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MODELING AND ANALYSIS OF A TUNABLE BISTABLE MEMS FOR OPTO-MECHANICAL COMPUTATIONAL LOGIC ELEMENTS

M. Taher A. Saif*
University of Illinois
Mechanical and Industrial Engineering
1206 W. Green St.
Urbana, Illinois 61801, USA
Email: saif@uiuc.edu

Norm R. Miller
University of Illinois
Mechanical and Industrial Engineering
1206 W. Green St.
Urbana, Illinois 61801, USA
Email: nr-millr@uiuc.edu

ABSTRACT

We present the design, analysis, and testing of a tunable bi-stable MEMS device that changes state when subjected to a threshold force in the range of pN to μN. It consumes power during change of state, but not while maintaining a state. Depending on the design and tuning, the state change can be initiated by even a moderate power laser beam that generates force by the impacts of its photons. The device presents itself as a potential bistable multivibrator (flip-flop), as well as the basic component of combinational logic elements, such as AND and OR gates, thus opening the door for low power digital opto-mechanical computing in normal to harsh environments. Several designs of the combinational logic elements are provided for optomechanical computing. In addition to bi-stability, the device shows a rich and complex set of dynamical behavior (e.g., chaotic response) when subjected to certain parametric excitations.

INTRODUCTION

Bistable systems have drawn attention from various disciplines. Bistable models have been used to explain a wide range of physical phenomena including earth's temperature change between ice ages (Benzi, 81) and firing patterns of sensory neurons (Longtin, 94). Texas Instruments has developed a bistable micro mirror system for digital light display (Hornbeck, 97). A mechanical bistable system can be developed by applying a compressive force on a long slender micromechanical beam to buckle it. After buckling the beam has two possible stable equilibrium states provided the compressive force is less than the second critical buckling load. Buckling of micro mechanical beams fixed at both ends has been studied in (Lindberg, 93; Chiao, 97; Fang, 94). The load in these studied are residual or thermal. Micro mechanical beams subjected to a known axial compressive force and a transverse force is studied in (Vangbo, 98). Here the ends of the beam are clamped, but free to translate. Clearly, a general study of the mechanical bistable system requires two actuators, one applies an axial tunable force on a micromechanical beam, and the other applies a transverse force. These two actuators offer a wide range of forcing, both static and dynamic, for a detailed parametric study of the bistable system. Here the force of the actuators are known, but the axial force on the beam needs to be evaluated by solving a non-linear coupled actuator-buckling beam problem which is presented in this paper. The general model of the bistable system presented includes the fixed end case as well as the case where the compressive force is known as special cases. The model is verified by experiments on a MEMS bistable system. Excellent correspondance is found between the theory and experiments. The model predicts that the state of a buckled beam can be changed by a force as low as several pNs which can be achieved from the photonic impacts of a moderate power laser beam. This gives rise to a new possibility: optomechanical computing using buckled beams for harsh environment applications. Here, the buckling beams can serve as bistable multivibrators (flip-flops), the laser beams are the source of transverse force used to switch the device by means of the impact of their photons (a) to cause change of state or (b) to heat a MEMS beam (which is already subjected to an axial compressive force slightly

*Address all correspondence to this author.

less than the buckling force) locally to increase the axial compressive force and cause buckling along the direction of the laser beam. The buckling beam and the laser beam may also be used to build the basic combinational logic elements AND, OR, and NOT which form the basis of any digital computing machine.

The paper presents the theoretical model of the bistable system, experimental verification, and the design of the basic digital logic elements using the buckling beams.

THE MODEL OF THE BISTABLE SYSTEM

Figure 1 shows the schematic of the bistable system consisting of an electrostatic actuator A that applies a compressive force on the beam AB. Part of the actuator force is shared by the its own springs determined by its axial displacement. The rest is applied on AB. The axial displacement and hence the force applied on AB needs to be solved from the actuator-beam coupled problem. The following notations are used for the model:

K = net linear spring constant of all the springs of the actuator
F = total force generated by the actuator
P = axial force experienced by AB
$2R$ = transverse force applied on AB at the middle
P_{cr} = critical buckling load of AB for the first mode of buckling
D = transverse deformation of AB at the middle
δ_e = elastic axial compression of AB
δ_b = rigid body translation of the end A due to buckling of AB retaining the length fixed. Thus the total displacement of the actuator is $\delta = \delta_e + \delta_b$. D and δ_b are related by (Saif, 96)

$$\delta_b = \pi^2 D^2/4L \tag{1}$$

E = the modulus of elasticity of AB.

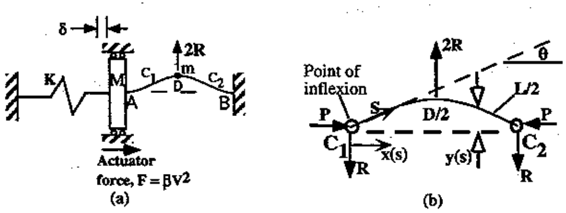

Figure 1. SCHEMATIC OF THE BISTABLE SYSTEM (a) AND THE FREE BODY OF THE BEAM BETWEEN THE POINTS OF INFLEXION (b).

In the following, the beam AB is first analyzed for a known force P. Next, P will be solved from the displacement compatibility between AB and the actuator A.

AB Subjected To Known Forces P And $2R$

Figure 1b shows the free body of the buckled beam AB between its points of inflexion, C_1C_2, along with the forces. The problem can be solved by considering only C_1C_2 due to symmetry. Let s and $y(s)$ denote the ordinate along the axis of C_1C_2 and the transverse deformation of C_1C_2 respectively. The moment curvature relation gives

$$\frac{-Py - Rx}{EI} = \frac{d\theta}{ds} = \frac{y''}{\sqrt{1 - y'^2}} \tag{2}$$

where the prime denotes the derivative with respect to s. Here x is the horizontal projection of s. Let

$$y = \frac{D}{2} \sin \frac{2\pi s}{L} \tag{3}$$

represent the deformation of AB. Multiplying both sides of Eq. 2 by $\sin(2\pi s/L)$ and integrating from 0 to $L/4$ we get

$$\frac{P}{P_{cr}} \frac{D_1}{L} + \frac{R}{P_{cr}} \left[\frac{2}{\pi^2} - \frac{8}{3} (D_1/L)^2 \right] \tag{4}$$

$$= \frac{D_1}{L} \left[1 + \frac{\pi^2}{2} (D_1/L)^2 \right] \tag{5}$$

where $D_1 = D/2$ which gives the transverse deformation of C_1C_2 for a known compressive force P and the transverse force $2R$.

AB coupled with actuator A

The actuator force F is balanced by the restoring force, $K(\delta_e + \delta_b)$ of the actuator spring and AB. Thus the axial force, P, on AB is

$$P = F - K(\delta_e + \delta_b) \tag{6}$$

The elastic axial stiffness of AB is AE/L. Hence $AE\delta_e/L$ gives the axial force on AB. Thus, $P = AE\delta_e/L$, which together with Eq. 6 gives

$$P = (F - K\delta_b) \left[\frac{AE/L}{K + AE/L} \right] \tag{7}$$

in terms of δ_b. Replacing P of Eq. 5 by the right side of Eq. 7 we have

$$\frac{2R}{P_{cr}} = -K_{b1}\xi + K_{b3}\xi^3 \tag{8}$$

where $\xi = D/2L$, $\gamma = K/(AE/L)$, and the non-dimensionalized linear and cubic spring constants K_{b1} and K_{b3} are given by

$$K_{b1} = \left(\frac{F_0}{P_{cr}} + \frac{F}{P_{cr}}\frac{1}{1+\gamma} - 1\right)\pi^2 \quad (9)$$

$$K_{b3} = \left(\frac{\gamma}{1+\gamma}\frac{AE}{P_{cr}} + \frac{1}{2}\right)\pi^4 - \frac{4}{3}\left(\frac{F_0}{P_{cr}} + \frac{F}{P_{cr}}\frac{1}{1+\gamma} - 1\right)\pi^4 \quad (10)$$

The details of the derivation can be found in (Saif, 99). Eq. 8 gives, in closed form, the relation between the equilibrium transverse displacement ξ (nondimensionalized) of AB due to the actuator force F and the transverse force $2R$ which is the mathematical description of the bistable system. Figure 2 shows $2R/P_{cr}$ as a function of ξ for three levels of the actuator force F such that the axial force on AB is less then P_{cr} ($K_{b1} = -.1$), equals P_{cr} ($K_{b1} = 0$) and greater than P_{cr} ($K_{b1} = +.1$). $K_{b3} = 4000$ is used to plot the figure. Note that the system switches from the right buckled state to the left when the threshold force exceeds the value F_{th}.

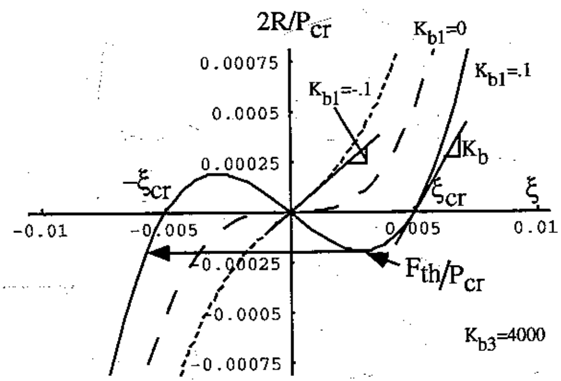

Figure 2. LOAD-DISPLACEMENT FOR THE BISTABLE SYSTEM. IT IS MONOSTABLE WHEN THE AXIAL FORCE ON THE BUCKLING BEAM IS $< P_{cr}$ ($K_{1b} = -.1$), AND BISTABLE WHEN THE AXIAL FORCE IS $> P_{cr}$ ($K_{1b} = .1$).

The threshold force for state change can be obtained from Eq. 8 by evaluating the maxima of $2R/P_{cr}$:

$$\frac{F_{th}}{P_{cr}} = \frac{2}{3}\sqrt{\frac{K_{b1}}{3K_{b3}}}K_{b1} \quad (11)$$

Let us look at a numerical example for the threshold force. Let the buckling beam be made of Single Crystal Silicon (SCS) and is $1000\mu m$ long, $1\mu m$ wide and $10\mu m$ deep. It is aligned with the [110] direction. Thus $E = 170GPa$, and $P_{cr} = 5.593\mu N$. Let the actuator force $F = 5.6\mu N$ and the spring constant $K = .17\mu N/\mu m$ so that $\gamma = .0001$. Then $K_{b1} = .0118$, $K_{b3} = 3009$, and the buckled displacement $D = 4\mu m$. The required threshold force, F_{th}, to switch the beam from $D = 4\mu m$ to $D = -4\mu m$ is $F = 50pN$. This force can be obtained from the radiation pressure of a moderate power laser beam of $15mW$ (Radiation force of a laser beam $= W/c$ in absence of any reflection or transmission, where c is the speed of light and W is the laser intensity).

EXPERIMENTAL VALIDATION

Figure 3 shows the fabricated bistable system. It consists of an actuator A, and two buckling beams clamped at both ends. The beams are attached at the middle for enhanced stability against rotation. The length of the beams, excluding the frame at the middle is $1000\mu m$. The beams are made of SCS, $1\mu m$ wide and $20\mu m$ deep. A 3-comb actuator at the middle applies the transverse force $2R$. The combs are $3\mu m$ wide and $20\mu m$ deep. The gap between the combs is $3\mu m$. The actuator A has 800 comb drives. Each comb is $2\mu m$ wide, with a gap of $2\mu m$ between them. The device is made by the SCREAM process (Shaw, 94).

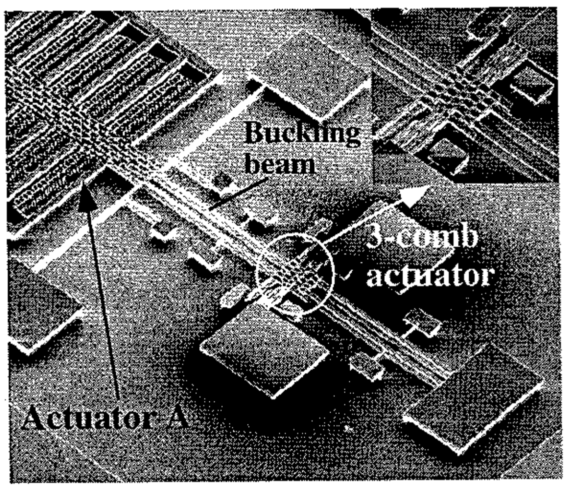

Figure 3. BISTABLE DEVICE: ACTUATOR A, 3-COMB ACTUATOR AND TWO BUCKLING BEAMS FOR INCREASED STABILITY AGAINST ROTATION.

We find that the beams buckle when the voltage applied on the actuator A is around $13.5V$. In order to carry out a switching experiment, the beams are first buckled by a distance D by the actuator A. D is measured using a fabricated vernier scale. The 3-comb actuator is then employed to apply the threshold transverse force to change the state. The corresponding threshold voltage is recorded. Figure 4 shows the buckled displacements cubed, D^3, as a function of the threshold voltage squared, V_{th}^2. The dotted line is a best fit straight line through the data. The linearity is expected because of the following.

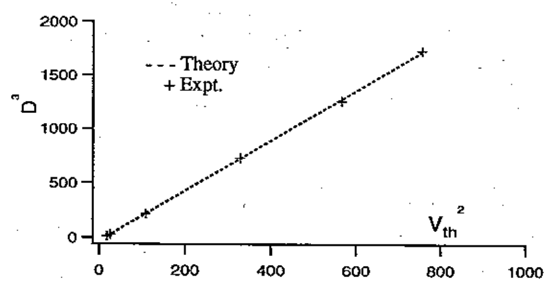

Figure 4. EXPERIMENTAL RESULT ON SWITCHING OF STATES. D IS THE BUCKLED DISTANCE AND V_{th} IS THE THRESHOLD VOLTAGE ON THE 3-COMB ACTUATOR THAT CAUSES THE CHANGE OF STATE. THE LINEAR DEPENDENCE OF D^3 ON V_{th}^2 IS PREDICTED BY THE THEORETICAL MODEL. THE 3-COMB ACTUATOR FORCE IS $F_{th} = \beta V_{th}^2$, WHERE $\beta = .00018 \mu N/V^2$.

From Eq. 8 the buckled displacement $\xi = D/2L$ can be obtained when $R = 0$:

$$\xi|_{R=0} = \sqrt{K_{b1}/K_{b3}} \tag{12}$$

Then from Eq. 11

$$\frac{F_{th}}{P_{cr}} = \frac{2}{3^{3/2}} \left(\frac{K_{b1}}{K_{b3}} \right)^{3/2} K_{b3} = \frac{2}{3^{3/2}} K_{b3} \xi|_{R=0}^3 \tag{13}$$

K_{b3} for the experimental device varies negligibly with F and hence can be considered as a constant (a function of geometry of the device). Thus F_{th} varies as the cube of the the buckled displacement. But the actuator force varies as the square of the applied voltage. Hence V_{th}^2 is linearly dependent on D^3 as observer by the experimental result of Figure 4.

Remark 1

The excellent correspondence between theory and experiment not only verifies the model presented, but also has an important implication. MEMS devices offer a unique test bed for carrying out fundamental experimental studies, both static and dynamic, of bistable systems which is otherwise difficult by macro systems.

Remark 2

If the buckling beam is fixed at both ends then $K \to \infty$ and hence $\gamma \to \infty$ in the model, but K_{b1} and K_{b3} remain bounded and well defined. Thus the fixed boundary condition is a special case of the model presented. The buckling force for the fixed boundary condition case can be contributed by residual stress or thermal stress. For example a coating of thermal oxide (of appropriate thickness) on silicon will give rise to compressive force to buckle the beam. The bistability of the system will be characterized by Eq. 8.

In the following, possible designs of basic computational elements using the bistable system are provided.

BASIC MEMORY ELEMENTS FOR OPTOMECHANICAL COMPUTING

Modern digital computing machines use synchronous clocked logic. These machines depend on memory elements (both long term and short term) as well as combinational elements to implement Boolian logic. We will focus first on the design of a basic opto-mechanical memory element, a simple, asynchronous, reset-set flipflop (RSFF) or bistable multivibrator.

The RSFF must be capable of being set and reset by separate external inputs. In this case the inputs are two separate laser beams. It must maintain its state, either set or reset, until another signal is used to change its state. Finally, it must be possible to determine the state of the device and derive an output signal. In Figure 5 we show a buckled beam device in its two possible states. We have arbitrarily chosen buckling to the right as the set state and buckling to the left as the reset state. The figure also demonstrates how two independent laser beams can be used to switch the device from its set to its reset state and the reverse.

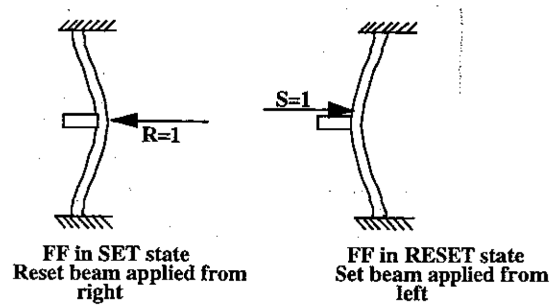

Figure 5. A BUCKLED BEAM USED AS A RESET-SET FLIP FLOP. BUCKLING TO THE RIGHT HAS BEEN ARBITRARILY CHOSEN AS THE SET STATE. THE ARROWS REPRESENT THE TWO SEPARATE LASER BEAMS USED TO SWITCH THE DEVICE BETWEEN ITS TWO STABLE STATES.

Figure 6 demonstrates how a continuous interrogation laser beam can be used to determine the state of the flip-flop. When the flip-flop is reset, the interrogation beam is blocked, but when the device is set, the beam passes and can be routed to other combinational or sequential logic elements

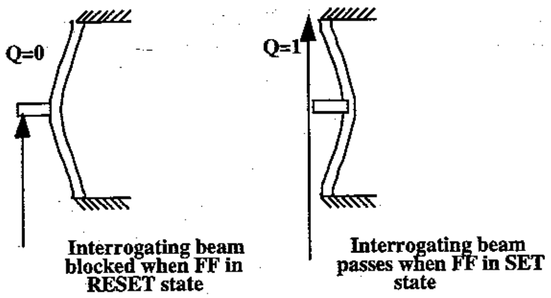

Figure 6. SENSING THE STATE OF THE RSFF USING AN INTERROGATION LASER BEAM. THE INTERROGATION BEAM WILL PASS TO ANOTHER DEVICE OR COULD BE ROUTED TO AN OPTO-ELECTRIC RECEIVER FOR OUTPUT.

Designs for Combinational Logic Devices

The basic compressively loaded beam can also be used to implement the fundamental operations of combinational logic (AND, OR, and NOT). The basic difference between combinational logic and sequential logic elements is that combinational logic elements react only to current inputs and do not have any memory function. As a result, micro mechanical beams need to be fabricated with less residual compressive stress so that they will move easily under the influence of external forces, but will return to their initial configuration when the external force is removed.

Figure 7 illustrates the NOT operation. When the laser beam A is on and strikes the mechanical beam, it is moved to the left and blocks the interrogating beam C. Thus C=NOT(A). (When A is on, C is off.) The beam element needs to be fabricated so that light beam A has enough thrust to move it far enough to block light beam C.

Figure 8 illustrates the AND operation. The interrogating beam is blocked whenever either light beam A or B or both A and B are off. Only when both beams A and B are on does beam C pass. This is the logic AND operation. (C=A AND B.) Notice that the device can be easily extended to accommodate more than two inputs simply by changing the design of the MEMS beam. Since we are using light to actuate and interrogate our device, beam C is able to cross beams A and B without interaction. This is a distinct advantage of optical operation.

The OR operation can be implemented in much the same way. The mechanical beam would be fabricated so that it would bend sufficiently to block the interrogating laser beam if either light beam A or B was on. Then if beam A or beam B or both beams A and B are on, then beam C is unblocked and thus on.

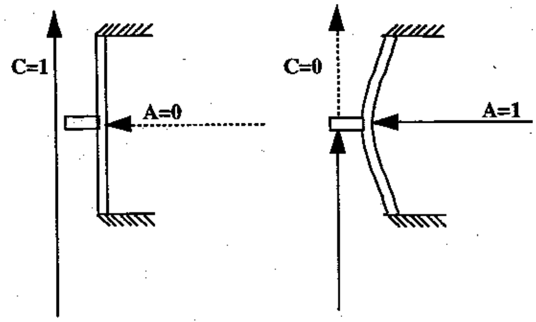

Figure 7. THE NOT OPERATION. LIGHT BEAM C IS BLOCKED (OFF) WHEN LIGHT BEAM A IS ON.

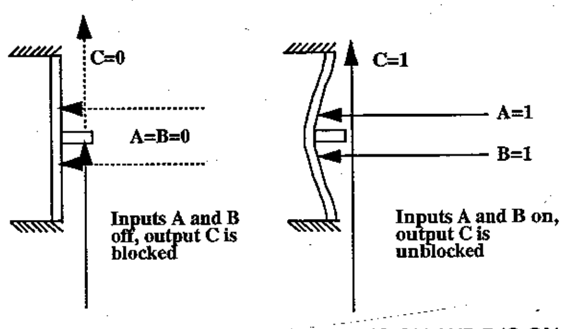

Figure 8. THE AND OPERATION. WHEN A IS ON AND B IS ON THEN C IS ON.

This is the logic OR operation. (C=A OR B.) Once again, more than two inputs are easily accommodated.

Other derived logic operations can be implemented as well. For example, in Figure 9, the NAND operation is shown. It is the logic NOT of the AND operation.

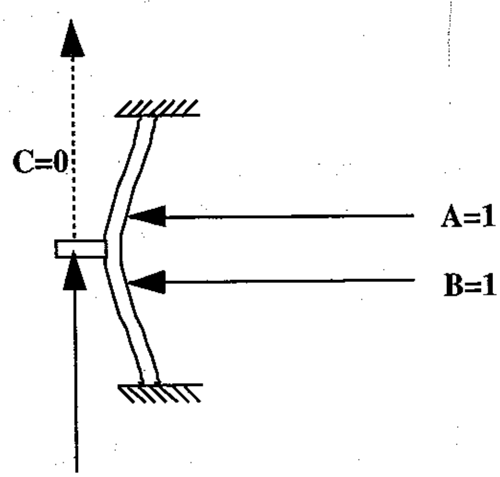

Figure 9. THE NAND OPERATION. IF A IS ON AND B IS ON THEN C IS OFF. IT IS THE NOT OF THE AND OPERATION.

In Figure 12, the exclusive or (XOR) operation is shown. The exclusive or operation says if A is true or B is true but not both, then C is true. This figure illustrates how multiple logic elements can be configured to work as a system. Notice that this logic family is extensible. At each stage, optical energy from an external source (the optical "power bus") is applied to the system. In theory, there is no limit to the complexity of the switching logic circuit that can be implemented using these devices.

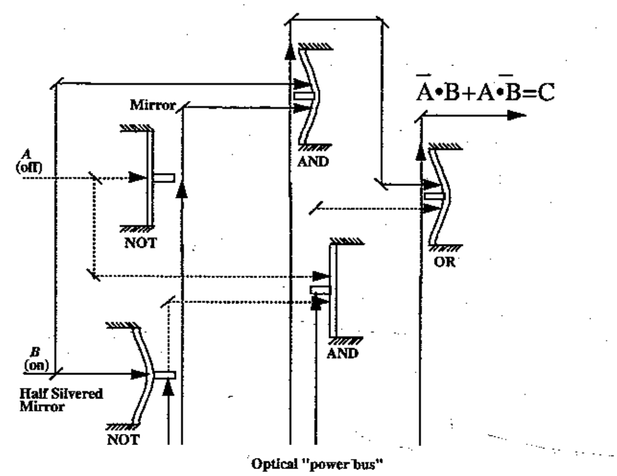

Figure 10. THE XOR OPERATION. IF A IS ON OR B IS ON, BUT NOT BOTH, THEN C IS ON. AS SHOWN, B IS ON AND A IS OFF RESULTING IN C BEING UNBLOCKED OR ON. DOTTED LINES REPRESENT THE OPTICAL PATH OF LIGHT BEAMS WHICH ARE NOT ON.

CONCLUSIONS

We have presented a model of a micro mechanical bistable system based on buckling of a long slender beam by an actuator. The beam is subjected to a transverse force at the middle. We show that depending on the level of tuning, the force required to switch the system from one buckled state to the other can be as low as *pNs*, but the corresponding displacement may be on the order of several μms. Thus the state change can be initiated by the radiation pressure of a moderate laser beam, giving rise to the possibility of employing such bistable systems for developing optomechanical computing elements for operation in harsh environments. The model is verified by experimental demonstration. The paper is concluded by several designs of basic optomechanical computational elements.

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