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TRANSPORT IN A POLARIZATION-INDUCED 2D ELECTRON GAS

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AlGaN/GaN structures constitute a new class of 2D systems in that a large population of electrons can be produced without doping as a result of spontaneous and strain-induced polarization. Electron transport can, in principle, be mediated solely by phonon scattering and, for the first time, it is possible to realistically envisage the formation of a drifted Maxwellian or Fermi-Dirac distribution in hot-electron transport. We first describe a simple model that relates electron density in a heterostructure to barrier width and then explore electron-electron (e-e) energy and momentum exchange in some depth. We then illustrate the novel hot-electron transport properties that can arise when only phonon and e-e scattering are present. These include S-type NDR, electron cooling and squeezed electrons.

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1. Introduction

The spontaneous and piezoelectric polarization exhibited by semiconductors that crystallize in the wurtzite structure can be exploited to induce large, quasi two-dimensional (2D) electron populations [1, 2] which, because they do not depend for their existence on donor impurities constitute, in fact, a new class of 2D electron systems. It is the purpose of this chapter to give an account of the physics that enters the description of these new properties. Inevitably, given the current state of the art of crystal growth, experiments on electron transport that have been carried out have been influenced by the presence of unwanted impurities and a high concentration of dislocations and, not surprisingly, there have been no reports of properties that we will be describing. Contact of theory with experiment must therefore await the production of purer and more perfect crystals and heterostructures. Since these are likely to be in the III-V nitride systems, given the technological interest in visible lasers and in high-power microwave transistors, we focus on these systems and, in particular, GaN and its alloys with AlN, though our analysis will apply to II-VI and other semiconductors.

2. Polarization-Induced Electrons

Wurtzite and other low symmetry crystal systems can exhibit spontaneous polarization directed along the c-axis as well as polarization arising out of barrier strain in heterostructures through the piezoelectric effect, and this has been quantified for the III-V nitrides [1, 2] and exploited to produce a quasi-2D electron gas without deliberate doping in AlGaN/GaN heterostructures leading to successful FET devices [3, 4]. The questions arise: where do the induced electrons come from, and how is their density related to the polarization?

The first task is, therefore, to explain the origin of polarization-induced electrons in undoped structures that are grown along the C-axis. (Growth along the A-axis would eliminate the effect of spontaneous polarization.) It has been argued that the source of electrons is the surface charge acquired by the structure from the adsorption of ambient ions and from changes in the charge trapped in surface states [5, 6, 7] and this explanation will be adopted here. Thus, in this model, a free-standing slab of GaN will have zero internal electric field, the large field associated with spontaneous polarization having been entirely neutralized by surface charges. In the case of a heterostructure consisting of an AlGaN barrier on a GaN buffer layer, both grown along the C-axis, the upper and lower surfaces have different total polarization charges induced due to the spontaneous polarization and piezoelectric strain field and, consequently, surface charges cannot entirely eliminate internal fields. In general, the surface charge on the upper surface will be different from that at the lower surface, and it is this difference that determines the induced electron density. Specifically, the upper Ga-face, AlGaN surface turns out to have a negative polarization charge which will attract a positive surface charge density, $+\sigma_{S1}$, and the lower, GaN surface will have a positive polarization charge which will attract a negative surface charge density, $-\sigma_{S2}$; the induced electron density is then equal to $(\sigma_{S1} - \sigma_{S2})$ for overall charge neutrality to obtain. The surface charge densities can be obtained from the known polarization charge densities by first deducing the fields from Gauss's equation and then requiring that the voltage difference across the structure vanishes. The voltage difference across the GaN will include the depth of the quantum well at the AlGaN/GaN interface which, in turn, will depend on the induced electron density. The solution, therefore, will have to satisfy a self-consistency condition.

In typical practice at the present time, an AlGaN/GaN structure is grown on a thin nucleation layer of AlN deposited on a sapphire or SiC substrate. The lack of a lattice-matched substrate means that the nucleation layer is highly dislocated, and dislocations thread their way to the top surface, their density being of order 10^9 cm^{-2}. These dislocations introduce deep states that capture electrons from donors inadvertently introduced during growth that will affect the mobility of electrons if not screened [8, 9]. As a consequence of this acceptor-like action, acting along with other acceptors inadvertently introduced, the heterostructure is heavily compensated with a mobile electron density of less than 10^{14} cm^{-3} [10]. In nominally undoped material, therefore, screening of the sheet charges introduced by polarization, other than by the induced 2D gas, can be ignored. In some cases the barrier is doped n-type and, in general, this situation needs to be considered, though adding impurities destroys the unique features introduced by polarization.

We illustrate our elementary electrostatic model for the case of an n-type, uniformly doped barrier that is entirely depleted of electrons [9]. Figure 1 depicts the electron-energy diagram. We assumed that the structure is in thermodynamic equilibrium with zero electric fields applied outside. We also assumed that the characteristic size of the area of surface and interface greatly exceeds the thickness of the layers, so that the electrostatic problem reduces to a one-dimensional one. The Fermi level is chosen to be $e\phi$ below the conduction band in the GaN buffer layer. In the case of a free GaN surface measurements of Schottky barriers suggest that the Fermi level is pinned by surface states such that $\phi \approx 1$V. Though deviations from this magnitude may be expected to occur at the internal surface of GaN and the nucleation layer we will assume that the Fermi level remains deep in the forbidden gap, in accord with observation [10]. At the lower boundary of the 2D gas the transition from quantum well to bulk will be relatively rapid. For simplicity we assume that the 2D gas acts like a sheet charge of zero width exactly like all the other charges in the problem.

Beginning on the left, at the surface, we assume without loss of generality, the existence of a surface charge $+\sigma_{S1}$ (adsorbed ions/surface state population charge) in response to the polarization charge of the Ga face AlGaN, $-\sigma_{p1}$ (spontaneous plus piezoelectric), and there will also be a surface-state charge $-\sigma_{D1}$ arising from the electrons from the donors in the barrier. If ε_1 is the permittivity of the barrier, the field E_{11} immediately in the barrier is given by: $\varepsilon_1 E_{11} = \sigma_{s1} - \sigma_{p1} - \sigma_{D1}$. Solving Gauss's equation for the depleted barrier gives the field E_{12} at the interface:

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$\varepsilon_1 E_{12} = \varepsilon_1 E_{11} + en_D a_1$, where n_D is the donor density and a_1 is the barrier width. The field E_2 in the buffer layer is then given by: $\varepsilon_2 E_2 = \varepsilon_1 E_{12} + \sigma_{p1} - \sigma_{p2} - \sigma_{D2} - \sigma_n$.

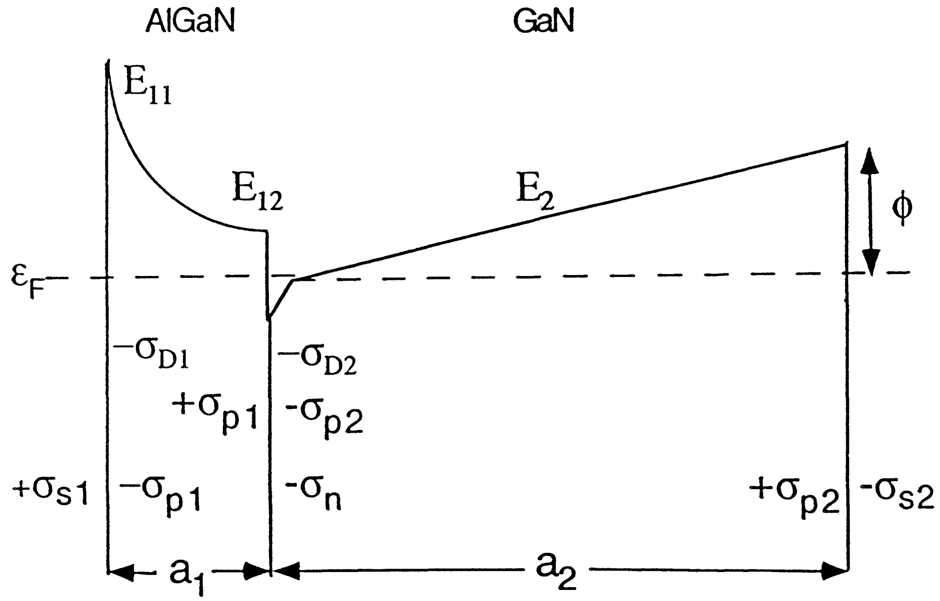

Figure 1. Schematic conduction-band energy dependence through a AlGaN/GaN heterostructure. The AlGaN barrier is doped n-type and completely depleted, and the GaN is free of mobile charge. The sheet-charge densities are defined in the text.

Here, ε_2 is the permittivity in the buffer layer, σ_{p2} is the magnitude of the polarization charge in GaN, σ_{D2} is the charge contributed by the donors, and σ_n is the magnitude of the induced charge. Assuming zero field beyond the buffer layer, we obtain another equation for E_2, i.e.: $-\varepsilon_2 E_2 = \sigma_{p2} - \sigma_{s2}$, where σ_{s2} is the magnitude of the negative charge at the lower boundary. It is then straightforward to show that $\sigma_n = \sigma_{s1} - \sigma_{s2}$, noting that $\sigma_{D1} + \sigma_{D2} = en_D a_1$.

The voltage change across the barrier is obtained by integrating Gauss's equation, thus: $V_{12} = V_{11} - E_{11}a_1 - (en_D a_1^2)/2\varepsilon_1$ in obvious notation. The voltage change across the buffer layer is: $-\Delta / e - E_2 a_2$ and $-E_2 a_2 = \phi$, where Δ is the magnitude of the energy depth of the quantum well which is determined by the electron density in the well, and a_2 is the width of the buffer layer. For simplicity, we assume all the electrons occupy the lowest sub-band, and obtain: $\Delta = (\sigma_{D2} + \sigma_n) / eN_s$ where $N_s = m^* / \pi\hbar^2$ is the density of states per unit energy and m* is the effective mass of the electrons in GaN. Constraining the total voltage change across the structure to be zero allows us to obtain all the unknowns. In particular, the total 2D electron density, $\sigma_{tot} = \sigma_{D2} + \sigma_n$ is given by:

$$\sigma_{tot} = (1+\eta)^{-1}\left[\sigma_{p1} - \sigma_{p2} + 0.5 en_D a_1 - (\varepsilon_2 \phi / a_2)(1+\alpha)\right], \tag{1}$$

where $\eta = \varepsilon_1 \pi\hbar^2 / e^2 m^* a_1$ and $\alpha = \varepsilon_1 a_2 / \varepsilon_2 a_1$.

This simple equation predicts that half of the doping electrons appear in the 2D gas and that the barrier width must exceed a minimum, given by the zero of Eq. (1), for a

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2D gas to be established. Figure 2 illustrates the barrier-width- and ϕ-dependences for doped and undoped, pseudomorphic $Al_{0.3}Ga_{0.7}N$ barrier on 2μm GaN with $\varepsilon_1 = 10.3\varepsilon_0$, $\varepsilon_1 = 10.4\varepsilon_0$, $\sigma_{p1} = \sigma_{p1}(spon) + \sigma_{p1}(piezo) = (0.045 + 0.11)Cm^{-2}$, $\sigma_{p2} = 0.029Cm^{-2}$.

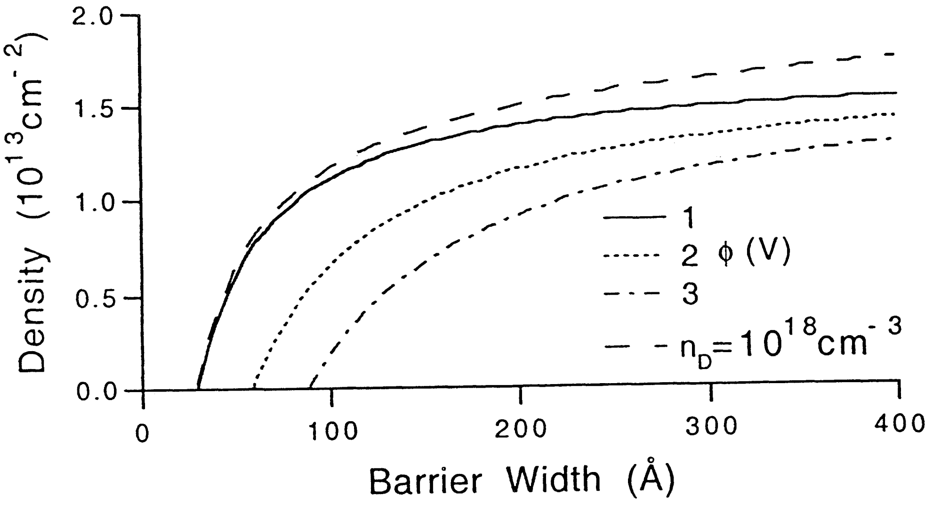

Figure 2. Dependence of electron density on barrier width and Fermi level. The effect of doping with 10^{18}cm^{-3} donors is shown for the case ϕ=1V.

Comparison with published data, e.g. Ref. 2, suggests that $\phi \approx 1$V. The important point here is that in the absence of doping the induced electron density is still large. In what follows we will assume that impurities are entirely absent.

In principle, therefore, we have a new situation in semiconductor physics, in which a substantial density of electrons can be created without adding donor impurities. As far as electron transport is concerned, this means that we can contemplate an ideal situation in which scattering by impurities and other defects is negligible, and only phonon scattering need be considered. Moreover, the presence at the same time of a large density of electrons means that transport will be markedly influenced by electron-electron (e-e) scattering. One interesting effect of this combination of circumstances is the possibility that under the influence of an applied electric field the distribution function of the electrons becomes a drifted Maxwellian, in the non-degenerate case, or a drifted Fermi-Dirac in the degenerate case. The usual case in semiconductor physics, where the effect of e-e scattering is countered by substantial impurity scattering, is one in which steady-state drifted distributions are unattainable, so the effect of polarization offers new possibilities. We need, therefore, to examine phonon scattering rates in order to compare them with the e-e scattering rate and in order to solve the Boltzmann equation.

3. The Boltzmann Equation

Provided that we are not interested in describing phenomena in ultra short times, over ultra short lengths or in very high electric fields, and provided that collisions are not too frequent, we can regard the electrons as occupying eigenstates of the unperturbed crystal, making infrequent transitions between those states as a response to perturbations associated with the vibrations of the crystal lattice. The statistical book-keeping can then be carried out via the Boltzmann equation which relates the rate of change of the

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occupation probability, f(**k**), of the state with wavevector **k** to the volume rate and to the divergence of the probability current:

$$\frac{\partial f(\mathbf{k})}{\partial \mathrm{t}} = \left(\frac{\partial f(\mathbf{k})}{\partial \mathrm{t}}\right)_{vol} - \nabla.\left(\mathbf{v}(\mathbf{k}) f(\mathbf{k})\right), \tag{2}$$

where **v**(**k**) is the group velocity. The last term vanishes in the absence of spatial non-uniformity and:

$$\left(\frac{\partial f(\mathbf{k})}{\partial t}\right)_{vol} = \left(\frac{\partial f(\mathbf{k})}{\partial t}\right)_{fields} + \left(\frac{\partial f(\mathbf{k})}{\partial t}\right)_{scat} + \left(\frac{\partial f(\mathbf{k})}{\partial t}\right)_{gen,rec} . \tag{3}$$

We can dispense with the generation and recombination rates in the present context. The field term is obtained from the probability current flow in **k**-space and the acceleration theorem, d**k**/dt= $\bar{e}$**F**/$\hbar$, where **F** is the electric field and $\bar{e}$ carries the sign of the charge:

$$\left(\frac{\partial f(\mathbf{k})}{\partial t}\right)_{fields} = -\nabla_{\mathbf{k}} \cdot \mathbf{j}_{\mathbf{k}} = -\nabla_{\mathbf{k}} \cdot \left(\frac{d\mathbf{k}}{dt} f(\mathbf{k})\right) = -\frac{d\mathbf{k}}{dt} . \nabla_{\mathbf{k}} f(\mathbf{k}). \tag{4}$$

The scattering rate has the form:

$$\left(\frac{\partial f(\mathbf{k})}{\partial t}\right)_{scat} = \sum_i \hat{S}_i = \sum_i (\hat{S}_i^- + \hat{S}_i^+), \tag{5}$$

where the sum is over all scattering processes and $\hat{S}^-$ is the scattering-out rate i.e. the rate at which an electron is scattered out of the state **k**, and $\hat{S}^+$ is the scattering-in rate. The Fermi Golden Rule applied to phonon scattering, in which the absorption or emission of a phonon causes a transition from state **k** to **k'** and vice versa, leads to the form:

$$\begin{aligned} \hat{S}_i = & \int W_i(\mathbf{k}',\mathbf{k})\left[\{n(\omega_q)+1\} f(\mathbf{k}')\{1-f(\mathbf{k})\} - n(\omega_q) f(\mathbf{k})\{1-f(\mathbf{k}')\}\right] \\ & \times \delta(E_{\mathbf{k}'} - E_{\mathbf{k}} - \hbar\omega_q) d\mathbf{k}' \\ & + \int W_i(\mathbf{k}'',\mathbf{k})\left[n(\omega_q) f(\mathbf{k}'')\{1-f(\mathbf{k})\} - \{n(\omega_q)+1\} f(\mathbf{k})\{1-f(\mathbf{k}'')\}\right] \\ & \times \delta(E_{\mathbf{k}''} - E_{\mathbf{k}} + \hbar\omega_q) d\mathbf{k}'', \end{aligned} \tag{6}$$

where ω_q is the phonon frequency, $n(\omega_q)$ is the phonon occupation factor, and **q** is the phonon wavevector (**q**= **k'**- **k** or **q** = **k** - **k"** in the first or second integral, respectively). There are three rates with this form, those associated with optical-phonon, piezoelectric and deformation-potential acoustic-phonon scattering. (There will be a fourth if intervalley scattering is relevant.)

Electron-electron scattering is different from the mechanisms just considered in that four electron states are involved: an incident electron with wavevector $\mathbf{k}_1$ collides with a target electron with wavevector $\mathbf{k}_2$ and after scattering they occupy states with wavevectors $\mathbf{k}_1'$ and $\mathbf{k}_2'$. If electrons were classical particles we would think of $\mathbf{k}_1$ going to $\mathbf{k}_1'$ and $\mathbf{k}_2$ going to $\mathbf{k}_2'$ and that would be it. But electrons are not distinguishable particles: the scattering event would look the same, ignoring spin, if the end states were exchanged, and this extra, possible process adds to the rate. But the exchange of fermions with the same spin changes the sign of the wavefunction, with the result that interference occurs between the two processes. The squared matrix element that enters the Fermi Golden Rule then consists of four components:

$$M^2 = \frac{1}{2}\left[(M_{12}^{\uparrow\downarrow})^2 + (M_{21}^{\uparrow\downarrow})^2 + (M_{12}^{\uparrow\uparrow} - M_{21}^{\uparrow\uparrow})^2\right], \tag{7}$$

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where the factor 1/2 takes into account the fact that in half of the collisions the spins are aligned and otherwise they are not aligned. The scattering rate is of the form:

$$\hat{S}_{ee} f(\mathbf{k}_1) = \iiint \left[W(\mathbf{k}'_1, \mathbf{k}'_2, \mathbf{k}_1, \mathbf{k}_2) f(\mathbf{k}'_1) f(\mathbf{k}'_2)\{1 - f(\mathbf{k}_1)\}\{1 - f(\mathbf{k}_2)\} \right. \\ \left. - W(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}'_1, \mathbf{k}'_2) f(\mathbf{k}_1) f(\mathbf{k}_2)\{1 - f(\mathbf{k}'_1)\}\{1 - f(\mathbf{k}'_2)\} \right] d\mathbf{k}_2 d\mathbf{k}'_1 d\mathbf{k}'_2 . \qquad (8)$$

At the steady state the distribution function is to be obtained from:

$$\frac{\bar{e}\mathbf{F}}{\hbar} . \nabla_{\mathbf{k}} f(\mathbf{k}) = \left(\frac{\partial f(\mathbf{k})}{\partial t} \right)_{scat} . \qquad (9)$$

Under the influence of the field the distribution function consists of the sum of a symmetric part, f^{+}, and an antisymmetric part, f^{-}, and Eq. (9) becomes:

$$\bar{e}\mathbf{F}.\mathbf{v}(\mathbf{k}) \frac{df^{+}(E)}{dE} = \hat{S} f^{-}(\mathbf{k}), \\ \frac{\bar{e}\mathbf{F}}{\hbar} . \nabla_{\mathbf{k}} f^{-}(\mathbf{k}) = \hat{S} f^{+}(\mathbf{k}), \qquad (10)$$

where $\mathbf{v}(\mathbf{k})$ is the electron group velocity.

Further progress requires the explicit scattering rates for the e-e and phonon processes. The general complexity of the problem has driven most workers in this area to resort to purely numerical techniques, in particular, Monte Carlo methods. While adopting such an approach is inevitable if a particular experimental situation is to be modelled, it is not useful if a detailed understanding of the operation of the various processes involved is required. A particular problem is the incorporation of e-e scattering into the Monte Carlo simulation. Unlike other scattering mechanisms which can be regarded usually as one-electron processes, e-e scattering is a two-electron process that entails keeping information about two electron trajectories. Furthermore, taking into account the long range coulomb interaction and the simulation of screening processes calls for extremely sophisticated molecular-dynamic techniques. In the light of such numerical complexities it is useful to adopt a complementary analytical approach even if this involves the sacrifice of some accuracy, and we take this approach. In order to highlight the new physics as clearly and as briefly as possible it is necessary to adopt some approximations. Accordingly, we assume that the conduction band is spherical and parabolic and, to avoid the complexity of intersubband transitions, we assume that the electrons are all in the lowest subband of the quantum well. We will further assume that the electron gas is non-degenerate, which will be reasonably valid for hot electrons even for relatively high electron densities. All interactions will be dynamically screened by the dielectric response of the polar lattice and of the electron gas. Screening is a complicated process that deserves a chapter on its own. We will return to this topic later, but for simplicity we will ignore screening entirely.

The scattering rates for phonons are reasonably straightforward provided the phonons are regarded as bulk-like and their interaction with electrons taken in an elastically isotropic, spherical approximation. The e-e scattering rates of interest are less familiar, and we begin with these.

4. Electron-Electron Scattering

Poisson's equation describes the interaction potential:

$$\nabla^2 V(\mathbf{R}) = -\frac{\rho(\mathbf{R})}{\varepsilon} = \frac{e}{\varepsilon} \delta(\mathbf{R} - \mathbf{R}_1) . \qquad (11)$$

If this is expanded in Fourier series we get:

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$$V(\mathbf{R}) = \int V(\mathbf{Q})e^{i\mathbf{Q}.(\mathbf{R}-\mathbf{R}_1)}d\mathbf{Q}\frac{\Omega}{(2\pi)^3},$$
$$\delta(\mathbf{R}-\mathbf{R}_1) = \int e^{i\mathbf{Q}.(\mathbf{R}-\mathbf{R}_1)}d\mathbf{Q}\frac{1}{(2\pi)^3}. \tag{12}$$

Inserting into Poisson's equation, we obtain:

$$V(\mathbf{Q}) = -\frac{e}{\varepsilon Q^2\Omega},$$
$$V(\mathbf{R}) = -e\int\frac{e^{i\mathbf{Q}.(\mathbf{R}-\mathbf{R}_1)}}{\varepsilon Q^2}d\mathbf{Q}\frac{1}{(2\pi)^3}. \tag{13}$$

The energy of interaction with an electron at $\mathbf{R}=\mathbf{R}_2$ is:

$$-eV(\mathbf{R}_2) = e^2\int\frac{e^{i\mathbf{Q}.(\mathbf{R}_2-\mathbf{R}_1)}}{\varepsilon Q^2}d\mathbf{Q}\frac{1}{(2\pi)^3}. \tag{14}$$

Introduce in-plane vectors $\mathbf{r}$ and $\mathbf{q}$, $\mathbf{Q}=(\mathbf{q}, q_z)$. Then:

$$-eV(\mathbf{R}_2) = e^2\int\frac{e^{i\mathbf{q}.\mathbf{r}_{12}}e^{iq_z z_{12}}}{\varepsilon(q^2+q_z^2)}d\mathbf{q}dq_z\frac{1}{(2\pi)^3}, \tag{15}$$

where $\mathbf{r}_{12} = \mathbf{r}_2 - \mathbf{r}_1$ and $z_{12} = z_2 - z_1$.

There is no conservation of crystal momentum in the z direction, so there is no special restriction on q_z. Integration over q_z can be carried out using:

$$\int_{-\infty}^{\infty}\frac{e^{iq_z z_{12}}}{q^2+q_z^2}dq_z = \frac{\pi}{q}e^{-q|z_{12}|}. \tag{16}$$

Thus:

$$-eV(\mathbf{R}_2) = \frac{e^2}{8\pi^2}\int\frac{e^{i\mathbf{q}.\mathbf{r}_{12}}e^{-q|z_{12}|}}{\varepsilon q}d\mathbf{q}. \tag{17}$$

Let the incident electron have in-plane wavevector $\mathbf{k}_1$ and wavefunction:

$$\psi_1(\mathbf{r},z) = A^{-1/2}e^{i\mathbf{k}_1.\mathbf{r}_1}\phi_1(z). \tag{18}$$

A is the area of the plane. The electron collides with an electron with wavevector $\mathbf{k}_2$. After the collision the wavevectors are $\mathbf{k}_1'$ and $\mathbf{k}_2'$. Following the Born approximation, we assume that the wavefunctions after collision are the unperturbed ones. For the moment, exchange and spin interference are ignored. The matrix element is:

$$M_{12} = \int\psi_{2'}^*(\mathbf{r}_2,z_2)\psi_{1'}^*(\mathbf{r}_1,z_1)\{-eV(\mathbf{R}_2)\}\psi_2(\mathbf{r}_2,z_2)\psi_1(\mathbf{r}_1,z_1)d\mathbf{r}_2d\mathbf{r}_1dz_2dz_1. \tag{19}$$

Transforming to centre-of-mass and relative coordinates gives:

$$\mathbf{r}_{cm} = \frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2),\ \mathbf{r}_{12} = \mathbf{r}_2-\mathbf{r}_1,\quad d\mathbf{r}_2d\mathbf{r}_1 = d\mathbf{r}_{cm}d\mathbf{r}_{12}. \tag{20}$$

$$M_{12} = \frac{e^2}{8\pi^2A^2}\int e^{i(\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_1'-\mathbf{k}_2').\mathbf{r}_{cm}}e^{-i(\mathbf{k}_1-\mathbf{k}_1'+\mathbf{k}_2'-\mathbf{k}_2).\mathbf{r}_{12}/2} \times\frac{F(q)e^{i\mathbf{q}.\mathbf{r}_{12}}}{\varepsilon q}d\mathbf{r}_{cm}d\mathbf{r}_{12}d\mathbf{q}. \tag{21}$$

The form-factor is given by:

$$F(q) = \int\phi_{2'}(z_2)\phi_{1'}(z_1)e^{-q|z_{12}|}\phi_2(z_2)\phi(z_1)dz_1dz_2. \tag{22}$$

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The integration over $\mathbf{r}_{cm}$ leads to the conservation of in-plane crystal momentum in the case of N-processes (which are the only ones we need consider). The integration over $\mathbf{r}_{12}$ specifies $\mathbf{q}$. Thus:

$$\begin{aligned} \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_1' - \mathbf{k}_2' &= 0, \\ \mathbf{k}_1 - \mathbf{k}_1' + \mathbf{k}_2' - \mathbf{k}_2 &= 2\mathbf{q}. \end{aligned} \tag{23}$$

Since $\mathbf{q}$ is defined, the integration over $\mathbf{q}$ yields a factor $4\pi^2/A$ and the other integrations yield a factor A^2. The matrix element becomes:

$$M_{12} = \frac{e^2 F(q)}{2A\varepsilon q}. \tag{24}$$

We consider the case in which both electrons, before and after colliding, are in the lowest subband:

$$W(\mathbf{k}_1, \mathbf{k}_2) = \frac{2\pi}{\hbar} \int |M_{12}|^2 \delta(E_2' + E_1' - E_2 - E_1) \mathbf{dk}_1' \mathbf{dk}_2' \frac{A^2}{(2\pi)^4}. \tag{25}$$

When the electron population is degenerate the integrand would contain the factor $\{1 - f(\mathbf{k}_1')\}\{1 - f(\mathbf{k}_2')\}$. We take $\mathbf{k}_2'$ to be fixed by momentum conservation, and so:

$$W(\mathbf{k}_1, \mathbf{k}_2) = \frac{e^4}{8\pi\hbar A} \int \frac{F^2(q)}{\varepsilon^2 q^2} \delta(E) \mathbf{dk}_1'. \tag{26}$$

where, for a parabolic band, $\delta(E) = \delta\left\{\left(\hbar^2 / 2m^*\right)\left(k_2'^2 + k_1'^2 - k_2^2 - k_1^2\right)\right\}$.

4.1. The scattering rate

At this point in the calculation it is usual to go on to obtain the bare scattering rate which is what is required as input to Monte Carlo simulations and to provide estimates of the dephasing rate. This is not the most useful rate for our purposes - we require energy and momentum exchange rates - but, for completeness, we continue along the usual lines.

We now define relative wavevectors:

$$\mathbf{g}_{12} = \frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2) \; and \; \mathbf{g}_{12}' = \frac{1}{2}(\mathbf{k}_1' - \mathbf{k}_2'), \tag{27}$$

and observe that:

$$\begin{aligned} &g_{12}^2 = \frac{1}{4}(k_1^2 + k_2^2 - 2k_1 k_2 \cos\phi), \quad |\mathbf{k}_1 + \mathbf{k}_2|^2 = k_1^2 + k_2^2 + 2k_1 k_2 \cos\phi, \\ &\therefore g_{12}^2 = \frac{1}{2}\left(k_1^2 + k_2^2 - \frac{1}{2}|\mathbf{k}_1 + \mathbf{k}_2|^2\right). \end{aligned} \tag{28}$$

Similarly,

$$g_{12}'^2 = \frac{1}{2}\left(k_1'^2 + k_2'^2 - \frac{1}{2}|\mathbf{k}_1' + \mathbf{k}_2'|^2\right), \tag{29}$$

whence it follows that the δ-function in Eq. (26) can be represented by:

$$\delta(E) = \delta\left\{(\hbar^2 / m^*)(g_{12}'^2 - g_{12}^2)\right\}. \tag{30}$$

Furthermore, $\mathbf{g}_{12}' = \frac{1}{2}\{\mathbf{k}_1' - (\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_1')\} = \mathbf{k}_1' - \mathbf{k}_{cm}$, and, since $\mathbf{k}_{cm}$ is a constant of the motion, we can change variables and replace $\mathbf{k}_1'$ by $\mathbf{g}_{12}'$:

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$$W(\mathbf{k}_1,\mathbf{k}_2) = \frac{e^4}{8\pi\hbar A}\int \frac{F^2(q)}{\varepsilon^2 q^2}\delta\left\{\frac{\hbar^2}{m^*}(g_{12}'^2 - g_{12}^2)\right\} g_{12}' dg_{12}' d\theta$$

$$= \frac{e^4 m^*}{16\pi\hbar^3 A}\int_0^{2\pi} \frac{F^2(q)}{\varepsilon^2 q^2} d\theta, \qquad (31)$$

where θ is the angle between **g12'** and **g12**. Since $q^2 = g_{12}'^2 + g_{12}^2 - 2g_{12}'g_{12}\cos\theta$ and $g_{12}'^2 = g_{12}^2$, we obtain $q^2 = 4g_{12}^2\sin^2(\theta/2)$. The total rate is obtained by integrating over the target-electron states, taking into account spin degeneracy, weighted by the probability of occupancy.

In the case of the exchange process, $q^2 = 4g_{12}^2\cos^2(\theta/2)$. Strong interference effects, where the spins are parallel, will be confined to scattering angles around π/2. Ignoring interference altogether would double the rate in Eq. (31). However, when screening reduces the dependence on q, interference will be more important. Furthermore, many-body effects involving exclusion and correlation will limit the interaction between like spins. In view of these considerations, the contribution from collisions with like spins is often neglected, and it is assumed that $M^2 = M_{12}^2$, in which case Eq. (31) is the total rate including exchange. In what follows we focus on the squared matrix element M_{12}^2, keeping in mind that the total rate subsequently deduced will be a factor of two larger when interference effects are negligible.

4.2. Energy exchange

The theory so far has focused on the total rate of scattering, given the wavevectors of the incident and target electrons. It tells us nothing, however, about the rate at which energy is exchanged in electron-electron collisions. This rate is important for understanding how fast thermalization occurs in a quasi-2D electron gas. In order to calculate this rate we have to focus on the wavevectors of the incident and scattered electrons rather than on the wavevectors of the incident and target electrons, which means integrating over $\mathbf{k}_2$ rather than $\mathbf{k}_1'$. Given $\mathbf{k}_1$ and $\mathbf{k}_1'$, we must sum over all possible target states weighted by the probability that a target state is occupied. This statistical factor introduces the distribution function into the problem.

We will continue to assume non-degenerate statistics and take the distribution function to be isotropic and Maxwellian described by an electron temperature T_e:

$$f(E) = \frac{n}{N_d}e^{-E/k_BT_e},\ N_d = \frac{m^* k_BT_e}{\pi\hbar^2}, \qquad (32)$$

where n is the areal electron density and N_d is the effective areal density of states taking account of spin degeneracy. For intrasubband processes, Eq. (26) is replaced by:

$$W(\mathbf{k}_1,\mathbf{k}_1') = \frac{e^4 n}{8\pi\hbar A N_d}\int e^{-E_2/k_BT_e}\frac{F^2(q)}{\varepsilon^2 q^2}\delta(E)2d\mathbf{k}_2, \qquad (33)$$

where the factor 2 is included to account for spin degeneracy. The delta function conserving energy must now be expressed in a form convenient for integration over $\mathbf{k}_2$. This can be done by noting that Eq. (23) implies that:

$$\mathbf{q} = \mathbf{k}_2' - \mathbf{k}_2,$$

$$k_2'^2 = k_2^2 + q^2 + 2k_2q\cos\phi_{2q}, \qquad (34)$$

and so:

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$$\delta(E) = \delta\left\{\left(\hbar^2 / 2m^*\right)\left(k_1'^2 - k_1^2 + q^2 + 2k_2 q \cos\phi_{2q}\right)\right\} \tag{35}$$

Integrating over the angle, and noting that $\int_0^{2\pi} d\theta = 2\int_{-1}^{1} \frac{d(\cos\theta)}{\sin\theta}$, we obtain:

$$W(\mathbf{k_1}, \mathbf{k_1'}) = \frac{e^4 n m^*}{2\pi\hbar^3 A N_d} \int_{k_{2\min}}^{k_{2\max}} e^{-\hbar^2 k_2^2 / 2m^* k_B T_e} \frac{F^2(q)}{\varepsilon^2 q^3 \sin\phi_{2q}} dk_2,$$

$$\sin\phi_{2q} = \sqrt{1 - \left(\frac{k_1'^2 - k_1^2 + q^2}{2k_2 q}\right)^2}, \tag{36}$$

$$k_{2\min} = \frac{k_1'^2 - k_1^2 + q^2}{2q}, \; k_{2\max} = \infty.$$

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Note that $\vec{\mathbf{q}} = \vec{\mathbf{k}}_1 - \vec{\mathbf{k}}_1'$ and so it is independent of k_2. Integration over k_2 is straightforward:

$$W(\mathbf{k_1'}, \mathbf{k_1}) = \frac{e^4 n(\pi m^*)^{1/2}}{2^{3/2}\hbar^2 (k_B T_e)^{1/2} A} \frac{F^2(q)}{\varepsilon^2 q^3} \exp\left\{-\frac{\hbar^2}{8m^* k_B T_e}\left(\frac{k_1'^2 - k_1^2 + q^2}{q}\right)\right\}. \tag{37}$$

This rate is dependent on the angle between **k1'** and **k1** through **q**. What is required is an average over angle:

$$W(k_1', k_1) = \frac{1}{\pi}\int_0^{\pi} W(\mathbf{k_1'}, \mathbf{k_1}) d\varphi. \tag{38}$$

Clearly, the angle dependence is not straightforward, especially as screening, in general, is angle-dependent. Esipov and Levinson [11] approach the problem by putting F(q)=1, assuming static screening by the lattice ($\varepsilon \equiv \varepsilon_s$), and defining the variable u as follows:

$$u = \frac{q}{(k_1^2 - k_1'^2)^{1/2}}, \tag{39}$$

and we consider the case $k_1 > k_1'$ for the present. The relation of u to the angle is obtained from:

$$q^2 = k_1^2 + k_1'^2 - 2k_1 k_1' \cos\varphi = (k_1 - k_1')^2 + 4k_1 k_1' \sin^2(\varphi/2), \tag{40}$$

i.e.:

$$u^2 = \gamma^2 + \frac{4k_1' k_1}{k_1^2 - k_1'^2}\sin^2(\varphi/2), \qquad \gamma = \left|\frac{k_1 - k_1'}{k_1 + k_1'}\right|^{1/2}. \tag{41}$$

The energy difference can be embodied in the symbol $\varpi = (E_1 - E_1') / k_B T_e$. Substitution into Eq. (38) and noting that:

$$4k_1' k_1 = (k_1^2 - k_1'^2)\left(\gamma^{-2} - \gamma^2\right), \tag{42}$$

gives:

$$W(k_1^{'},k_1) = W_0 \int_{\gamma}^{\gamma^{-1}} \frac{e^{\varpi/2}}{\varpi^{3/2}} \frac{\exp\left\{-\frac{\varpi}{4}\left(u^2+\frac{1}{u^2}\right)\right\}}{u^2\left\{\left(u^2-\gamma^2\right)\left(\frac{1}{\gamma^2}-u^2\right)\right\}^{1/2}} du, \tag{43}$$

$$W_o = \frac{e^4 n\hbar}{8\pi^{1/2}\varepsilon_s^2 m^* (k_B T_e)^2 A}.$$

So far, $\varpi > 0$. The rate for $\varpi < 0$ can readily be obtained by using the principle of detailed balance (consistent with the equilibrium implied by the existence of an electron temperature). The expression for both cases is then:

$$W(k_1^{'},k_1) = W_0 \int_{\gamma}^{\gamma^{-1}} \frac{e^{\varpi/2}}{|\varpi|^{3/2}} \frac{\exp\left\{-\frac{|\varpi|}{4}\left(u^2+\frac{1}{u^2}\right)\right\}}{u^2\left\{\left(u^2-\gamma^2\right)\left(\frac{1}{\gamma^2}-u^2\right)\right\}^{1/2}} du. \tag{44}$$

Following Esipov and Levinson, we have put F(q)=1 and assumed only static lattice screening. We note that for strictly 2D electrons the form-factor is unity, but it will be close to unity for quasi-2D electrons for quasi-elastic collisions. The rate becomes:

$$W(k_1^{'},k_1) = W_0 \int_{\gamma}^{\gamma^{-1}} W(u)du, \tag{45}$$

where:

$$W(u) = \frac{e^{\varpi/2}}{|\varpi|^{3/2}} \frac{\exp\left\{-\frac{|\varpi|}{4}\left(u^2+\frac{1}{u^2}\right)\right\}}{u^2\left\{\left(u^2-\gamma^2\right)\left(\frac{1}{\gamma^2}-u^2\right)\right\}^{1/2}}. \tag{46}$$

In such a case, Esipov and Levinson have shown that the integral in Eq. (44) can be obtained in terms of a modified Bessel function of the second kind, provided that $\gamma^2 << 1$ and $\gamma^2 / |\varpi| << 1$,

$$W(k_1^{'},k_1) = W_0 \frac{e^{\varpi/2}}{|\varpi|^{3/2}} \gamma K_1(|\varpi|/2). \tag{47}$$

In terms of energy, the parameter γ is given by:

$$\gamma = \frac{|\varpi|^{1/2}}{(E/k_B T_e)^{1/2} + (E/k_B T_e)^{1/2}}. \tag{48}$$

Thus, for quasi-elastic scattering, $|\varpi|<<1$, $K_1(|\varpi|/2) \approx 2/|\varpi|$, and Eq. (47) becomes:

$$W(k_1^{'},k_1) = W_0 \frac{1}{|\varpi|^2} \frac{1}{(E/k_B T_e)^{1/2}}. \tag{49}$$

In the case of a lower energy incident electron, the rate increases rapidly. For a thermal electron ($E/k_B T_e \approx 1$), the quasi-elastic rate is obtained from an integrand that is concentrated around $u \approx \gamma$, where, under these conditions, $\gamma^2 = |\varpi|/(4E/k_B T_e) << 1$. If,

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once again, we take the 2D limit of the integral in Eq. (38) and ignore screening, we obtain the result of Esipov and Levinson:

$$W(k_1^{'},k_1) = 2W_0 \frac{e^{\varpi/2}}{|\varpi|^2} e^{-E/k_BT_e} \int_0^{\sqrt{E/k_BT_e}} e^{t^2}\,dt. \tag{50}$$

As in the case of a fast electron, the rate diverges as $|\varpi|^{-2}$.

However, quasi-elastic scattering does little for energy and momentum exchange. Rates for these are going to be associated with strongly inelastic scattering in which $|\varpi| >> 1$. In this case γ is no longer small. When $\varpi(1-\gamma)^2 >> 1$, most of the integral comes from the region around u=1 and when E, E_1', $|E-E_1'| >> k_BT_e$:

$$W(k_1^{'},k) = W_0 \frac{F^2(q_1)e^{(\varpi/2-|\varpi|/2)}}{|\varpi|^{3/2}(\gamma^{-1}-\gamma)} \frac{\pi^{1/2}}{2}\left[erf[\varpi^{1/2}(\gamma^{-1}-1)] + erf[\varpi^{1/2}(1-\gamma)]\right], \tag{51}$$

where $q_1 = \sqrt{\varpi(2m^*k_BT/\hbar^2)}$, so for the downward transition ($\varpi > 0$):

$$W(k_1^{'},k_1) = W_0 \frac{\pi^{1/2}}{2} \frac{F^2(q_1)}{|\varpi|^{3/2}(E_1^{'}/k_BT_e)^{1/2}}, \tag{52}$$

and for the upward transition ($\varpi<0$):

$$W(k_1^{'},k_1) = W_0 \frac{\pi^{1/2}}{2} \frac{e^{\varpi}}{|\varpi|^{3/2}} \frac{F^2(q_1)}{(E_1/k_BT_e)^{1/2}}. \tag{53}$$

Note that downward transitions are significantly emphasised via the Maxwellian factor. The form-factor has been taken to be determined by the condition u = 1.

The energy-relaxation rate, Q, is obtained by integrating Eq. (45) over k_1':

$$Q = -\int (E_1 - E_1^{'})W(k_1^{'},k_1)k_1^{'}dk_1^{'}A/2\pi = -\frac{e^4 n}{32\varepsilon_s^2\hbar}F^2(\overline{q}_1), \tag{54}$$

where we have introduced a mean value $\overline{q}_1$ of q_1 for the form-factor in order to facilitate comparison with the result of Esipov and Levinson. Since small energy-transfers are favoured, the form-factor is very roughly unity. An energy-relaxation time can be defined by $Q = -E/\tau_{ee}$:

$$\frac{1}{\tau_{ee}} = \frac{e^4 n}{32\varepsilon_s^2\hbar E}F^2(\overline{q}_1). \tag{55}$$

With a dielectric constant equal to 9, and taking the form-factor to be unity, the rate is 2.09n/E s^{-1}, where n is the areal number of electrons per cm^2 and E is the electron energy in eV.

4.3. Momentum exchange

Esipov and Levinson do not discuss momentum relaxation, but this is readily derived. Returning to Eq. (44), we can define a momentum-relaxation rate, W_m, by weighting the integrand by (1-cosθ), where, from Eq. (40):

$$1-\cos\theta = \frac{2(u^2-\gamma^2)}{\gamma^{-2}-\gamma^2}, \quad \gamma << 1, \quad \gamma^2/|\varpi| << 1, \tag{56}$$

so that:

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$$W \approx 2W_0\gamma^3 \frac{e^{\varpi/2}}{|\varpi|^{3/2}} \int_0^\infty \frac{1}{u} e^{-\frac{|\varpi|}{4}\left(u^2+\frac{1}{u^2}\right)} du. \tag{57}$$

The integral can be evaluated by changing the variable to:

$$z = \frac{1}{2}\left(u^2 + \frac{1}{u^2}\right) \; and \; \int_0^\infty = \int_0^1 + \int_1^\infty, \tag{58}$$

whence:

$$W_m = 2W_0\gamma^3 \frac{e^{\varpi/2}}{|\varpi|^{3/2}} K_0(|\varpi|/2) \xrightarrow{\varpi \approx 0} W_0 \frac{\ln\left(\frac{4}{\varpi}\right)}{4(E/k_BT_e)^{3/2}}. \tag{59}$$

where $K_0(x)$ is the modified Bessel function. This is the momentum-relaxation rate for quasi-elastic scattering which, in the absence of screening, diverges logarithmically. With screening, quasi-elastic collisions will be inhibited. For strongly inelastic scattering, less affected by screening, the momentum-relaxation rate coincides with the energy-relaxation rate given by Eq. (55). Thus, we take the Esipov-Levinson expression of Eq. (55) to quantify both energy and momentum exchange rates.

5. Phonon Scattering

The strength of e-e scattering plays an important rôle in determining the form of the distribution function under the influence of an electric field, but e-e scattering cannot relax momentum and energy of the whole electron system. These relaxation processes depend on there being other scattering mechanisms. Whereas all scattering mechanisms (apart from e-e scattering) can relax momentum, only inelastic processes can relax energy and, of these, phonon processes are in nearly all cases the most important. Here, we limit attention to the momentum and energy relaxation rates associated with scattering by phonons.

There is first the problem of describing phonons in a quantum well, where the discontinuity of electrical and mechanical properties across the heterojunction between well and barrier materials affects the spectrum of lattice vibrations. This is a well-known problem that has been discussed at length elsewhere [12]. It has been found that, provided the well is not too narrow, an approximation can be adopted based on the assumption that the scattering rates are given without much error if the spectrum of phonons in the well is taken to that of the bulk material. Such an assumption would be wholly invalid for describing situations where individual phonon modes are observed, as in Raman scattering, but it works quite well for calculating scattering rates which entails summing over all modes. In what follows we exploit this approximate sum rule and work with bulk phonon modes.

The net scattering rates that affect the occupation probability of a given electron state depend upon the occupation probabilities of states involved in the processes of absorption and emission of phonons. For the moment we will ignore this dependence and we will only consider scattering-out events.

5.1. Polar optical phonons

Because of the large optical-phonon energy, scattering by optical phonons is highly inelastic. Moreover, the net scattering rate associated with a particular electron state depends on scattering rates of states a phonon energy above and below. A ladder of scattering with rungs a phonon energy apart is therefore involved in order to calculate the momentum and energy relaxation rates associated with a given state, and this requires a knowledge of the distribution function. In the presence of an electric field, the distribution

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function, in general, cannot be found without taking into account momentum and energy relaxation, so the problem is one of finding a self-consistent solution. This problem is usually solved by using Monte Carlo methods, but in some cases it is possible to use an analytical approach, and this will be adopted here.

The scattering rate associated with an electron in the state with wavevector k and energy E is:

$$W=\frac{1}{2}W_0\left(\frac{\hbar\omega}{E}\right)^{1/2}\left[\left(n(\omega)+1\right)\int_{q_1}^{q_2}\frac{F(q)}{q\sin\theta_+}dq+n(\omega)\int_{q_3}^{q_4}\frac{F(q)}{q\sin\theta_-}dq\right], \tag{60}$$

where n(ω) is the optical phonon occupation factor and:

$$W_0=\frac{e^2}{4\pi\hbar}\left(\frac{2m^*\omega}{\hbar}\right)^{1/2}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_s}\right),$$

$$q_1=k\left[1-\left(1-\frac{\hbar\omega}{E}\right)^{1/2}\right],\quad q_2=k\left[1+\left(1-\frac{\hbar\omega}{E}\right)^{1/2}\right],$$

$$q_3=k\left[\left(1+\frac{\hbar\omega}{E}\right)^{1/2}-1\right],\quad q_4=k\left[\left(1+\frac{\hbar\omega}{E}\right)^{1/2}+1\right], \tag{61}$$

$$\sin\theta_\pm=\sqrt{1-\left(\frac{m^*\omega}{\hbar kq}\pm\frac{q}{2k}\right)^2}.$$

The form factor is given by:

$$F(q)=\int_{-a/2}^{a/2}\int_{-a/2}^{a/2}dzdz'\,\psi^2(z)\psi^2(z')e^{-q|z-z'|}, \tag{62}$$

where ψ(z) is the electron wavefunction, and we make the assumption that scattering is confined to the lowest subband in a square quantum well of width *a*. The estimation of the corresponding momentum relaxation rate can be obtained by weighting the emission integrand by $(q/k)\cos\theta_+$ and the absorption integrand by $-(q/k)\cos\theta_-$, thus:

$$W_m=\frac{1}{2}W_0\left(\frac{\hbar\omega}{E}\right)^{1/2}$$

$$\times\left[\left(n(\omega)+1\right)\int_{q_1}^{q_2}\frac{F(q)}{q\sin\theta_+}\left(\frac{q}{2k}+\frac{m^*\omega}{\hbar kq}\right)dq+n(\omega)\int_{q_3}^{q_4}\frac{F(q)}{q\sin\theta_-}\left(\frac{q}{2k}-\frac{m^*\omega}{\hbar kq}\right)dq\right]. \tag{63}$$

The energy-loss rate is:

$$W_E=\frac{1}{2}W_0\left(\frac{\hbar\omega}{E}\right)^{1/2}\hbar\omega\left[\left(n(\omega)+1\right)\int_{q_1}^{q_2}\frac{F(q)}{q\sin\theta_+}dq-n(\omega)\int_{q_3}^{q_4}\frac{F(q)}{q\sin\theta_-}dq\right]. \tag{64}$$

These integrals must be evaluated numerically. They involve scattering-out rates only, and they are strictly valid only for a drifted Maxwellian distribution.

5.2. Piezoelectric scattering

In strongly polar materials the most powerful interaction with acoustic phonons at low energies is via the piezoelectric effect. The phonon energy in this case is small and

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in most cases we can adopt a quasi-elastic approximation, and except at very low temperatures we can assume that equipartition holds for the phonon occupation factor. The scattering rate is then given by:

$$W = \frac{e^2K^2m^*k_BT}{2\pi\varepsilon_s\hbar^3k}\int_0^{2k}\frac{F(q)}{q\sqrt{1-(q/2k)^2}(1+q_s/q)^2}dq, \tag{65}$$

where K is the electromechanical coupling coefficient averaged over direction, T is the lattice temperature, F(q) is the form factor of Eq. (62) and q_S is the static screening factor, which for non-degenerate statistics is:

$$q_s = \frac{e^2nF(q)}{2\varepsilon_sk_BT}, \tag{66}$$

and n is the areal density of electrons. The momentum relaxation rate is obtained by weighting the integrand in Eq. (65) by $(q/2k)^2$:

$$W_m = \frac{e^2K^2m^*k_BT}{8\pi\varepsilon_s\hbar^3k^3}\int_0^{2k}\frac{qF(q)}{\sqrt{1-(q/2k)^2}(1+q_s/q)^2}dq. \tag{67}$$

In order to obtain the energy relaxation rate we must take into account the small but finite energy of the acoustic phonon, which means treating the emission and absorption integrals separately. Thus:

$$W_E = \frac{e^2K^2m^*v_s}{4\pi\varepsilon_s\hbar^2k}\int_{-\infty}^{+\infty}dq_z\left[\begin{array}{l}\displaystyle\int_0^{2k-\eta}\frac{\hbar\omega\{n(\omega)+1\}J(q,q_z)}{\sqrt{1-[(q/2k)+\eta\}^2}(1+q_s/q)^2}dq\\ \displaystyle-\int_0^{2k+\eta}\frac{\hbar\omega n(\omega)J(q,q_z)}{\sqrt{1-[(q/2k)-\eta\}^2}(1+q_s/q)^2}dq\end{array}\right]. \tag{68}$$

In this equation $J(q, q_z)$ is given by:

$$J(q,q_z) = \frac{1}{\sqrt{q^2+q_z^2}}G^2(q_z), \tag{69}$$

$$G^2(q_z) = \left|\int_{-a/2}^{a/2}\psi^2(z)e^{iq_zz}dz\right|^2, \tag{70}$$

and $\eta=2m^*v_S/\hbar$, where v_S is the averaged velocity of the acoustic waves. Once more one is faced with numerical integrations.

For quasi-elastic processes such as scattering by acoustic phonons, involving only the scattering-out rates in the derivation of momentum and energy relaxation rates is more generally justified than it is for optical phonon scattering.

5.3. Deformation-potential scattering

In the case of non-polar scattering by acoustic phonons it is possible to obtain analytical solutions. The scattering rate is:

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$$W = \frac{\Xi^2 m^* k_B T}{2\pi^2 \hbar^3 v_s^2 \rho k} \int_{-\infty}^{\infty} \int_{0}^{2k} \frac{G^2(q_z)}{\sqrt{1-(q/2k)^2}\,(1+q_s/q)^2} dq_z dq, \tag{71}$$

where ρ is the mass density of the lattice, Ξ is the deformation potential. For a deep well such that $\psi(z)=\cos(\pi z/a)$:

$$\int_{-\infty}^{\infty} G^2(q_z) dq_z / 2\pi = \frac{3}{2a}, \tag{72}$$

and the scattering rate is:

$$W = \frac{3\Xi^2 m^* k_B T}{2\hbar^3 v_s^2 \rho a} \left[1 - \frac{q_s}{\pi k} \left(L(\alpha) \frac{2-\alpha^2}{1-\alpha^2} - \frac{1}{1-\alpha^2} \right) \right], \tag{73}$$

where $\alpha = q_s/2k$ and $L(\alpha) = \left[\ln(\sqrt{1-\alpha^2} + 1) \right] / \sqrt{1-\alpha^2}$.

The momentum relaxation rate, calculated as before, is:

$$W_m = \frac{3\Xi^2 m^* k_B T}{2\hbar^3 v_s^2 \rho a} \left[1 - \frac{8\alpha}{\pi} + 6\alpha^2 - \frac{4\alpha^3}{\pi(1-\alpha^2)} \left(L(\alpha)\{4-\alpha^2\} - 1 \right) \right]. \tag{74}$$

For the energy relaxation rate we need:

$$\begin{gathered} \int_{-\infty}^{\infty} (q+q_z)^2 G^2(q_z) dq_z / 2\pi = \frac{3q^2}{2a} + \frac{2\pi^2}{a^3}, \\ \int_{-\infty}^{\infty} (q+q_z) G^2(q_z) dq_z / 2\pi = \frac{3q}{2a}. \end{gathered} \tag{75}$$

Then the energy relaxation rate is:

$$W_E = \frac{3\Xi^2 m^{*2}}{\hbar^4 \rho a} \times$$

$$\left[\begin{aligned} & E + \frac{2}{3}E_0 - \frac{2\alpha}{\pi} \left(4E + \frac{2}{3}E_0 \left\{ L(\alpha) - \frac{L(\alpha)-1}{1-\alpha^2} \right\} - 2k_B T \right) + 6\alpha^2 E \\ & - \frac{8\alpha^3}{\pi} \left(\frac{E}{2} \left\{ 3L(\alpha) + \frac{L(\alpha)-1}{1-\alpha^2} \right\} + \frac{k_B T}{4\pi(1-\alpha^2)} \left\{ \frac{4-\alpha^2}{1-\alpha^2} L(\alpha) - \frac{1-\sqrt{1-\alpha^2}}{1+\sqrt{1-\alpha^2}} \right\} \right) \end{aligned} \right], \tag{76}$$

where $E_0 = \hbar^2 \pi^2 / 2m^* a^2$ is the subband energy.

6. Electron-Electron Scattering Dominated Transport of 2D Gas in GaN/AlGaN Quantum Wells

In this section we will consider behaviour of a 2D electron gas in a square infinite quantum well (QW) in the presence of strong longitudinal electric field **F**. In line with previous discussions we deal with an ideal GaN/AlN QW as this double heterostructure is best suited in order to observe the some new physical effects which have not been discussed or studied before. Since the depth of the GaN/AlN QW is about 2 eV it can accommodate a high density electron gas and the free electrons can be supplied not only via modulation doping but also via the doping effect of the intrinsic polarisation fields. Apart from this doping effect, polarisation fields will be ignored. The latter

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property creates an exceptional situation when there is no direct correlation between the number of free electrons in the QW and the number of background and remote charged impurities. In such circumstances the scattering by the deformation acoustic (DA) phonons, the piezoacoustic (PA) phonons, and the polar optical (PO) phonons are the main scattering mechanisms besides the e-e scattering. In real structures it would be necessary to include scattering by background impurities, charged dislocations and interface roughness which usually determine the low-field electron mobility, but as here we mostly will focus on the high-field transport we do not include them. Also each one of these can be eliminated, whereas the scattering mechanisms we consider cannot be eliminated, although inclusion of the these mechanisms in our theory has no principal difficulties.

It is useful to appreciate the magnitude of each of the main intrasubband scattering rates. The most rapid is that for PO phonon emission ($\sim 10^{14}\ s^{-1}$) when the electron energy is above the PO phonon energy; below the PO phonon energy, however, the rate is determined by the PO phonon absorption which becomes weak toward low lattice temperatures $k_B T << \hbar\omega_o$. Due to high polarity of the III-V nitrides the PO phonons mediated scattering rate is about an order of magnitude higher in GaN than in GaAs. Because of this the PO phonon absorption considerably contributes to the electron mobility even at relatively low temperatures when the above condition is satisfied, for example at room temperature [13]. We will include the PO phonon absorption in calculating the PO phonon mobility. PA phonon scattering is always significant in wurtzite GaN especially at low temperatures (at T=50 K the rate is about $5\times 10^{12}\ s^{-1}$ for energies about $k_B T$, decreasing with increasing energy). DA phonon scattering is slightly weaker, the corresponding rate being about $10^{12}\ s^{-1}$). The e-e scattering rate depends on the electron density. At energy equal to the PO phonon energy (the worst case in the range we consider) the rate in a gas of density $10^{11}\ cm^{-2}$ is about $4\times 10^{12}\ s^{-1}$. In the range of the electron energies 0 - 100 meV it is easy for the e-e scattering to dominate both the energy and momentum rates randomisation at densities above $10^{12}\ cm^{-2}$. Such densities are easy obtained in GaN/AlGaN structures [1, 2]. Here we will deal with the case in which the electrons occupy only the ground QW state with the quantization energy E_o, and the lattice temperature T is small in comparison with the PO phonon energy $\hbar\omega_o$, $k_B T << \hbar\omega_o < 3E_o$. These conditions are easily satisfied within a wide range of lattice temperature in GaN-based QWs, where $\hbar\omega_o = 92.8\ meV$, and for the QW thickness d=70 Å. We ignore the electron gas degeneracy. Of course, at high electron densities this effect is important, but with increasing electric field its importance will not be so significant as the electron gas will occupy the high energy states, and therefore the degeneracy will be partially or completely removed.

The strength of the electron-electron (e-e) interaction is a key parameter which defines the distinctively different regimes of energy and momentum relaxation and non-equilibrium electron kinetics in semiconductors [14]. Because the integral operator of the inter-electron scattering $\hat{I}_{ee}\left\{f(\bar{k}), f(\bar{k}')\right\}$ is a bilinear functional of the electron distribution function $f(\bar{k})$ of the interacting electrons in the states with the wavevectors $\bar{k}$, $\bar{k}'$, the magnitude of the transition probability $W_{ee}(\bar{k},\bar{k}')$ is proportional to the electron density n, as it can be seen from Eq. (33). Screening can be expected to modify the linear dependence. Indeed, for collisions involving small energy exchanges, which can be taken to be screened statistically, an increase of rate with density is countered by an increase of screening, so little dependence on density occurs. But, as discussed in

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discussed in Section 4.2, small quasi-elastic collisions are not important for relaxing energy. Highly inelastic collisions are the more significant, and for these, screening is no longer a static process but rather a dynamic one. To give full treatment of the dynamic screening of the e-e interaction is beyond the scope of this work, but preliminary indicators are that anti-screening effects become important at energy exchange of order of the optical phonon energy. The interplay of screening and anti-screening is therefore complex in the dynamic regime, which is just the regime we are interested in. Pending a full study we will assume that there is sufficient balance between screening and anti-screening for us to ignore the screening of the e-e interaction altogether. Therefore the e-e scattering contribution to the relaxation processes can be ignored only when the electron density is very low, otherwise e-e scattering will control the electron energy relaxation (at intermediate electron densities) or both the energy and momentum relaxations (at high electron densities). In the last two cases the electron system can be described by means of the electron temperature T_e.

Due to dependence of the e-e scattering operator on the electron density it is possible in principle to distinguish between three physically different situations [15]. Case I, which can be called the partial energy control case, takes place at the intermediate electron densities when the e-e scattering controls the energy relaxation only within the passive energy region $E < \hbar\omega_o$. In the active energy region $E > \hbar\omega_o$ the PO phonon scattering is stronger than the e-e scattering. Case II, which can be called the full energy control, takes place at higher electron densities when the e-e scattering is responsible for the energy relaxation at all energies, but the electron momentum relaxation in the active region is still controlled by the PO phonon scattering. Case III, which can be called the electron momentum-energy control, takes place at yet higher electron densities when the e-e scattering controls both the electron energy and the momentum relaxation at all energies. The most interesting physical situation belongs to the case III which is characterised by a unique strongly non-linear regime with a non-monotonous behaviour of the electron temperature T_e as a function of an applied electric field F. But for completeness we carry out below the kinetic equation based analysis of the all above three cases.

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6.1. Electron kinetics and the electron temperature at the partial energy relaxation control by the e-e scattering

In this case in the passive energy region the energy relaxation rate due to the e-e scattering is higher than the energy relaxation rate due to any other scattering mechanism, but the electron momentum relaxation is controlled by other mechanisms (in our case by the DA and PA phonon scattering). It is obvious physically that the distribution function in this region will be very close to the Maxwellian function, $f(E) \equiv F_T(E) = A_o \exp(-E / k_B T_e)$, with the electron temperature T_e (A_o is a normalisation constant, see Eq. (32)). Since the deviation from the Maxwellian function will take place near the PO phonon energy only, as the PO scattering dominates the e-e scattering at this energies, it may appear that since this region is quite small it would be possible to neglect the effect of the PO phonon scattering altogether. But such a neglect would be wrong. This is because the interaction with PO phonons is extremely inelastic process which results in large change in the electron energy. It is physically obvious that the intensity of this interaction depends strongly on the electron population (electron distribution function) at the energies near the threshold energy $E = \hbar\omega_o$. Therefore much care should be taken in calculating the distribution function at these energies even if the majority of the electrons still are in the passive energy region. The emission of the PO phonons even by the relatively small number of the electrons could be very effective channel of the electron energy loss for the whole electron gas due to large magnitude of the PO phonon energy. Although the emission of the PO phonons takes place only at the energies $E > \hbar\omega_o$ this process will effect the electron distribution function not only at these energies but it will also have a profound effect on the electron distribution function at the energies $E \le \hbar\omega_o$ just below the PO phonon energy. This is because any electron which is in the state just below the threshold energy may acquire the necessary excess of the energy from the other electrons due to the e-e scattering. In this case this "lucky" electron will leave the passive energy region and will never come back into this region by means of losing the excess of the energy due to e-e scattering. This is because the PO scattering is stronger than the e-e scattering in the active energy region and any electron is transferred from the active energy region into the passive energy region due to the PO phonon scattering. Therefore, the e-e scattering acts as some kind of pump which supplies the electron into the active energy region but which does not transfer them back. As a result of this asymmetry the electron distribution function will be depleted at the energies just below the PO phonon energy.

This case was analysed in detail by Levinson and Esipov [11, 16] for photoexcitation but in the absence of the electric field. They shown that the competition between the e-e and the PO phonon scattering near $E = \hbar\omega_o$ can be described by the parameter λ_o which is proportional to the ratio of the characteristic e-e scattering rate and the PO phonon scattering rate near the threshold, $\lambda_o = 2\sqrt{\hbar\omega_o / \pi k_B T_e}\, \nu_{ee}(\hbar\omega_o) / \nu_{po}(\hbar\omega_o)$, where $\nu_{ee}(\hbar\omega_o) = \pi^2 e^4 n / \varepsilon_s^2 \hbar^2 \omega_o$ is the e-e scattering frequency (ε_s is the static dielectric constant), $\nu_{po}(\hbar\omega_o) = \pi\alpha_F\omega_o$ is the PO phonon collision frequency (α_F is the Fröhlich constant). The regime in question takes place if $\lambda_o << 1$. For a GaN QW $\lambda_o \approx 0.02$ when $n = 10^{11}\ cm^{-2}$.

The determination of the distribution function $f(E)$ in this case requires the solution of an integral kinetic equation [16] which explicitly includes the e-e and the PO phonon scattering operators:

$$\int_0^\infty dE' N_s(E')\left[W(E',E)f(E') - W(E,E')f(E)\right] \tag{77}$$
$$= \nu_{po}(\hbar\omega_o)(N_o + 1) f(E)\Theta(E - \hbar\omega_o),$$

where W(E', E) is given in Eq. (44), $N_s(E) = m^* A / 2\pi\hbar^2$ is a 2D density of states (A is the area of the QW), $N_o = \left[\exp(\hbar\omega_o / k_B T) - 1\right]^{-1}$ is the PO phonon distribution function at equilibrium, and $\Theta(x)$ is the step-function. It is necessary to note that in deriving the above kinetic equation we assume that the rate of the electron transfer from the passive into the active energy region due to gain of the energy directly from the electric field is small in comparison with the rate of the transfer due to the e-e scattering near the threshold energy. This assumption imposes an additional limit on the strength of the electric field F. By the direct comparison of the corresponding terms in the kinetic equation we obtain the following approximate criterion for the electric field:

$$\frac{e^2 F^2 \tau_p(\hbar\omega_o)\tau_{ee}(\hbar\omega_o)}{m^* \hbar\omega_o} << 1, \tag{78}$$

where $\tau_p(\hbar\omega_o)$ and $\tau_{ee}(\hbar\omega_o)$ are the electron momentum scattering time due to DA and PA phonons and the e-e scattering time, respectively, at the PO phonon energy.

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A considerable complication in a 2D case in comparison with the 3D case arise because the e-e scattering in a 2D gas cannot be considered as a diffusion in the energy axis [16]. As a result of this the above integral equation cannot be transformed to the differential form as it was the case in a 3D electron gas.

The integral equation (77) has been solved analytically in Ref. 16 using the Wiener-Hopf method. The obtained solution is:

$$f(E) = A_o^+ \times \begin{cases} (2\lambda_o)^{1/2}, & |t| << \lambda_o, \\ 2\pi^{-1/2}\lambda_o t^{-1/2} e^{-t}, & t > 0, \ t >> \lambda_o, \\ e^{-t} erf(\sqrt{|t|}), & t < 0, \ |t| >> \lambda_o, \end{cases} \tag{79}$$

where $t = (E - \hbar\omega_o) / k_B T_e$, and $A_o^+ = F_T(E = \hbar\omega_o) = A_o \exp(-\hbar\omega_o / k_B T_e)$. It follows from Eq. (79) (the third line) that deep into the passive energy region (|t| >> 1) the distribution function is indeed equal to the Maxwellian function $F_T(E)$ as we pointed out earlier. In the important part of the active energy region the distribution function can be presented as (second line in Eq. (79)) $F^+(\varepsilon) = F_T(\varepsilon) 2\lambda_o \sqrt{(\varepsilon - \hbar\omega_o) / \pi k_o T_e}$.

The distribution functions obtained above should be used in the energy balance equation for the electron temperature T_e and for the calculation of the electron drift velocity v_d. The energy balance equation is derived from the kinetic equation (9) by multiplying it by the electron energy E(**k**) and summing up over all **k**. For a 2D electron gas interacting with unscreened DA, PA, and PO phonons in the square QW the balance equation is

$$\tilde{F}^2 \tilde{\mu}(T_e) = \frac{T_e}{T}\left[\left(1 - \frac{T}{T_e}\right)\left[\left(1 + \frac{2}{3}\frac{E_o}{k_B T}\frac{T}{T_e}\right) + \frac{2}{5}\frac{s_T^2}{s_L^2}\frac{\lambda a}{s_T \tau_{pa}}\frac{T}{T_e}\right]\right.$$
$$\left. + \frac{8}{3\pi}\lambda_o \frac{v_{op}(\hbar\omega_o)\lambda_a}{v_Q}\frac{k_B T}{2m^* s_L^2} P(w_o)(N_o + 1)\left(\frac{1}{2} + \frac{\hbar\omega_o}{k_B T_e}\right)\left(e^{-\frac{\hbar\omega_o}{k_B T_e}} - e^{-\frac{\hbar\omega_o}{k_B T}}\right)\right]. \tag{80}$$

As we see the last term in Eq. (80) is proportional to the parameter λ_o which in its turn is proportional to the rate of the e-e scattering, i.e. the e-e scattering rate is explicitly enters the balance equation. Here we introduced the following notations for the dimensionless electric field $\tilde{F}$ and dimensionless electron mobility $\tilde{\mu}$, and the form-factor $P(w_o)$ for interaction between the 2D electrons and the PO phonons, respectively:

$$\tilde{F}^2 = \frac{(eF\lambda_a)^2}{9m^* s_L^2 E_o}, \quad \tilde{\mu}(T_e) = \left(1 + \gamma_{op}\right)^{-1} \int_0^\infty \frac{x^{3/2} e^{-x} dx}{R(T_e) + x^{1/2}}, \quad R(T_e) = \frac{2}{3}\frac{\gamma_{pa}\sqrt{\frac{E_o}{k_B T_e}}}{1 + \gamma_{po}}, \tag{81}$$

$$P(w_o) = \frac{w_o}{4}\left[\frac{2}{w_o^2} + \frac{1}{1 + w_o^2} + \frac{1}{\pi}\frac{1 - e^{-2\pi w_o}}{w_o^3(1 + w_o^2)^2}\right], \quad w_o = \sqrt{\frac{\hbar\omega_o}{4E_o}}. \tag{82}$$

The electron mobility is:

$$\mu(T_e) = (1 + \gamma_{op})\frac{e\tau_{da}}{m^*}\tilde{\mu}(T_e). \tag{83}$$

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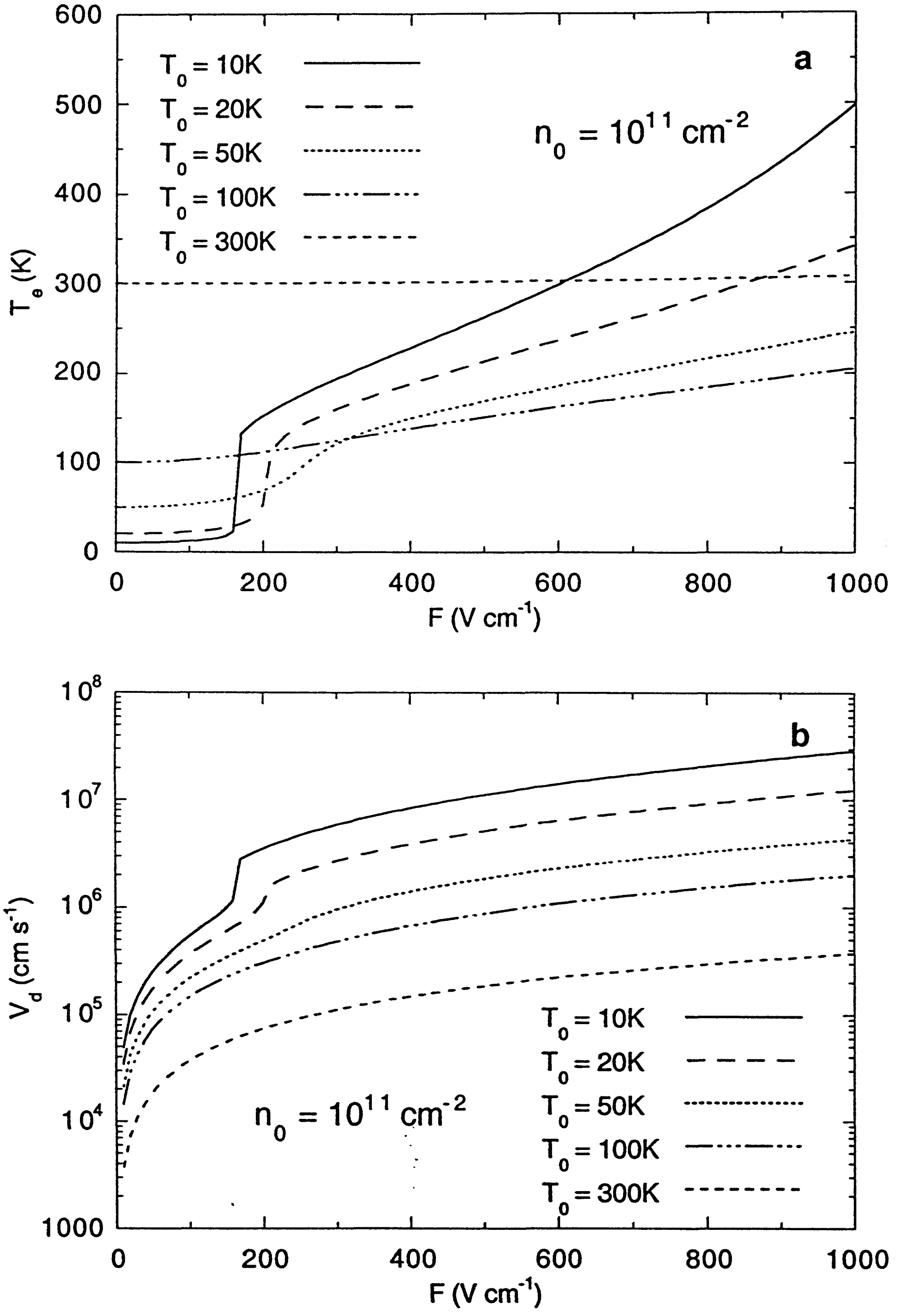

Figure 3. Variation of (a) electron temperature T_e and (b) drift velocity v_d with electric field F in a GaN square QW at the intermediate electron densities n_o corresponding to the partial control of the electron energy relaxation by the e-e scattering for different lattice temperatures T_o.

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In Eq. (83) $\tau_{da} = 3v_Q / 2\lambda_a$ is the DA phonon scattering time for the 2D electrons, $\lambda_a = \pi\hbar^4 \rho s_L^2 / m^{*2} \Xi^2 k_B T$ is the electron mean free path in a bulk material due to DA phonon scattering, ρ is the material density, Ξ is the DA potential constant, s_λ is the longitudinal (λ = L) or transverse (λ = T) acoustic velocity, $v_Q = \sqrt{2E_o / m^*}$, $\gamma_{op} = 2N_o v_{op}(\hbar\omega_o)\lambda_a / 3v_Q$, and $\gamma_{pa} = \left(k_B T \lambda_a / E_o s_T \tau_{pa}\right)\left(16 + 12 s_T^2 / s_L^2\right) / 35$ are the dimensionless coefficients, $\tau_{pa} = 2\pi\rho\hbar^2 s_T / m^* e^2 h_{14}^2$ is the characteristic scattering time for the bulk material due to PA phonons, h_{14} is the piezoelectric constant.

The balance equation (80) has been solved numerically in order to obtain the electric field dependences of the electron temperature $T_e(F)$ and the drift velocity $v_d(F) = \mu(T_e)F$ using the following parameters for the electrons in a square GaN/AlN QW (the well width was 70 Å with the ground state energy E_o=70 meV): $m^* = 0.21 m_o$, Ξ=10.1 eV, $h_{14} = 4.24 \times 10^7\ V/cm$, $\rho = 6.1\ g/cm^3$, $\alpha_F = 0.45$, $\omega_o = 1.41 \times 10^{14}\ s^{-1}$, $s_L = 4.57 \times 10^5\ cm/s$, and $s_T = 2.68 \times 10^5\ cm/s$. The obtained dependences are shown in Figures 3 (a, b).

The most remarkable feature of these dependences is the sharp increase of T_e and v_d with F at low lattice temperatures ($T_o < 50\ K$) and the smooth behaviour of these dependences at higher temperatures. This is the result of competition between PA phonon and DA phonon scattering where the PA scattering dominates at low temperatures and the DA scattering dominates at high temperatures.

6.2. Electron temperature of the hot electrons for the case of full energy control by the e-e scattering

This case means that the e-e scattering dominates the energy relaxation at all electron energies. This takes place in a QW when $n_o \geq 10^{12}\ cm^{-2}$. The distribution function is $F(E) = F_T(E)$ in the whole energy region.

It is necessary to note however, that the real material parameters of the GaN QW are such that probably this regime cannot be realised in practice. The necessary increase in the electron density leads to fast increase of the e-e scattering rate in such a way that at the above densities the e-e scattering will control not only the energy relaxation but also the momentum relaxation as well. (This case is considered in the next Section). For consider this case here for completeness only. The energy balance equation corresponding to this case can be obtained in the same way as the Eq. (80) in the previous section. The only difference is that in this case the e-e scattering rate does not enter the balance equation. Direct calculation shows that the balance equation in this case has the same form as the balance Eq. (80) with only one difference: the parameter λ_o in the last term in Eq. (80) has to be formally substituted by the factor (1/2).

The electron temperature and the drift velocity are shown in Figures 4 (a, b) as functions of F (solid lines). For comparison, we also show the $T_e(F)$ and $v_d(F)$ dependences for the previous case ($\lambda_o \ll 1$) for two different densities $n_o = 10^{11}\ cm^{-2}$ and $n_o = 5 \cdot 10^{11}\ cm^{-2}$. The main difference between cases I and II is that the electron temperature and the drift velocity in case I do depend explicitly on the electron density n_o while in case II there is no explicit dependence on n_o.

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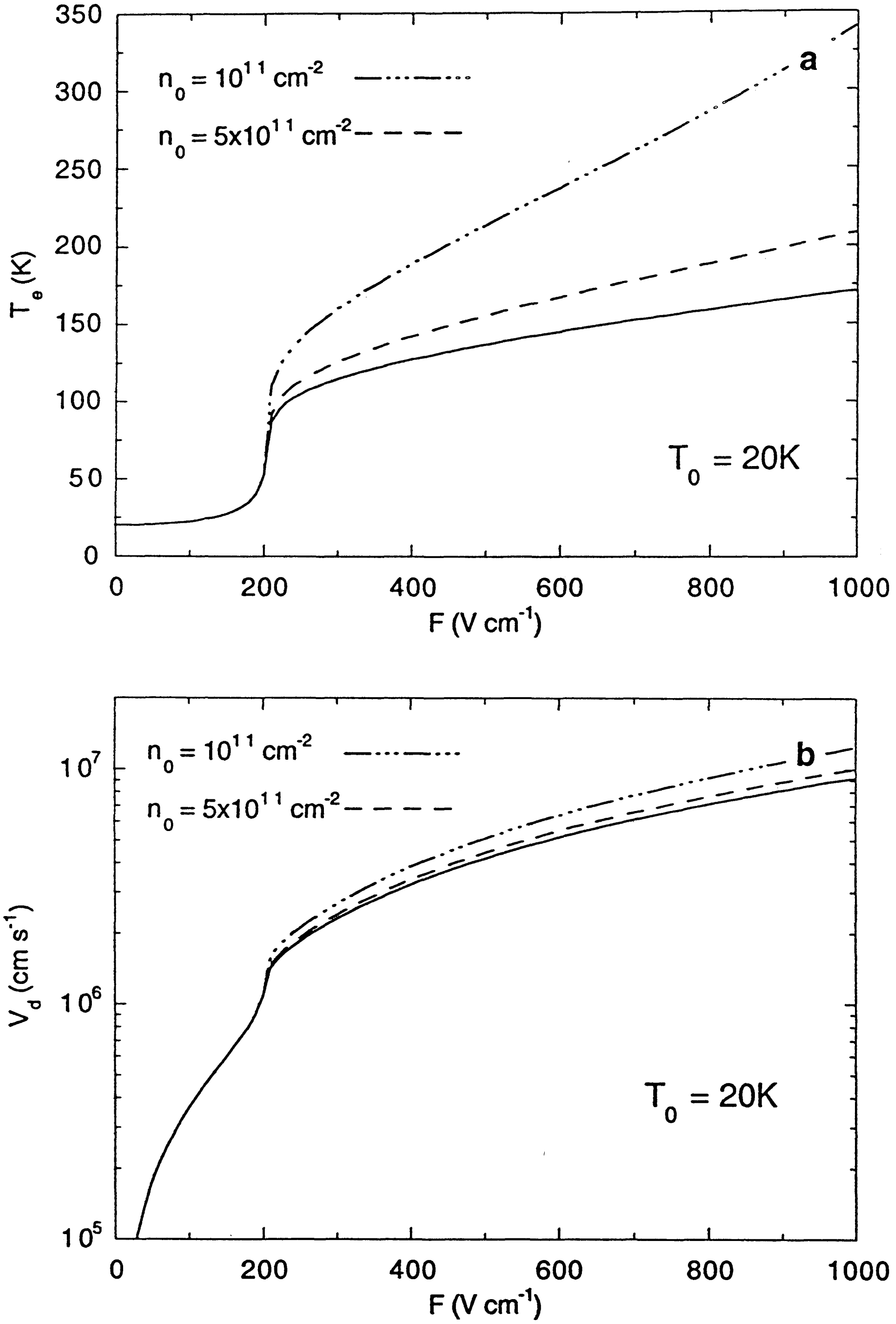

Figure 4. Variation of (a) electron temperature T_e and (b) drift velocity v_d with electric field F in a GaN square QW at the electron densities corresponding to the full control of the electron energy relaxation by the e-e scattering for different lattice temperatures T_0 (solid lines). The rest of the curves correspond to the case of the partial energy control.

6.3. The 2D electron gas cooling and squeezed electron distributions

Physically most interesting and unique case arises at high electron densities which are sufficient o allow the e-e collisions to control both the energy and the momentum relaxation. Our calculation shows that this takes place if $n_o \geq 5 \cdot 10^{12}\ cm^{-2}$. The distribution function in this case is a Maxwellian drifted function $F(\vec{k}) = A_o \exp\{-\varepsilon(\vec{k} - \vec{\kappa}) / k_o T_e\}$, where $\vec{\kappa}$ is the displacement wavevector. It is necessary to note that usually the drifted distributions are inhibited by the presence of impurity and other scattering mechanisms which tend to control the electron momentum relaxation. This is because in order to obtain the necessary high electron density one has to use the highly doped materials. In this case the high electron density comes together with the high ionised impurities density and both the electron-electron and the electron-impurity scattering rates will have the same order of the magnitude. As result of this the e-e scattering alone is unable to dominate the electron momentum relaxation. (At the same time the e-e scattering is able to control the energy relaxation due to considerably lower rate of the energy relaxation in comparison with the momentum relaxation rate. Physically this is because it is enough only a few collisions in order to change the electron momentum direction, but it is necessary considerably more collisions in order to change the electron's energy if the collisions are quasielastic.) So far the only experimental possibility to obtain the drifted distributions were realised at the intensive photoexcitation conditions where the high electron density can be obtained through the interband excitation. This usually correspond to the transient regime. Here we want to point out that the piezoelectric field doping in III-V nitride-based heterostructures open physically new possibility to establish the electron drifted distributions at the steady-state conditions. And this leads to novel transport properties, including absolute cooling and a squeezing of the distribution in the direction of drift. Due to strong interaction with the PO phonons the electrons encounter the strong phonon emission barrier when the average kinetic energy approaches the PO phonon energy.

In order to investigate the non-equilibrium electron kinetics with drifted distribution it is necessary to derive two balance equation (the energy conservation and the momentum conservation). In the presence of the external electric field F the electron system gains from the electric field both the momentum and the energy. It is important to point out that since the e-e scattering is the fastest scattering mechanism in the system, the energy and the momentum gained from the electric field by each individual electron will be first distributed within the whole electron gas without substantial loss to the other scatterers. At some point the balance will be established between the whole electron gas and the thermal bath. As a result the electron gas will acquire the drifted (macroscopic) momentum $\hbar\vec{\kappa}$. The average kinetic energy of the gas, which is described by the electron temperature T_e, will also change. Usually both these parameters increase when the electric field increases. Here we want to point out that this is not the case any more if a strong inelastic scattering mechanism is present. In the case considered such a mechanism is mediated by the interaction with the PO phonons. Due to large magnitude of the PO phonon energy in GaN, the majority of the electrons will interact with the PO phonons only at relatively high electric field (~1 kV/cm). Until these fields will be reached the electron gas will interact mainly with the PA and DA phonons. This interaction is quasielastic [17] and it does not prevent both $\hbar\vec{\kappa}$ and T_e to grow when F increases. At higher electric fields, when the interaction with the PO phonons dominates over the PA and DA phonon interaction, the electron gas looses its energy by large portions ($\sim \hbar\omega_o$). This effectively hinders the further growth of the electron temperature. Due to the large coupling constant in GaN the optical phonon energy level acts as a "hard wall" for the electrons, which prevent the electrons from penetrating to the higher energy. In general

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the above parameters $\vec{\kappa}$ and T_e are found from the system of two balance equations which describe the momentum and the energy conservation. Using the above drifted Maxwellian function we have derived two balance equations for 2D electrons. The only assumption that has been made is that $k_o T_e << \hbar\omega_o$. The equations in question are:

$$\frac{eF}{\hbar} = \kappa\left[\frac{1}{\tau_{da}} + \frac{\sqrt{\pi}}{\tau_{pa}}\frac{k_B T}{\sqrt{2m^* s_T^2 k_B T_e}}\Psi_1(\kappa)\right. \\ \left. + \frac{2}{\tau_{po}}(N_o+1)P(w_o)\left(e^{-\frac{\hbar\omega_o}{k_B T}} + \Psi_2(\kappa)e^{-\frac{\hbar\omega_o+\varepsilon_\kappa}{k_B T_e}}\right)\right], \tag{84}$$

$$\frac{e\hbar}{m^*}\kappa F = \frac{4m^* s_L^2 E_o}{3k_B T_e}\left(\frac{1}{\tau_{da}} + \frac{9}{10}\frac{1}{\tau_{pa}}\sqrt{\frac{2(s_T k_B T)^2}{m^* s_L^4 E_o}}\right)\left(1 - \frac{T}{T_e}e^{-\frac{\varepsilon_\kappa}{k_B T_e}}\right) \\ + \frac{2}{\tau_{po}}\hbar\omega_o(N_o+1)P(w_o)\left(\Psi_3(\kappa)e^{-\frac{\hbar\omega_o+\varepsilon_\kappa}{k_B T_e}} - e^{-\frac{\hbar\omega_o}{k_B T}}\right). \tag{85}$$

Here $\varepsilon_\kappa = \hbar^2\kappa^2 / 2m^*$ is the electron drift energy and $\tau_{po} = \nu_{po}^{-1}(\hbar\omega_o)$. The function $\Psi_i(\kappa)$ in Eqs. (84), (85) is defined as $\Psi_i(\kappa) = 4/\pi\int_0^1 \varphi_i(u)\sqrt{1-u^2}du$, where $\varphi_i(u)$ are given by the expressions $\varphi_1(u) = \exp(-\varepsilon_\kappa u^2 / k_B T_e)$, $\varphi_2(u) = ch\left(2u\sqrt{\varepsilon_\kappa \hbar\omega_o} / k_B T_e\right)$, and $\varphi_3(u) = \varphi_2(u)/2(1-u^2)$. The above equations have been solved numerically with the same parameters which we used in the previous sections in order to obtain $\vec{\kappa}$ and T_e as a function of F.

First we calculate the electric field dependence of the electron temperature T_e and the drift velocity $v_d = \hbar\kappa / m^*$ of 2D electrons which are shown in Figure 5 for different lattice temperatures T.

The most interesting feature of these dependences is that the electron temperature T_e is a *non-monotonous* function of the electric field. Another interesting results concerns the field dependence of the drift velocity: $v_d(F)$ has the regions which obey an S-type dependence. These regions exist only at low lattice temperatures (T~10-20 K) and they disappear when T increases. This behaviour is a result of a complicated T_e-dependent competition between PA and DA phonon scattering [18]. At higher lattice temperatures the DA scattering dominates over the PA scattering and the S-type regions disappear. Another interesting feature is a saturation of v_d at high electric field (F order of 1-10 kV/cm). This effect is completely due to the e-e and PO phonon scattering. The PO phonon scattering effectively limits any further increase of the drift electron momentum since every time that an electron emits the optical phonon it loses almost all the energy and the momentum.

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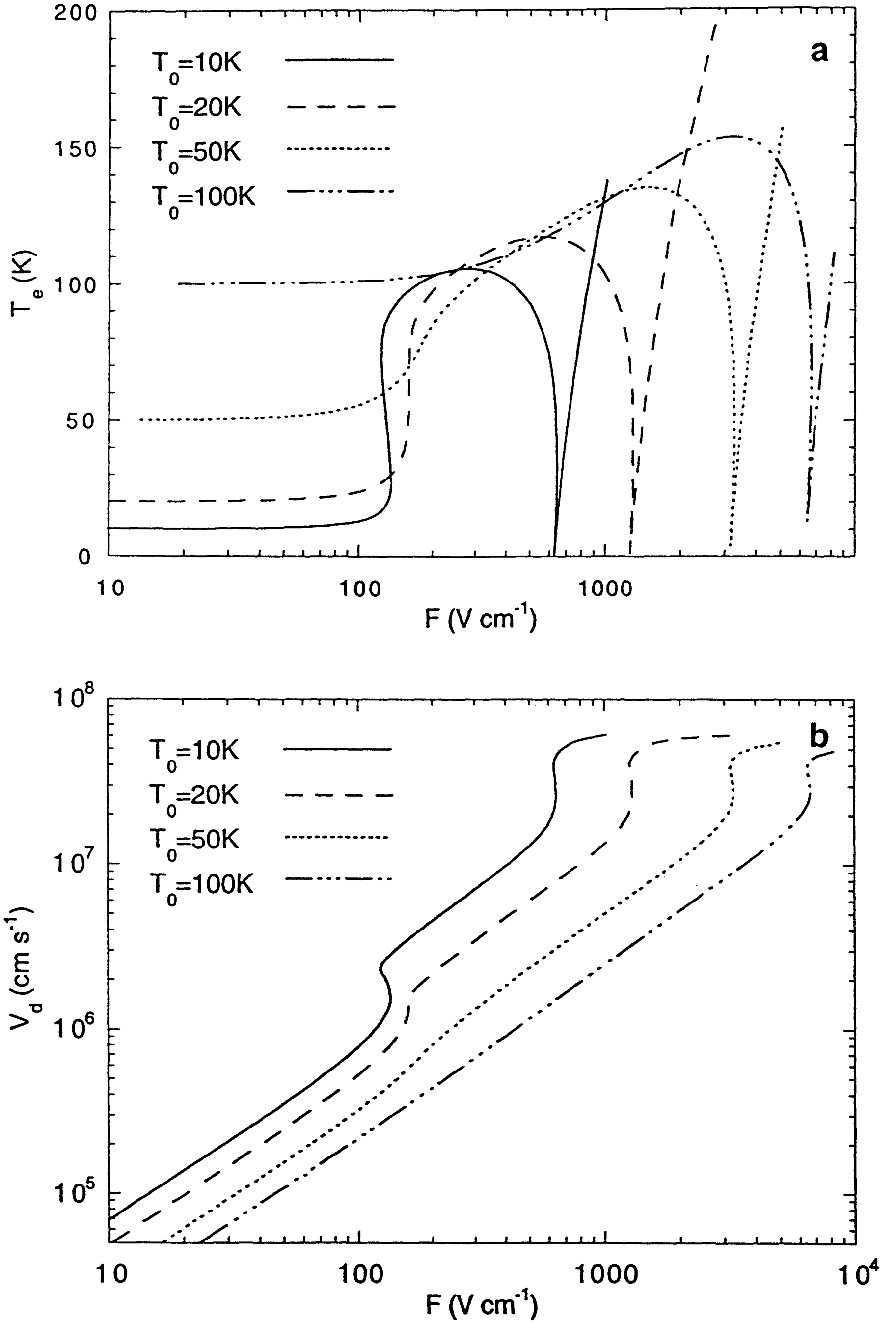

Figure 5. Variation of (a) electron temperature T_e with drift energy $\varepsilon_K = \hbar^2 K^2 / 2m^*$ and (b) drift velocity v_d with electric field F for different lattice temperatures T_0.

The existence of the S-type regions and the saturation of the drift velocity is evident also from the electric field dependence of the electron mobility $\mu(F) = v_d(F) / F$, which is shown in Figure 6.

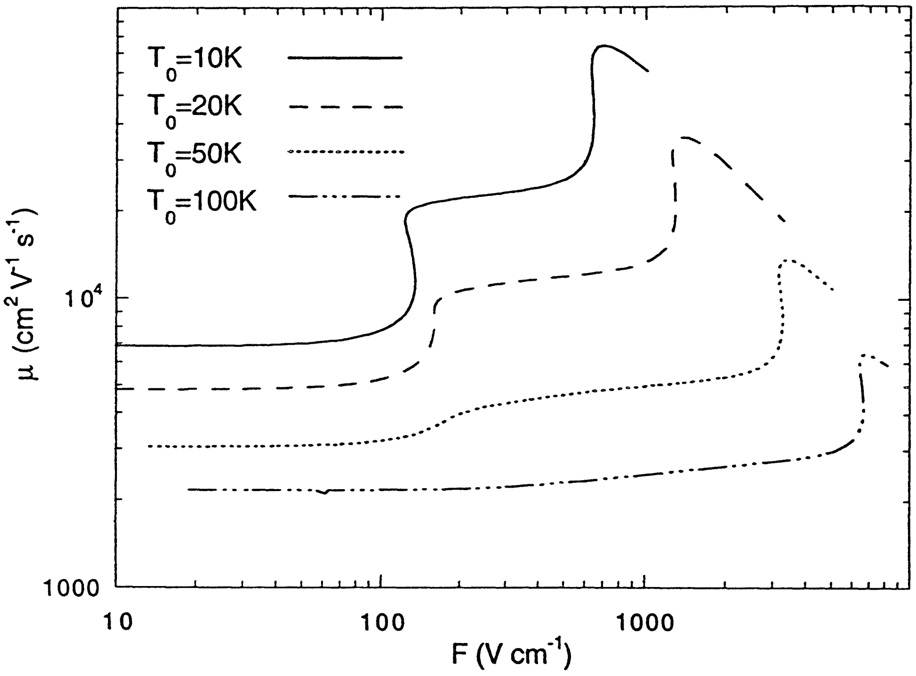

Figure 6. Mobility as a function of the electric field for different lattice temperatures T_o.

It is necessary to note that usually the drift velocity saturation and decrease of the mobility at high electric field take place in the streaming regime [19], when the electrons move ballistically in the momentum space until they reach the optical phonon energy, emit the optical phonon and repeat the ballistic motion again. But in our case the streaming regime does not take place because for the range of electric field considered the acceleration time [19] $\tau_F = \sqrt{2m^*\hbar\omega_o} / eF$ necessary to reach the PO phonon energy, is much longer ($\tau_F \sim 5\times10^{-12}s^{-1}$) in comparison with the e-e scattering time.

The electric field dependence of the total mean electron energy <E> is shown in Figure 7. The total mean energy of the electron is a sum of the mean kinetic energy $< E_k >= k_B T_e$ and the drift energy ε_K: $< E >= k_B T_e + \varepsilon_K$.

We see that at low T the electric field dependence of <E> has more complicated character than at higher T. At low T the PA phonon scattering is very strong in GaN and it suppresses increase of <E>. When F increases the electrons penetrates into the higher energy region where the PA scattering is weak. This results in a steep increase of <E> when F increases. At higher T the intensity of the PA scattering is small in comparison with the DA scattering and the region of steep increase of <E> disappears. Note that this region corresponds to the same range of F where the drift velocity obeys the S-type dependence as was shown in Figure 5 (b), and Figure 6.

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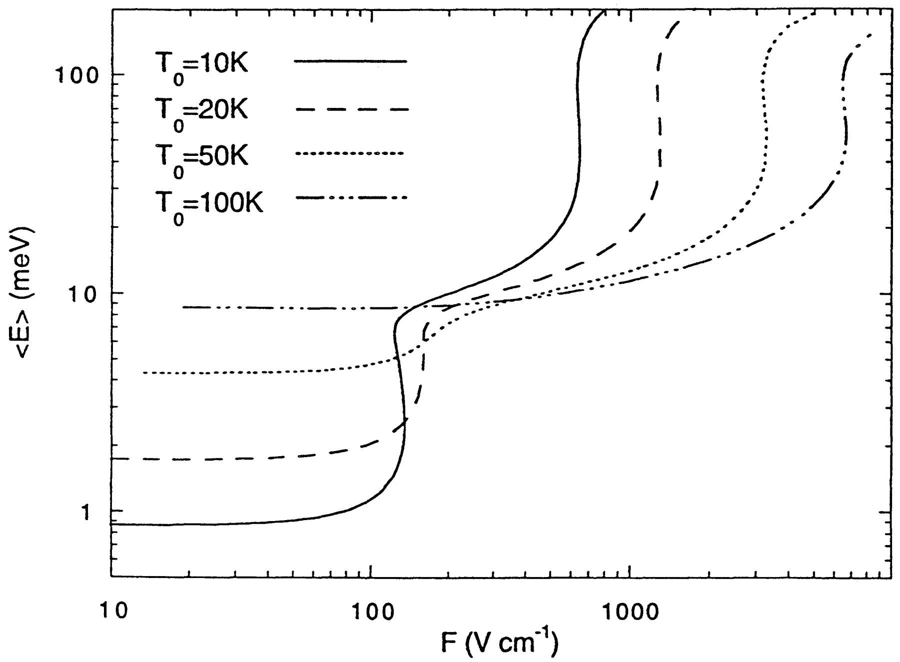

Figure 7. Variation of the total mean electron energy <E> with electric field F for different lattice temperatures T_0.

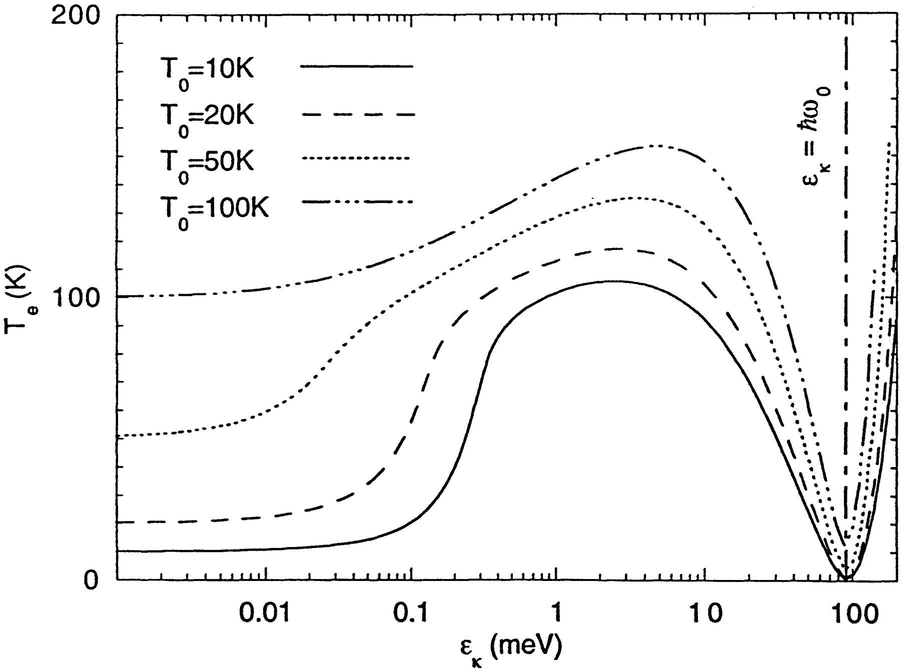

Figure 8. Variation of the electron temperature T_e with the drift energy ε_K for different lattice temperatures T_0 .

Increase in the total energy <E> does not mean that the electron temperature T_e increases as well when F increases. Figure 8 shows variation of the electron temperature T_e with the drift energy ε_K for different lattice temperatures T.

As we see this dependence is a non-monotonous function which has a region where the electron temperature decreases. This region corresponds to the electron cooling effect because the electron temperature T_e decreases with increase of the electric field. It is even possible to obtain at high electric field an electron temperature T_e which is smaller than the lattice temperature T - the absolute cooling effect. Of course, the total energy of the electron gas increases, as it should be, due to increase of the drift energy ε_K. The physical reason of the electron gas cooling is the intensive emission of the optical phonons when the total energy of the majority of the electrons is close to the PO phonon energy $\hbar\omega_O$.

It is interesting to investigate behaviour of the electron distribution function with the increase of the electric field F. This is shown in Figure 9 for two different lattice temperature T=10 K and T=100 K. The numbers near each curve are the values of the drift energy ε_K.

As the drift energy is a monotonous function of the electric field the higher drift energy corresponds to the higher electric field. Figure 9 shows that at very small electric fields the electron distribution function is close to the Maxwellian equilibrium distribution function which is a maximum at zero kinetic energy. When the electric field increases the distribution becomes wider in the momentum space. This corresponds to an increase of the electron temperature T_e. At the same time the distribution function is no longer centered at zero energy but has shifted along the electric field, a shift that corresponds to the drift of the electron gas as a whole. This behaviour continues with increase of the electric field until the electrons start to penetrate to the optical phonon energy. Strong inelastic scattering prevents the electrons from any further increase of their kinetic energy. As a result the electron distribution becomes more narrow or squeezed. This corresponds to a decrease of the electron temperature. At the same time the centre of the distribution function continues its shift when the electric field increases, which means increase of the electron drift energy. The most interesting physical consequence of this behaviour is that the electron distribution function is inverted in the momentum space ñ a majority of the electrons populate the high-energy region. Another interesting consequence of the decrease of the electron temperature with increase of the electric field is that the non-equilibrium electron gas becomes "less randomized". This should give, for example, a decrease of the electron noise temperature.

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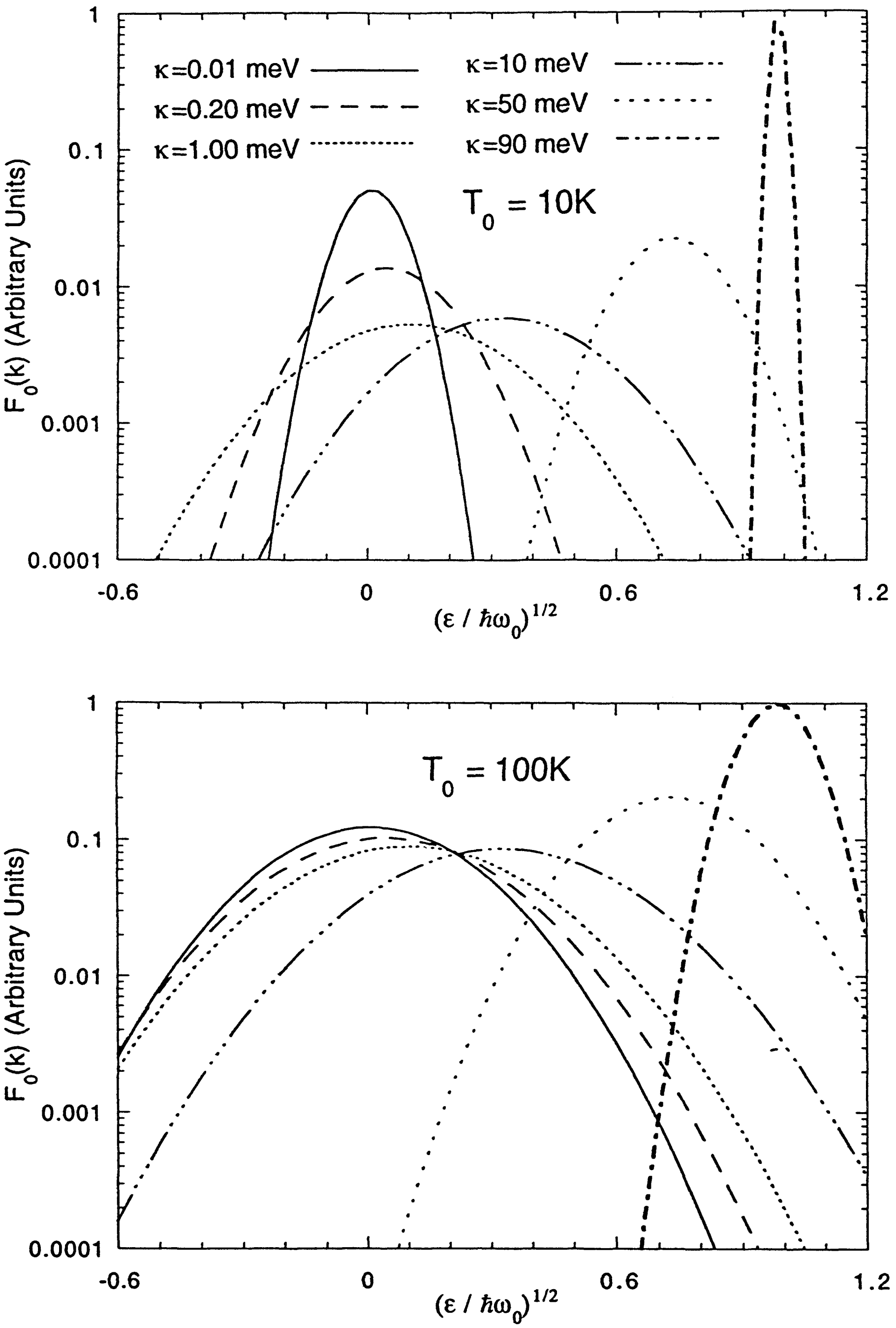

Figure 9. Squeezing of the distribution function $F_0(\vec{k})$ of hot electron gas for two different lattice temperatures T_0=10 K and T_0=100 K at different values of the drift electron energy ε_K (numbers near each curve).

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7. Summary

AlGaN/GaN structures constitute a new class of 2D systems in that a large population of electrons can be produced without doping as a result of spontaneous and strain-induced polarization. We have shown how a simple electrostatic model can describe the dependence of the induced electron density on barrier width in a AlGaN/GaN heterostructure. Large electron densities mean that a complete description of electron transport must include the effects of degeneracy, electron-electron scattering and dynamic screening. Such a description does not exist as yet, but an approach that ignores degeneracy and screening has a certain validity in the hot-electron regime. The effect of electron-electron scattering, in the absence of scattering by impurities and other defects, can then be regarded as establishing a drifted Maxwellian distribution. Accordingly, we have illustrated some consequences of the possibility of impurity-free hot-electron transport in perfect AlGaN/GaN heterostructures. These include S-type negative differential resistance, carrier cooling and squeezed electrons, novel properties that appear most strongly in the temperature range 100K and below. These properties should become accessible to experiment as material quality improves.

8. Acknowledgements

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9. References

1. R. Oberhuber, G. Zandler, and P. Vogl, Appl. Phys. Lett. **73** 818 (1998).
2. O. Ambacher, J. Smart, J.R. Shealy, N.G. Weimann, K. Chu, M. Murphy, W.J. Schaff, L. Eastman, R. Dimitrov, L. Wittmer, M. Stitzmann, W. Reiger, and J. Hilsenbeck, J. Appl. Phys. **85** 3222 (1999).
3. J. Burm, W.J. Schaff, L.F. Eastman, H. Amano, and I. Akasaki, Appl. Phys. Lett. **68** 2849 (1996).
4. Y-F. Wu, B.P. Keller, S. Keller, D. Kapolnek, P. Kozodoy, S.P. Denbaars, and U.K. Mishra, Appl. Phys. Lett. **69** 1438 (1996).
5. B.K. Ridley, O. Ambacher, and L.F. Eastman, Semicond. Sci. Technol. **15** 270 (2000).
6. B.K. Ridley, Appl. Phys. Lett. **77** 1 (2000).
7. J.P. Ibbetson, P.T. Fini, K.D. Ness, S.P. Denbaars, J.S. Speck, and U.K. Mishra, Appl. Phys. Lett. **77** 250 (2000).
8. N.G. Weimann, L.F. Eastman, D. Doppalapudi, H.M. Ng, and T.D. Moustakas, J. Appl. Phys. **83** 3656 (1998).
9. H.M. Ng, D. Doppalapudi, T.D. Moustakas, N.G. Weimann, and L.F. Eastman, Appl. Phys. Lett. **73** 821 (1998).
10. M.J. Murphy, K. Chu, H. Wu, W. Yeo, W.J. Schaff, O. Ambacher, L.F. Eastman, T.J. Eustis, J. Silcox, R. Dimitrov, and M.S tutzmann, Appl. Phys. Lett. **75** 3653 (1999).
11. S.E. Esipov and I.B. Levinson, Zh. Eksp. Teor. Fiz. **90** 330 (1986); (Sov. Phys. JETP. **63** 191 (1986).
12. B.K. Ridley, *Electrons and Phonons in Semiconductor Multilayers* (Cambridge University Press, Cambridge, 1997).
13. N.A. Zakhleniuk, C.R. Bennett, B.K. Ridley, and M. Babiker, Appl. Phys. Lett. **73** 2485 (1998).
14. B.K. Ridley, J. Phys. C: Solid State Phys. **17** 5357 (1984).
15. N.A. Zakhleniuk, B.K. Ridley, M. Babiker, and C.R. Bennett, Physica **B 272** 309 (1999).
16. S.E. Esipov and Y.B. Levinson, Advances in Physics **36** 331 (1987).
17. B.K. Ridley, *Quantum Processes in Semiconductors*, 4th edition, Oxford Press, 1999.
18. N.A. Zakhleniuk, B.K. Ridley, M. Babiker, and C.R. Bennett, Physica **B272** 309 (1999).
19. V.E. Gantmakher and Y.B. Levinson, *Carrier Scattering in Metals and Semiconductors*, North-Holland, 1987.

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