MEASUREMENT OF RAYLEIGH BACKSCATTER-INDUCED LINEWIDTH REDUCTION

Indexing term: Optics

We present measurements of the linewidth reduction for a semiconductor laser exposed to the Rayleigh backscatter from a single-mode fibre. The linewidth is measured as a function of fibre length and laser to fibre coupling efficiency and shows good agreement with the estimate from a simple statistical model.

In this letter we report on a measurement of the extreme linewidth reduction that can be obtained by exposing a semiconductor laser to the Rayleigh backscatter from a single-mode fibre.[1] We present the first measurement of the linewidth reduction as a function of fibre length and laser-to-fibre coupling efficiency. The results are shown to be in good agreement with the predictions from a simple statistical model.

A compound cavity laser oscillates at an angular frequency ω which is a solution to a frequency condition of the form[2,3]

$$\omega_0 = \omega + g(\omega) \tag{1}$$

where ω_0 is the angular frequency of the solitary laser and $g(\omega)$ is given in terms of the effective reflectivity of the laser mirror facing the external cavity. A stable solution requires $dg/d\omega > 0$. The linewidth $\Delta\nu$ (FWHM) is[3]

$$\Delta\nu = \Delta\nu_0\left(\frac{d\omega_0}{d\omega}\right)^{-2} = \Delta\nu_0\left(1 + \frac{dg}{d\omega}\right)^{-2} \tag{2}$$

where $\Delta\nu_0$ is the linewidth of the solitary laser. For a semiconductor laser exposed to the Rayleigh backscatter from a single-mode fibre, the function $g(\omega)$ is fluctuating with Gaussian distribution. The positive slopes $g' \equiv dg/d\omega$ at the solutions to eqn. 1 for constant ω_0 will therefore follow a Rayleigh distribution:[4]

$$P(g') = \frac{g'}{R_2}\exp\left[-\frac{(g')^2}{2R_2}\right] \tag{3}$$

where R_2 is obtained from the second-order derivative of the autocorrelation of $g(\omega)$. This gives[5]

$$R_2 = \frac{(1+\alpha_e^2)\alpha_s S}{3}\left(\frac{1-r_2^2}{v_g \tau_{in} r_2}\right)^2 a^2 L_e^3 \tag{4}$$

where a is the laser-to-fibre power coupling efficiency, L_e is the effective fibre length, given by $L_e^3 = \frac{3}{4}(e^x - 1 - x - (x^2/2))e^{-x}\alpha^{-3}$, $x = 2\alpha L$, α is the fibre attenuation and L is the fibre length. α_e is the linewidth enhancement factor, α_s is attenuation due to Rayleigh scattering, S is the recapture coefficient, r_2 is the amplitude reflectivity of the laser mirror facing the fibre, v_g is the group velocity in the fibre, and τ_{in} is the laser cavity round-trip time. By eqns. 2 and 3 the most frequent linewidth is $\Delta\nu = \Delta\nu_0/R_2$ for $R_2 \gg 1$. We therefore expect the linewidth to be proportional to a^{-2} and L_e^{-3}.

We have measured the linewidth of a 1·3 μm laser diode (NEC 5003) coupled to a single-mode fibre with attenuation α = 0·55 dB/km at 1·3 μm. To obtain single-mode operation the laser was coupled to a 28 mm grating cavity through the back facet. The Fresnel reflection from the fibre far end was suppressed by using an index-matching cell. The coupling between laser and fibre was realised with a drawn taper mounted on a PZT positioner. The coupling efficiency was varied by misaligning the fibre and was determined by measuring the output power from the fibre.

The output light from the feedback fibre was analysed with a Fabry–Perot interferometer and a delayed self-heterodyne linewidth measurement set-up. For the latter the time delay difference was τ = 30 μs. The power spectrum for the detector current generated by the beat signal is

$$S(f) \propto \frac{2\,\Delta\omega}{\omega^2 + (\Delta\omega)^2}\left[1 - e^{-\Delta\omega\tau}\left(\cos\omega\tau + \frac{\Delta\omega}{\omega}\sin\omega\tau\right)\right] + e^{-\Delta\omega\tau}\delta(f) \tag{5}$$

Here $\omega = 2\pi f$ is the angular frequency deviation from the beat frequency and $\Delta\omega = 2\pi\,\Delta\nu$. The linewidth $\Delta\nu$ can therefore be determined by fitting the expression (eqn. 5) to the measured spectrum.[6] In Fig. 1 we show the average power spectrum for the case with L_e = 760 m and a = 0·14. The spectrum is well approximated by eqn. 5 for $\Delta\nu$ = 10 Hz. The δ function in eqn. 5 is modified by the filter of the spectrum analyser. The level of the shoulder at $f = 0$ is a factor $\Delta\omega\,\Delta f\tau^2$ below the maximum of the central peak, Δf being the effective filter bandwidth. If the laser oscillates stably in a single external cavity mode the linewidth obtained by this method is the linewidth due to spontaneous emission phase noise. However,

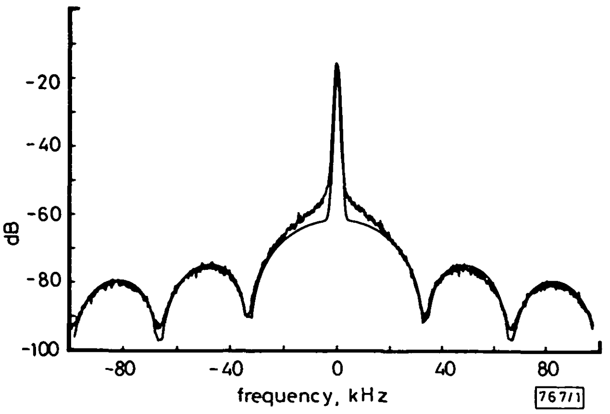

Fig. 1 *Averaged spectrum analyser signal (noisy trace) and theoretical spectrum (eqn. 5) for $\Delta\nu$ = 10 Hz (solid line)*

L_e = 760 m, a = 0·14

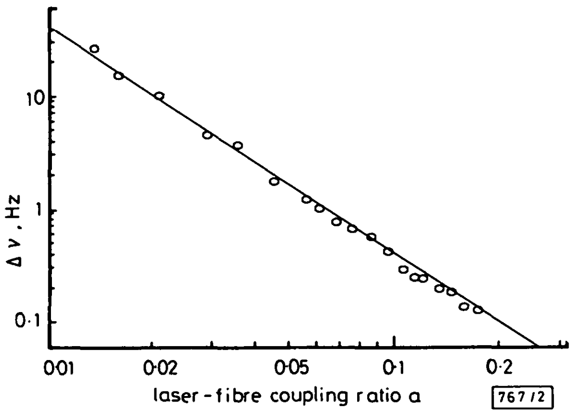

Fig. 2 *Linewidth against coupling efficiency for L_e = 1420 m*

○ experiment
—— theory

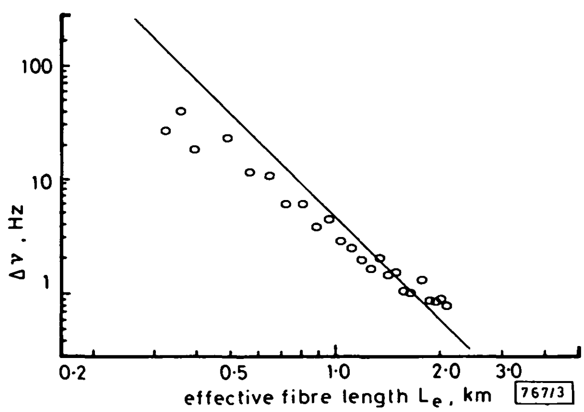

Fig. 3 *Linewidth against effective fibre length for a = 0·04*

○ experiment
—— theory

during the averaging process the laser frequency jumps between external cavity modes and the measured linewidth is therefore an average linewidth. To compare with theory, the logarithm of the spectrum (eqn. 5) must be averaged over the distribution (eqn. 3). For $2\pi\,\Delta\nu_0\,\tau/R_2 \ll 1$ this gives the prediction $\Delta\nu \simeq \frac{1}{2}e^{\gamma}\,\Delta\nu_0/R_2 = 0{\cdot}89\,\Delta\nu_0/R_2$ (γ is Euler's constant) for the measured linewidth.

Figs. 2 and 3 show the measured linewidth as a function of the coupling efficiency a and the effective fibre length L_e, respectively. The various fibre lengths were obtained by cutting the feedback fibre. The solid lines are the theoretical predictions for $r_2 = 0{\cdot}6$, $\alpha_e = 6{\cdot}8$, $\alpha_s = 0{\cdot}35$ dB/km and $S = 1{\cdot}22 \times 10^{-3}$. $\Delta\nu_0$ was measured to be 17 MHz. For a long fibre Fig. 2 shows good agreement between theory and experiment over a range of linewidths from 0·1 Hz to 20 Hz. The discrepancy for short fibre lengths in Fig. 3 may be due to the influence of polarisation or the correlation between g and g' for the lasing modes.

In conclusion, we have demonstrated significant line narrowing for a laser exposed to Rayleigh backscattering from a single-mode fibre. A good estimate is obtained from a simple analytical expression.

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27th August 1985

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UNIFORM BEND LOSS OF THE TE_{0q} MODE IN CIRCULAR WAVEGUIDES WITH SMALL SURFACE IMPEDANCE AND LARGE SURFACE ADMITTANCE

Indexing terms: Waveguides, Electromagnetic wave theory

The bending losses of the TE_{0q} mode in circular hollow waveguides are obtained with small surface impedance and large admittance based on the exact vector analyses for Maxwell's equations. Considerable difference is found for a special case between the present result and that obtained previously by using the coupled-mode theory.

Introduction: The coupled-mode theory[3] is a very powerful method to analyse many problems relating to guided modes in dielectric or hollow waveguides. As the theory itself is based on the completeness of the modes, results predicted are accurate when all modes including radiation modes are taken into account. However, to obtain explicit expressions for propagation properties, modes are restricted to several or only two modes. Therefore, the results should be checked carefully.

In this letter we first derive an explicit bending loss formula of the TE_{0q} mode in oversized circular hollow waveguides with small normalised surface impedance and a large admittance based on the theory of Reference 4. Next, we apply the formula to a special case of a circular cylinder embedded in a medium with a complex refractive index and show that a large difference exists between the present result and that obtained previously by using the coupled-mode theory.[1,2]

Analysis: In a previous paper[4] we derived vectorially a general formula to describe a uniform bend loss in circular oversized hollow waveguides characterised by a normalised surface impedance z_{TE} and admittance y_{TM},[5,6] at $r = T$ (a hollow core radius):

$$\left.\frac{E_\theta}{H_z}\right|_{r=T} = \frac{\omega\mu_0}{n_0 k_0} z_{TE} \qquad \left.\frac{H_\theta}{E_z}\right|_{r=T} = -\frac{n_0 k_0}{\omega\mu_0} y_{TM} \tag{1}$$

where n_0 ($\simeq 1$) is a refractive index in the hollow core, k_0 is the wavenumber in vacuum, and we assume that $n_0 k_0 T \gg 1$.

Let a waveguide be bent uniformly with a large bending radius R and the axial phase constant β be expressed by

$$\beta = \beta_0 + \frac{\delta\beta}{R^2} + \ldots \tag{2}$$

We hereafter consider the case

$$|z_{TE}| \ll n_0 k_0 T/u_0 \qquad |y_{TM}| \gg n_0 k_0 T/u_0 \tag{3}$$

where $\beta_0 = \beta_\infty - j\alpha_\infty$ is the axial phase constant in a straight waveguide and u_0 is the root of $J_1(u_0) = 0$.

By applying eqn. 45 of the expression of $\delta\beta$ in Reference 4 to the TE_{0q} mode under the condition eqn. 3, we have deduced $\delta\beta$ after many calculations as follows:

$$\delta\beta = -j\frac{(n_0 k_0 T)^2 T}{2u_0^2} y_{TM} \tag{4}$$

Therefore, noticing that α_∞ is expressed by

$$\alpha_\infty = n_0 k_0 \frac{u_0^2}{(n_0 k_0 T)^3} \mathrm{Re}\,(z_{TE}) \tag{5}$$

we express the attenuation constant α in the bent circular waveguide as follows:

$$\alpha = \alpha_\infty\left[1 + \frac{1}{2}\left(\frac{n_0 k_0 T}{u_0}\right)^4\left(\frac{T}{R}\right)^2 \frac{\mathrm{Re}\,(y_{TM})}{\mathrm{Re}\,(z_{TE})}\right] \tag{6}$$

Now, we apply eqn. 6 to a circular hollow waveguide where the outer region ($r > T$) consists of an infinite single layer with a large complex refractive index of $n_0(n - j\kappa)$. As the normalised surface impedance z_{TE} and admittance y_{TM} are

$$z_{TE} = [(n - j\kappa)^2 - 1]^{-1/2} \simeq (n - j\kappa)^{-1} \tag{7}$$

$$y_{TM} = (n - j\kappa)^2/[(n - j\kappa)^2 - 1]^{1/2} \simeq n - j\kappa \tag{8}$$

we can express α as follows:

$$\alpha = \alpha_\infty\left[1 + \frac{1}{2}\left(\frac{n_0 k_0 T}{u_0}\right)^4\left(\frac{T}{R}\right)^2 (n^2 + \kappa^2)\right] \tag{9}$$

On the other hand, the coupled-mode theory was used previously to predict bending losses of the TE_{0q} mode which only considers the coupling between the TE_{0q} mode and a degenerate mode of the TM_{1q} mode. By using the fact that the coupling coefficient c between the two modes is given by[1]

$$c = \frac{n_0 k_0 T}{\sqrt{(2)} u_0 R} \tag{10}$$