

Nonlinear dynamics of MEMS systems

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Abstract. Parametric resonance and bifurcation sensing has been utilized for a number of applications in MEMS (microelectromechanical systems), including mass detection, RF oscillators/filters and inertial sensing. Nonlinearities are very important in these applications, where often the device is actually operating in a nonlinear regime. Microcantilevers which monitor mass change as a way perform chemical detection is not a new idea. In the linear, harmonic implementation, this has been demonstrated by many. We present a sensor that utilizes nonlinear stability behavior to achieve reliable quantitative chemical gas sensing and threshold detection in a noisy environment. This sensor, while comparable in ultimate sensitivity in vacuum environments, demonstrates the benefit of nonlinear operation when operating in noisy environments. Sensor feasibility is demonstrated experimentally in air through the sensing and detection of methanol. Additionally we discuss the effects of noise, and the strategies to improve sensitivity through the use of nonlinear devices and improved sweeping strategies. These strategies can be employed in any application where bifurcation tracking is necessary.

Keywords: MEMs, Nonlinear

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INTRODUCTION: MASS SENSING

MEMS chemical gas sensors utilizing microcantilevers and the mass loading effect are widely studied and used in the recognition of various chemical substances [1,2]. Most of these sensors operate in harmonic resonance mode which tracks the shift in the resonant frequency due to mass change from the adsorption of the chemical vapors to be sensed [3]. The sensitivity is compromised by the addition of noise processes that cause the resonant frequency to fluctuate, leading to uncertainty in the identification of the chemical. Many methods have been proposed to improve the sensitivity [4,5,6], including use of the parametric resonance phenomenon [7]. In this paper, we experimentally validate the advantage of a sensor operating in parametric resonance mode by showing the sensitivity is largely unaffected by increase in noise level. Sensing is achieved by tracking the frequency shift at the stability boundary of the first-order parametric resonance region where a sharp transition from zero to large response amplitude occurs. Threshold detection is realized by measuring this sharp change in amplitude. Another advantage of the parametric resonance-based sensor is that the setup is also simpler as it does not require a phase-locked controller to keep track of the resonant frequency [8].

EXPERIMENT: MASS SENSING

The sensor is comprised of a fixed-fixed beam. Base excitation from an externally mounted shear piezo provides actuation in both the out-of-plane and in-plane directions. Thus, by applying the appropriate driving voltage, the sensor can be made to operate in either the harmonic or parametric resonance mode. Sensor response is measured using a laser vibrometer through an optical microscope. Fig. 1 depicts the sensor and a schematic of the experimental setup.

In experiment, three types of measurements were conducted. First, we operated the sensor in a traditional harmonic resonance mode by actuating at the resonant frequency ω , and tracking its shift using a phase-locked loop. We then operated the sensor in parametric resonance mode by actuating at 2ω , and tracking the frequency shift at the stability boundary by sweeping down the driving frequency until a large change in response amplitude occurred. Finally, threshold detection was measured by actuating at a fixed driving frequency near the stability boundary and recording the resultant response amplitude. Fig. 2 shows the results from two experiments. The first experiment (left column) shows methanol sensing with a low noise level of $6.2 \mu V / \sqrt{Hz}$. The second experiment (right column) shows methanol sensing with a high noise level which was achieved by decreasing the laser signal strength (focus?) and characterized to be $60.4 \mu V / \sqrt{Hz}$. For sensing in harmonic resonance mode (Fig. 2(a)-2(b)), a tenfold increase in noise level significantly impairs the sensor's ability to recognize the presence of methanol. However, for sensing in parametric resonance mode (Fig. 2(c)-2(d)), the sensor's ability to distinguish methanol is unaffected by the additional noise. Furthermore, threshold detection (Fig. 2(e)-2(f)) remains consistent despite the increase in noise. (can you elaborate that this type of sensor would respond with a less-sensitive on-chip detection scheme than an equivalent harmonic....thus looking more favorable in scaling arguments, where smaller size leads to less capacitance./piezoresistance/etc)?

DISCUSSION: MASS SENSING

The experimental results show that the sensitivity of a parametric resonance-based chemical sensor is less susceptible to noise than conventional harmonic resonance-based sensors. This makes it extremely practical for operation in noisy environments such as sudden pressure changes or mechanical vibrations. Its threshold detection ability provides an additional useful feature in applications where actions may be taken, such as activating an alarm or security system, upon the detection of certain chemicals.

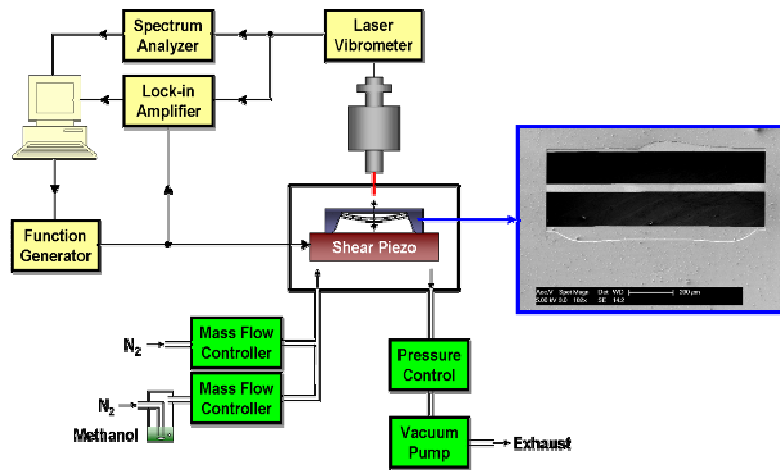


FIGURE 1: Schematic showing the experimental setup. Sensor dimensions are $986\ \mu\text{m} \times 40\ \mu\text{m} \times 2.5\ \mu\text{m}$. The measured resonant frequency and quality factor in air are $32.24\ \text{kHz}$ and 73, respectively. The lock-in amplifier is used in the phased-locked loop to track the resonant frequency when operating the sensor in harmonic resonance mode.

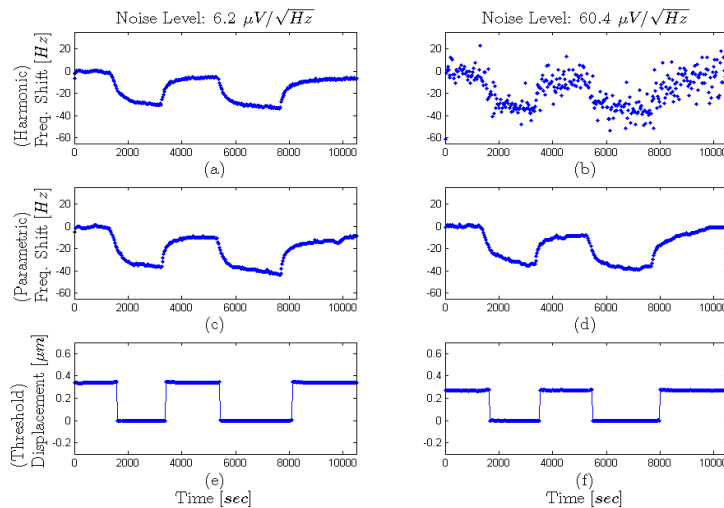


FIGURE 2: Sensing and threshold detection of methanol. A concentration of $2900\ \text{ppm}$ results in a frequency shift of $35\ \text{Hz}$. For harmonic resonance mode, the applied driving voltage is $2\ \text{V}$. For parametric resonance mode, the applied driving voltage is $20\ \text{V}$.

INTRODUCTION: NOISE EFFECTS

A variety of sensing strategies have been proposed that involve tracking changes in parameters at which dynamic bifurcations occur in MEMS, and relating these to measurements of environmental effects [9]. An example is that of mass sensing using the sharp jumps in response that occur in parametrically excited devices [10]. In order

to accurately determine parameter values at which bifurcations occur, it is essential to understand the effects and interplay of noise and sweep rate on the bifurcations [11,12]. Previous analytical work on this topic considered the limiting case of small values of noise and sweep rates [11,12], whereas the results obtained herein are valid for broad ranges of these parameters. These results can be used to solve the practically important problem of determining the sweep rate that allows for the most rapid and precise determination of bifurcation points for a given level of noise, thereby optimizing sensor response time.

EXPERIMENT: NOISE EFFECTS

The relevant mathematical problem is that of escape from a time-varying potential well in the presence of noise [11,12]. When parameters are swept slowly toward a bifurcation point the system may jump due to noise before the critical point is reached; this is known as noise-activated escape. If the system is swept quickly it may not noticeably respond until well after the critical point has passed; this is known as delayed bifurcation. The deciding parameter separating these cases is the ratio of noise strength to sweep rate. In order to consider the entire range of possibilities, one must numerically solve the Fokker-Planck equation derived for the bifurcation at hand. For the present study such a solution is obtained for sweeping through a subcritical subharmonic bifurcation. This solution is found to be in good agreement with limiting case analytical solutions for slow and fast sweep approximations, as shown in Fig. 3.

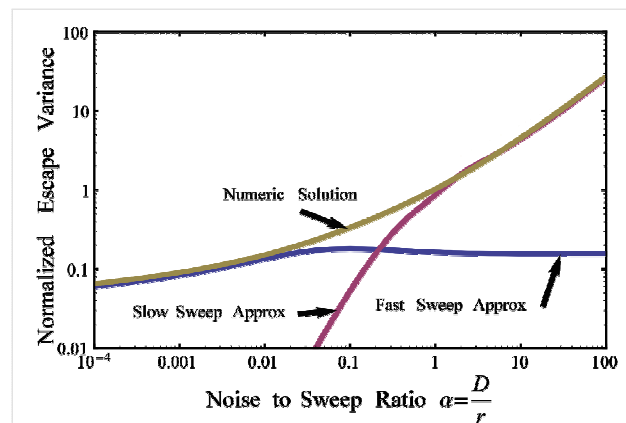


FIGURE 3. Comparison of the slow (previously known) and fast sweep approximations with the numeric solution to the Fokker-Planck equation, in terms of the variance of jump events versus parameter α , the ratio of noise intensity to sweep rate. Note that the numerical solution spans a wide range of parameter values. The mean values of the distribution are similarly accurate.

DISCUSSION: NOISE EFFECTS

These results have broad applicability, for example in Josephson junctions [13], and are applied here to a parametrically excited MEMS resonator. The device

employed is the drive mode of the rate gyro described in [14]. The experimental setup is shown in Fig. 4, and extensive sweep tests were carried out at ambient noise intensity. The distribution of frequencies at which jumps to nonlinear response were recorded for different sweep rates, shown in Fig. 5, and were compared against the theoretical results, shown in Fig. 6. The theoretical results are based on an assumed device model and are shown to accurately predict the qualitative behavior of the device, specifically, that faster sweeps lead to increased shifting of the jump values (leftward in Figures 5 and 6), and a spreading and slight skewing of their distribution.

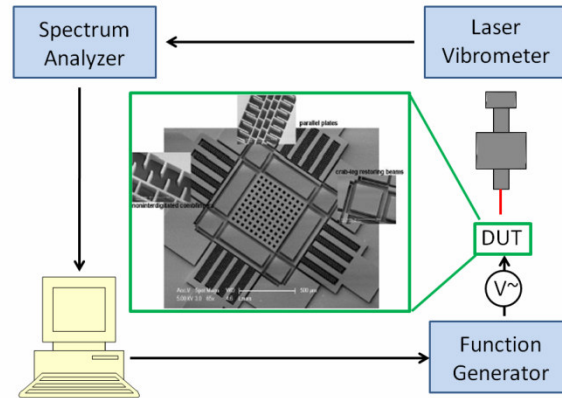


FIGURE 4. SEM of the fabricated device shown in center. Parametric resonance is excited in the drive mode using noninterdigitated comb drives [6]. Experimental setup consists of a laser Doppler vibrometer for velocity measurements, spectrum analyzer to detect bifurcation jumps, and a function generator, all controlled in MATLAB.

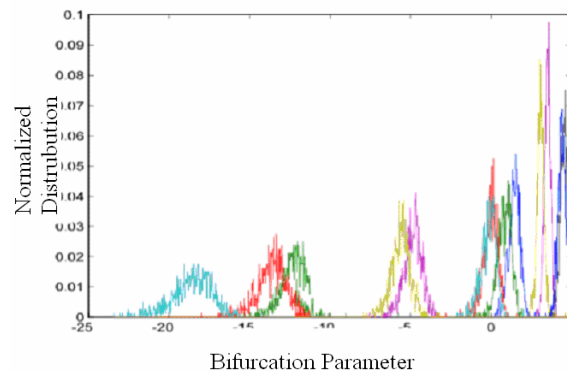


FIGURE 5. Experimentally measured jump distributions for a range of sweep rates at ambient noise intensity, showing a qualitative match with the theoretical results of FIGURE 6. Bifurcation point is at zero, sweeping is to the left. As the sweep rate is increased the distributions move leftward, become more broad, and slightly skewed.

One can use the theory to determine the optimal sweep rate to provide the most rapid and precise detection of a bifurcation parameter condition for a given noise level, as shown in Figure 7. This is important in the application of bifurcation sensors

for real-time detection of chemical threats. More detail on this experiment can be found in [15].

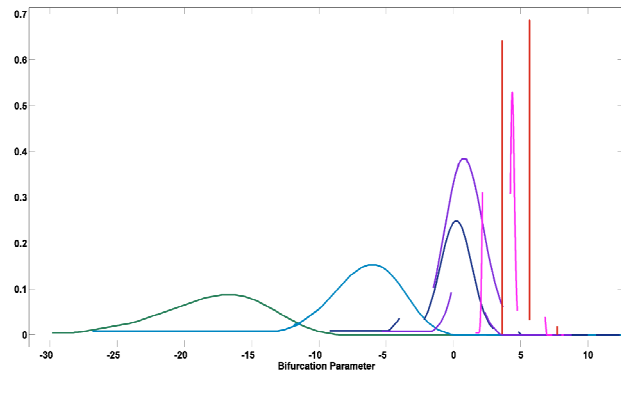


FIGURE 6. A set of jump distributions created from the numerical solution of the Fokker-Planck equation for a range of sweep rates, for a given noise level and device model. Inset shows the steady-state frequency response amplitude and behavior (arrows) as frequency is swept downwards until jump events occur near the bifurcation point; pink represents noise activated escape and green represents delayed bifurcation (both exaggerated for clarity).

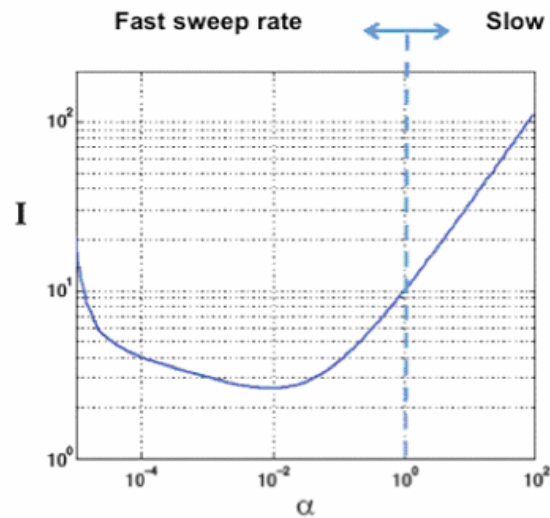


FIGURE 7. For a given noise level parameter α can be chosen at the smallest value of I (defined below) such that the bifurcation parameter confidence interval is minimum, providing the most precise measurement. Conversely, given a required confidence interval and noise level, an optimal value of α can be chosen to minimize the measurement time. Note that the optimal value for α falls in a regime not described by the previously known slow sweep approximations. Here the parameter I is proportional to the confidence interval in the case that the number of escape events used to make a measurement is equal to $r \cdot T$ where r is the sweep rate and T is the measurement time.

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