PHASE MATCHING BY PERIODIC MODULATION OF THE NONLINEAR OPTICAL PROPERTIES‡

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Periodic modulation of the optical nonlinear coefficient of a propagation medium is proposed as a new method for phase matching. This proposal is examined in some detail in the case of a thin film waveguide where the prospects of its implementation seem favorable.

The problem of phase matching in nonlinear optical interactions in thin films has received considerable attention recently. This is due, in part, to the fact that in the thin film configuration the large power concentration can give rise to efficient interactions even at moderate total powers. Secondly, materials which are not birefringent and cannot, consequently, be phase-matched by the conventional technique [1] can be phase-matched by either dimensional [2] control or by periodic perturbation of the dielectric constant [3] or the boundary [4]. In this paper we wish to discuss a new approach to phase matching which involves a periodic modulation of the nonlinear optical properties of the propagating medium. This approach is capable in principle of yielding effective nonlinear coefficients approaching the bulk value while affecting little the propagation characteristics of the propagating modes. A technique for implementing this idea in a thin dielectric waveguide is also described.

Let us consider, for the sake of simplicity, a second harmonic generation in the dielectric waveguide shown in fig. 1. The electric field of the nth TE guided mode, as an example, is given by the following expression:

$$E_n^\omega(x, z, t) = A_n^\omega \mathcal{E}_n^\omega(x) \exp[\mathrm{i}(\omega t - \beta_n^\omega z)] \ . \tag{1}$$

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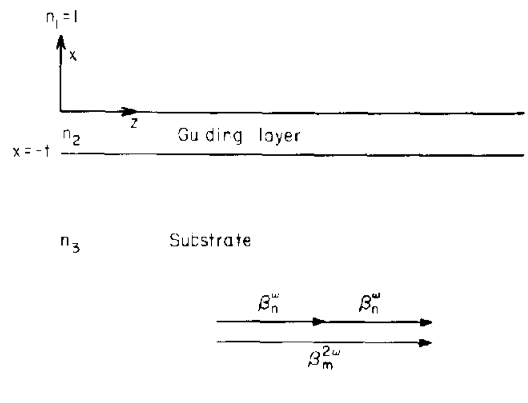

Fig. 1. The basic configuration of a dielectric waveguide and the required phase matching condition for second harmonic generation.

The propagation phase constant β_n^ω as well as the lateral mode profile $\mathcal{E}_n^\omega(x)(\int_{-\infty}^{\infty} |\mathcal{E}_n^\omega(x)|^2\, \mathrm{d}x = 1)$ are determined by the frequency ω, the mode number n, the guide index $n_2(\omega)$, the substrate index $n_3(\omega)$ and the guide thickness t. The mode amplitude A_n^ω represents the power P_n^ω carried by the mode and is given by

$$|A_n^\omega|^2 = 2\omega\mu P_n^\omega / W\beta_n^\omega \ , \tag{2}$$

where W is the width of the waveguide in the y direction. β_n^ω varies between the bulk guide and substrate wave numbers

$$n_3(\omega)\,k_0 < \beta_n^{\omega} < n_2(\omega)\,k_0\,, \tag{3}$$

where $k_0 \equiv 2\pi/\lambda_0$ is the free space wave number. For large t and small mode number β_n^{ω} approaches the upper limit, while the lower limit is approached by reducing the thickness t or choosing a higher number mode. The electric field of the second harmonic mth mode is given similarly:

$$E_m^{2\omega}(x,z,t) = A_m^{2\omega}\,\mathcal{E}_m^{2\omega}(x)\exp\left[\mathrm{i}(2\omega t - \beta_m^{2\omega} z)\right] \tag{4}$$

and the value for $\beta_m^{2\omega}$ is confined between the two limits

$$n_3(2\omega)\,2k_0 < \beta_m^{2\omega} < n_2(2\omega)\,2k_0\,. \tag{5}$$

The second harmonic polarization generated by the fundamental $E_n^{\omega}(x,z,t)$ is taken as

$$\mathcal{P}^{2\omega}(x,z,t) = d_{\mathrm{NL}}(x)\,(A_n^{\omega})^2\,[\mathcal{E}_0(x)]^2 \exp\left[\mathrm{i}(2\omega t - 2\beta_n^{\omega} z)\right]\,, \tag{6}$$

where $d_{\mathrm{NL}}(x)$ is the appropriate bulk nonlinear tensor element. This polarization drives the second harmonic radiation, thus the rate of growth of the average power in the mth second harmonic mode is given by:

$$\mathrm{d}P_m^{2\omega}/\mathrm{d}z = \omega W \,\mathrm{Im} \int_{\infty}^{\infty} E_m^{2\omega}(\mathcal{P}^{2\omega})^*\,\mathrm{d}x\,. \tag{7}$$

Substituting (4) and (6) in (7) yields

$$\mathrm{d}P_m^{2\omega}(z)/\mathrm{d}z = \omega W\,\mathrm{Im}\left\{(A_n^{\omega})^2 A_m^{2\omega}(z)\exp\left[-\mathrm{i}(\beta_m^{2\omega} - 2\beta_n^{\omega})z\right] \times \int_{-\infty}^{\infty} d_{\mathrm{NL}}(x)\,[\mathcal{E}_n^{\omega}(x)]^2\,\mathcal{E}_m^{2\omega}(x)\,\mathrm{d}x\right\}. \tag{8}$$

Eq. (8) readily gives the two requirements needed for effective nonlinear interaction. The first requirement is the well known phase matching condition which arises from the need to eliminate the oscillating exponential term in the differential equation (8). The condition is:

$$\Delta\beta = \beta_m^{2\omega} - 2\beta_n^{\omega} = 0\,. \tag{9}$$

The second requirement for a high rate of second harmonic power growth is a large value for the overlap integral of the fundamental intensity mode profile and the second harmonic field profile in eq. (8)

$$\int_{\infty}^{\infty} d_{\mathrm{NL}}(x)\,|\mathcal{E}_n^{\omega}(x)|^2\,\mathcal{E}_m^{2\omega}(x)\,\mathrm{d}x\,. \tag{10}$$

Under certain conditions, namely, $n_2(\omega) > n_3(2\omega)$ it is possible to compensate for the normal dispersion of the material and to phase match by choosing the right thickness t and the mode numbers n and m. However, this usually requires accurate control of the thickness [2] and for $n \neq m$ involves a large decrease in the magnitude of the overlap integral (10) thus reducing the effective nonlinear coefficient for the interaction.

To overcome the problem of $\Delta\beta \neq 0$ in a thin film structure, let us consider the waveguide shown in fig. 2. The nonlinear coefficient of the guiding layer is modulated periodically with a period Λ, while the index of refraction is assumed to remain unchanged. To analyze the new situation we note that the non-

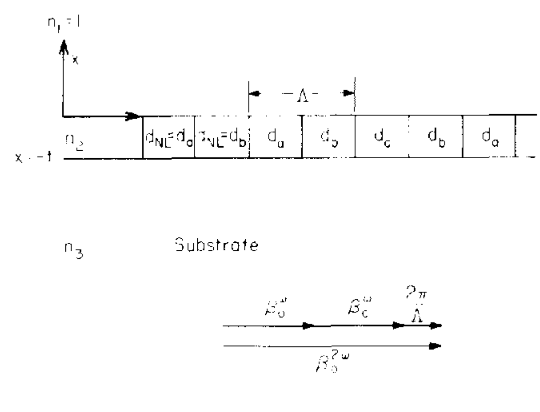

Fig. 2. A waveguide with a periodic modulation of the nonlinear coefficient d_{NL}, and the required phase matching condition for second harmonic generation.

linear coefficient d_{NL} in (8) is now a function of z as well as of x. We limit ourselves to the case where the fundamental and the second harmonic are well confined zero order modes. This makes it possible to neglect the x dependence of d_{NL} so that (8) can be written as

$$\mathrm{d}P_0^{2\omega}(z)/\mathrm{d}z = \omega W \,\mathrm{Im}\left\{ (A_0^{\omega})^2 A_0^{2\omega}(z) \times \left[\int_0^t [\mathcal{E}_0^{\omega}(x)]^2\, \mathcal{E}_0^{2\omega}(x)\,\mathrm{d}x \right] \times \exp(\mathrm{i}\Delta\beta z)\, d_{NL}(z) \right\} . \tag{11}$$

If the spatial modulation period Λ is chosen equal to

$$2\pi/\Lambda = \Delta\beta , \tag{12}$$

the fundamental component in the Fourier expansion of $d_{NL}(z)$ will provide a term with an exponential dependence of $\exp(-\mathrm{i}\Delta\beta z)$. This term, by multiplying the $\exp(\mathrm{i}\Delta\beta z)$ in (11) gives rise to a synchronous contribution that allows the cumulative buildup of the second harmonic power. The amplitude of this particular term in the expansion of $d_{NL}(z)$ determines the effectiveness of the interaction. To be specific, consider an example with

$$\mathrm{d}_a = d_{NL} , \quad d_b = 0 , \tag{13}$$

where d_{NL} is the original nonlinear coefficient of the guiding layer, and where the period Λ is chosen so that (12) is satisfied. The Fourier expansion of this rectangular form nonlinear coefficient is:

$$d_{NL}(z) = \tfrac{1}{2} d_{NL} + \sum_{m=\mathrm{odd}} \frac{2 d_{NL}}{m\pi} \sin\left(\frac{2\pi m}{\Lambda} z\right) . \tag{14}$$

Using (14) and (12) in (11) and keeping the synchronous term only leads to

$$\mathrm{d}P_0^{2\omega}/\mathrm{d}z = \omega W (A_0^{\omega})^2 A_0^{2\omega}(z) \times \left[\int_0^t [\mathcal{E}_0^{\omega}(x)]^2\, \mathcal{E}_0^{2\omega}(x)\,\mathrm{d}x \right] \frac{d_{NL}}{\pi} . \tag{15}$$

The modes overlap integral reaches an optimum value when the modes are well confined and equals approximately $t^{-1/2}$. Using (2) to express A_0^{ω} and $A_0^{2\omega}$ in terms of the respective mode powers (15) becomes

$$\frac{\mathrm{d}P_0^{2\omega}(z)}{\mathrm{d}z} = \frac{4\omega^{5/2}\mu^{3/2} P_0^{\omega}}{\beta_0^{\omega}(\beta_0^{2\omega})^{1/2}(Wt)^{1/2}} \frac{d_{NL}}{\pi} [P_0^{2\omega}(z)]^{1/2} . \tag{16}$$

A simple manipulation of (16) where β_0^{ω} and $\beta_0^{2\omega}$ are assumed to be equal to the bulk propagation constants gives in the nondepleted pump approximation:

$$\frac{P_0^{2\omega}(l)}{P_0^{\omega}} = \frac{2\omega^2 d_{\mathrm{eff}}^2 l^2}{[n_2(\omega)]^2 n_2(2\omega)} \left(\frac{\mu_0}{\epsilon_0}\right)^{3/2} \frac{P^{\omega}}{Wt} . \tag{17}$$

This result is of a form identical to the bulk interaction [5] except that here the effective nonlinear coefficient is

$$d_{\mathrm{eff}} = d_{NL}/\pi . \tag{18}$$

The conversion efficiency from ω to 2ω is seen to be proportional to the mode power density P_0^{ω}/Wt. Since W and t can be made comparable to λ this power density can become very large even for small power input. The penalty for modulating d_{NL} in order to phase match is a reduction of the effective nonlinear coefficient by a factor of $1/\pi$.

A physical picture of the way in which the spatial modulation of d_{NL} overcomes the problem of phase matching is the following. When $\Delta\beta \neq 0$ the generated second harmonic wave and the second harmonic polarization driving it drift gradually (with distance) apart in phase. When $\Delta\beta z = \pi$ the accumulated phase shift is $\pi/2$ and power begins to flow back from the second harmonic to the fundamental. This happens after one coherence length $l_c \equiv \pi/\Delta\beta$. By having d_{NL} equal to zero between $z = l_c$ and $z = 2l_c$ the reversal of power flow is prevented. By $z = 2l_c$ the accumulated phase shift has returned to the favorable region $(-\tfrac{1}{2}\pi < \phi < \tfrac{1}{2}\pi)$ and the nonlinear interaction is "turned on" ($d_{NL} \neq 0$) again‡. The reduced value of d_{eff} as given by (18) reflects the fact that not all the

‡ It is clear from this picture that a reversal of the sign of d_{NL} in the second half of each period will lead to a doubling in the value of d_{eff}.

physical length of the structure partakes in the interaction.

The practical implementation of the modulation of d seems feasible with presently available techniques, since the guiding layer extends only a few microns below the surface and therefore is readily accessible. One approach is to use ion-milling to fabricate a series of grooves normal to the propagation direction in a single crystal thin film guide‡ and then sputter-fill the grooves with a polycrystalline form of the film material (for which $d_{NL} = 0$) or with some other material with a similar index of refraction.

In summary: recent experimental developments in thin film technology make it possible to control the dispersion characteristics of the propagating modes by means of spatial periodic modulation. The possibility of phase matched nonlinear interactions using a periodic modulation of the nonlinear d_{NL} coefficient is examined. It is shown that interaction strengths approaching the bulk phase matched value are possible.

‡ The fabrication of such gratings was reported by Garmire, Somekh, Stoll, Yariv, Garvin, and Wolf and is described in the Report on the Meeting on Integrated Optics by Pole et al. [6]. Further details are given in ref. [7].

References

[1] P.D. Maker, R.W. Terhune, M. Nisenoff and C.M. Savage, Phys. Rev. Letters 8 (1962) 21;
J.A. Giordmaine, Phys. Rev. Letters 8 (1962) 19.
[2] D.B. Anderson and J.T. Boyd, Appl. Phys. Letters 19 (1971) 266;
P.K. Tien, Appl. Opt. 10 (1971) 2395.
[3] A. Ashkin and A. Yariv, Bell Telephone Laboratories Technical Memorandum, November 13, 1961;
N. Bloembergen and A.J. Sievers, Appl. Phys. Letters 17 (1970) 483.
[4] S. Somekh and A. Yariv, Appl. Phys. Letters 21 (1972) 140.
[5] A. Yariv, Introduction to optical electronics (Holt, Rinehart and Winston, New York, 1971) p. 190.
[6] R.V. Pole, S.E. Miller, J.H. Harris and P.K. Tien, Appl. Opt. 11 (1972) 1675.
[7] H.L. Garvin, E. Garmire, S. Somekh, H. Stoll and A. Yariv, Appl. Opt., to be published.