Equation (15) in the far field becomes the equation for …eported, for example, in Ref. 2 and differs from Eq. … that it has no exponent. Indeed, Ref. 2 has demon… that the maximum value of the index through which (15) is summed is $l_{max} \sim \rho$. Hence Eq. (15) becomes …ression for $T(\varphi)$ for $kr \gg \rho^2$, which is analogous to the …d condition $r \gg a^2/\lambda$.

…el zone, the exponential term in Eq. (15) …ed for $kr \gg \rho$, although it is not significant …

…the error in deriving Eqs. (14). When … used the Hankel function can be repre…ing formula:

$$\cdot e^{-i(kr-(l\pi/2)-(\pi/4)+(l^2/2kr))}.$$

… analysis reveals that the relative error of the equation is less than $l/10kr$. Since $l \leqslant \rho$, the error of Eq. (16) attributable to this asymptotic Hankel function will not exceed $\rho/10kr$. This is a rather soft condition. Thus, even for $r \geqslant 10a$, the error will be less than 1%.

The error from integral estimation by the stationary phase method can be ignored. Indeed, this error, according to the theory of the method, will be of the order of $1/kr$ in our case which, for example, for $\lambda = 1\ \mu$m and $r = 10$ cm, is less than 0.002%.

Equation (4) for the optical theorem is therefore applicable in the case of a circular cylinder not only in the far field but also in the Fresnel zone. The error of this formula will be less than $\rho/10kr$.

In estimating the error we used criteria of the type $kr \gg \rho^2$, $kr \gg \rho$ with a clear physical meaning. These criteria represent the boundaries on the far field and the Fresnel zone; however, they are identical for bodies of any shape. Size a entering into parameter ρ represents the characteristic size of the body. Hence Eq. (3) for bodies of arbitrary shape and finite size will be applicable in the Fresnel zone, while its error will also be less than $\rho/10kr$.

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Measurement of polarization anisotropy of Rayleigh scattering in a quartz lightguide

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Polarization effects in a lightguide under Rayleigh scattering are studied. A expression is found for the dependence of the response of a fiber-optic reflectometer on the ellipticity of radiation propagating in the lightguide and the scattering parameter. The parameter of polarization anisotropy of Rayleigh scattering in a single-mode quartz lightguide is found to be $\rho = 0.05 \pm 0.01$.

INTRODUCTION

The phenomenon of partial depolarization of radiation by Rayleigh scattering in fused quartz is well known.[1] According to this principle, part of the power in a single-mode fiber lightguide (SFL) is scattered into an orthogonally polarized mode. This effect determines the limiting length of preservation of the state of polarization in the SFL, which is inversely proportional to the parameter of Rayleigh scattering anisotropy ρ.[2]

The value of parameter ρ was determined experimentally in Refs. 3 and 4 using a time domain reflectometer in a ratio of the powers scattered backward into orthogonally polarized modes of an anisotropic SFL. This ratio depends not only on the anisotropy parameter but also on the state of polarization of the radiation propagating in the lightguide and, consequently, on dichroism and random coupling between the polarized modes. Since the resolution of the reflectometer (in tens of meters) did not allow us to track the evolution of the state of polarization in anisotropic SFL, interpretation of the results was a complicated problem, and the considerable spread in the values of $\rho = 0.04$–0.08 obtained in Refs. 3 and 4 was possibly due to that.

Parameter ρ was determined in our paper using a frequency polarization reflectometer,[5] in which we used autodyne of reception radiation scattered in a slightly anisotropic SFL. The interval of spatial resolution of the reflectometer (of the order of 1 cm) was shorter than the beat length of polarized modes of the SFL, so that SFL anisotropy did not have any effect on the results of our experiment.

THEORY

The response of an autodyne reflectometer is proportional to the projection of the wave amplitude scattered from the resolution interval Δz of the reflectometer[5] onto the state of polarization of the laser radiation. The dependence of the complex amplitude $\mathbf{E}$ of wave scattered backward on the

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amplitude $\mathbf{E}_+$ of wave propagating forward in the Cartesian ...tem, the z axis of which is directed along the ..., is determined by the set of equations[6]

$$\hat{n}\mathbf{E}_+ = i\hat{r}\mathbf{E}_-, \tag{1}$$

$$\ldots_- = -i\hat{r}\mathbf{E}_+, \tag{2}$$

...trix of coupling coefficients of polarized ...uperscript T denotes the matrix transposition operation, and $\hat{r}$ is a matrix of coupling coefficients for the counterpropagating waves.

We find the amplitude $\mathbf{E}_+$ of wave propagating forward to first-order perturbation theory with initial condition $\mathbf{E}_-(L) = 0$ from Eq. (1), neglecting multiple Rayleigh scattering, i.e., the right-hand side of Eq. (1), is

$$\mathbf{E}_+(z) = \hat{M}\mathbf{E}_+(0)\exp(i\beta z). \tag{3}$$

Here, β is the propagation constant of radiation in a lightguide, $\hat{M}$ is the Jones matrix of the SFL, satisfying the matrix differential equation

$$\frac{\partial}{\partial z}\hat{M} = i(\hat{m} - \beta\hat{I})\hat{M}, \quad \hat{M}(0) = \hat{I}, \tag{4}$$

where $\hat{I}$ is the unit matrix. We find the complex amplitude of the counterpropagating wave from the inhomogeneous differential equation (2). For this we multiply both sides of Eq. (2) by $\hat{M}^T\exp(i\beta z)$ and integrate it over the length of the lightguide

$$\mathbf{E}_-(0) = i\int_0^l \hat{M}^T\hat{r}\hat{M}\mathbf{E}_+(0)\exp(2i\beta z)\,dz. \tag{5}$$

The reflectometer selects partial contribution $\mathbf{E}_z$ from the reflected wave $\mathbf{E}_-(0)$, due to scattering in a lightguide segment $[z; z+\Delta z]$,

$$\mathbf{E}_z = i\hat{M}^T\hat{c}(z)\hat{M}\mathbf{E}_+(0). \tag{6}$$

Here

$$\hat{c}(z) = \int_z^{z+\Delta z} \hat{r}(u)\exp(2i\beta\mu)\,du.$$

Since the resolution interval of the reflectometer is less than the characteristic length of variation of the state of polarization of light in the SFL, in deriving Eq. (6) we have neglected dependence $\hat{M}(z)$ and $\hat{M}_z^T(z)$ on z in the interval Δz. Equation (6) has a simple physical meaning: $\hat{M}\mathbf{E}_+(0)$ is the amplitude of the direct wave at point z, $\hat{r}\hat{M}\mathbf{E}_+(0)$ is its contribution to the backward wave, and matrix $\hat{M}^T$ describes conversion of the wave back scattered from point z to the entrance of the lightguide.

We find the projection $\mathbf{E}_z$ onto the linear polarization state of laser radiation

$$E_z = (\mathbf{E}_+^T(0)\mathbf{E}_z) = i\mathbf{E}_+^T(0)\hat{M}^T\hat{c}\hat{M}\mathbf{E}_+(0) = i\xi^T\hat{c}\xi, \tag{7}$$

where

$$\xi = \hat{M}\mathbf{E}_+(0) = \hat{R}(\theta)\begin{pmatrix}\cos\varepsilon \\ i\sin\varepsilon\end{pmatrix}$$

is the Jones vector of the state of polarization of radiation at the back-scattering point. $\hat{R}$ is the rotation matrix, θ is the azimuth, and ε is the angle of ellipticity of the state of polarization at point z.[7] In deriving Eq. (7), it was taken into account that the vector $\mathbf{E}_+(0)$ is real in the Cartesian system.

For further treatment it is necessary to specify the relationship between the random matrix elements of $\hat{r}$. As is well known, polarization anisotropy of Rayleigh scattering in fused quartz is caused by microscopic fluctuations of birefringence,[1] i.e., it is due to the real part of the permittivity tensor of the SFL material. Then, the mode-coupling-coefficient matrix of a weakly guiding lightguide is real and symmetric,[6] and can be represented in the form

$$\hat{r} = p\left[\hat{I} + \hat{R}(\alpha)\begin{pmatrix} q & 0 \\ 0 & -q\end{pmatrix}\hat{R}(-\alpha)\right], \tag{8}$$

where the coefficient p characterizes a portion of the wave scattered by a single Rayleigh center and falling backward into an SFL spatial mode, while q and α are, respectively, the magnitude and direction of local scattering anisotropy. Owing to the microscopic homogeneity and isotropy of fused quartz, p, q, and α are independent random stationary functions of z with zero mean value, so that α is distributed uniformly in the interval $[-\pi/2; \pi/2]$. Taking this into account, we determine from Eqs. (6) and (8) the nonzero statistical second-order moments of the matrix $\hat{c}$

$$\langle c_{11}c_{11}^*\rangle = \langle c_{22}c_{22}^*\rangle = \frac{\varkappa}{1-\rho}, \quad \langle c_{11}c_{22}^*\rangle = \varkappa,$$
$$\langle c_{12}c_{12}^*\rangle = \langle c_{21}c_{21}^*\rangle = \frac{\varkappa\rho}{1-\rho}, \tag{9}$$

where $\varkappa$ is the coefficient of backscattering from the reflectometer resolution interval; $\rho = \langle c_{12}^2\rangle/\langle c_{11}^2\rangle$ is the fraction of power back scattered into a polarization state orthogonal to the initial; and the angle brackets denote averaging over an ensemble of statistically homogenous SFL. We find from Eqs. (7) and (8) using Eqs. (9) that

$$\langle|E_z|^2\rangle = \langle\varkappa\rangle\left\{\cos^2 2\varepsilon(z) + \frac{\rho}{1-\rho}[1+\sin^2 2\varepsilon(z)]\right\}. \tag{10}$$

Equation (10) connects the response of the autodyne reflectometer to the evolution of ellipticity of the state of polarization of light in the SFL and the parameter of polarization anisotropy of Rayleigh scattering (Fig. 1). The value of ρ can be determined from the relation $2\rho = \langle|E_z|^2\rangle_{\min}/\langle|E_z|^2\rangle_{\max}$, where $\langle|E_z|^2\rangle_{\min}$ and $\langle|E_z|^2\rangle_{\max}$ are standard deviations of reflectometer signals for, respectively, circular ($\varepsilon = \pi/4$) and linear ($\varepsilon = 0$) polarizations at the scattering point.

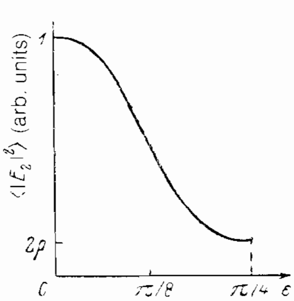

FIG. 1. Reflectometer response vs the angle of ellipticity of the state of polarization at the scattering point.

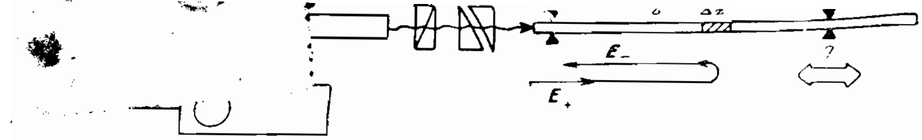

FIG. 2. Schematic of experimental setup: 1—laser, 2—photodiode, 3—spectrum analyzer, 4—polarizer, 5—Babinet-Soleil compensator, 6—SFL, 7—SFL length modulator.

EXPERIMENT

A schematic of the experimental setup is shown in Fig. 2. Radiation from a He–Ne laser with 0.63-μm wavelength was introduced into a slightly anisotropic, irregular SFL with fused-quartz lightguiding core (typical polarization beat length $\sim$20 cm). Laser intensity variation under the action of radiation scattered in the lightguide was recorded by a photodiode, the signal from which was processed by a low-frequency spectrum analyzer.[5]

Resolution of the autodyne polarization reflectometer is determined by the modulation index of the phase shift in the SFL. Modulation can be achieved both by scanning the laser lasing frequency and by changing the temperature or tension of the lightguide. Our studies have shown that we can achieve the highest resolution with a He–Ne laser by stretching the lightguide. Periodically linear modulation of the phase shift necessary for operating the reflectometer was realized using a magnetodynamic system, stretching a segment of the SFL of length $L = 35$ cm. The system ensured a modulation index $\Delta\varphi = 1500$ rad, and this corresponds to a limiting spatial resolution of $\Delta z = 2\pi L/\Delta\varphi = 1.5$ mm.

A typical response of the polarization reflectometer to the Rayleigh scattering signal is given in Fig. 3. The peak of the photocurrent spectrum, corresponding to $z = 35$ cm, was due to scattering from a finite SFL segment 6 cm long, which was not stretched. The halfwidth of this peak was 12 mm, and this also characterizes the actual spatial resolution of the reflectometer. Resolution was limited by the nonlinear magnetodynamic modulator and string vibrations of the SFL in the stretching process.

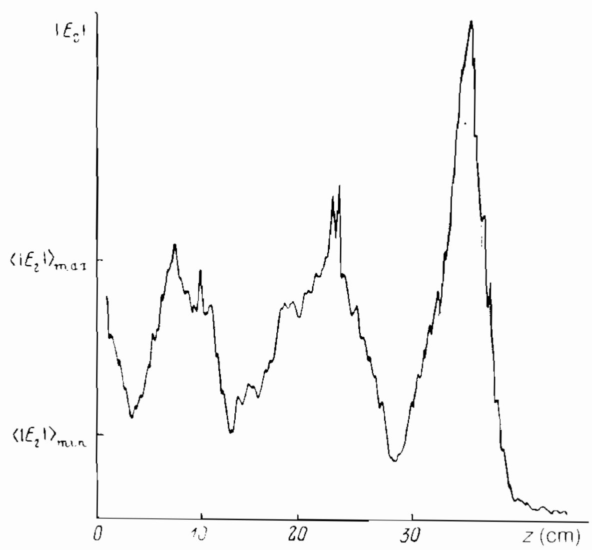

FIG. 3. Response of autodyne polarization reflectometer.

Polarization filtering in the setup was achieved by a linear polarizer, whose transmission axis was matched to the polarization plane of the laser radiation. The polarization state at the entrance to the SFL was selected using a Babinet-Soleil compensator in such a manner that we achieved maximum modulation depth in the reflectometer response. The polarization anisotropy parameter was calculated as the mean value of the $|E_z|^2_{\min}/2|E_z|^2_{\max}$ over several realizations of the reflectometer response and was $\rho = 0.05 \pm 0.01$. The value obtained agrees within the limits of error with measurement results of the degree of depolarization of radiation in bulk samples of fused quartz $\rho = 0.043$.[8]

We note that polarization anisotropy of Rayleigh scattering can have a significant effect on the noise of fiber-optic sensors with a laser source. Depolarization decouplings are used in high-precision sensors for removing the effect of reflected signals on the laser.[9] However, as Fig. 3 shows, the action of the scattered signal on the laser can be reduced by no more than triple decoupling.

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