HOPPING CONDUCTION IN SEMICONDUCTORS SUBJECTED TO A STRONG ELECTRIC FIELD

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A theory of hopping conduction is developed for compensated crystalline and amorphous semiconductors subjected to a strong electric field. It is shown that at sufficiently low temperatures the current–voltage characteristics should be given by the equation $I \propto \exp[-(E_0/E)^{1/4}]$. The theory is compared with the experimental data for amorphous germanium films. The value of E_0 is deduced from the experimental data and used to estimate the characteristic length of the fall of the wave functions of the states located near the Fermi level. The paper is concluded with a discussion of the influence of longitudinal and transverse magnetic fields on the hopping conduction in a strong electric field.

The theory of hopping conduction in semiconductors subjected to weak electric fields corresponding to the ohmic region has a long history, and it has achieved considerable success (see reviews [1, 2]). The interest in hopping conduction in strong electric fields is much more recent. This is due to the fact that the first experimental investigations have been concerned with the hopping conduction between impurity levels located quite close to the allowed band. In this case an increase in the electric field results in fairly rapid ionization of impurities (impurity breakdown) so that conduction ceases to be of hopping nature. However, in the case of strongly compensated semiconductors which are now used very widely [3, 4], only the deepest impurity levels are occupied by electrons at low temperatures, and these levels appear because of fluctuations in the charged-impurity potential [2, 5] (Fig. 1). In this case the separation of the Fermi level from the nearest allowed band is considerably greater than the binding energy of isolated impurities, and this means that the ionization of impurities is a difficult process, so that there is a wide range of fields in which nonohmic hopping conduction can be observed.

In recent years an increasing amount of work has been done on electronic properties of amorphous semiconductors. According to modern ideas [6], the forbidden band in these substances is replaced by a band of energies in which the density of states is low but finite. The Fermi level is located somewhere near the middle of the band. Electron states close to the Fermi level overlap weakly and are localized. Thus, the energy-level schemes of amorphous and compensated crystalline semiconductors are similar near the Fermi level.

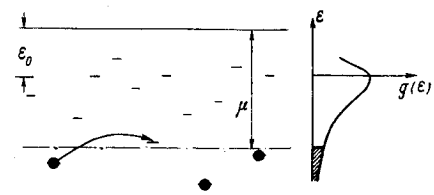

Fig. 1. Energy-level scheme of a compensated semiconductor. The continuous line represents the bottom of the conduction band and the chain line is the Fermi level. Short horizontal dashes are the energy levels of impurities and the black dots are electrons occupying these levels; ε_0 is the binding energy of an isolated impurity. The energy dependence of the density of states is shown on the right. The filled states are shown shaded.

The large gap between the Fermi level and the allowed bands makes an amorphous semiconductor the most convenient material for the study of nonohmic hopping conduction in a wide range of fields and currents. A crystalline semiconductor is a suitable material only if it is almost completely compensated so that the Fermi level is depressed by an amount comparable with the forbidden-band width [7].

At low temperatures the ohmic conductivity σ of amorphous and compensated crystalline semiconductors is due to the jumps of electrons between states located in a narrow band of energies $\Delta\varepsilon$ near the Fermi level [2, 6, 8] (Fig. 1). For this reason the value of σ is independent of the distribution of the density of states in the forbidden band, but is governed by two quantities, which are the density of states at the Fermi level $g(\mu)$ and the characteristic length a of the fall of these states. Mott [8] showed that when the temperature is lowered

the conductivity σ should decrease in accordance with the law

$$\sigma = \sigma_0 \exp\left\{-\left(\frac{T_0}{T}\right)^{1/4}\right\}, \quad (1)$$

where $T_0 \approx 16 / g(\mu)a^3$. Dependences close to Eq. (1) have been reported for crystalline and amorphous semiconductors [2, 6, 8]. A superlinear rise of the current I in a wide range of strong electric fields has been reported for amorphous films of germanium, silicon, and carbon [9] and for strongly compensated crystalline germanium [4]; this rise has been observed at temperatures in the range of validity of Eq. (1). Figure 2 gives, by way of example, the current–voltage characteristics of an amorphous germanium film obtained at various temperatures [10].

The present paper is devoted to the nonohmic dependence I(E) in the range of temperatures in which the ohmic conductivity is governed by Eq. (1). We shall assume that the temperature is so low that all the effects associated with the transfer of carriers to the allowed bands (for example, the Poole–Frenkel effect) can be ignored. However, we must bear in mind that in most cases the superlinear part of the current–voltage characteristic is followed by breakdown and a negative resistance region. These phenomena are obviously due to the transfer of carriers to the allowed bands and will not be considered here. We shall discuss only the lower, positive, part of the current–voltage characteristic sufficiently far from the turnover field. We shall restrict our discussion to the nonlinearities of electronic origin and we shall ignore the heating of the lattice and the establishment of equilibrium in the phonon system.

The problem considered here was first discussed by Mott [11]. According to Mott, the rapid rise of I in strong electric fields is due to the following physical factors. As is known, the hopping conduction in weak electric fields involves phonon absorption and emission (Fig. 1). In a weak field, when the slope of the Fermi level can be ignored, the absorption and emission of phonons should occur equally frequently. The absorption of phonons is responsible for the exponential temperature dependence of the conductivity given in Eq. (1). In a strong electric field, when the fall of the potential energy of an electron eER(T) over a typical length of a jump R(T) becomes comparable with the width of the band of energies around the Fermi level $\Delta\varepsilon$(T), an electron can move along the field, emitting phonons at each jump (Fig. 3). According to Mott [11], the current in this case is independent of the temperature and it rises with the field in accordance with the law

$$I(E) \propto \exp\left\{-\frac{1}{E^{1/3}}\right\}. \quad (2)$$

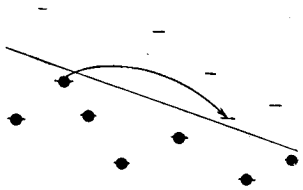

Fig. 3. Hopping conduction accompanied by phonon emission. The inclined line represents the position of the quasiequilibrium Fermi level in a strong electric field.

Although no objection can be raised to the idea of activation-free phonon-emission-assisted hopping conduction in strong fields, Eq. (2) seems to be in error. We shall show that in strong fields the current should obey the law

$$I(E) \propto \exp\left\{-\left(\frac{E_0}{E}\right)^{1/4}\right\}, \quad (3)$$

where

$$E_0 = \frac{\alpha T_0}{ea}, \quad (3a)$$

α is a coefficient of the order of unity (our theory is insufficiently refined to give its value) and T_0 is the same temperature as in Eq. (1).

We note, first of all, that an electron can be transferred from a filled to an empty state, located "down the field" at a distance r, without absorption of a phonon, provided the difference between the energies of these states does not exceed eEr (Fig. 3). Therefore, the effective density of states involved in jumps over distances of the order of r is

$$N_{\text{eff}}(r) = g(\mu)\, eEr, \quad (4)$$

and the average distance r to one such state is given by

$$N_{\text{eff}}(\bar{r})\, \bar{r}^3 \simeq 1, \quad (5)$$

which leads to

$$\bar{r} \simeq (eEg(\mu))^{-1/4}. \quad (6)$$

The probability of an electron jump over a distance r is [1, 2]

$$W(r) \propto e^{-\frac{2r}{a}}. \quad (7)$$

Substituting Eq. (6) into Eq. (7), we obtain Eq. (3).

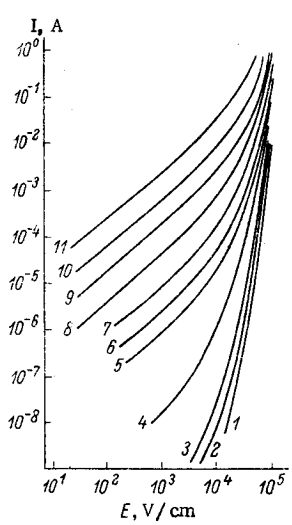

Fig. 2. Current–voltage characteristics of an amorphous germanium film, taken from [10]. T (°K): 1) 4.2; 2) 12; 3) 15; 4) 23.3; 5) 34; 6) 38; 7) 43; 8) 50; 9) 58; 10) 66; 11) 79.

We have mentioned earlier that the transition from Ohm's law and the conductivity of Eq. (1) to the process described by Eq. (3) occurs in an electric field E_c which can be found by equating $eE_cR(T)$ and $\Delta\varepsilon(T)$. According to [2, 6, 8], $R(T) \simeq a(T_0/T)^{1/4}$, $\Delta\varepsilon(T) = T^{3/4}T_0^{1/4}$, and E_c is given by

$$E_c \simeq \frac{T}{ea}. \qquad (8)$$

We can easily show that in this field the principal exponential terms in the expression for the ohmic current σE and in Eq. (3) become equal.

We shall now discuss the question of comparison of Eq. (3) with the experimental results. We shall use the data for an amorphous germanium film [10] plotted in Fig. 2. We can see that at low temperatures the current is independent of the temperature of the sample and that it rises very rapidly with increasing field, which is in qualitative agreement with Eq. (3). The curve I(E) at 4.2°K (with the possible exception of the lowest part) corresponds to the limit of the low-temperature range. We shall compare it with Eq. (3) by plotting log I as a function of $E^{-1/4}$ (Fig. 4). We can see that the experimental dependence has a slight but finite curvature. Deviations from Eq. (3) on the high-field side (low values of $E^{-1/4}$) may be due to prebreakdown effects, whereas on the low-field side the deviations may be due to a temperature-dependent contribution to the current. For this reason it would be interesting to compare Eq. (3) with the data obtained at still lower temperatures. The results considered here can be described satisfactorily by the expression $I \propto E^8$ (see [10]), but this expression has no theoretical justification. The low-temperature dependences I(E) obtained for other substances can also be approximated satisfactorily by an expression of the type $I \propto E^n$, where n can have all possible values right up to 15-16 [12]. It is likely that this range of values of n is wide because the law (3) is obeyed in a relatively narrow range of fields centered on values of E_0/E which differ greatly from one substance to another.

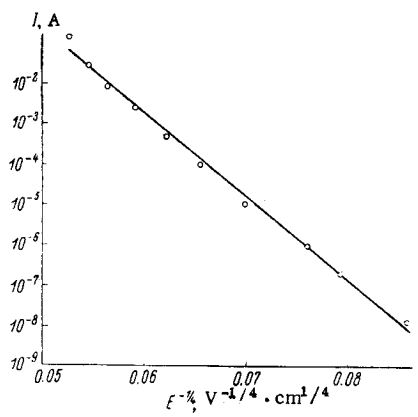

Fig. 4. Open circles represent the experimental dependence of the current on $E^{-1/4}$ in the case of amorphous germanium at 4.2°K [10]. The straight line gives the best approximation to the experimental results by Eq. (3).

Equation (3) can be used to estimate the value of a. This can be done by finding E_0 from Fig. 4. Next, the temperature dependence of the ohmic conductivity can be used to determine T_0 and both these quantities can be substituted in Eq. (3a). Such an analysis of the data for amorphous germanium gives $E_0 = 4.8 \cdot 10^{10}$ V/cm, $T_0 = 1.1 \cdot 10^8$ °K, and $a = 20\alpha$ Å.

We have considered so far the high-field range $E > E_c$, where the current is completely independent of the temperature. Mott [13] and Hill [14] considered the influence of an electric field in the case when the drop in the electron energy in a typical jump, eER(T), is less than the average energy of the emitted and absorbed phonons ($E < E_c$). In this case the electric field reduces by eER(T) the energy of the phonons which are absorbed in jumps along the field and increases by the same amount the energy in jumps against the field. Mott [13] and Hill [14] conclude that if $eER(T) \gg T$, the argument in the exponential function in the equation for the conductivity should be corrected by a term which is linear in E. For example, if the ohmic conductivity σ is given by Eq. (1), the current density j calculated by Hill [14] is

$$j = \sigma E \exp\left\{\frac{eER(T)}{T}\right\} = \sigma E \exp\left\{\frac{eEaT_0^{1/4}}{T^{5/4}}\right\}. \qquad (9)$$

This result is based on the assumption that an electron jumps mainly along the field in fields $E < E_c$ and in fields $E > E_c$. It is difficult to agree with this assumption. In weak electric fields an electron moves along percolation paths which are zigzag-shaped [2]. An electric field which satisfies the condition $eER(T) \ll \Delta\varepsilon(T)$ cannot alter a percolation path because this would reduce the current by a much larger factor than the increase implied in Eq. (9). Therefore, electrons are equally likely to jump along and against the field. The net result of the application of an electric field is an increase in the probability of some of the jumps and a reduction in the probability of others. In view of our limited knowledge of percolation paths we cannot say whether these two effects will be compensated exactly. However, we cannot exclude the possibility that as a result of such compensation the correlation to the argument of the exponential function in Eq. (1) is proportional not to E, but to E^2. In any case, Eq. (9) and the corresponding results obtained by Mott [13] cannot be regarded as justified.

We shall conclude this discussion by considering the influence of a magnetic field on the hopping conduction in a strong electric field. It is known [2] that a magnetic field compresses the wave functions of electrons in the transverse direction and reduces the hopping conductivity. In the case of relatively short distances between electron states $r < \lambda^2/a$, where $\lambda = (c\hbar/eH)^{1/2}$ is the magnetic length, such compression reduces the probability of a jump at right angles to the magnetic field by a factor $\exp(-sr^3a/\lambda^4)$, where s is a numerical coefficient of the order of 0.1. If $\bar{r} <$

λ^2/a, i.e., $H < H_c = (c\hbar a/e)(E_0/E)^{1/4}$, this reduction in the probability applies to a considerable number of jumps in $\mathbf{H} \perp \mathbf{E}$ and $\mathbf{H} \parallel \mathbf{E}$. In fact, in the absence of the field the jumps leading to the dependence given by Eq. (3) occur in the right-hand half-space, and the average distance of a jump is $\bar{r}$. The frequency of the jumps depends naturally on the angle ϑ between the "jump vector" $\mathbf{r}$ and the electric field $\mathbf{E}$. However, this frequency decreases considerably only in the angular range $\pi/2 - \vartheta \ll \pi/2$. A magnetic field $H < H_c$ simply alters slightly the argument of the exponential function in the jump probability (7) and, therefore, it has practically no influence on the electron path. Thus, electron jumps are characterized by vectors $\mathbf{r}$ with large projections along $\mathbf{E}$ and at right angles to this field. Thus, we find that in the $\mathbf{H} \perp \mathbf{E}$ and $\mathbf{H} \parallel \mathbf{E}$ configurations the influence of a magnetic field smaller than H_c can be described by the equation

$$I(H) = I(0) \exp\left\{-s \frac{e^2 a H^2}{\hbar^2 c^2} \left(\frac{E_0}{E}\right)^{3/4}\right\}. \quad (10)$$

Some anisotropy in the probability of a jump in $H = 0$ simply means that the numerical coefficients in the argument of the exponential function in Eq. (10) may differ slightly for $\mathbf{H} \perp \mathbf{E}$ and $\mathbf{H} \parallel \mathbf{E}$ ($s_\perp > s_\parallel$).

In stronger magnetic fields $H > H_c$ the main term in the argument of the exponential function in the jump probability becomes strongly anisotropic and electron jump paths change considerably [2]. In this case we can use the method for calculation of the hopping conductivity developed in [2, 15] for strongly anisotropic jump probabilities and we can show that the current, considered as a function of E and H, varies in accordance with the law

$$I(H, E) \sim \exp\left\{-\left(\frac{Hea^2}{c\hbar}\right)^{3/5} \left(\frac{E_0}{E}\right)^{2/5}\right\} \quad (11)$$

in the $\mathbf{H} \perp \mathbf{E}$ configuration and in accordance with the law

$$I(H, E) \sim \exp\left\{-\left(\frac{Hea^2}{c\hbar}\right)^{1/5} \left(\frac{E_0}{E}\right)^{1/5}\right\} \quad (12)$$

in the $\mathbf{H} \parallel \mathbf{E}$ case.

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