Seeing Quantum Fractals

Gregory A. Fiete, *et al.*

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PHYSICS

Seeing Quantum Fractals

Scanning tunneling microscopy reveals indications of fractal electronic structure in a magnetic semiconductor.

Gregory A. Fiete and Alex de Lozanne

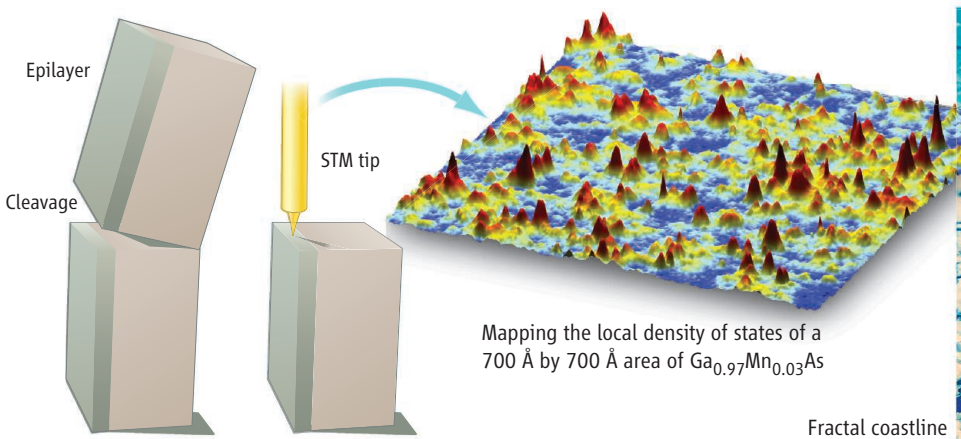
If you have been fortunate enough to have been on a flight that lands near a beautiful sunny coastline, you will no doubt have taken in the scenery on the way down, and perhaps even marveled at the winding of the shore into the horizon. What you may not have realized is that the coastline is actually a fractal—an object that appears the same at all length scales (perhaps only statistically). Fractals actually abound in nature: Galaxies, clouds, mountains, trees, and broccoli are all familiar examples. But fractals can occur in the quantum realm as well, even though they have never been observed there, until, perhaps, now. On page 665 of this issue, Richardella *et al*. (*1*) report direct measurements of quantum mechanical electron waves that indicate that they may also possess a fractal nature.

Quantum particles, such as the electrons surrounding the nuclei of atoms, have a wave nature associated with them that gives rise to interference phenomena much like that found in waves on the surface of water, light waves, or sound waves. Ordinarily these quantum mechanical electron waves are extremely difficult to see because their ripples occur on the scale of the spacing of individual atoms in a solid. However, the scanning tunneling microscope (STM) has the capability to locally image these quantum waves on the surface of a material (*2*, *3*). As the STM can measure these atomic-scale waves over a wide range of energies and with good energy resolution, it is an ideal tool to probe quantum mechanical effects in solid materials.

The crucial physical ingredient that gives rise to fractals in electron waves is the phenomenon of localization of waves in a disordered medium (*4*). A light wave traveling in a vacuum will continue traveling indefinitely. However, waves traveling in a random environment can get "stuck" or "localized" in certain positions due to the complicated pattern of constructive and destructive interference taking place from random scattering events. It turns out that in many situations, a weakly random background will still allow wave propagation. However, when the fluctuations in the random environment become too large the waves localize, and are no longer spread over the entire system but occupy only a small region. There is thus a special value of the randomness where the waves make the transition from being extended over the entire system to being localized. At this transition point, the waves are fractal (*5*, *6*). In the context of electron waves in a solid, the transition from extended waves to localized waves as a function of disorder is called the metal-insulator transition, and its physics is the focus of the study by Richardella *et al*. (see the figure).

The material chosen for their study was $Ga_{1-x}Mn_xAs$, a ferromagnetic semiconductor intensely studied over the last decade because of its potential use in spintronics—electronic devices based on the quantum mechanical spin of the electron. Whereas the nonmagnetic semiconductor GaAs has been an industry standard for decades, the field of ferromagnetic semiconductors opened up when it was shown that doping the large–spin transition metal Mn into GaAs could result in a ferromagnetic material (*7*) that could readily be interfaced with GaAs to allow the efficient injection of spin, a requirement for spin-based devices (*8*, *9*). The Mn introduced into GaAs disorders it, and at the same time supplies free charged carriers that form the quantum waves undergoing a metal-insulator transition as a function of Mn concentration, x (*10*). While the magnetic properties of $Ga_{1-x}Mn_xAs$ were not the focus of the work of Richardella *et al*., they are intimately related to the electrical properties because the carriers mediate the magnetic interaction and therefore play a direct role in the magnetism itself. Obtaining a detailed understanding of the electronic states and their precise relationship to the ferromagnetic properties is the central challenge in this field. The metal-insulator transition presents an extra degree of difficulty due to the inherent large fluctuations, but also grants an opportunity for intriguing quantum phenomena to emerge.

Richardella *et al*. provide an energy-resolved real-space measurement of the quantum wave properties near the metal-insulator transition. Although some features of their data are suggestive of fractal aspects of the quantum waves, a number of questions are raised by their experiments. For example, why does the carrier concentration appear to be nearly independent of the Mn concentration? What is the physics that leads to this preferred carrier concentration? Why do the strongest



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Coming into view. Cleaving the GaAs wafer in situ and finding the thin $Ga_{1-x}Mn_xAs$ layer near the surface is challenging (*15*), but the wealth of tunneling data that can be obtained are worth the effort. As reported by Richardella *et al*., the STM data may share the fractal nature observed in many coastlines.

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signatures of the metal-insulator transition appear at one particular energy, independent of Mn concentration? How much do these surface measurements reveal about the bulk physics? Taken as a whole, the data indicate that interactions among the carriers play an important role in this system. Complementary studies of the metal-insulator transition in electrical transport have reached a similar conclusion (*11*). It thus appears likely that interaction effects at the metal-insulator transition are here to stay, and may even lead to novel effects like self-organized quantum criticality (*12*) on the surface of a recently discovered class of materials known as topological insulators (*13*, *14*). For the time being, it looks like we might have to be content with a clear view of fractals from 35,000 feet up.

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BIOCHEMISTRY

An Ensemble View of Allostery

Proteins existing as ensembles of conformations may be key to understanding signal transduction processes.

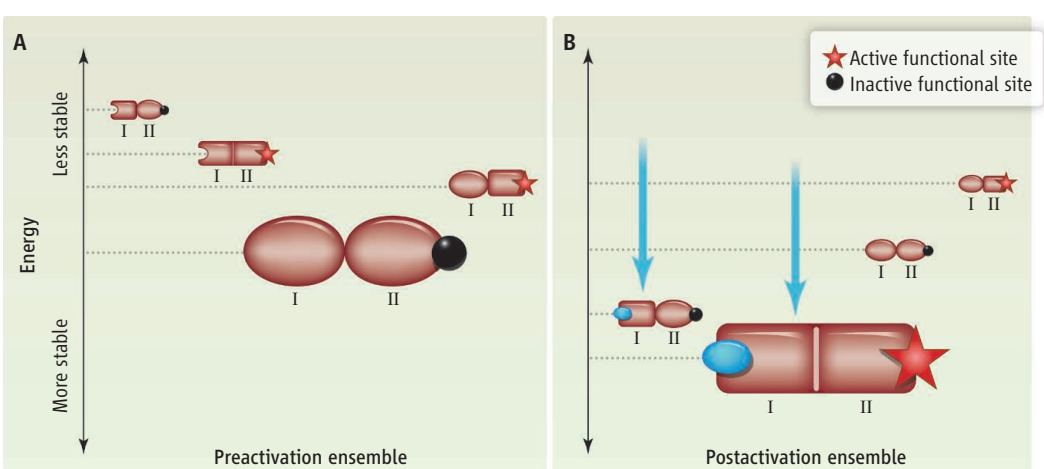
Vincent J. Hilser

Allostery is the process by which biological macromolecules (mostly proteins) transmit the regulatory effects induced by the binding of a ligand at one site to a different, often distant, functional site. This process governs the function of almost all metabolism and gene regulation (*1*, *2*). Despite the importance of this process, a general understanding of the structural and energetic basis for allostery has been elusive. On page 685 of this issue, Bai *et al.* (*3*) shed light on a key obstacle to understanding allosteric control by showing how proteins existing as ensembles of multiple conformations affect the allosteric signal transduction process.

For the past 40 years, allostery has usually been interpreted through either of two classic models: the concerted model (*4*) or the sequential model (*5*). The former model treats the conformational coupling between different parts of the molecule as absolute, whereas the latter assumes a fixed coupling between ligand binding and conformational changes (*6*). The practical consequence of these assumptions is that without knowing how probable the allosterically activated states are in the absence of the allosteric ligand, it is difficult to know what structural and/or energetic features of a protein must be present in order for signal transduction to occur.

Attempts to resolve this problem have relied most heavily on high-resolution structural analysis (such as x-ray crystallography) of the various liganded species (*7*, *8*). Although such analysis can show what bonds are formed and lost between the liganded and unliganded species, the underlying assumption is that the functional states of the molecules—the so-called tensed (T) or relaxed (R) states—are well represented by single static structures. This may not be the case, as Bai *et al.* show in their analysis of allostery in the bacterial flagellar motor (*3*). Their work shows that the presence of conformational heterogeneity among the subunits in the native state ensemble of the protein is key both to the allosteric mechanism and to the cooperativity of the motor. Thus, a functional state cannot be obligatorily associated with a single static structural state of the protein.

Recent studies investigating the importance of conformational dynamics (*9*–*12*), local unfolding (*13*–*15*), and intrinsic disorder (*16*) to allosteric signaling support the conclusion that regional changes in the conformational heterogeneity of proteins often accompany allosteric transitions. Thus, allosteric proteins may often rely on a mechanism in which multiple conformations must retain significant probabilities in order to function and transmit signal. In such cases, structural studies that rely on a single static structure to describe the activated or inactivated states of the protein may only provide limited insight into allosteric mechanisms. This potential setback, however, is accompanied by a more important opportunity: Knowledge of the conformational states in the native state ensemble can provide vital



The ensemble model of allostery reveals the energetic basis of coupling. The ensemble for a protein with an effector (I) and a functional (II) subunit is shown (**A**) before and (**B**) after activation with an effector ligand (blue circle). Prior to activation, the most probable state has an inactive functional subunit (II with black circle). Stabilization of the active conformation of the effector subunit (blue arrows) populates the active conformation of the functional subunit (II with red star).

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