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A Study of Fluid Squeeze-Film Damping

The fluid squeeze-film produced by relative axial or tilting motion of two closely spaced plates provides viscous damping action over certain ranges of operation. When gas is the working fluid, a damper can be realized which is operable over a wide frequency range in the presence of extreme temperatures and intense radiation. A linearized analysis and approximate design equations, verified by a limited experimental program, are presented for several useful damper configurations.

Introduction

Fluid power, control, and instrumentation systems which employ gases as the working media are highly suited to the increasingly severe environments of temperature and radiation associated with many modern aerospace systems and military weapons. In addition to the ability to tolerate extreme temperatures and intense radiation, pneumatic systems are often simpler, more reliable, and capable of better performance than other systems which might be considered for a given application. For these reasons, pneumatic systems are beginning to find widespread use in industrial and military application and, in many cases, are replacing more conventional hydraulic systems.

An often difficult problem which arises frequently in the design of pneumatic devices, particularly of pneumatic instruments and signal processing components, is that of providing damping to flexibly supported mechanical elements. Examples of such elements are flapper-type valves, jet-pipe valves, gyroscope rotors, or accelerometer masses. Conventional dampers which depend on direct viscous shear of a fluid are impractical because of the low viscosity of gases, and piston-and-cylinder dashpots which depend on forcing of fluid through a restriction in the piston or cylinder are useful only at very low frequencies of motion because of the gas compressibility. At high frequencies (very rapid relative velocity between piston and cylinder), the latter damper acts almost entirely as a spring and hence does not perform its intended damping function [1].[1]

If the maximum relative displacements of the elements to be damped in a pneumatic system are small (of the order of several thousandths of an inch or less), as they will often be in gas-operated instruments and miniature control devices, it is possible to derive substantial damping forces even at very high frequencies of motion by squeezing a thin film of gas from between two parallel flat plates. The damping forces thus produced will be similar to those of a conventional viscous damper (force proportional to relative velocity) over certain ranges of operation, will be almost unaffected by radiation, and will be increased with increasing temperature level. This type of damper, termed a "squeeze-film damper," is discussed in this paper, and results are presented which can be used to design three basic types of squeeze damper. Limited experimental results which confirm the linearized equations derived are also presented.

[1] Numbers in brackets designate References at end of paper.

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Analysis

Fig. 1 shows two configurations of squeeze-film dampers which will be discussed initially; flat rectangular plates having width w long compared with length L, and circular parallel plates of radius R.

The following assumptions are made:

(a) The separation h of the plates is very small compared with the linear dimensions of the plates.

(b) The gas flow between the plates is assumed to be laminar and primarily viscous. This assumption implies low Reynolds numbers and parabolic velocity distributions across the gas film.

(c) The relationship between pressure and density at any point in the gas film is assumed described by a polytropic process with exponent n:

$$\frac{p}{\rho^n} = \text{const} \tag{1}$$

Nomenclature

A_k, B_k = Fourier coefficients
b = damping coefficient
b_ϕ = angular damping coefficient
c = initial squeeze-film thickness
e = perturbation of film thickness
e_0 = amplitude of step change of e
h = squeeze-film thickness
j = $\sqrt{-1}$
k = ratio of specific heats
L = length of squeeze plate
m = integer
M = tilting moment
n = polytropic coefficient
p = fluid pressure
P_a = ambient pressure
P = dimensionless pressure function
R = radius of circular pad
R_i = inner radius of annulus
R_o = outer radius of annulus
s = Laplace transform variable
t = time
W_D = squeeze-film force
w = plate width
z = axial coordinate
α_m = characteristic number
β = R_o/R_i
β_f = fluid bull modulus
$\delta(\ )$ = perturbation
ϵ = base of natural logarithms
η_m = mth zero of Bessel function of zero order
θ = circumferential angle
μ = gas viscosity
ξ = z/L
ρ = fluid density
σ = squeeze number = $12\mu L^2\omega/nc^2P_a$
ϕ = tilt angle
ω = angular frequency, rad/sec
ω_c = cutoff frequency, rad/sec

where p is the local pressure, ρ is the local density, and n is a constant depending on the process. If the plates are of metal having high thermal conductivity and if the relative velocities of the plates are relatively low, the film will be nearly isothermal and $n \approx 1$. If the plates are poor thermal conductors—for example, ceramics—or if the relative velocities are very high, the process will approach an adiabatic condition and $n \approx k$, the ratio of specific heats.

(*d*) The variation of plate spacing is assumed to be small compared with the mean spacing c:

$$h = c + e \tag{2}$$

where $e \ll c$.

The application of equation (1), the continuity relationships, and the momentum equations to an element of fluid in the squeeze-film yields the well-known differential equation for gas-film lubrication [2, 3]:

$$\frac{\partial}{\partial t}(hp^{1/n}) = +\frac{h^3}{12\mu}\frac{n}{(n+1)}\nabla^2\left(p^{\frac{n+1}{n}}\right) \tag{3}$$

where μ is the fluid viscosity, t is time, and ∇^2 is the Laplacian operator.

Equation (3) is a nonlinear partial differential equation which can be solved only for special cases by numerical methods. However, by virtue of assumption (*d*), the variations in p will be small compared with the ambient pressure level P_a:

$$p = P_a + \delta p \tag{4}$$

where $\delta p \ll P_a$.

When equation (4) is substituted into equation (3) and second-order terms are neglected, a linear equation for δp results:

$$\frac{c^2P_a}{12\mu}\nabla^2\left(\frac{\delta p}{P_a}\right) - \frac{1}{n}\frac{\partial}{\partial t}\left(\frac{\delta p}{P_a}\right) = +\frac{\partial}{\partial t}\left(\frac{e}{c}\right) \tag{5}$$

This equation can be solved for the pressure δp for any specified displacement function $e(t)$, providing the plate geometry is specified. The pressure must then be integrated over the plate area to determine the squeeze-film force. Solutions of equation (5) for the two geometries shown in Fig. 1 will be discussed.

Squeeze-Film Force for Infinitely Wide Parallel Plates. In the case of plates whose width w is long compared with their length L, as illustrated in Fig. 1(*a*), the gas flow occurs primarily in one direction, the z-direction in Fig. 1(*a*). The geometry is then primarily one-dimensional and equation (5) becomes

$$\frac{c^2P_a}{12\mu L^2}\frac{\partial^2}{\partial \xi^2}\left(\frac{\delta p}{P_a}\right) - \frac{1}{n}\frac{\partial}{\partial t}\left(\frac{\delta p}{P_a}\right) = +\frac{\partial}{\partial t}\left(\frac{e}{c}\right) \tag{6}$$

where

$$\xi = \frac{z}{L} \tag{7}$$

A product solution of equation (6) is assumed of the form

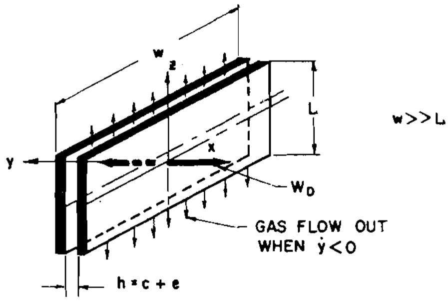
$$\frac{\delta p}{P_a} = P(\xi)\epsilon^{-\alpha t} \tag{8}$$

where ϵ is the base of natural logarithms, and α is an undetermined characteristic number. The displacement function $e(t)$ is now assumed to be a sudden or step change of magnitude e_0 which occurs at $t = 0$. From the results produced by this function, the response to any $e(t)$ may be deduced. For all $t > 0$, equation (6) then becomes

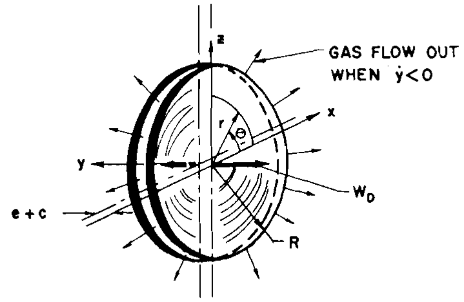
$$\frac{c^2P_a}{12\mu L^2}\frac{d^2P}{d\xi^2} + \frac{\alpha}{n}P = 0 \tag{9}$$

The solution of equation (9) is of the form

$$P = A_k \sin\sqrt{\sigma}\xi + B_k\cos\sqrt{\sigma}\xi \tag{10}$$



a) LONG PARALLEL PLATES



b) CIRCULAR PARALLEL PLATES

Fig. 1 Parallel-plate squeeze dampers

where A_k and B_k are constant; and σ, termed the squeeze number, is defined as follows:

$$\sigma = \frac{12\mu L^2\alpha}{nc^2P_a} \tag{11}$$

Equation (10) must satisfy the boundary conditions that $P = 0$ when $\xi = 1/2$ and $\partial P/\partial \xi = 0$ when $\xi = 0$. This requires that $A_k = 0$ in equation (10) and that

$$\sigma = (2m - 1)^2\pi^2;\ m = 1, 2, 3, \ldots . \tag{12}$$

Equation (10) is now substituted into equation (8). The B_k must be determined so that equation (8) satisfies the initial condition at $t = 0$ of δp. For a step change in displacement, the gas film is compressed or expanded without leakage flow according to the equation of state, equation (1). Hence, for small e_0,

$$\left(\frac{\delta p}{P_a}\right)_{t=0} = -n\frac{e_0}{c} \tag{13}$$

In order that equation (8) equal equation (13) at $t = 0$, B_k in equation (10) must be the Fourier coefficients of a square wave. Hence, using equations (8), (10), and (11),

$$\frac{\delta p}{P_a} = -\frac{ne_0}{c}\sum_{m=1}^{\infty}\frac{4(-1)^{m-1}}{(2m-1)\pi}[\cos(2m-1)\pi\xi]\epsilon^{-\alpha_m t} \tag{14}$$

where

$$\alpha_m = \frac{nc^2P_a(2m-1)^2\pi^2}{12\mu L^2} \tag{15}$$

To obtain the force $W_D(t)$ acting on the squeeze plates, the pressure is integrated over the plate surface:

$$W_D(t) = 2wL\int_{\xi=0}^{\xi=1/2}\delta p(t, \xi)d\xi \tag{16}$$

Equation (14) is substituted into equation (16) and the integration performed to determine $W_D(t)$ for the specified step change in e. It is convenient then to take the Laplace transform of $W_D(t)$

$$\bar{W}_D(s) = \int_0^\infty W_D(t)\epsilon^{-st}dt \qquad (17)$$

The transform of W_D due to any displacement function $e(t)$ is then obtained by dividing $\bar{W}_D(s)$ by the transform of the step function and multiplying the result by the transform of the desired displacement function $\bar{e}(s)$ [4]. The result of these operations is [5]

$$\frac{\bar{W}_D(s)}{\bar{e}(s)} = \frac{96\mu w L^3}{\pi^4 c^3} \sum_{m=1}^{\infty} \frac{1}{(2m-1)^4} \frac{s}{\left(1 + \dfrac{s}{\alpha_m}\right)} \qquad (18)$$

Although equation (18) is an infinite series, it converges very rapidly. The second term, corresponding to $m = 2$, is 81 times smaller than the first, and the third term is 2401 times less than the first. For most practical purposes a one-term approximation to equation (18) is sufficiently accurate. Thus

$$\frac{\bar{W}_D(s)}{\bar{e}(s)} \cong \frac{bs}{\left(1 + \dfrac{s}{\omega_c}\right)} \qquad (19)$$

where

$$b = \frac{96\mu w L^3}{\pi^4 c^3} \qquad (20)$$

$$\omega_c = \frac{\pi^2 n c^2 P_a}{12\mu L^2} \qquad (21)$$

Any displacement function $e(t)$ may be considered to be composed of a series of sinusoids of various frequencies. For any particular frequency ω, $s = j\omega$ in equation (19), and $\bar{W}_D$ and $\bar{e}$ then are phasors representing the force and displacement sinusoids, respectively,

$$\frac{\bar{W}_D(j\omega)}{\bar{e}(j\omega)} = \frac{bj\omega}{\left(1 + j\dfrac{\omega}{\omega_c}\right)} \qquad (22)$$

If the frequency ω is such that

$$\omega \ll \omega_c \qquad (23)$$

the denominator term in equation (22) is approximately equal to unity. If equation (23) holds for all frequencies of interest, then equation (19) reduces to $\bar{W}_D(s) = bs\bar{e}(s)$ or

$$W_D(t) = b\,\frac{de(t)}{dt} \qquad (24)$$

Thus the squeeze plates act as a viscous damper having a damping constant b. The frequency range below which equation (24) is substantially valid is thus determined by the "cutoff frequency" ω_c. In the case of a steady sinusoidal displacement, equation (22) shows that, when $\omega = \omega_c$, there is a phase lag of 45 deg between the force and the velocity sinusoids. At large frequencies $\omega \gg \omega_c$, force and displacement are in phase with each other, and the squeeze plates act as a spring.

Equation (20) shows that the low-frequency damping constant b is independent of ambient pressure. Also, if L/c is large as assumed, it is clear that relatively large values of damping are possible. If, for example, $L = 0.4$ in., $w = 2$ in., and $c = 10^{-3}$ in., $b = 0.32$ lb-sec/in. In addition, equation (21) shows that the cutoff frequency may be very large. For these dimensions, $n = 1$ or isothermal conditions, and $P_a = 1$ atmosphere, $\omega_c = 4680$ cycles/sec.

Squeeze-Film Force for Thin Annular Parallel Plates. The preceding result for infinitely wide rectangular plates may be applied to find approximately the squeeze-film force for parallel annuli. R_o is the outer and R_i the inner radius of the annulus, whe R_o/R_i is near unity, equations (20) and (21) become

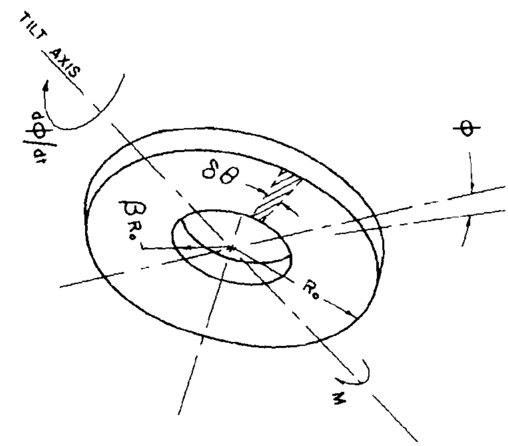


Fig. 2 Tilting-plate annular damper

$$(b)_{\text{annulus}} = \frac{48\mu R_o^4(1+\beta)(1-\beta)^3}{\pi^3 c^3} \qquad (25$$

where

$$\beta = R_i/R_o$$

and

$$\omega_c = \frac{\pi^2 n c^2 P_a}{12\mu R_o^2(1-\beta)^2} \qquad (26)$$

An exact analysis for circular annuli is given in reference [6].

Squeeze-Film Force for Parallel Circular Plates. By a derivation similar to that used in arriving at equation (18), the squeeze-film force for the circular plates shown in Fig. 1(*b*) can be obtained [6]:

$$\frac{\bar{W}_D(s)}{\bar{e}(s)} = \frac{48\pi\mu R^4}{c^3} \sum_{m=1}^{\infty} \frac{s}{\eta_m^4(1 + s/\gamma_m)} \qquad (27)$$

where R is the plate radius, η_m is the mth zero of the Bessel function of zero order, and

$$\gamma_m = \frac{nc^2 P_a \eta_m^2}{12\mu R^2} \qquad (28)$$

A good approximation to equation (27) is obtained by taking only the first term of the series. In that case, the film force has the form of equation (19) with

$$(b)_{\text{circular}} = \frac{4.45\mu R^4}{c^3} \qquad (29)$$

$$\omega_c = \frac{nc^2 P_a}{2.07\mu R^2} \qquad (30)$$

Squeeze-Film Torque for Thin Annuli in a Tilting Mode. Equation (19) for long rectangular plates may be used to derive an approximate relation for the torque produced by the relative tilting motion of two initially parallel annular plates. This configuration, shown in Fig. 2, is very difficult to analyze in an exact fashion.

The approximation is made that flow in the circumferential direction is negligible in comparison to the flow in the radial direction. The annulus can then be broken into a series of infinitesimal elements having the dimensions

$$L = R_o(1 - \beta) \qquad (31)$$

$$w = R_o\,\frac{(1+\beta)}{2}\,\delta\theta \qquad (32)$$

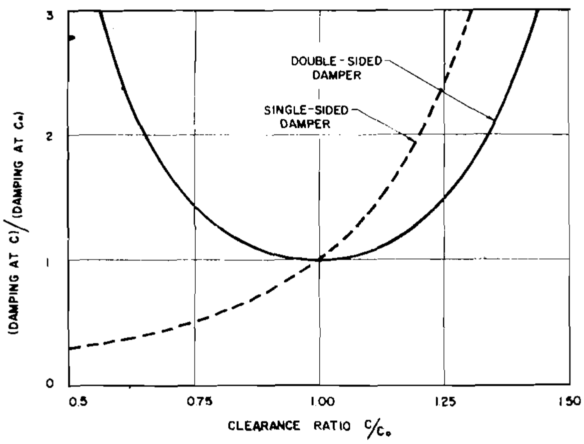


Fig. 3 Nonlinearity of squeeze-film dampers

By summing the moments of the forces given by equation (19) due to each element about the diameter, the following expression is obtained:

$$\frac{\bar{M}(s)}{\bar{\phi}(s)} \cong \frac{b_{\phi}s}{(1 + s/\omega_c)} \tag{33}$$

where

$\bar{M}(s)$ = Laplace transform of moment $M(t)$

$\bar{\phi}(s)$ = Laplace transform of angle $\phi(t)$

and

$$b = \frac{12\mu R_o^6(1 - \beta^2)^3}{\pi^3 c^3} \tag{34}$$

$$\omega_c = \frac{\pi^2 n c^2 P_a}{12\mu R_o^2(1 - \beta)^2} \tag{35}$$

Nonlinearity of Squeeze-Film Damper. Since the preceding solutions assumed only small displacements compared with the mean fluid film thickness, and since this condition rarely will be met in practice, it is important to give some attention to the effect of large displacements on the damper performance.

An exact solution for the squeeze-film forces is extremely difficult if the percentage changes of clearance c are large. However, if the damper plate motions occur at frequencies considerably lower than the cutoff frequencies ω_c, the behavior of the film can be assumed quasi-static. At any instant, the film force can then be obtained from equation (22) and the appropriate expression for b [equations (20), (23), or (27)]. The variation of the instantaneous damping coefficient with clearance c is shown in Fig. 3 for a single pair of plates and for two pair of plates arranged in push-pull so that the sum of their clearances is constant. The single-sided configuration is seen to have a large variation in damping constant with clearance while, in the double-sided arrangement, the variation is much less. Moreover, the damping coefficient always increases in the latter arrangement as the central element moves away from its midposition between the plates.

Experimental Results

A limited experimental program was conducted [6] to evaluate the accuracy with which the preceding linearized equations predicted the average damping forces in the squeeze-film damping process. The configurations studied were circular plates and thin annuli in relative translation.

The experimental apparatus, shown in Figs. 4 and 5, consisted of a rigid steel cantilever beam supported at one end by an elastic flexure and acted upon near the opposite end by steel squeeze-film damping plates. The beam was deflected to one side and released. The transient oscillation of the mass-spring-damper system composed of the beam mass, flexure spring, and squeeze-film pads was observed by a linear differential transformer pickoff attached directly to the beam. From oscilloscope photographs, of which a typical example is given in Fig. 6, the average damping coefficient was computed. Tests were run for squeeze-film thickness of 10^{-3} to 10×10^{-3} in. at ambient pressures between 0.15 and 1.0 atm. A circular pad of radius R = 0.039 in. and an annulus of outer radius R_o = 0.309 in. and inner radius R_i = 0.191 in. were tested.

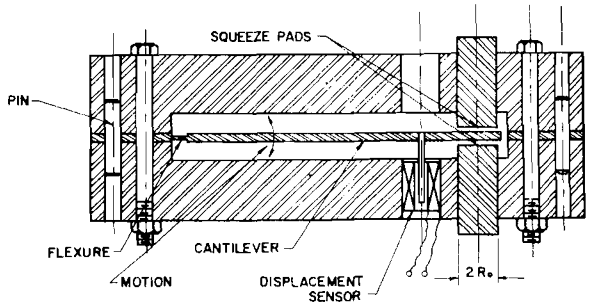


Fig. 4 Schematic diagrams of test apparatus

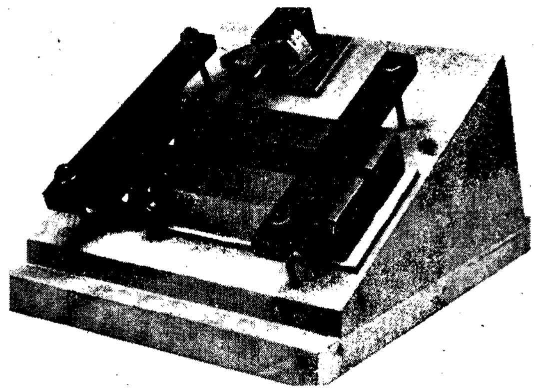


Fig. 5 Photograph of test apparatus

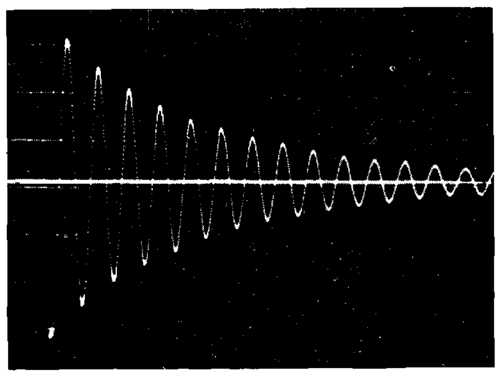


Fig. 6 Typical transient response of squeeze-film damped system

Figs. 7 and 8 show experimental and analytical results for the circular and annular pads, respectively. The theoretical results shown are the low and high-frequency asymptotes computed from the single-term approximate equations, assuming an isothermal condition, i.e., n = 1. Generally, if the squeeze plates are of metal or other material of high thermal conductivity, isothermal conditions are likely to prevail.

It was not possible with the apparatus used to approach the cutoff frequency ω_c by reducing the ambient pressure. The pres-

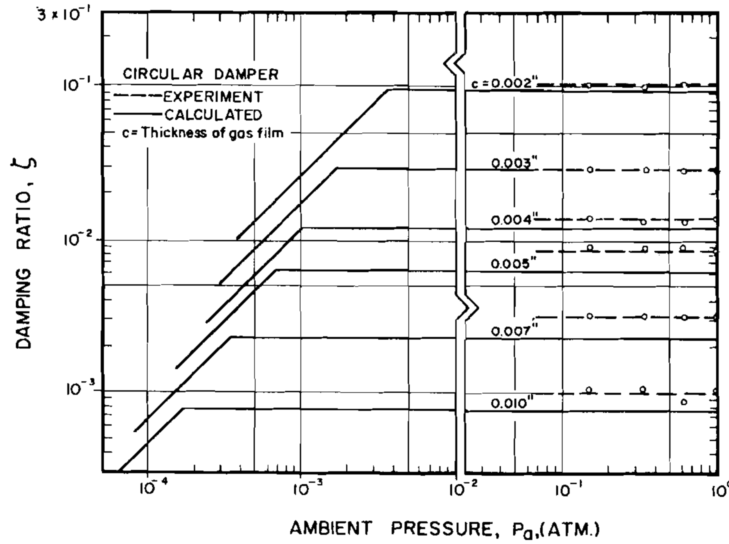


Fig. 7 Experimental results for circular squeeze pads compared with calculated asymptotic damping ratios

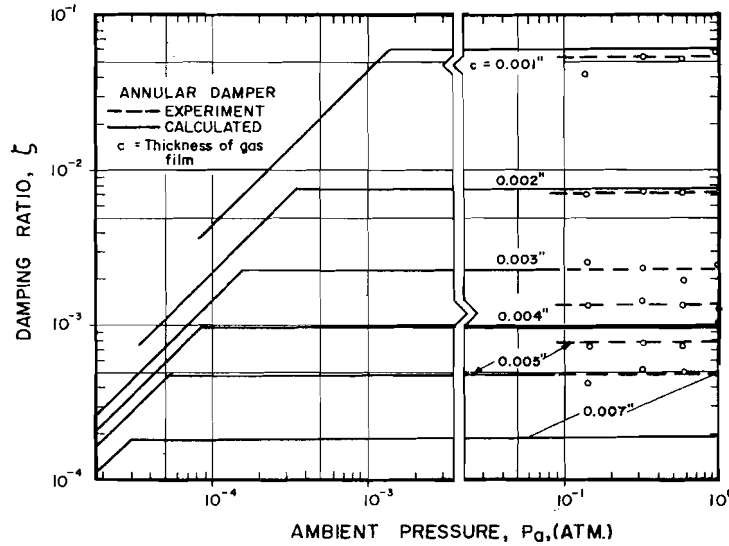


Fig. 8 Experimental results for annular squeeze pads compared with calculated asymptotic damping ratios

sure levels at which the cutoff frequency would be reached are the points at which the curves of Figs. 7 and 8 change slope. In the range covered, experiment showed the damping ratio to be independent of ambient pressure, as predicted by theory.

For the smaller clearances, as shown in Figs. 7 and 8, the agreement between theory and experiment was excellent. At larger clearances, there was a systematic trend for the theory to underestimate the measured damping ratio. In these cases, however, the damping ratio was extremely small, of the order of 10^{-3}, and other sources of damping in addition to the squeeze pads were becoming significant. In addition, the length-to-clearance ratios of the flow passages were also approaching magnitudes where the assumption of laminar viscous flow in the gas films is questionable (the experimental data indicate that a length-to-clearance ratio greater than 30 is necessary).

An indirect check of expression (34) for the damping contributed by a tilting annulus, Fig. 2, was made in reference [5]. Although the measurements were approximate, the results tended to confirm equation (34).

Conclusions and Discussion

The fluid squeeze film produced by the relative axial or tilting motion of two closely spaced flat plates produces the action of viscous damping (force proportional to relative velocity) over certain ranges of operation. When the working fluid is a gas, the squeeze film acts as a damper for frequencies of motion small compared with the first cutoff frequency ω_c. At high frequencies, the gas film behaves as a spring (force proportional to relative displacement). The squeeze-film damping effect is nonlinear with respect to displacement and, when two plates are used in push-pull, the incremental damping increases with displacement away from the centered position.

The gas squeeze-film damper can be used effectively in damping mechanical elements whose maximum displacements are of the order of thousandths of an inch, such as pneumatic instrument components and valve elements. The gas damper is insensitive to extreme temperatures and intense radiation flux.

Experimental results indicate that the single-term approximation to the linearized squeeze-film damping equations adequately predicts the low-frequency average damping forces. The damping forces were found to be independent of ambient pressure from 0.15 to 1.0 atm.

The results presented in this paper for gas films may be applied to liquid films (when no cavitation occurs) if np_a is replaced by β_f, the bulk modulus of the fluid, in all equations.

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