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Pressure-Dependent Dissipation Effect at Multiple Cantilever Resonant Modes

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Based on the optical deflection method, the resonant characteristics of a microcantilever under various pressure have been observed at room temperature to understand the pressure-dependent dissipation effect. Especially, the quality factor of the cantilever has been measured for up to fourth harmonic mode of cantilever resonance as a function of pressure between 0.1 and 1000 Torr. By considering the intrinsic dissipation present in the system at 0.1 Torr, the pressure-dependent fluidic quality factors were determined for the multiple cantilever resonant modes. The inverse of the fluidic quality factor appears to follow two different asymptotic behaviors at high and low pressure limits, which indicates that the dynamics of the fluid, due to the oscillating cantilever, changes from Newtonian to non-Newtonian with decreasing pressure. The experimentally observed transition of the fluidic dissipation effect agrees well with the recently proposed rapidly oscillating flow model based on the Boltzmann equation, regardless of the different mode shapes.

Keywords: Dissipation, Fluidic Quality Factor, Weissenberg Number, Microcantilever.

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1. INTRODUCTION

Recently, miniaturized mechanical systems such as nanoelectromechanical systems (NEMS) and microcantilevers have regained much attention along with the advances in fabrication techniques.[1–3] They posses much improved mechanical properties, such as high resonance frequencies and quality factors in their resonant modes, compared to previously-studied bulk mechanical systems. Based on these superior mechanical characteristics, these miniaturized systems have been proven to be promising in many of technological applications. For example, the micro-fabricated AFM cantilever has been most commonly used for the precise determination of various surfaces.[4] Also, the microcantilevers and NEMS devices have been explored as sensing elements in sensor applications.[5,6]

The energy dissipation effects are always present in these mechanical systems and there have been theoretical and experimental attempts to address this issue originating from various mechanisms.[7–10] Typically, when one uses the mechanical sensing element, the detection sensitivity is closely related to the quality factor of the mechanical system.[11] Therefore it is imperative to understand the dissipation effect in these miniaturized mechanical systems operating under various gas or liquid flow to achieve the optimized detection sensitivity under various conditions. Under moderate vacuum, the damping of these mechanical systems will be dominated by the intrinsic dissipation effect. However, with increase in flow, one has to assess the additional fluidic dissipation factor affecting the motion of the mechanical system.

The fluid dynamics coupled with an oscillating mechanical element can be characterized with the oscillation frequency (ω) of the mechanical element and the relaxation time (τ) of the fluid. For $\omega < 1/\tau$, the flow due to the motion of the mechanical element follows the Newtonian approximation, but for $\omega > 1/\tau$, this approach is no longer valid. The expected break-down of the Newtonian approximation has been considered with a model of an rapidly oscillating plate based on the Boltzmann equation, where the fluidic dissipation factor has been given as a universal function of the Weissenberg number $\omega\tau$.[12] Following this theoretical description, the pressure-dependent dissipation effect in a wide range of $\omega\tau$ has been investigated experimentally using doubly-clamped beams and cantilevers, mostly in their fundamental resonance modes.[13]

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Here, we present an investigation of the dissipation effect in a microcantilever under various pressures ranging from 0.1 to 1000 Torr. The changes in the quality factor due to the fluidic dissipation are measured up to the fourth harmonic of the resonant mode. To see the effect of different mode shapes on the fluidic dissipation, the experimentally observed dissipation factors in five resonant modes are compared to the recently proposed dissipation model in non-Newtonian limit.

2. EXPERIMENTAL DETAILS

The resonant modes of a silicon microcantilever with dimensions ($l \times w \times t$) of 520 μm $\times$ 40 μm $\times$ 4 μm have been identified up to fourth harmonic by measuring the amplitude of the photodiode output due to the change in the position of the optical beam on a dual-element photodiode, reflected from the cantilever inside of a small vacuum chamber as it moves.[14] This optical deflection measurement was carried out at room temperature through an optical window in front of the vacuum chamber with a He-Ne laser with a wavelength of 632 nm, and the pressure inside was adjusted by carefully introducing N_2 gas into the chamber. The cantilever was placed on top of a piezoelectric actuator and actuated by applying ac voltage to the actuator near the resonance. The resonance frequency (ω_{res}) and the quality factor (Q) for each mode were determined by fitting the resonance spectrum to Lorentzian curve as shown in the inset of Figure 1 and the values of ω_{res} and Q for each mode at 1000 Torr are listed in Table I. The values of ω_{res} are close to the theoretically predicted values for a monolithic cantilever beam based on the observed resonance frequency at the fundamental mode.[15] Once the fundamental and higher harmonic resonance modes of the cantilever were identified, we have

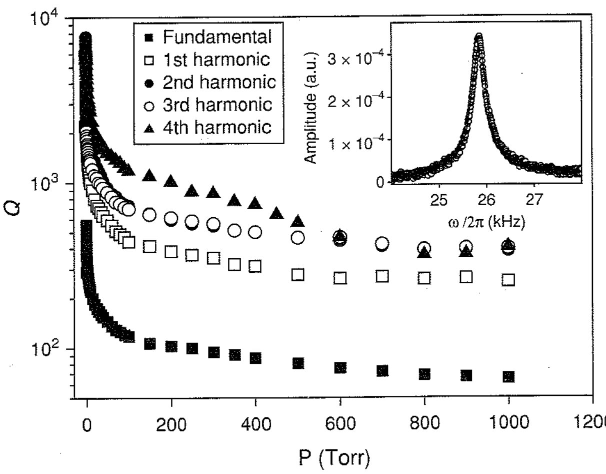

Fig. 1. Quality factor Q of a microcantilever at multiple resonant modes as a function of pressure. The inset shows the fundamental resonance line shape at 1000 Torr. The solid line is a Lorentzian fit with $\omega_{res}/2\pi$ of 25.9 kHz and Q of 65.

Table I. The observed resonance frequencies ω_{res}^{exp} and the values of quality factor Q of five resonant harmonic modes at 1000 Torr. ω_{res}^{theo} is the predicted resonance frequency at each harmonic mode, calculated for a monolithic cantilever beam based on the experimentally determined fundamental resonance frequency. Also, the fluid-dependent coefficient α in Eq. (1), obtained from the fit, and the predicted transition point p_c ($\omega\tau = 1$) are shown.

Mode	$\omega_{res}^{exp}/2\pi$ (kHz)	$\omega_{res}^{theo}/2\pi$ (kHz)	Q	α	p_c (Torr)
Fund.	25.9	—	65	0.18	0.3
1st	162.9	162.0	250	0.17	1.9
2nd	456.1	453.7	384	0.20	5.3
3rd	894.0	889.0	398	0.22	10.4
4th	1475.3	1469.6	405	0.20	17.1

measured the resonance frequency and quality factor for each resonance mode as a function of N_2 gas pressure between 0.1 and 1000 Torr.

3. RESULTS AND DISCUSSION

The observed changes in quality factor Q at multiple resonance modes are shown as a function of pressure in Figure 1. With decrease in pressure, the value of Q increases due to the diminishing damping effect, as expected. However, the experimentally determined Q-values do not fully describe the pressure-dependent dissipation effect present in this rapidly oscillating gas flow, since the observed Q-values contain both intrinsic and fluidic dissipation terms. The motion of the cantilever under the presence of the gas can be simplified as of a forced harmonic oscillator with a corresponding equation of motion of $\ddot{x} + (\gamma_0 + \gamma_{gas})\dot{x} + \omega_{res}^2\, x = f$, where the γ_0 and γ_{gas} are intrinsic and pressure-dependent damping factors, and f is the drive force per unit mass. Since the damping factor is inversely proportional to the quality factor, the fluidic quality factor Q_{gas}, which is the measure of the damping of the motion of the cantilever due to the surrounding gas, can be determined from $1/Q_{gas} = 1/Q - 1/Q_0$ where Q_0 is the pressure-independent intrinsic quality factor, present regardless of the external environment, and Q is the total value measured at each pressure. Here, Q_0 is taken to be the value measured at 0.1 Torr, where the damping due to the surrounding gas is negligible.

Figure 2 shows the inverse of the fluidic quality factor $1/Q_{gas}$ at multiple resonant modes as a function of pressure between 0.1 and 1000 Torr. They exhibit similar pressure dependencies with two different asymptotic behaviors at low and high pressure limits. A recent theory by Yakhot and Colosqui[12] predicts that this apparent change in the pressure dependence is due to the transition of the fluid flow from viscous flow at high pressure with small τ to viscoelastic fluid flow at low pressure with large τ, where the Newtonian approximation breaks down.

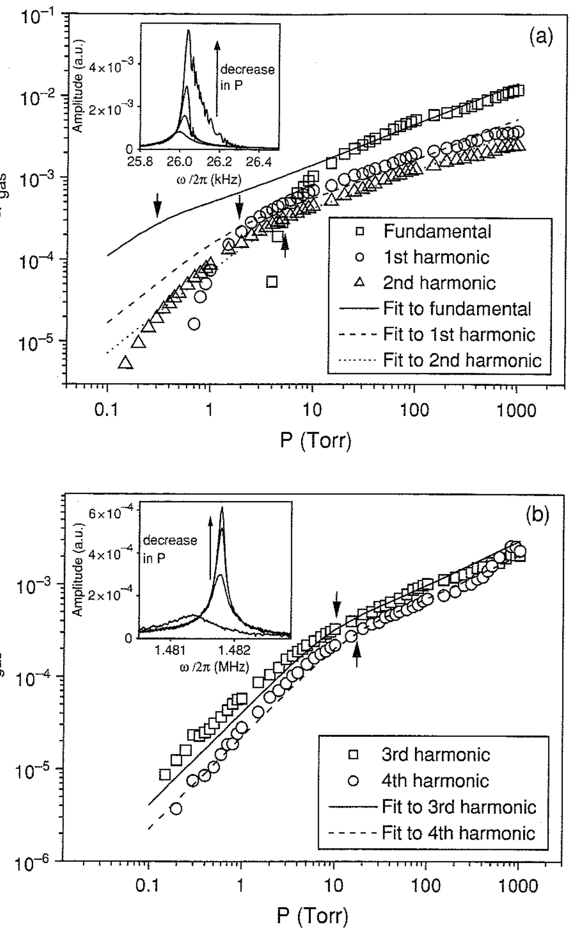

Fig. 2. Inverse of the fluidic quality factor $1/Q_{gas}$ as a function of pressure for (a) fundamental, first and second harmonic modes, and (b) third and fourth modes. The lines are fits to Eq. (1) and the expected transition points p_c are indicated with the arrows for the corresponding resonant modes. The insets in (a) and (b) show the fundamental and fourth harmonic resonances at 0.1, 1, 5, and 50 Torr. The nonlinear response of the fundamental resonance at low pressure can be seen with the broadened resonance line shape.

The details of the pressure-dependent fluidic dissipation effect can be further described with this rapidly oscillating flow model based on the solution of Boltzmann equation for an infinite plate. In this model, $1/Q_{gas}$, corresponding to the fluidic damping coefficient, at pressure p is given by

$$\frac{1}{Q_{gas}} = \alpha\sqrt{\frac{p}{\omega}}(1+\omega^2\tau^2)^{-3/4}\left[(1+\omega\tau)\cos\left(\frac{\tan^{-1}\omega\tau}{2}\right) - (1-\omega\tau)\sin\left(\frac{\tan^{-1}\omega\tau}{2}\right)\right] \quad (1)$$

where α is a fluid-dependent coefficient. The experimentally determined $1/Q_{gas}$ are fitted with Eq. (1) as shown in Figure 2. The fits are obtained by varying α and considering the relation between the relaxation time and the pressure (in units of nanoseconds and Torr), reported to be $\tau \sim 1850\ p^{-1}$.[13] We have also observed the decrease of ω_{res} with pressure due to the mass-loading effect. Since this change in ω_{res} was less than 1% in all the modes within the range of pressure studied here, we have used ω_{res} at 1000 Torr for ω in Eq. (1). The resulting α from the fits are listed in Table I and all the values from the different modes are close since they only depend on the surrounding fluid, which is N_2 gas in this work.

However, in Figure 2(a) we have noticed that at low pressure, $1/Q_{gas}$ of the first two modes of resonance drop much faster than expected from Eq. (1), while $1/Q_{gas}$ at the second mode only shows a slight deviation below 0.3 Torr. At low pressure, diminishing damping leads to the larger displacement amplitude of the cantilever and the cubic term in the equation of motion is no longer negligible. Above the critical amplitude, the motion becomes that of a non-linear Duffing resonator, accompanying the broadening of the resonance spectrum as well as asymmetric line shape due to the bistability. The presence of the strong non-linear response, even at the lowest actuation power tried here, in the low resonant modes (shown in the inset of Fig. 2(a)) causes the deviation of $1/Q_{gas}$ from Eq. (1) since the resulting value of Q at 0.1 Torr would be much smaller than the intrinsic Q-value due to the broadened spectrum. As the cantilever is driven into higher resonant modes, this non-linear effect gets weaker (inset of Fig. 2(b)), resulting in better fit to Eq. (1). The rather smaller values of α observed in first two modes, where the non-linearity is most apparent, are consistent with the broadening of the spectrum, since the nonlinear effect tends to lower the entire $1/Q_{gas}$ curve in Figure 2.

Besides the deviation due to the appearance of the non-linear resonance, all of our data obtained from different resonant modes are in a good agreement with the theory by Yakhot and Colosqui. In this theoretical attempt of explaining the fluidic energy dissipation effect, the dimensionless Weissenberg number is the only relevant dynamical parameter and Eq. (1), derived from this rapidly oscillating flow model, is not expected to be a function of additional factors such as the geometrical dimensions and the mode shape of the mechanical resonator. The fact that the observed pressure-dependency of $1/Q_{gas}$ closely follows the relation of Eq. (1), regardless of the mode shapes of the oscillating cantilever, confirms the universality of the theory. Also, the theory predicts that the transition between the Newtonian and non-Newtonian regimes—shown as the change in the slope of $1/Q_{gas}$ curve—occurs at $\tau = 1/\omega$. Since $\tau \sim p^{-1}$, one can determine the expected transition point p_c in pressure, which would be proportional to the oscillation frequency ω. These expected values of p_c for each harmonic mode are listed in Table I. Accordingly, Figure 2 further shows that the observed transitions appear at the expected transition points, shifting toward higher pressure with increasing resonance mode number.

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4. CONCLUSIONS

We have studied the pressure-dependent dissipation effect in a microcantilever under various N_2 gas pressure. The observed $1/Q_{gas}$ exhibits two distinct behaviors, suggesting that the nature of the flow, generated by the oscillating cantilever, changes. This pressure dependence in fluidic energy dissipation shows the transition from non-Newtonian to Newtonian fluid as $\omega\tau$ goes to zero and by investigating them at multiple resonant modes, we have been able to observe a range of transition points. Our results agree well with the recently proposed model of rapidly oscillating flow generated by a plane oscillator, regardless of the cantilever mode shape, confirming the universality of the theory. This study clearly demonstrates the dramatic response of a miniaturized mechanical system even to a slight external perturbation, which will allow exploring the fundamental aspects of nanofluidics and related applications.

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