [69] the expression

$$\chi_i'' = \frac{26\kappa^2}{135\pi^2} n_i \left(\frac{\mu^2 \Omega}{v_0 T^2}\right) \left(\frac{J_0}{\theta_N}\right)^2 \left(\frac{B}{\theta_N}\right)^2 \left(\frac{\omega_H}{\theta_N}\right)^2 C(\delta_1) \tag{4.24}$$

where $\kappa = (1+\sigma)/(1-\sigma)$, σ is the Poisson coefficient, $B = 4M_0^2 \lambda_3 v_0$, λ_3 is the magnetostriction constant,

$$C(\delta_1) = \int_{\delta_1}^{\infty} \frac{dx\, (x^2 - \delta_1^2)}{x^4 (e^x - 1)^2}\,.$$

Magnon scattering on dislocations plays an essential role in the relaxation processes in antiferromagnets with the magnetic anisotropy of the "light plane"-type [129]. In a variety of cases it is magnon-dislocation interactions that determine the effective damping of spin waves. These interactions certainly play a significant part as the mechanism of stochastization of magnons under nonresonance parallel pumping.

In the model of chaotically distributed ring dislocations for the contribution to χ'' we have [69]

$$\chi_d'' = \frac{\nu}{35\pi^2} n_d \left(\frac{J_0}{\theta_N}\right)^2 \left(\frac{B}{\theta_N}\right)^2 \left(\frac{\omega_H}{\theta_N}\right) \left(\frac{\mu^2 \Omega}{v_0 T^2}\right) \left(\frac{b}{a}\right)^2 \frac{R^3}{a} C(\delta_1) \tag{4.25}$$

where

$$\nu = 13(\gamma^2 - 1)(2\gamma^2 - 3) + 2,\ \gamma^2 = \eta/(\lambda + 2\eta)\,,$$

λ, η are the Lame coefficients.

For the antiferromagnet $MnCO_3$, beginning with very low concentrations $n_d \sim 10^8$ and $b/a \sim 3 \div 10$, $R \sim 10^{-3}$ cm, $\chi'' \sim 10^{-3} \div 10^{-2}$. For the increased concentrations when $n_d \sim 10^9 \div 10^{10}$, the value of χ_d'' amounts to the values observed during parametric resonance. As for absorption in an ideal (without defects) antiferromagnet, the values of χ'' are essentially smaller than those for a ferromagnet (section 4.2). This may be attributed to the fact that, firstly, the interaction with the nonresonance field in an antiferromagnet is weaker than in a ferromagnet and, secondly, antiferromagnets are ordinarily studied at lower temperatures.

4.5. Damping of ultrasound in nonequilibrium antiferromagnets

In this section we will supply an example where the high-frequency magnetic field ($\Omega\tau \gg 1$) can change radically the kinetic properties of phonons interacting with magnons in the antiferromagnet having the magnetic anisotropy of the "light-plane" type. We will demonstrate that in the antiferromagnets in which the sound velocity c exceeds the spin wave velocity s with the nonresonance alternating magnetic field applied parallel to the constant field ($\Omega > 2\Delta$, $\Delta \equiv \Delta_1$), the phonon damping due to the phonon–magnon interactions can be negative [134]. In other words, amplification of ultrasound phonons is perceivable under nonresonance parallel pumping.

It is well-known [91] that in antiferromagnets of the light-plane type the sound damping with the frequency $\omega_q > 2\Delta$ (where $\omega_q = cq$) assuming that $c > s$ (let us remember that the spin wave velocity enters the magnon spectrum in the following fashion, $\varepsilon_k = \sqrt{\Delta^2 + (sk)^2}$) stems only from phonon–phonon interactions. It is attributed to the fact that the magnon–phonon processes of the type $\omega_q \to \varepsilon_k + \varepsilon_{q-k}$ are forbidden by the laws of energy conservation ($\omega_q < 2\Delta$) and as for the processes of the type $\omega_q \pm \varepsilon_k \to \varepsilon_{q\pm k}$, the laws of energy and momentum conservation are incompatible because of the condition $c > s$ ($MnCO_3$ is an example of such an antiferromagnet).

The impact of the external alternating magnetic field (for $\Omega\tau \gg 1$, τ is the magnon relaxation time) manifests itself in the development of new processes in which the external field quanta participate (section 2.4). In the single-quantum approximation such impact on the magnon–phonon interactions consists in the development of processes of the type $\omega_q \pm \Omega \to \varepsilon_k + \varepsilon_{q-k}$ and $\omega_q + \varepsilon_k \pm \Omega \to \varepsilon_{q+k}$. Note that if $\Omega < 2\Delta$ and $\omega_q < 2\Delta$, processes of the type $\omega_q \pm \Omega \to \varepsilon_k + \varepsilon_{q-k}$ and $\omega_q + \varepsilon_k + \Omega \to \varepsilon_{q+k}$ are forbidden, whereas the process $\omega_q + \varepsilon_k - \Omega \to \varepsilon_{q+k}$ may result in damping other than zero even on the condition that $c > s$.

In the situation when the external nonresonance ($\Omega > 2\Delta$) alternating magnetic field applied parallel to the constant field lying in the basal plane of the crystal, the equation for the phonon damping coefficient ($ql \gg 1$) of the type below follows from the kinetic equation for the phonon distribution function which is written similarly to the one for magnons (see section 2.4)

$$\alpha_q = 4\pi \sum_n \sum_p |\varphi(q, p, p+q)|^2 I_n^2(\lambda_{p,p+q})(f_p - f_{p+q})\,\delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q + n\Omega) \tag{4.26}$$

where $\varphi(q, p, p+q)$ is the amplitude of the magnon–phonon interactions,

$$\lambda_{p,p+q} = \frac{\mu h_0 \omega_H}{\Omega}\left(\frac{1}{\varepsilon_{p+q}} - \frac{1}{\varepsilon_p}\right).$$

Further on we will analyze the single-quantum limit $\lambda \ll 1$ (though $\mu h_0 \gtrsim \Omega$ is also possible). We will remark here also that in the external field square approximation the equilibrium distribution functions for magnons must be substituted in expression (4.26), the phonon damping at the expense of magnon–phonon processes being determined only by (4.26) (certainly on the condition that $\omega_q < 2\Delta$).

The expression for damping is simplified to the following type under the above cited conditions

$$\alpha_{\boldsymbol{q}} = \frac{J_0^2 B^2 \omega_H^2 \theta_D}{2^5 \pi \theta_F \theta_N^3} \left(\frac{\mu h_0}{\Omega}\right)^2 \int_{\varepsilon_*}^{\infty} d\varepsilon \, \frac{(\Omega - \omega_{\boldsymbol{q}})^2}{\varepsilon^2(\varepsilon + \omega_{\boldsymbol{q}} - \Omega)^2} [f_\varepsilon - f_{\varepsilon + \omega_{\boldsymbol{q}} - \Omega}] \tag{4.27}$$

where

$$\varepsilon_* = \tfrac{1}{2}(\omega_{\boldsymbol{q}} - \Omega) + \tfrac{1}{2}\alpha\omega_{\boldsymbol{q}} \sqrt{1 + \frac{4\Delta^2}{\alpha^2\omega_{\boldsymbol{q}}^2 - (\Omega - \omega_{\boldsymbol{q}})^2}}, \qquad \alpha = \frac{s}{c} < 1 .$$

This expression is valid if $\alpha^2\omega_{\boldsymbol{q}}^2 > (\Omega - \omega_{\boldsymbol{q}})^2$, which defines the interval of phonon energies at which the process in question is permitted by the laws of energy and momentum conservation:

$$\Omega(1+\alpha)^{-1} < \omega_{\boldsymbol{q}} < \Omega(1-\alpha)^{-1} .$$

As is evident from expression (4.27) within the interval of phonon energies $\Omega(1+\alpha)^{-1} < \omega_{\boldsymbol{q}} < \Omega$ the magnitude $\alpha_{\boldsymbol{q}}$ is negative, while within the interval $\Omega < \omega_{\boldsymbol{q}} < \Omega(1-\alpha)^{-1}$, $\alpha_{\boldsymbol{q}} > 0$. The change of sign of $\alpha_{\boldsymbol{q}}$ takes place at the point $\omega_{\boldsymbol{q}} = \Omega$.

When $\omega_{\boldsymbol{q}}$, $\Omega < \Delta < T$ and $\Omega(1+\alpha)^{-1} < \omega_{\boldsymbol{q}} < \Omega$ in the order of magnitude we have [134]

$$\alpha_{\boldsymbol{q}} = -\frac{J_0^2 B^2 \omega_H^2}{5 \cdot 2^3 \pi \theta_F \theta_N^3} \left(\frac{\theta_D}{\theta_N}\right) \left(\frac{\mu h_0}{\Omega}\right)^2 \frac{(\Omega - \omega)^3 T}{\Delta^5} \varphi^2(\boldsymbol{q}^0)$$

where

$$\varphi(\boldsymbol{q}^0) = e_s^x q_x^0 - e_s^y q_y^0 + \tfrac{1}{2}\eta(e_s^z q_y^0 + e_s^y q_z^0) ,$$

e_s^i is the vector of phonon polarization, $\boldsymbol{q}^0 = \boldsymbol{q}/q$, η is the ratio of the striction constants.

So, we see that nonresonance parallel pumping may result in the effect of sound amplification if the above mentioned conditions are satisfied and $|\alpha_{\boldsymbol{q}}| > \gamma_{\boldsymbol{q}}^{ph}$ ($\gamma_{\boldsymbol{q}}^{ph}$ is the total sound damping in respect of all possible mechanisms). In the case of $\Omega - \omega_{\boldsymbol{q}} \sim \alpha\omega_{\boldsymbol{q}}$ we get

$$\alpha_{\boldsymbol{q}} = -\frac{1}{2^3\pi} \frac{J_0^2 B^2 \omega_H^2}{\theta_F \theta_N^3} \left(\frac{\mu h_0}{\Omega}\right)^2 \varphi^2(\boldsymbol{q}^0) \frac{[\alpha^2\omega_{\boldsymbol{q}}^2 - (\Omega - \omega_{\boldsymbol{q}})^2]^2}{\alpha^2 \omega_{\boldsymbol{q}} \Delta^4} \exp\{-\alpha\omega_{\boldsymbol{q}}\Delta / T\sqrt{\alpha^2\omega_{\boldsymbol{q}}^2 - (\Omega - \omega_{\boldsymbol{q}})^2}\} . \tag{4.28}$$

In conclusion, we will provide numerical estimates. For example, for $\mu h_0 \sim \Omega \sim \Delta$, $B \sim T \sim 1$–10 K, $\theta_D/\theta_N \sim 3$, also for $\omega_{\boldsymbol{q}} \sim \Omega \cdot 0.75$, $|\alpha_{\boldsymbol{q}}| \sim 10^3$–$10^4\ \mathrm{sec}^{-1}$ which under the given conditions is much bigger than at least the damping caused by the phonon anharmonisms. Note that the effect described here is largely analogous to the one surveyed in section 3.3.

5. Kinetics of bosons interacting with electrons in a strong constant electric field

As has already been said in section 2.5, the impact of a strong constant electric field on the electron and bosons interacting with them (phonons, magnons, plasmons) is far more complicated than it is inferred from a usual kinetic equation of the Boltzmann type. Electric field affects electrons not only between collisions, but also during their interaction with other quasiparticles. The latter consideration that has long been ignored in the studies covering kinetics in a strong electric field, can play an essential part while investigating the peculiarities of nonequilibrium states. The thing is that for the time of interaction of the electron with some other quasiparticle the electron changes its state appreciably because of the acceleration in the field. Even if the energy acquired by the electron in the field for the time of interaction is much smaller than its change in the energy owing to the emission (absorption) of, say, a phonon or a magnon, all the same, the account taken of the explicit dependence of the collision integrals (2.54) and (2.55) on the field is essential. And such an account does not imply very often only small corrections for the already known effects. For the sake of illustration we may supply the following example. Let some process with the electron participating in it be forbidden in the absence of the field due to the incompatibility of the laws of energy and momentum conservation. The very law of energy concentration of the interacting quasiparticles is the consequence of the assumption about the instantaneous character of the interaction. It is for this reason that the collision integrals which appear while constructing the theory in the second order of the perturbation theory with respect to interaction, contain the corresponding δ-functions. In the absence of the field such an approximation is justifiable in those cases when the widening of energy caused by the interaction itself can be neglected. Otherwise, as is known, instead of δ-functions there appear Lorenz's functions. The effect involving the widening will not be considered here assuming that they are negligibly small. We will study only the field impact. The electric field affecting the process of interaction, as is clear, for example, from the equations (2.54) and (2.55), permits only those processes that could be forbidden without field. In the situation like this, certainly, even a very weak field plays a cardinal role in the kinetics involving the "forbidden" processes and defines the possibility of the kinetic processes themselves.

Unfortunately, in view of the complexity of the kinetic equations (2.54) and (2.55), there is not any possibility as yet of studying consistently the properties determined by electrons. We mean here such properties as the phenomenon of transport, high-frequency effects, etc. The theory still lacks a serious study of "electronic effects" in wide-band semiconductors. The problem of investigating the electronic properties lies mainly in the determination of the nonequilibrium distribution function satisfying an equation of the type (2.54). However, as for the kinetic properties of phonons [56, 57] and magnons [57], a number of basic conclusions can be made without having to solve the kinetic equation for electrons.

In this section we will tackle only one problem, namely, the conditions of instability of acoustic, optical phonons, magnons interacting with electrons under a strong constant electric field. It will be demonstrated that owing to the impact of the electric field on the interactions for the time of their duration, a new type of instability is possible which, regarding its physical mechanism, is close to instability with respect to the Ginzburg–Frank transition emission (see, for example, [135]). We will show that such non-Cherenkovian mechanism of instability may prove to be much more effective than the Vavilov–Cherenkov mechanism during the amplification of sound waves, of the flux of optical phonons, of spin waves.

The results presented below will pertain, as before, only to the case when one can neglect the interband tunnelling of electrons [31] and the effect of Stark's quantization [51]. For wide-band conductors these two effects are usually realized at "superhigh" intensities of the electric field ($E > 10^5$–10^6 V/cm). As for the effect of acceleration of electrons for the time of interactions, it can be substantial, as has been stated, for any, even small, value of the field.

5.1. *Instability of acoustic phonons in semiconductors and semimetals*

As is well-known, the analysis of the conditions for instability is based on the calculation of the corresponding coefficient of damping. Let us see how the effect of electronic acceleration for the time of its interaction with a phonon and the effect of time nonlocalities of these interactions influence the sign of this magnitude.

According to the kinetic equation for plasmons (2.54), the phonon damping γ_q may be represented by

$$\gamma_q = |M(q)|^2 \sum_p \int_{-\infty}^{0} d\tau\, e^{\eta\tau} (f_{p+eE\tau-q} - f_{p+eE\tau}) \cos\left[(\varepsilon_{p-q} - \varepsilon_p + \omega_q)\tau - \frac{eEq}{2m}\tau^2\right] \tag{5.1}$$

where $M(q)$, for example, in the model of deformation interaction, is equal to

$$\Lambda\sqrt{q/V\rho s}\,;$$

here Λ is the deformation potential constant, V is the volume of the system, and ρ is the density.

When calculating the damping, it is convenient to make a formal substitution in the expression (5.1): $p + eE\tau \to p$. As can be checked easily, in the cosine argument the sign of the electric field volume E changes to the opposite. The main problem now is to choose the form of the, generally speaking, nonequilibrium distribution function of electrons f_p. In the present section we will first consider the weak field limit, in particular,

$$\bar{p}\sqrt{\frac{q}{m|e|E}} \gg 1 \tag{5.2}$$

where $\bar{p} = m\bar{v}$, $\bar{v}$ is the mean electron velocity.

For choosing the model of the distribution function f_p in the weak field limit, we will make the following simplifying assumptions. Firstly, we will assume that we deal with the steady regime ($t \gg \tau_0$, see section 2.5) and therefore neglect the nonlocality in respect of the explicit dependence of functions f. We will also assume that the collisions of electrons with each other are the most frequent ones. Since owing to the law of momenta conservation of electrons during the electron–electron collisions, the field drops out explicitly from the kernels of the electron–electron collision integral, the zeroth approximation is the quasi-equilibrium distribution function with the drift velocity u [56]. One may presume here that the value of the drift velocity is determined through the electron–impurity collision integral. Assumptions like these are not necessary if the field under study is a pulsed one with the pulse duration τ_p satisfying the inequality

$$\tau \gg \tau_p \gg \omega^{-1}$$

where τ is the electronic relaxation time. Under the given condition the field for the phonons may be regarded as constant ($\tau_p \gg \omega^{-1}$), on the other hand, for the time τ_p ($\tau \gg \tau_p$) the quasi-equilibrium distribution of electrons does not manage to vary. This signifies that one may use the equilibrium form of the distribution function when calculating (5.1). Actually, as will be seen below, the phonon damping dependent of the electric field impact on the elementary acts of the electron–phonon interactions is weakly sensitive to the form of the function f_p and the condition of nonequilibraty is defined not by the value of the drift velocity, but explicitly by the magnitude and the direction of the electric field.

We will first analyze the degenerate case. After integrating in (5.1), the result for γ_q can be represented by the difference of two expressions of the same kind containing the functions Φ and Φ_1 akin to the Fresnel integrals [57]

$$\gamma_{\boldsymbol{q}} = \frac{V}{(2\pi)^2}|M(q)|^2 \frac{m}{2q}[A_+ - A_-],$$

$$A_\pm = (p_0^2 - \kappa_\pm^2)[|\Phi(p_0 - \kappa_\pm)|^2 - |\Phi(-p_0 - \kappa_\pm)|^2] - \frac{1}{\nu}\operatorname{Im}\Big\{(p_0 + \kappa_\pm)\,\Phi_1(p_0 - \kappa_\pm) + (p_0 - \kappa_\pm)\,\Phi_1(-p_0 - \kappa_\pm) - \int_0^{p_0} [\Phi_1(p_0 - \kappa_\pm) + \Phi_1(-p_0 - \kappa_\pm)]\,\mathrm{d}p\Big\}, \tag{5.3}$$

$$\Phi(x) = \int_{-\infty}^{\nu x} \mathrm{e}^{\mathrm{i}t^2}\,\mathrm{d}t, \qquad \Phi_1(x) = \mathrm{e}^{\mathrm{i}\nu^2 x^2} \int_{-\infty}^{\nu x} \mathrm{e}^{-\mathrm{i}t^2}\,\mathrm{d}t,$$

$$\nu = \frac{q}{\sqrt{2m|e\boldsymbol{E}\boldsymbol{q}|}}, \qquad \kappa_\pm = \pm\frac{q}{2} + m\left(\frac{\boldsymbol{u}\boldsymbol{q}}{q} - s\right),$$

p_0 is the Fermi momentum.

The result (5.3) is rather complicated for the analysis, however, it may be simplified [57] in the weak field limit. As is evident from (5.2) and (5.3), the asymptotics of the expressions for $\Phi(x)$ and $\Phi_1(x)$ at $|\nu x| \to \infty$ correspond to this limit. For realizing this limit, obviously, one needs the simultaneous fulfilment of our inequalities

$$|p_0 \pm \kappa_\pm|\nu \gg 1\,. \tag{5.4}$$

If the condition (5.2) is satisfied, inequalities (5.4) can be fulfilled beyond the vicinity of the solutions to the equations $p_0 \pm \kappa_\pm = 0$ for $\boldsymbol{q}$. The zeroth term of the asymptotic expansion $\gamma_{\boldsymbol{q}}^{(0)}$ in respect of the parameter (5.4) will take on the form

$$\gamma_{\boldsymbol{q}}^{(0)} = \frac{V|M(q)|^2}{8\pi}\frac{m}{q}[(p_0^2 - \kappa_+^2)\,\theta(p_0^2 - \kappa_+^2) - (p_0^2 - \kappa_-^2)\,\theta(p_0^2 - \kappa_-^2)] \tag{5.5}$$

where, as before, $\theta(x)$ is the step function, and with $p_0 > |\kappa_\pm|$ (as a rule, it is an ultrasound region) we have the well-known expression

$$\gamma_{\boldsymbol{q}}^{(0)} = \frac{|M(q)|^2 V m^2}{4\pi q}(\omega - \boldsymbol{q}\boldsymbol{u})\,. \tag{5.6}$$

The next term of the asymptotic expansion $\gamma_{\boldsymbol{q}}^{(1)}$ is proportional to the first degree of the electric field $\boldsymbol{E}$ (it should be noted here that the terms proportional to $\sqrt{E}$ cancel each other during the expansion of (5.3)):

$$\gamma_q^{(1)} = -\frac{|M(q)|^2 V}{(2\pi)^2} \frac{m^2}{q^3} (e\boldsymbol{E}\boldsymbol{q})\, p_0 \left[\frac{1}{p_0 - \kappa_+} - \frac{1}{p_0 + \kappa_+}\right.$$

$$\left. - \frac{1}{p_0 - \kappa_-} + \frac{1}{p_0 + \kappa_-} - \frac{1}{p_0} \ln\left|\frac{(p_0 - \kappa_+)(p_0 + \kappa_-)}{(p_0 + \kappa_+)(p_0 - \kappa_-)}\right|\right]. \tag{5.7}$$

As is seen from (5.5), formula (5.7) describes in the case of $p_0 > |\kappa_\pm|$ only a small correction for the "basic" damping (5.6) which describes the possibility of the Cherenkovian method of amplification of sound waves: $\gamma_q^{(0)} < 0$ if $\boldsymbol{q}\boldsymbol{u} > \omega$. Another opportunity immediately attracts attention when $p_0 < |\kappa_\pm|$. In this case $\gamma_q^{(0)} = 0$ and the "conventional" expression for damping (5.6) is nonexistent. Thus, we face an extremely interesting situation when the phonon kinetics dictated by the electron–phonon interactions is defined by the field presence in the system. Before embarking on the analysis of a case like this, we will dwell in more detail on the "conventional" situation $p_0 > |\kappa_\pm|$.

The result (5.7) has the simplest form if the inequality $p_0 \gg |\kappa_\pm|$ [56] is satisfied

$$\gamma_q^{(1)} = -\frac{|M(q)|^2 V}{(2\pi)^2} \frac{m}{q^2 v_0} (e\boldsymbol{E}\boldsymbol{q}) \left\{1 + \frac{2}{3} \frac{1}{(q v_0)^2} \left[3(\omega - \boldsymbol{q}\boldsymbol{u})^2 + \left(\frac{q^2}{2m}\right)^2\right]\right\} \tag{5.8}$$

where v_0 is the Fermi velocity.

Expressions (5.6) and (5.8) indicate that if the time nonlocal effect of the electric field on the acts of the electron–phonon interactions (i.e. the electronic acceleration for the time of its interaction with a phonon) is taken into account, a new condition for instability [56] consequently arises

$$\boldsymbol{q}\boldsymbol{u} + \frac{2}{\pi} \frac{e\boldsymbol{E}\boldsymbol{q}}{m v_0 q} > \omega\,. \tag{5.9}$$

It differs formally from the Cherenkovian condition. We would like to emphasize that in spite of the fact that we are concerned here with a small correction for damping, we can speak of a new type of phonon instability since for $\boldsymbol{q}\boldsymbol{u} < \omega$ when the Cherenkovian instability is not observed, the sign of the damping can become negative precisely due to the effect of nonlocality of the interaction and the electronic acceleration for the time of its interaction with a phonon. As might be expected, the relative value of the shift of the Cherenkovian instability point is small.

We will supply now the results for the nondegenerate case assuming that $p \gg |\kappa_\pm|$,

$$\gamma_q^{(1)} = \frac{|M(q)|^2 V}{q^2 T^2} n_e (e\boldsymbol{E}\boldsymbol{q}) \tag{5.10}$$

where n_e is the concentration of the electrons.

The expression for $\gamma_q^{(0)}$ has the standard form

$$\gamma_q^{(0)} = \frac{|M(q)|^2 V}{q} \left(\frac{m}{2\pi T}\right)^{1/2} n_e \exp\left\{-\frac{m}{2Tq^2}\left[(\omega - \boldsymbol{q}\boldsymbol{u})^2 + \left(\frac{q^2}{2m}\right)^2\right]\right\}$$

$$\times \left\{\exp\left[\frac{1}{2T}(\omega - \boldsymbol{q}\boldsymbol{u})\right] - \exp\left[\frac{-1}{2T}(\omega - \boldsymbol{q}\boldsymbol{u})\right]\right\}. \tag{5.11}$$

Before proceeding to the study of the strong field limit and of the situation when $\gamma_q^{(0)} = 0$, we should note an important consideration. The shift value of the curve $\gamma_q = \gamma_q(\boldsymbol{E})$ in reference to the point $\omega = \boldsymbol{qu}$, as is clear, for example, from expression (5.8), is inversely proportional to v_0, i.e. the shift increases with the decreased concentration of conduction electrons. This fact has long been known experimentally and for its explanation rather artificial hypotheses have been advanced. The asymmetry of the curve $\gamma_q = \gamma_q(\boldsymbol{E})$ in the suggested approach develops naturally and is easily explained by the effect considered above.

Of great interest is the case when we are concerned with the maximum nonconservation of energy in the acts of electron–phonon collisions, i.e. when the field cannot basically be neglected in the collision integrals [56].

In contrast to the case of the weak field which actually corresponds to the expansion of the collision integral in powers of the small value of the field, we will study the opposite limiting case

$$\bar{p}q/\sqrt{m|e\boldsymbol{Eq}|} \ll 1 . \tag{5.12}$$

It may seem initially that such a condition is inconsistent with the case of a strong field since, if, for example, $\bar{v} \sim u$ the right-hand side of the inequality decreases with the increased field and there is no physically interesting interval for q. However for the reasonable values of the field (smaller than Ziener's breakdown) this is not the case. In fact, in the model $u = \mu E$ (μ is the mobility), when $\bar{v} \sim u$ one can study the fields

$$E \ll \frac{|e|}{mq\mu^2}$$

which at $q \sim 10^2\,\text{cm}^{-1}$, $\mu \sim 10^4\,\text{cm}^2/\text{sec}\cdot\text{V}$, $m \sim 10^{-28}\,\text{gr}$ amounts to $E \ll 10^6\,\text{V/cm}$, i.e. smaller than the value of Ziener's breakdown.

Under the condition (5.12) a fundamental question arises concerning the form of the strongly nonequilibrium electron distribution function. Here, in contrast to the previous section, there are not any grounds to be restricted to the simple drift approximation, though, as earlier, we may take advantage of the considerations about the pulsed field. The method of integration of the expression (5.1), considered above and using the electron distribution function in its explicit form, will not be applied here. It turns out that in the calculation of γ_q in the limit (5.12) the nonequilibrium distribution function enters the results only in the form of its moments [56]: the concentration of electrons n_e, the drift velocity $\boldsymbol{u}$, the mean electron energy $\bar{\varepsilon}$, etc.

$$n_e = \frac{1}{V}\sum_{\boldsymbol{p}} f_{\boldsymbol{p}}, \qquad m\boldsymbol{u} = \frac{1}{Vn_e}\sum_{\boldsymbol{p}} \boldsymbol{p}f_{\boldsymbol{p}}, \qquad \bar{\varepsilon} = \frac{1}{Vn_e}\sum_{\boldsymbol{p}} \frac{p^2}{2m} f_{\boldsymbol{p}} .$$

As can be easily verified, such moments appear in the course of series expansion of the parameter (5.12) of Fresnel's integrals (see formula (2.56)) obtained in the τ-integration of expression (5.1).

Let us write out the result restricting ourselves to the first two terms in the expansion γ_q [56],

$$\gamma_q = -\frac{|M(q)|^2 Vn_e}{e\boldsymbol{Eq}}\, q^2\left[1 - \sqrt{\frac{\pi m}{|e\boldsymbol{Eq}|}}\,(\omega - \boldsymbol{qu})\right]. \tag{5.13}$$

We will emphasize here that the second term in brackets is the result of the expansion and therefore it is by far smaller than a unit.

Thus, in the case (5.12) the magnitude $\gamma_{\boldsymbol{q}}$ is determined predominantly not by the drift velocity, but by the electric field. The damping becomes dependent of the drift velocity, as is clear from (5.13), only in the terms of high order of smallness. In the situation considered the instability ($\gamma_{\boldsymbol{q}} < 0$) that occurs at $e\boldsymbol{Eq} > 0$, cannot be called, in principle, a Cherenkovian one since the condition of instability does not contain the relation between the wave velocity.

Here are some numerical estimates. Inequality (5.12) for parallel and antiparallel directions of $\boldsymbol{q}$ in respect of $\boldsymbol{E}$ can be re-written as

$$E[\mathrm{V/cm}] > 10^{-3}\bar{\varepsilon}[\mathrm{K}]\, q[\mathrm{cm}^{-1}]\,.$$

For $q^2\bar{\varepsilon}/|e\boldsymbol{Eq}| \sim 0.1$, $\omega \sim 5\times 10^8\,\mathrm{sec}^{-1}$, $s \sim 5\times 10^5\,\mathrm{cm\,sec}^{-1}$ the value of the deformation potential $\Lambda \sim 10^{-11}\,\mathrm{erg}$, $n_e \sim 10^{19}\,\mathrm{cm}^{-3}$, $\bar{\varepsilon} \sim 300\,\mathrm{K}$, we obtain $|\gamma| \sim 10^6\,\mathrm{sec}^{-1}$. This corresponds to the field $E \sim 300\,\mathrm{V/cm}$.

We have shown above that if the relation between u and s is arbitrary, but the inequality (5.12) is fulfilled, the phonon instability is governed by the effect of electronic acceleration by the field for the time of the electron–phonon interaction. The inequality (5.12) is formally possible even with $u < s$ (for sufficiently small q), then the only mechanism of instability is the one determined by the effect of electronic acceleration. Though, for the interval of wave vectors (ultrasound and upwards), interesting from the physical point of view, formula (5.13) describes only the behaviour of $\gamma(E)$ under strong fields preceded by the proper Cherenkovian phonon instability. That is why the result (5.13) is of no importance in terms of the practical application of non-Cherenkovian mechanisms of phonon instability in not too strong fields. Let us return to the case of weak fields.

We have attracted the reader's attention above (formula (5.5)) to the fact that within the weak field limit, but with $|\kappa_\pm| > p_0$ the damping $\gamma_{\boldsymbol{q}}$ is different from zero only due to the explicit dependence of the collision integral on the electric field. Formally this may be seen from the fact that in the given conditions the expression for $\gamma_{\boldsymbol{q}}^{(0)}$ vanishes whereas the first term of the field expansion (5.7) is not equal to zero. Let us analyze this case in more detail. The inequality

$$\left|\frac{\boldsymbol{uq}}{q} - s \pm \frac{q}{2m}\right| > v_0 \tag{5.14}$$

can be realized, in principle, in a wide interval of wave vectors $\boldsymbol{q}$ if $s > v_0$. However in a more real experimentally situation ($v_0 > s$) the condition (5.14) is associated with the hypersound region. In a sufficiently weak field when $u \ll s$, the condition (5.14) is perceivable if, for example, $q \gtrsim 10^5\,\mathrm{cm}^{-1}$, $m \sim 10^{-28}\,\mathrm{gr}$, $v_0 \sim 10^6\,\mathrm{cm\,sec}^{-1}$.

We will now show that a substantial increment may be obtained for the phonons satisfying the inequality (5.14) in rather weak fields. We will simplify formula (5.7) for the case $|\kappa_\pm| \gg p_0$. Then we have [57]

$$\gamma_{\boldsymbol{q}} \simeq \frac{2}{3}\frac{\overline{|M(q)|^2 V}}{(2\pi)^2}(e\boldsymbol{Eq})\frac{\overline{mv_0^3[3(\omega - \boldsymbol{qu})^2 + (q^2/2m)^2]}}{q^4[(q/2m)^2 - (s - \boldsymbol{uq}/q)^2]^3}\,. \tag{5.15}$$

Before proceeding to numerical estimates we will analyze the sign of the magnitude $\gamma_{\boldsymbol{q}}$. As is evident from formula (5.15), with the given sign of the magnitude $e\boldsymbol{E}\boldsymbol{q}$ (for definiteness we will speak so far about the positive value of this magnitude), in principle, both a positive and a negative value of γ_q is possible, keeping to the limits of the inequality (5.14). This depends on the relation between the change in the electron velocity while interacting with a phonon and the wave velocity (including the Doppler shift). If the change in the electron velocity owing to acceleration in the field for the time of the electron–phonon interaction exceeds the wave velocity, the wave as though breaks off backwards (in reference of $e\boldsymbol{E}$), so we have to deal with instability with respect to the "backward" irradiation. In this case the sign of γ_q is determined by the signs of $e\boldsymbol{E}\boldsymbol{q}$. If the change in the electron velocity is inferior to the wave velocity, the wave breaks away "forward" (in reference to the direction of $e\boldsymbol{E}$), we confront instability with respect to the forward irradiation, the sign being dictated by the sign of $-(e\boldsymbol{E}\boldsymbol{q})$. The considerations given above are analogous to those which usually explain the effect of instability as related to the transition irradiation (see [136]) for spatially homogeneous systems. Therefore the instability considered here and the new method, related to it, of amplification of hypersound phonons (formula (5.15)) may be treated as another kind of instability in reference to transition irradiation.

We will emphasize again that amplification of hypersound in semiconductors ($q > 2p_0$) exposed to a constant electric field, is not, possibly, associated with the sign of the magnitude $(\omega - \boldsymbol{q}\boldsymbol{u})$ even if $u \ll s$. In the model of deformation potential in (5.15) for $\Lambda \sim 5\times 10^{15}\,\mathrm{sec}^{-1}$, $s \sim 5\times 10^{5}\,\mathrm{cm}\cdot\mathrm{sec}^{-1}$, $\rho \sim 4\,\mathrm{gr}\cdot\mathrm{cm}^{-3}$, $m \sim 10^{-28}\,\mathrm{gr}$, $v_0 \sim 10^{6}\,\mathrm{cm}\cdot\mathrm{sec}^{-1}$, $q \sim 3\times 10^{5}\,\mathrm{cm}^{-1}$, $E \sim 100\,\mathrm{V/cm}$, the value of the increment of γ_q in its order of magnitude equals $10^{8}\,\mathrm{sec}^{-1}$. As is clear, the method of amplification of hypersound based on the use of instability close to its mechanism to the one related to transition irradiation, has great advantages over the Cherenkovian method. The main asset is that we do not need here great mobilities of current carriers. It is obligatory only to fulfil the inequality (5.14) as well as the condition for $|\gamma_q|$ be superior to the phonon damping caused by the phonon–phonon anharmonisms.

5.2. The problem of amplification of optic phonons

For the realization of the Cherenkovian method of instability in a conductor normally great mobilities of current carriers and high intensities of the constant electric field are needed to attain the drift whose velocity exceeds that of the wave. For the case of acoustic phonons whose phase velocity s, as a rule, is of the order of $10^{5}\,\mathrm{cm\,sec}^{-1}$, the appropriate fields and materials (semiconductors) have long been created and used (see, for example, [137]). At the same time, for other types of waves whose phase velocities may be far superior to that of sound, the problem of their amplification arises since mobility of carriers in solids is limited and, hence, very high intensities of the field are necessary which may simply destroy the sample. One of the examples where we face the problem of realization of the Cherenkovian mechanism of amplification, is the optic phonons. Let us see what possibilities may be gained from the mechanism of instability suggested in section 5.1 for amplification of optic phonons flux in semiconductors exposed to a strong constant electric field. We will note that we are not familiar so far with the experiments aimed at the amplification of optic phonons.

Similar to the case of acoustic phonons, the damping of optic phonons is calculated using formula (5.1) where, though, one must bear in mind that ω_q is the spectrum of optic phonons. The weak field limit is represented by the inequality

$$\bar{v} \gg \frac{|e|E}{m}\tau_0\,, \qquad \tau_0 = \max\{(\bar{v}q)^{-1}, \omega_q^{-1}\}\,. \tag{5.16}$$

We will study this case in more detail. As a model for the electron distribution function we may choose the quasi-equilibrium function with drift velocity when the frequency of electronic collisions is the highest one in the system (see also the analogous considerations in section 5.1). Let us neglect the phonon dispersion assuming that $\omega_{\boldsymbol{q}} = \omega_0$ and restrict ourselves to the analysis of the degenerate case [138]. The result for the first two terms of the expansion (5.1) with respect to the parameter (5.16) is similar to the formulae (5.5) and (5.7) from the previous subsection where, however, by the magnitude s we mean the phase velocity of optic phonons $s = \omega_0/q$. We will dwell on the most interesting case when $p_0 < |\kappa_\pm|$ and, therefore $\gamma_{\boldsymbol{q}}^{(0)} = 0$. For $q \ll m\omega_0/p_0$ we have [138]

$$\gamma_{\boldsymbol{q}}^{(1)} = -|M(q)|^2 \frac{V}{\pi^2} \frac{q^2}{m^2} p_0^3 \frac{1}{\omega_0^4} (e\boldsymbol{E}\boldsymbol{q}) . \tag{5.17}$$

If the inequality $q \gg p_0$ [138] is fulfilled,

$$\gamma_{\boldsymbol{q}}^{(1)} = \frac{16}{3} |M(q)|^2 \frac{V}{\pi^2} \frac{m^2}{q^6} p_0^3 (e\boldsymbol{E}\boldsymbol{q}) . \tag{5.18}$$

Thus, in the case of even small drift velocities (we must stress here that in the limiting cases (5.17) and (5.18) the magnitude u does not even enter these results) and not too strong fields there is a possibility of amplification of optic phonons ($\gamma_{\boldsymbol{q}} < 0$), this possibility being due to the mechanism considered in section 5.1. Here are some numerical results, for instance, for the deformation potential of the electron interaction with optic phonons

$$|M|^2 = \mathscr{D}^2/2\rho\omega_0 V$$

where $\mathscr{D}$ is the deformation potential. Assuming that $\mathscr{D} = 10^9\,\mathrm{eV/cm}$, $\rho = 5\,\mathrm{gr/cm^3}$, $q = 10^5\,\mathrm{cm^{-1}}$, $\omega_0 = 10^{13}\,\mathrm{sec^{-1}}$, $p_0 = 10^6\,\mathrm{cm^{-1}}$, we obtain in the case (5.17)

$$|\gamma_{\boldsymbol{q}}| \sim 10^6 E[\mathrm{V/cm}]\,\mathrm{sec^{-1}} ; \tag{5.19}$$

under these same conditions but with $q = 10^7\,\mathrm{cm^{-1}}$ when the situation (5.18) is realized, we have

$$|\gamma_{\boldsymbol{q}}| \sim 10^4 E[\mathrm{V/cm}]\,\mathrm{sec^{-1}} . \tag{5.20}$$

5.3. Damping of spin waves in ferromagnetic semiconductors

We will supply one more example of instability of waves interacting with electrons under a strong constant electric field. Let us examine the spin waves damping in a ferromagnetic semiconductor taking into account the field effect on the electron–magnon interactions [57, 139], using extensively the results of sections 2.3 and 2.5.

Generalizing equation (2.54) for the case of magnon–electron interactions we will write the kinetic equation for magnons in the form

$$\frac{\partial N_q}{\partial t} = \sum_p \int_{-\infty}^{0} d\tau\, e^{\eta\tau} |\psi_3|^2 \{(1+N_q)(1-f_{p+eE\tau-q,\uparrow}) f_{p+eE\tau,\downarrow} - N_q(1-f_{p+eE\tau,\downarrow}) f_{p+eE\tau-q,\uparrow}\}$$
$$\times \cos[(\varepsilon_{p-q\uparrow} - \varepsilon_{p\downarrow} + \omega_q)\tau - eEq\tau^2/2m], \qquad \eta \to +0 \tag{5.21}$$

where the notations of the magnitudes that appear here are given in section 2.3.

As in the preceding examples, our main interest will be concentrated on the peculiarity of the magnon damping due to the electric field. Therefore, as in sections 5.1 and 5.2, where the phonon damping has been discussed, we will need here the kinetic equation for electrons. We will only take advantage of the fact that in a sufficiently weak field in the case of most effective electron–electron collisions the distribution function will be chosen by the quasi-equilibrium one with the drift velocity $\boldsymbol{u}$ whose value is defined by the electron–impurity collisions. As follows from (5.21), the collision integral being explicitly dependent on the field, which is the case for phonons, too, brings about the situation when, generally speaking, the laws of energy and momentum conservation should not be necessarily observed simultaneously in the elementary acts of interaction, i.e. the electric field allows only those processes which would be forbidden in a certain region of the $\boldsymbol{q}$-space in the absence of the field. For the first time the problem of the spin waves damping was discussed in [140] where within the framework of drift approximation for a nondegenerate case the authors obtained the condition of instability of the Cherenkovian type: $\omega_q < \boldsymbol{qu}$. Below we will give some results including the spin splitting between the subbands. We will also consider a degenerate semiconductor involving the difference in the electron occupation numbers in the subbands. In so doing, we will, similar to sections 5.1 and 5.2, take into account the effect of electron acceleration for the time of the electron–magnon interactions and their time nonlocality. The damping coefficient for a spin wave after the substitution of the derivatives $\boldsymbol{p} + e\boldsymbol{E}\tau \to \boldsymbol{p}$ is given by

$$\gamma_q = |\psi_3|^2 \sum_p \int_{-\infty}^{0} d\tau\, e^{\eta\tau} (f_{p-q,\uparrow} - f_{p,\downarrow}) \cos[(\varepsilon_{p\downarrow} - \varepsilon_{p-q\uparrow} - \omega_q)\tau - eEq\tau^2/2m]\,. \tag{5.22}$$

Repeating all the reasoning and making the calculations, analogous to those from section 5.1, we arrive at the formula coinciding with (5.3). The only difference is that the matrix element of the electron–phonon interaction should be replaced by the matrix element of the s–d exchange interaction, the expression for $\kappa_\pm$, by

$$\kappa_\pm = \pm\frac{q}{2} + \frac{m\Delta}{q} + m\left(\frac{\boldsymbol{uq}}{q} - s\right) \tag{5.23}$$

where s is the spin wave velocity. Instead of the magnitude p_0 in the expressions for A_+ and A_-, we have respectively

$$p_\pm = \sqrt{2m(\varepsilon_0 \pm \tfrac{1}{2}\Delta)}\,, \qquad (\varepsilon_0 > \tfrac{1}{2}\Delta) \tag{5.24}$$

where ε_0 is the Fermi level.

When the inequalities are fulfilled [57]

$$|p_\pm \pm \kappa_\pm|\nu \gg 1 \tag{5.25}$$

the first two terms of the asymptotic expansion γ_q are given by [57]

$$\gamma_q^{(0)} = \frac{V|\psi_3|^2}{8\pi}\frac{m}{q}\left[(p_+^2-\kappa_+^2)\,\theta(p_+^2-\kappa_+^2)-(p_-^2-\kappa_-^2)\,\theta(p_-^2-\kappa_-^2)\right], \tag{5.26}$$

$$\gamma_q^{(1)} = -\frac{V|\psi_3|^2}{4\pi^2}\frac{m^2}{q^3}(e\boldsymbol{E}\boldsymbol{q})\left\{\frac{p_+}{p_+-\kappa_+}-\frac{p_+}{p_++\kappa_+}-\frac{p_-}{p_--\kappa_-}+\frac{p_-}{p_-+\kappa_-}-\ln\left|\frac{(p_+-\kappa_+)(p_-+\kappa_-)}{(p_++\kappa_+)(p_--\kappa_-)}\right|\right\}. \tag{5.27}$$

In the region where both θ-functions are other than zero, we get a conventional result [140]

$$\gamma_q^{(0)} = \frac{V}{4\pi}|\psi_3|^2\frac{m^2}{q}(\omega-\boldsymbol{q}\boldsymbol{u}). \tag{5.28}$$

The expression (5.27) in this case describes the shift of the Cherenkovian instability point [140]. Let us dwell in more detail on the situation when $\gamma_q^{(0)}=0$. Formula (5.27) under the condition $|\kappa_\pm| \gg p_\pm$ is greatly simplified [57, 139]. If the inequalities

$$\omega_0 \ll \Delta, \qquad u \ll \Delta/q$$

are fulfilled that are reduced to the conditions $q \gg p_\pm$ or $q \ll m\Delta/p_\pm$ we obtain expressions of the type

$$\gamma_q = \frac{8|\psi_3|^2 V}{3\pi^2}\frac{m^2}{q^6}(e\boldsymbol{E}\boldsymbol{q})(p_+^3+p_-^3), \qquad q \gg p_\pm \tag{5.29}$$

$$\gamma_q = \frac{|\psi_3|^2 V}{3\pi^2}m^2(e\boldsymbol{E}\boldsymbol{q})\frac{p_+^3-p_-^3}{(m\Delta)^3}, \qquad q \ll \frac{m\Delta}{p_\pm}. \tag{5.30}$$

The physical interpretation of the mechanism of instability (which, as is clear from (5.29) and (5.30), under the present conditions is possible only for $(e\boldsymbol{E}\boldsymbol{q})<0$) and of the sign of the coefficient γ_q cannot be presented so vividly as in section 5.1. This is attributed to the fact that the classical language that we have used to explain the phonon instability is not, generally speaking, applicable here (at least for the case (5.30)) since electron–magnon processes are accompanied with the change in a purely quantum magnitude (spin flip). Still, if we determine the change in the electron velocity proceeding from the change in its energy while interacting with a magnon, then, as both in the case (5.29) and in the case (5.30), we will find that the change in the electron velocity (along $e\boldsymbol{E}$) exceeds the wave velocity, i.e. the instability in reference to the "backward" irradiation of spin waves (relative to the direction $e\boldsymbol{E}$) takes place.

Here are some numerical estimates for $CdCr_2Se_4$, for example, in the conditions (5.30). For $\varepsilon_0 \sim 10^{14}\,\mathrm{sec}^{-1}$, $\Delta \sim 10^{13}\,\mathrm{sec}^{-1}$, $m \sim 10^{-28}$ gr, $q \sim 10^5\,\mathrm{cm}^{-1}$, $E \sim 10^2$ V/cm, $|\gamma| \sim 10^5\,\mathrm{sec}^{-1}$ which under the chosen conditions is much superior to the damping due to the magnon–magnon processes.

6. Kinetic theory of the phenomena in tunnel junctions as an example of exact account of intense external excitation

Another system that may be related to the problem discussed in the given report is a tunnel junction of two metal films separated by a thin insulating layer (N_1–I–N_2). In a system like this tunnelling current flows if the voltage V is applied to the junction. As was established in [141], as a result of electron tunnelling from one electrode to the other, nonequilibrium distributions of electrons come to be realized in N_1 and N_2. The character of these distributions is determined by the transparency of the tunnel barrier, the voltage applied and the electron relaxation times [49].

To describe the tunnelling processes in nonequilibrium states of the electron and phonon subsystems of metal films, kinetic equations can be formulated (see [49, 142]) which take exact account of the external excitation acting on the system. This excitation is produced by the voltage applied to the tunnel junction. As this takes place, one can derive equations resembling very much those which were constructed in the study of the high-frequency field effect on the processes of interaction between quasiparticles. It is easy to see the analogy if one analyzes the Hamiltonian of the system in question.

While constructing kinetic equations we can make use of the tunnel Hamiltonian [143]. Let us assume that the complete Hamiltonian of the system may be represented as [49]

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_V + \mathcal{H}_T + \mathcal{H}_{T1} + \mathcal{H}_c \tag{6.1}$$

where $\mathcal{H}_1$ and $\mathcal{H}_2$ are complete Hamiltonians of the left and right electrodes,

$$\mathcal{H}_1 = \sum_{p\sigma} \varepsilon_p^{(1)} \alpha_{p\sigma}^+ \alpha_{p\sigma} + \sum_{p'-p=\kappa,\sigma,\lambda} M_\lambda^{(1)}(p,p')\, \alpha_{p\sigma}^+ \alpha_{p'\sigma} (\overset{+}{b}{}_{\kappa\lambda}^{(1)} + b_{-\kappa\lambda}^{(1)}) + \sum_{\kappa\lambda} \omega_{\kappa\lambda}^{(1)} \overset{+}{b}{}_{\kappa\lambda}^{(1)} b_{\kappa\lambda}^{(1)} ,$$

$$\mathcal{H}_2 = \sum_{q\sigma} \varepsilon_q^{(2)} \beta_{q\sigma}^+ \beta_{q\sigma} + \sum_{q'-q=\kappa,\sigma,\lambda} M_\lambda^{(2)}(q,q')\, \beta_{q\sigma}^+ \beta_{q'\sigma} (\overset{+}{b}{}_{\kappa\lambda}^{(2)} + b_{-\kappa\lambda}^{(2)}) + \sum_{\kappa\lambda} \omega_{\kappa\lambda}^{(2)} \overset{+}{b}{}_{\kappa\lambda}^{(2)} b_{\kappa\lambda}^{(2)} ,$$

(α^+, α), (β^+, β) are electron creation and annihilation operators of the left and right film, respectively, σ is the spin index, $(\overset{+}{b}{}^{(i)}, b^{(i)})$ are phonon operators of each film, λ is the phonon polarization index, $M_\lambda^{(i)}(k, k')$ are the matrix elements of the electron–phonon interaction in each film (1, 2), $\varepsilon_k^{(i)} = E_k^{(i)} - \mu_i$, where $E_k^{(i)}$ characterizes the law of electron dispersion in the i-film, μ_i is the electron chemical potential, the Hamiltonian $\mathcal{H}_V$ describes the voltage applied to the junction:

$$\mathcal{H}_V = eV(t) \sum_{p\sigma} \alpha_{pq}^+ \alpha_{p\sigma} , \tag{6.2}$$

the elastic tunnelling channel [143] is described by the term

$$\mathcal{H}_T = \sum_{p,q,\sigma} T_{pq}^\sigma \alpha_{p\sigma}^+ \beta_{q\sigma} + \text{h.c.}$$

where T_{pq} is the matrix element of the tunnelling interaction, $\overset{*}{T}_{p\uparrow q\uparrow} = T_{-p\downarrow -q\downarrow}$, the momenta $\boldsymbol{p}$ and $\boldsymbol{q}$ will refer afterwards to the left and right films, respectively. The inelastic tunnelling channel is described as [144]

$$\mathcal{H}_{T1} = \sum_{p,q,k,\sigma,\lambda} T_{qp\lambda}^{(1)\sigma}(k)\, \alpha_{p\sigma}^+ \beta_{q\sigma} \frac{1}{\sqrt{2}} (\overset{+}{b}{}_{k\lambda}^{(1)} + \overset{+}{b}{}_{k\lambda}^{(2)}) + \text{h.c.}$$

where $\boldsymbol{k}$ is the phonon wave vector; the matrix element of the inelastic tunnelling interaction is defined as

$$T_{pq}(k) = \int \mathrm{d}r\, \exp(\mathrm{i}kr)\, \overset{*}{\psi}{}^{(1)}_{p}(r)\, \psi^{(2)}_{q}(r)$$

where the single-particle wave functions $\psi^{(1,2)}(r)$ are those for electrons incident on the barrier from the left and right, respectively.

The Hamiltonian $\mathscr{H}_c$ describing the external circuit is usually discarded assuming that the main effect is to conserve the electric neutrality of the electrodes (with $eV \neq 0$) by the transfer of charge excess in the external circuit. Let us also note that the separation of phonons into "left" and "right" introduced above refers only to phonons of frequency $\omega > \omega_0 \sim 2\pi s/d$ (s is the sound velocity, d is the film thickness). However, precisely these phonons are also affected by the "redressing" of electrons through tunnelling, i.e., it is reasonable to consider the change of electron and phonon states in each film under the action of the source due to the inelastic tunnelling channel. It follows from the results [144] obtained in the determination of the dependence of $T^{(1)}_{pq}$ on k that the contribution of phonons with wave vectors $k < 2\pi/d$ can be neglected.

As is clear, the Hamiltonian (6.1) refers precisely to the case that has been considered in section 2 and for which the "external field" $V(t)$ can be exactly allowed for in the kinetic equations. The result (2.27) is completely applicable to the case (6.1), (6.2). We will pass over to the description of the programme for constructing kinetic equations which was accomplished in [49]. We will start with the case of constant voltage applied to the junction.

6.1. *Elastic tunnelling channel under constant voltage*

We will first consider the kinetic equations without taking $\mathscr{H}_{T1}$ into account. For making calculations in (2.27) with the accuracy of up to the second order of the perturbation theory in the interaction $\mathscr{H}_T$, the following pairings between operators must be regarded as different from zero:

$$\underbrace{\overset{+}{\alpha}_{p\sigma}\alpha_{p'\sigma}} = f^{\alpha\sigma}_{p}\,\delta_{pp'}\,, \qquad \underbrace{\overset{+}{\beta}_{q\sigma}\beta_{q'\sigma}} = f^{\beta\sigma}_{q}\,\delta_{qq'}\,, \qquad \underbrace{\overset{+}{b}{}^{(i)}_{\kappa\lambda}b^{(i)}_{\kappa'\lambda}} = N^{(i)\lambda}_{\kappa\kappa'}\,.$$

Further on we should take into consideration that in the approximation of "specular" penetration of particles through the tunnel barrier the component of the momentum parallel to the barrier is preserved

$$T_{pq} = T_{p_z q_z}(p_{11})\,\delta_{p_{11},q_{11}}\,.$$

In view of the fact that the Hamiltonian $\mathscr{H}_V$ commutes with $\mathscr{H}^{\mathrm{cl}}_0$, the electron–phonon collision integrals for each electrode have the usual form. Therefore it is sufficient to consider only the collision integrals due to the tunnelling. These collision integrals may be given by

$$\left(\frac{\partial f^{\alpha}_{p}}{\partial t}\right)_T = -2\pi \sum_q |T_{pq}|^2\,(f^{\alpha}_{p} - f^{\beta}_{q})\,\delta(\varepsilon^{(1)}_{p} - \varepsilon^{(2)}_{q} + eV) \tag{6.3}$$

$$\left(\frac{\partial f^{\beta}_{q}}{\partial t}\right)_T = -2\pi \sum_p |T_{pq}|^2\,(f^{\beta}_{q} - f^{\alpha}_{p})\,\delta(\varepsilon^{(1)}_{p} - \varepsilon^{(2)}_{q} + eV)\,. \tag{6.4}$$

The formulae given above enable the judgement about the character of nonequilibrium states emerged during tunnelling [49, 142].

To investigate the nonequilibrium states of the electron–phonon system of a metal induced by the tunnelling current, it is necessary, as is evident, to solve an extremely complex system of equations. Specific difficulties are also associated with taking account of boundary conditions. In the present section we only show that tunnelling between two films of normal metals can lead to essentially nonequilibrium electron states. We assume that the thickness of the films d_1, d_2 is less than the phonon mean free path λ_{ph} and consequently we are justified to consider only the electron equation. Let the film thickness also satisfy the inequality $\lambda_p \ll d_i \ll \lambda_\varepsilon$ (λ_p is the electron mean free path with change of momentum, λ_ε is the diffusion length). The right side of the inequality permits the study of the spatially homogeneous problem without taking account of the boundary conditions, whereas the left side admits an approximation isotropic in momenta

$$f_k = f(\varepsilon_k) .$$

In passing from summation to integration in (6.3), (6.4) we take into account the fact that when $eV \ll \mu$ the momentum dependence of the matrix elements T_{pq} can be neglected and $\langle |T|^2 \rangle$ may be taken at the Fermi surface. In this case the energy dependence of the electron density of states (at distances eV from the Fermi surface) can also be neglected. Finally we obtain a system of equations in the form [49]

$$\partial f^\alpha(\varepsilon)/\partial t = -I_{01}\{f^\alpha(\varepsilon) - f^\beta(\varepsilon + eV)\} + \mathscr{L}_\varepsilon^{ep}\{f^\alpha, N\} \tag{6.5}$$

$$\partial f^\beta(\varepsilon)/\partial t = -I_{02}\{f^\beta(\varepsilon) - f^\alpha(\varepsilon - eV)\} + \mathscr{L}_\varepsilon^{ep}\{f^\beta, N\} \tag{6.6}$$

where $I_{0i} \sim 2Dv_{0i}/d_i$, $i = (\mathrm{i}, 2)$, v_0 is the Fermi velocity, $\mathscr{L}_\varepsilon^{ep}$ are the electron–phonon collision integrals.

The first of the equations, (6.5) describes the injection of holes below the Fermi surface (extraction of electrons from the vicinity of the Fermi surface) in the first electrode, while the second refers to the injection of electrons in the region above the Fermi surface of the right film. In the first case, due to the electron neutrality condition, states above the Fermi surface are rearranged too, while in the second case the occupation number of electron states below μ_2 is changed. Stationary solutions of the system of nonlinear integral equations (6.5), (6.6) can be found only by numerical methods. Therefore, we consider below the case of that "tunnelling nonequilibrium" which in limiting cases admits exact analytical solutions.

We will consider the tunnelling structure N_1–I–N–I–N_1 [142] in which N_1 are thick electrodes (and, according to (6.5), (6.6), in equilibrium), while the central N-film is separated from the outer ones by identical insulators. Let the same voltage eV be applied to each of the functions. Then the kinetic equation for the distribution function of the central film $f(\varepsilon)$ has the form [142]

$$\partial f(\varepsilon)/\partial t = -I_0\{2f(\varepsilon) - F(\varepsilon + eV) - F(\varepsilon - eV)\} + \mathscr{L}_\varepsilon^{ep} \tag{6.7}$$

where $F(\varepsilon) = (e^{\varepsilon/T} + 1)^{-1}$ is the equilibrium distribution function. We will solve the equation obtained for the case $T = 0$. According to the conditions enumerated above, the stationary solutions of (6.7) belong to a class of functions symmetrical in ε, namely, $f(-\varepsilon) = 1 - f(\varepsilon)$, and, therefore, it is sufficient to seek a solution for $\varepsilon > 0$. Introducing the dimensionless quantities $\nu = I_0/c_{el}(eV)^3$, $x = \varepsilon/eV$ (here

$c_{el} = \Lambda^2/4\pi s^4 \rho v_0$, Λ is the deformation potential constant, s is the sound velocity, ρ is the density (the matrix element of the phonon interaction is expressed in the Bardeen–Pines model)), for $f(x)$ whose values different from zero lie below $x = 1$, eq. (6.7), taking into account the explicit form of the electron–phonon collision integral, can be written in the form

$$\nu(2f(x) - 1) = (1 - f(x)) \int_x^1 \mathrm{d}x' \, (x' - x)^2 f(x') - f(x) \left(\int_0^1 \mathrm{d}x' \, (x + x')^2 f(x') + \int_0^x \mathrm{d}x' \, (x - x')^2 (1 - f(x')) \right).$$

This equation may be reduced to a differential equation with boundary conditions. We denote by $\psi(x)$ the quantity

$$\nu + \int_x^1 \mathrm{d}x' \, (x' - x)^2 f(x') + \frac{x^3}{6} + 2ax$$

where $a = \int_0^1 \mathrm{d}x \, x f(x)$. A second order equation for $\psi(x)$ may then be represented in the form

$$\psi \psi''_{xx} - \tfrac{1}{2}(\psi'_x)^2 = \frac{x^4}{24} + ax^2 + \nu - 2a^2 .$$

The boundary conditions for $\psi(x)$ have the form $\psi'_x(0) = 0$, $\psi'_x(1) = \frac{1}{2} + 2a$ (the magnitude a must be determined self-consistently), $\psi''_{xx}(1) = 1$. Evidently, $f(x) = \frac{1}{2} - \frac{1}{2}\psi'''_{xxx}$. We will present only the results. When $\nu \ll \frac{1}{6}$ a result is obtained which in fact corresponds to the generalized τ-approximation of eq. (6.7)

$$f(x) = \frac{1}{2 + x^3/3\nu} . \tag{6.8}$$

When $\nu \gg 1$ we obtain a solution which is described by the emergence of "terraces" at the Fermi surface [142]

$$f(x) = \tfrac{1}{2}\left[1 - \frac{1}{6\nu} x(x^2 + 3)\right].$$

The development of nonequilibrium electron distributions leads to some peculiar phenomena in tunnel junctions. For example, in [141] the authors discovered experimentally the effect of self-blocking of tunnelling current at low voltages.

6.2. *Inelastic tunnelling*

We will now see to what effects the allowance for tunnelling accompanied by the emission (absorption) of phonons may lead. The electron collision integrals corresponding to these types of interactions are given by [49]

$$\left(\frac{\partial f_p^\alpha}{\partial t}\right)_{T1} = \pi \sum_{i=1,2} \sum_{q=p+k} |T_{pq}^{(1)}(k)|^2 \{f_q^\beta(1-f_p^\alpha) - N_k^{(i)}(f_p^\alpha - f_q^\beta)\}\, \delta(\varepsilon_p^{(1)} + eV - \varepsilon_q^{(2)} + \omega_k^{(i)}), \tag{6.9}$$

$$\left(\frac{\partial f_q^\beta}{\partial t}\right)_{T1} = -\pi \sum_{i=1,2} \sum_{q=p+k} |T_{pq}^{(1)}(k)|^2 \{f_q^\beta(1-f_p^\alpha) - N_k^{(i)}(f_p^\alpha - f_q^\beta)\}\, \delta(\varepsilon_p^{(1)} + eV - \varepsilon_q^{(2)} + \omega_k^{(i)}) \tag{6.10}$$

where the phonon polarization index is included in the notation of the wave vector.

It is clear that only the inelastic tunnelling channel can serve as the source of phonon pumping in each of the films. The calculation of the phonon collision integrals provides the following results for kinetic equations

$$\begin{aligned}\frac{\partial N_k^{(i)}}{\partial t} = \pi \sum_{p=q-k} |T_{pq}^{(1)}(k)|^2 \{(f_q^\beta - f_p^\alpha)\, N_k^{(i)} + (1-f_p^\alpha)\, f_q^\beta\} \\ \times\, \delta(\varepsilon_p^{(1)} + eV - \varepsilon_q^{(1)} + \omega_k^{(i)}) + \mathscr{L}_k^{\text{pe}}\{f^{(i)}, N^{(i)}\}\end{aligned} \tag{6.11}$$

where $\mathscr{L}_k^{\text{pe}}$ are the usual electron–phonon collision integrals.

Equations (6.3), (6.4), (6.9), (6.10) and (6.11), supplemented by boundary conditions, as well as by the condition of electric neutrality of the films (determining their chemical potentials) constitute the complete system for the solution of the problems of electron tunnelling in normal metals in the case when nonequilibrium effects are important.

6.3. Oscillating voltage at tunnelling junction

We will now study modified kinetic equations for the situation where

$$V(t) = V_0 + V_1 \cos \Omega t\,. \tag{6.12}$$

For the sake of brevity we will demonstrate here the results concerning only the elastic tunnelling channel described by the Hamiltonian $\mathscr{H}_{\text{T}}$. The term in the kinetic equation for f_p^α, for example, depending on the voltage according to (2.27) is written in the form [49]

$$\begin{aligned}\left(\frac{\partial f_p^\alpha}{\partial t}\right)_T = -2 \sum_q |T_{pq}|^2 \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{\eta\tau}\, (f_p^\alpha(t+\tau) - f_q^\beta(t+\tau)) \\ \times \cos\left\{[\varepsilon_p^{(1)} + eV_0 - \varepsilon_q^{(2)}]\, \tau + \frac{eV_1}{\Omega}[\sin \Omega(t+\tau) - \sin \Omega t]\right\}.\end{aligned} \tag{6.13}$$

Let us note that effects of the type (6.13) will not appear in the electron–phonon collision integrals since in the Hamiltonian of the electron–phonon interaction the operators α_p enter in the combination $\alpha_p^+ \alpha_{p'}$, while the oscillating part (6.12) does not depend on the electron momenta.

If voltage oscillations at the junction are sufficiently large ($\Omega\tau \gg 1$, τ is the electron relaxation time), the kinetic equation for the α-film takes on the form

$$\frac{\partial f_p^\alpha}{\partial t} = -2\pi \sum_n \sum_q |T_{pq}|^2 I_n^2\left(\frac{eV_1}{\Omega}\right) (f_p^\alpha - f_q^\beta)\, \delta(\varepsilon_p^{(1)} + eV_0 - \varepsilon_q^{(2)} + n\Omega) + \mathscr{L}_p\{f^\alpha, N^{(1)}\} \tag{6.14}$$

where $\mathscr{L}_p$ are collision integrals describing the relaxation processes in the α-electrode.

The kinetic equation for f^β is obtained from (6.14) by a simple substitution $\alpha \rightleftarrows \beta$. As is evident from (6.14), the oscillating voltage is a tunnelling channel associated with the "emission and absorption quanta" of voltage oscillations. On the basis of such analogies, the phenomenon under consideration was called the "quantization" of voltage oscillations at a tunnel junction of normal metals [49]. This "quantization" does not only determine the value of the tunnelling current, but affects also significantly other properties dependent, generally speaking, of the electron distribution function satisfying a kinetic equation of type (6.14). Here is an example of one of the possibilities of analyzing the nonequilibrium distribution.

Let the constant component of the voltage vanish ($V_0 = 0$), the left film be thin (of thickness d) and the right film thick. At the same time we assume that the thick film is at potential zero, so that only the left film experiences chemical potential oscillations. The massiveness of the right film enables us to assume that, owing to the efficient spatial diffusion of the electrons, it is in the equilibrium state. Let us consider first the "single-quantum" limit $eV_1 \ll \Omega$. In the isotropic momentum approximation the kinetic equation (6.14) is represented by

$$\frac{\partial f_\varepsilon}{\partial t} = -I_0\left\{\left[1 - \frac{1}{2}\left(\frac{eV_1}{\Omega}\right)^2\right](f_\varepsilon - F_\varepsilon) + \frac{1}{4}\left(\frac{eV_1}{\Omega}\right)^2 [(f_\varepsilon - F_{\varepsilon+\Omega}) + (f_\varepsilon - F_{\varepsilon-\Omega})] + \mathscr{L}_\varepsilon^{\mathrm{ep}}\{f_\varepsilon, N\}\right\}. \tag{6.15}$$

The kinetic equation (6.15) results in the stationary case, for example, at $T = 0$, where $I_0\tau(eV_1/\Omega)^2 \ll 1$, as was shown in [49], in the development of strongly nonequilibrium distributions – in the formation of two "terraces" on the Fermi step.

The experimental research into nonequilibrium phenomena in normal tunnel junctions encounters great difficulties since in order to obtain effects anywhere near appreciable, junctions with sufficiently small tunnel barrier transparencies must be available. This problem is rather complex, therefore the number of experiments dealing with nonequilibrium effects is so far rather scarce.

7. Conclusion

In this report some new results obtained recently in the theory of kinetic phenomena in solids exposed to strong external fields have been surveyed. The approach based on the accurate account of the field taken in kinetic equations for quasiparticles proves, as has been demonstrated, to be successful and yields some new and sometimes unexpected results. There is every ground to believe now that this trend in solid state physics will culminate in the nearest future in a number of new and promising results. The author has attempted to show that the development of the theory of nonlinear and nonequilibrium phenomena in solids under intense external fields affecting substantially the quasiparticle interactions, is interesting not only from the point of view of finding the possible mechanisms causing nonequilibrium states and nonlinear properties but also as to the practical application of the peculiarities of these nonequilibrium states for various purposes: for generation and amplification of waves, for obtaining the necessary parameters of the systems, etc. The latter, to our mind, is very important for the now rapidly developing trend of creating new nonlinear elements in electronics and of elaborating basically new technical devices.

It is not accidental that the theoretical research that has lately been carried out while investigating the problems of the external field effects on the elementary acts of interactions, is mainly focused on those systems (normal and ferromagnetic semiconductors, ferro- and antiferrodielectrics, and superconductors) which are most promising for experimental studies and practical applications. Some problems concerning the experimental realization of quantum kinetic effects in strong fields and the use of these effects have already been dwelt upon in this report while supplying the theoretical results. We would like to make here some additional remarks on the experiments already available and on the prospects of the theoretical and experimental research into the phenomena arising under the influence of the external field on the quasiparticle interactions in solids.

As has already been noted in the Introduction, the theory is somewhat ahead of the experiment, as far as the problems covered in this report are concerned. On the one hand, this is likely to be associated with the fact that the understanding of the main peculiarities of quasiparticle interactions in external fields came comparatively not long ago and therefore the ways of the possible development of experiments were not known before then. On the other hand, in order to observe the phenomena owing its origin to the field action on the duration of elementary quasiparticle interactions, we need, as a rule, rather specific conditions of the experiment. For example, for the realization of a new mechanism of waves amplification in the conductors one needs in a strong constant electric field (section 5.1) a source of the gigacycle (or teracycle) band as well as sufficiently low temperatures (helium ones). However, in spite of these conditions being so specific, they are completely attainable at the present level of development of experimental technique. Nevertheless, despite the fact that some of the effects described in sections 2–6 still await experimental checks, one can already speak about substantial progress not only in the theoretical, but also in the experimental research into quantum kinetic effects under strong external fields.

One of the first experiments in which the role of the strong constant electric field was vividly elucidated in changing the probabilities of interband transitions of electrons with the absorption of the light field quanta were published in [145, 146]. Those studies corroborated Franz–Keldysh's theory [30, 31].

In narrow-band semiconductors there appears, due to the strong constant electric field a discrete term in the electron spectrum (Wannier–Stark levels [147]) and due to this the electron kinetics can acquire a number of interesting peculiarities*. At the resonance electron transitions nonmonotonous areas must appear between the Stark ladders in the field current dependence. This effect is described in detail in theoretical papers [51, 52] and was experimentally observed in [149] while examining ZnS. With the highly peculiar phenomena which could be observed when the electric field affected considerably the electron–phonon processes, can also be grouped negative absolute [150] and differential [151] conductivity of semiconductors and amplification of hypersound caused by the electron transitions along the Stark ladder with the emittance of phonons [152]. In particular, the last-mentioned effect seems to be promising for practical use in that frequency range ($q > 2p_0$) where the Cherenkovian mechanism of amplification is impossible. This, as has been noted in section 5.1, is associated with the fact that at $q > 2p_0$ electron–phonon processes are forbidden** without taking into account the electric field effect. It should also be noted that, as has been determined, the indirect optic exciton transitions

* The experimental proof of Wannier–Stark levels was given in [148].

** The "cutout" of electron–phonon processes under such conditions was observed in the experiments [153, 154].

which are allowed only if the interaction with phonons is taken into consideration, can essentially alter their character in strong constant electric field. The theory of the effects elaborated in [155] enabled the detailed explanation of the experimental results [156].

In the case of wide-band semiconductors (or under not too strong constant electric fields: $E \lesssim 10^4$–10^5 V/cm), the main field effect on the processes of electron scattering is the acceleration of electrons for the duration of the interactions (see the intra-collision effect in sections 2 and 5). This mechanism appears to have already been observed in the experimental study of the polaron mobility in InSb at 77 K [157]. These results were discussed qualitatively and quantitatively in [53]. The influence of the strong constant electric field on the electron–phonon processes is evidently important not only for semiconductors, but also for metals. For example, in the experiments with microcontacts (see [158–160]) while studying the spectrum dependence of the electron–phonon interaction function $\alpha^2(\omega) F(\omega)$, the field intensity in the vicinity of the microcontact is as high as 10^4–10^5 V/cm. Under such conditions the non-Cherenkovian generation of phonons arises [56, 57] which is likely to result in the "backgrounds" observed in the experimental curves $\alpha^2(\omega) F(\omega)$ at $\omega > \omega_D$ [157]. Unfortunately, there are up till now no direct experiments studying intra-collision effects. However, the experiments [157–160], and the observation of the curve asymmetry of the sound damping (see [57]) in respect testify rather convincingly to the point $\omega_q = \boldsymbol{qu}$ in favour of the considerations given in section 5. In refs. [161–163] it was shown that the problems of the electric field effect on the collision of particles are exceedingly important and must be taken into account in connection with the modelling of small semiconductor devices. Basic results were also obtained in the recent work [164] which studied hot carrier microwave conductivity in the non-zero collision duration regime. The data obtained in [164] are very essential for analyzing the experimental results in the study of the transient response characteristics of semiconductor devices.

In section 4 we have described the results obtained by the present moment on the theoretical research into so-called nonresonance parallel pumping in ferro- and antiferromagnets. It has been shown that such a nonresonance way of excitation of spin waves is attainable only due to the influence of the alternating magnetic field on the elementary acts of interactions of quasiparticles of the magnetic subsystem. In spite of the fact that the problem of nonresonance parallel pumping is a comparatively new one and the first theoretical results have been obtained quite recently (for more detail see the literature cited in section 4), there are at present first experimental proofs of the theory. The experiment [165] confirmed the effect of nonequilibrium cooling of spin waves predicted theoretically (section 4.1). It was shown that the main mechanism to which the formation of nonequilibrium states is attributed, is the external field effect on magnon scattering on dislocations.

The impact of external electric fields (both constant and alternating ones) on the electron interaction with the potential barrier in metal–barrier–metal junctions may now be considered to be comparatively well studied experimentally. In the case of a constant electric field (constant voltage applied to the tunnelling junction) the experiment [141] vividly demonstrated that nonequilibrium states of electrons can be realized under such conditions (section 6.1). The effect of high-frequency fields on electron interactions with tunnel barriers leads also to a number of interesting properties (section 6.3). Some of these properties are already widely used in technical applications for constructing frequency multipliers, irradiation mixers of submillimeter and infrared bands and visible spectrum detectors [166, 167].

In this report we have not discussed the similar problems in the sphere of superconducting states of metals. It should, however, be noted that it is in these very systems that one of the most spectacular experimental proofs of the effect of external alternating electromagnetic fields on quasiparticle interactions have been produced. For example, the excitation of quasiparticles in thin superconducting films by a high-frequency field is mainly due to the field effect on electron–impurity collisions [71–73].

The processes like these may culminate in fascinating nonequilibrium effects. In particular, the experiment [168] discovered the nonthermal destruction of superconductivity by laser irradiation. The effect of increasing critical parameters of superconductors as against the thermodynamically equilibrium values was experimentally discovered in [169] (see ref. [170]).

Some of the examples given above speak for the fact that the experimental research into quantum kinetic effects caused by the influence of external fields on quasiparticles in solids promises much and it should be expected that very soon new experimental results and proofs of the theory will appear. However, this does not yet mean that all the related theoretical problems have been solved.

Alongside the progress already made in this sphere there is still quite a number of problems to be solved. Therefore, to avoid a false impression made on the reader that the theory of kinetic phenomena in strong fields may be complete, we will enumerate the problems that remain to be solved in future.

One of the principal questions in the study of nonequilibrium effects taking into account the field impact on the elementary acts of quasiparticle interaction is that of the form of the quasiparticle nonequilibrium distribution function since its specific properties are largely determined by the parameters of the nonequilibrium distribution. Unfortunately, up to now, except for the τ-approximation in the case of slightly nonequilibrium states, the form of the distribution functions satisfying generalized quantum kinetic equations has not yet been found. To solve this problem numerical methods can primarily be applied. However, even if the computer programme for the calculation of the distribution function is accomplished, some basic questions can be solved, for example, by the method suggested in [171].

Of great interest may be the study of collective effects and of the problems of screening in conductors exposed to a strong constant electric field allowing for the effects considered in section 5. They are not practically studied and one may expect here some intriguing effects.

The problem of investigation of the nonequilibrium initial stage may turn out to be of basic importance in the case where a strong external field is switched on and also in the case of the system evolution with time towards the nonequilibrium state. This problem has been considered so far only in the approximation of the instantaneous character of interactions between the quasiparticles. Also topical is the problem of spatially inhomogeneous systems in an intense external field. Finally, we think it to be important to apply the approach considered in this report to other systems which have not yet been studied. Some considerations on the indicated problems have already been expressed in the literature though the review of these studies would be premature.

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