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Curvature Induced Mode Coupling in Large Core Optical Fibres with Step Refractive Index Profiles

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ABSTRACT

A simple ray model is developed to approximate the output beam angle increase caused by a symmetric bend with variable curvature in a large core optical fibre beam delivery system of the type commonly used in laser material processing. The results are compared with experiments using curved fibres in the shape of half-cycle cosines with different rates of curvature change. The theoretical model was supported by experiment, and is sufficiently simple for application in the design of an optical fibre beam delivery system.

1 INTRODUCTION

Since optical fibres were first used as a means of beam delivery, bending properties have been studied extensively for both single mode and multimode fibre, mainly for telecommunications purposes. Mode power coupling in a multimode fibre affects the attenuation and bandwidth in telecommunications; in optical power beam delivery systems, mode coupling may change the near field and far field properties of the output beam and degrade the beam quality.

Previous work has used coupled power equations to derive the mode conversion coefficients between adjacent modes.[1–5] Geometric optics was also used to investigate mode coupling and predict output beam angle increase.[6,7] The effect of random mode coupling by microscopic bends, lack of uniformity of core diameter and irregularity of the core-cladding boundary were studied by using the above methods. These factors of mode coupling can be regarded as intrinsic, which may exist even without disturbance from outside the fibre. Extrinsic mode coupling factors, like macroscopic bending of the fibre and the effect of pressure exerted on the fibre also play a very important role in optical beam delivery systems[7,8] for material processing. In such systems, large core optical fibres are normally employed to accommodate high laser power, and the optical fibres are often subject to bending between the laser source and workpiece. Such flexibility is an advantage of an optical fibre over mechanical lens and mirror beam delivery systems. But a penalty for this advantage is the degradation of beam quality by mode coupling, generated by bending, which could cause changes of the near field beam profile and an increase in the output beam angle.[8] In the case when a fibre is mounted onto a robot arm, the change of output beam quality may vary as the movement of the robot arm changes the bending radius of the fibre. In the laser power delivery system where consistency of beam quality and the safety of the system is demanded, it is important to be able to estimate the angle increase in the output beam. The angle change should be within the limit of beam quality tolerance and the total beam NA should be smaller than the fibre NA, in order to reduce power loss within the fibre.

The approach by Shildback[7] for a bend with variable curvature gives a geometric model for the angle increase caused by a bend. Although the model is simple it still requires integration over the ray path inside the fibre bend. In this report we give a simpler geometric approach to the problem of beam NA increase by a symmetric bend with variable curvature, and a simplified formula to calculate the increase. The work concentrates on the beam NA increase instead of the mode power distribution. The model is compared with experiments for a specific bend configuration of cosine shape.

2 THEORY

Mode coupling in a multimode optical fibre leads to power redistribution between different modes as well as an increase in beam NA. Power coupling from lower to higher modes produces changes in the near field of the fibre output and larger divergence angles in the far field. It is the maximum divergence change which is of chief interest in this paper.

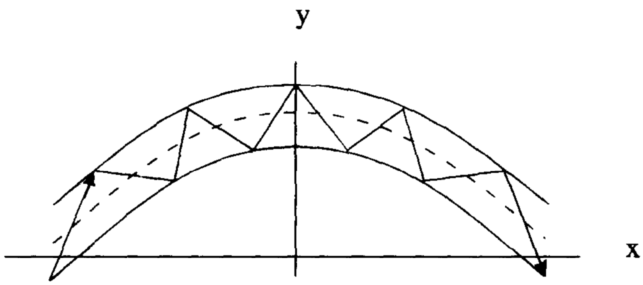

Fig. 1. Even number of reflections in a symmetric bend.

In this section we present a simplified ray mode of a symmetric bend with variable curvature and as an example we also give the result of a half-cycle cosine bend. For a symmetric bend, when the number of reflections is an even number, as shown in Fig. 1, there is no angle change by the bend because the rays on one side of the bend are a mirror image of the rays on the other side but travelling in different directions; for every angle increase there is always an angle decrease by the same amount and vice versa. Following this argument we concentrate on the situation where the reflections in the bend are an odd number as shown schematically in Fig. 2. The mirror image of the right hand side rays of the bend is also shown on the left hand side of the bend. The angle changes between the rays and the fibre axis in one reflection are the φ_is as in Fig. 2. If the ray travels towards the centre of curvature, the angle increases; otherwise the angle decreases. If we pair the two adjacent parallel rays as indicated by the same letter a, b, c, d in Fig. 2, with one angle increasing and one angle decreasing, and suppose the arc length between two reflection points is ΔS_i and the angle change between these reflections is φ_i, then we have the relation

$$\varphi_i = \frac{\Delta S_i}{R_i} = \Delta S_i C_i$$

where R_i is the local bend radius and C_i is the curvature. Suppose the fibre core radius is a; we use the following approximation when beam angle and core diameter are not very large,

$$\Delta S_i = \frac{2a}{\theta_i}$$

where θ_i is the angle the ray makes with the fibre tangent direction as

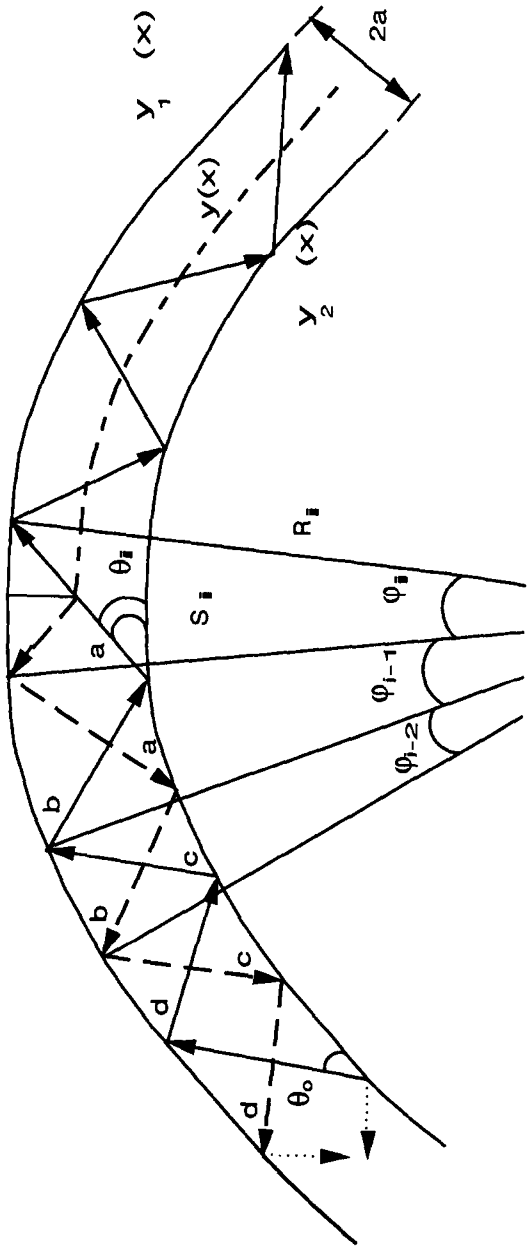

Fig. 2. Odd number of reflections in a symmetric bend.

shown in Fig. 2. The net angle increase $\Delta\varphi_i$ between each pair can be expressed as:

$$\Delta\varphi_i = \frac{2a}{\theta_i} C_i - \frac{2a}{\theta_{i-1}} C_{i-1}$$

$$\Delta\varphi_{i-1} = -\left(\frac{2a}{\theta_{i-1}} C_{i-1} - \frac{2a}{\theta_{i-2}} C_{i-2}\right)$$

The total angle change

$$\Delta\varphi = \sum \Delta\varphi_i$$

$$= 2a\left[\frac{C_i}{\theta_i} - \frac{C_{i-1}}{\theta_{i-1}} - \frac{C_{i-1}}{\theta_{i-1}} + \frac{C_{i-2}}{\theta_{i-2}} + \cdots\right]$$

As an approximation, we use an averaged angle $\theta_a(i)$ to replace the angles θ_i, θ_{i-1} and θ_{i-2}, then we have

$$\Delta\theta = 2a\left[\frac{C_i - 2C_{i-1} + C_{i-2}}{\theta_a(i)} + \frac{C_{i-2} - 2C_{i-3} + C_{i-4}}{\theta_a(i-2)} + \cdots\right]$$

$$= \frac{C_i - 2C_{i-1} + C_{i-2}}{\Delta S_a^2(i)} \cdot \Delta S_a(i)(\Delta S_a(i))^2$$

$$+ \frac{C_{i-2} - 2C_{i-3} + C_{i-4}}{\Delta S_a^2(i-2)} \Delta S_a(i-2)(\Delta S_a(i-2))^2 + \cdots$$

where

$$\Delta S_a(i) = \frac{2a}{\theta_a(i)}$$

If we substitute an average value ΔS_a for the $\Delta S_a(i)$ and $\Delta S_a(i-2)$ in the bracket in the above equation, then using the finite difference formula,

$$\Delta\varphi = \Delta S_a^2\left[\frac{d^2C_i}{dS_a^2(i)} dS_a(i) + \frac{d^2C_{i-2}}{dS_a^2(i-2)} dS_a(i-2) + \cdots\right]$$

When the total number of reflections is not small, that is the bend length is not too short, we can assume that

$$\frac{d^2C_i}{dS_a^2(i)} dS_a(i) + \frac{d^2C_{i-2}}{dS_a^2(i-2)} dS_a(i-2) + \cdots$$

$$= \frac{d^2C_{i-1}}{dS_a^2(i-1)} dS_a(i-1) + \frac{d^2C_{i-3}}{dS_a^2(i-3)} dS_a^{(i-3)} + \cdots$$

then

$$\Delta\varphi = \tfrac{1}{2}\Delta S_a^2\left[\frac{d^2C_i}{d^2S_a(i)}dS_a(i) + \frac{d^2C_{i-1}}{d^2S_a(i-1)}dS_a(i-1) + \cdots\right]$$
$$= \tfrac{1}{2}\Delta S_a^2 \sum_i \frac{d^2C_i}{d^2S_a(i)}dS_a(i)$$

For a large number of reflections we use an integral to replace the summation such that

$$\Delta\varphi = \tfrac{1}{2}\Delta S_a^2 \int_0^{S_o/2} \frac{d^2C}{d^2S}ds$$
$$= \tfrac{1}{2}\Delta S_a^2 \frac{dC}{dS}\bigg|_0^{S_o/2}$$
$$= \frac{2a^2}{\theta_a^2}\frac{dC}{dS}\bigg|_0^{S_o/2} \tag{1}$$

where S_o is the total bend length, θ_a is the average angle over the bend and

$$\Delta S_a = \frac{2a}{\theta_a}$$

The angle change $\Delta\varphi$ is the accumulated angle change of all the ray pairs. As an approximation to the average angle, we take θ_a as the average between the input angle and the possible maximum angle inside the bend. Suppose the minimum bend radius is R_m, which is the bend radius at the middle of the curve for a symmetric bend; the maximum angle change of the ray at R_m is the angle change caused by a bend with constant bend radius of R_m, which is[7,9]

$$\frac{a}{\theta_o R_m}$$

then the average angle is given by

$$\theta_a = \theta_o + \frac{a}{2\theta_o R_m}$$

where a is the fibre core radius and θ_a is the input ray angle at the beginning of the bend.

In order to quantitatively estimate the angle change, the strain-optic effect, which modifies the refractive index of the fibre, must also be taken into account. The elastic deformations of the fibre modifies the fibre refractive index, and as a result a straight ray when there is no bend would pursue a curved path in the presence of a bend.[10] This effect can be compensated by introducing an effective bend radius R_e[10] to substitute the geometric minimum bend radius R_m,

$$R_e = R_m/\gamma$$

where γ is a factor less than one. Generally speaking γ is a function of the angle between the ray and fibre axis, but within the practical NA range of large core multimode fibres γ is approximately constant. As given previously,[10]

$$\gamma = 1 - \frac{1}{6n_o^2}(1-2\sigma)(n_o 2 - 1)(n_o^2 + 2)$$

and for fused silica $\gamma = 0{\cdot}76$.

As an example and to compare with experiments, we calculate the angle change by a half cycle of cosine bend with the fibre axis taking the form $y(X)$, where

$$y(X) = A\cos LX \qquad X \in \left[\frac{-\pi}{2L}, \frac{\pi}{2L}\right]$$

The curvature as a function of X is

$$C(X) = \frac{AL^2 \cos LX}{[1 + A^2L^2 \sin^2 LX]^{3/2}}$$

Using the relation

$$\frac{dC}{dS} = \frac{dC}{dX}\frac{dX}{dS} = \frac{1}{\sqrt{(1+Y'^2)}}\frac{dC}{dX}$$

eqn (1) becomes

$$\Delta\varphi = \frac{2a^2}{\theta_a^2} \cdot \frac{AL^3}{[1 + A^2L^2]^2}$$

Using the relations

$$R_m = \frac{1}{AL^2}, \qquad X_o = \frac{\pi}{2L}$$

we find

$$\Delta\varphi = \frac{\pi a^2}{\theta_a^2 R_m X_o}\left[1 + \frac{4X_o^2}{\pi^2 R_m^2}\right]^{-2} \tag{2}$$

where R_m is the minimum geometrical bend radius and X_o is the quarter period of the consine curve. The total angle change can be calculated by replacing all the R_m by R_e and then using eqn (2).

3 EXPERIMENT

Experiments were carried out on a short step index silica fibre with NA of 0·22, core diameter of 0·6 mm and a length of 1·1 m. A half cycle of cosine bend was used with the amplitude and period of the curve as control parameters. It is known from the curvature expression that at the start of the bend the bend radius is infinite, and at the top of the curve the bend radius is a minimum. The aim of the experiment is to measure the relation between output beam NA, input beam angle and the rate of curvature change. To do this the fibre was designed according to a cosine function of

$$y(x) = \frac{4X_o^2}{\pi^2 R_m} \cos\left(\frac{\pi}{2X_o} X\right)$$

where R_m is the minimum bend radius and X_o was used to change the total length of the curved section of the bend. Meanwhile the maximum and minimum bend radii remain unchanged. Therefore as X_o changes, so does the rate of curvature. As the fibre was bent care was taken not to generate localised strain on the fibre which could produce strong mode coupling. The fibre was kept straight except for the curved part. The input and output angle of the laser beam were measured with a scanning photodiode and a pinhole.

The experimental arrangement is shown in Fig. 3. The photodiode output was recorded with a chart recorder. Since only the beam divergence angle was needed, the sensitivity was adjusted such that

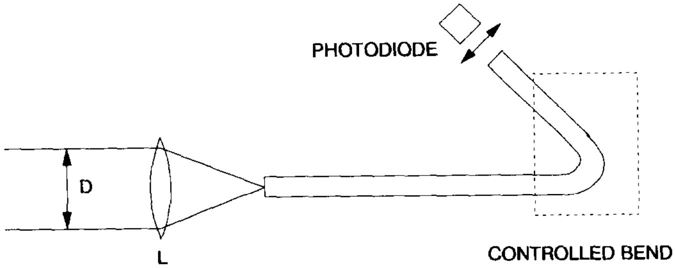

Fig. 3. Experimental arrangement.

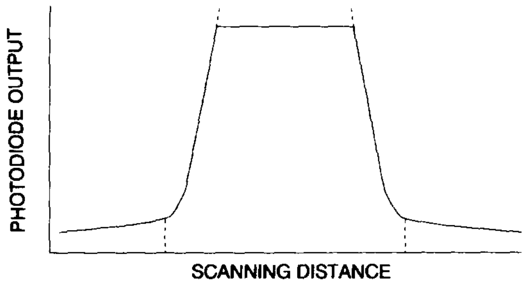

Fig. 4. Typical photodiode output recorded with a chart recorder.

when the photodiode is in the main beam, the chart recorder is saturated. The distance between the two rapid intensity changes was recorded and converted into a divergence angle; a typical curve is shown in Fig. 4.

The laser source used was a pulsed Nd:YAG laser (Lumonics JK702). The input angle of the beam was adjusted by changing the combination of launching lens L and beam diameter D which can be varied through the beam expanding telescope on the laser. The launching lenses used were $f = 60$ mm and 100 mm doublets. Five bends were used during the experiment with $X_0 = 36{\cdot}8$, 52·1, 73·7, 116·5 and 141·7 mm as shown in Fig. 5 and the input angles were 0·067, 0·098 and 0·135 rad.

4 COMPENSATION FOR INTRINSIC MODE COUPLING

Mode coupling caused by factors other than geometric bending also exist. This kind of coupling, also called intrinsic mode coupling, is fibre length dependent and could be stronger than the coupling by a

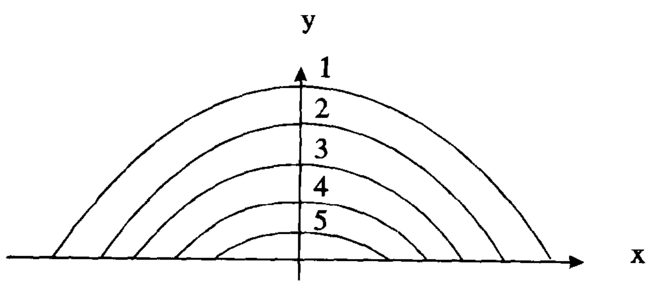

Fig. 5. Bend configuration in the experiment.

macroscopic bend.[11] Intrinsic mode coupling is caused, for example, by scattering at the core–cladding interface, and geometrical imperfections of the interface. The intrinsic mode coupling was measured with the fibre straight and compensated when the coupling caused by the bend is compared with theory. When there is no macroscopic bend the difference between the input and output beam angle was taken as intrinsic mode coupling. The fact was used that the intrinsic angle increase with fibre length is linear,[4] therefore the angle inside the fibre without a bend can be expressed as

$$\theta = \theta_{in} + \frac{\Delta\alpha}{L} . Z$$

where θ_{in} is the launching beam angle, L is the total length of the fibre, $\Delta\alpha$ is the output and input angle difference and Z is the distance from the input end of the fibre. The input angle θ_o to the bend was therefore calculated using

$$\theta_o = \theta_{in} + \frac{\Delta\alpha}{L} L_1 \tag{3}$$

where L_1 is the distance of the bend from input end; in this experiment $L_1 = 40$ mm.

To compare theory with experiment and take intrinsic mode coupling into account we use a modified formula for φ, such that

$$\varphi = \Delta\varphi + \left(1 - \frac{L_1}{L}\right) \Delta\alpha \tag{4}$$

where $\Delta\varphi$ is from eqn (2) and the second term of the right hand side is a correction for intrinsic mode coupling caused by the length of the fibre including the bend and after the bend.

As no coupling effect is considered between the intrinsic and bending mode coupling, the correction term is independent of bend parameters and is only related to the input angle via $\Delta\alpha$ which is a measured quantity and is a constant for a certain input angle. The angle φ is a theoretical angle difference between the output angle of the fibre and the input angle of the bend θ_o. In the experiment the output angle θ_{out} is measured and converted into the same quantity as φ by the formula

$$\theta = \theta_{out} - \theta_o$$

where θ_o is given by eqn (3).

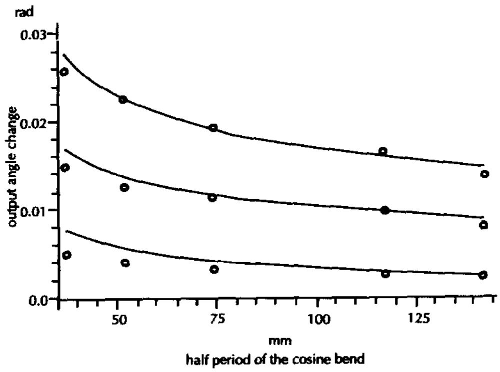

Fig. 6. Experimental results compared with theoretical curves (upper curve: input angle 0·067 rad; middle curve: input angle 0·098 rad; lower curve: input angle 0·135 rad).

The angles measured in air were converted into angles in the fibre by the refractive index of the fibre, when used in calculation, and then the calculated angle is converted back into the air angle when compared with experiment.

5 RESULTS AND DISCUSSION

The results of the experiment are shown in Fig. 6 and compared with theoretical curves for different bends. The x coordinate is the length of the half period of the cosine bend which indirectly represents the rate of curvature changes; and the y coordinate is the angle change. Each curve is for a different input angle. It can be seen that as the period of the cosine bend increases or as the rate of curvature change decreases for a certain input beam NA, the output angle change caused by the bend decreases; for a certain bend as the input beam NA increases, the output NA change decreases.

In a high average power beam delivery system, the poor beam quality leads to a relatively large value of input angle. In this work the angle range was between 67 and 135 mrad. For larger angles, the output beam NA change is very small and insignificant compared with the input angle. On the other hand it is not desirable to considerably underfill the fibre by launching a beam with a very small angle due to the mode coupling effect. In high power large core optical beam delivery systems safety and beam quality transfer are the main concern. Underfilling the fibre by appropriate design of the launching optics is normal practice to prevent power escape from the fibre, but there are also other factors like bending and intrinsic mode coupling which would introduce larger beam NA and bending loss.[12]

A sharp curvature change can give a large NA increase which overshadows the effect of intrinsic mode coupling, but in the case of a gradual curvature change, as in this experiment, the curvature induced angle change is smaller than the actual measured intrinsic mode coupling. The fact that the angle increase is a slow function of the curve span, for this specific cosine bend with a minimum bend radius of 110 mm, implies that using a large input angle is an effective way to reduce the mode coupling in a bend. This means that for a high beam quality delivery system, a large angle and small spot launching condition should be used, thus indicating the use of a small core optical fibre. Although a good fit is obtained for a cosine bend, the simplified model provides an easy way of estimating the beam NA increase by a symmetric bend with variable curvature in a beam delivery system, and yields information on systems design.

It is also shown in Fig. 6 that as the input angle and the bend length decrease, the difference between theory and experiment increases. This is expected due to the approximations of this model, i.e. small beam angle and long bend length.

6 CONCLUSION

We have modelled the effects of extrinsic mode coupling in a large core, step index optical fibre and calculated the angle increase of the output beam, resulting from a symmetric bend of variable curvature. We have tested the model by comparison with experiment for a bend of cosine shape. The results reveal that the smaller the rate of curvature change, the smaller the beam NA increase; the larger the input beam NA into the fibre, the smaller the output beam NA increase. The beam NA increase by a bend is smaller than the purely geometric effect due to the strain-optic effect which increases the geometric bend radius to a larger effective bend radius. The results of this work are intended to be useful in the design of high power optical fibre beam delivery systems.

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