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Localization Effects in Two-Dimensional Superconductors

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Effects of localization on two-dimensional superconductors have been investigated by taking into account quantum corrections to the theory of dirty superconductors in the order of $(\varepsilon_F\tau_0)^{-1}$ where ε_F and τ_0 are the Fermi energy and the relaxation time of an electron, respectively. The mean field superconducting transition temperature is seen to be reduced by localization. This is because the Coulomb repulsive interaction is enhanced and the density of states of electrons is depressed in dirty systems. Recent experiments by Kobayashi *et al.* on granular films of tin are discussed in this context.

§1. Introduction

It has been proposed[1,2] that two-dimensional metals are not truly metallic and that electrons are localized at absolute zero, $T=0$, in non-interacting systems no matter how weak the disorder is. The precursor effect of this complete localization at absolute zero is seen as a logarithmic dependence of resistance on temperature.[3-6] This logarithmic region can be called as weakly localized regime. Since perturbational treatments from the metallic limit are applicable in this regime,[7] microscopic calculations of the quantum corrections of transport coefficients have been performed with respect to $(\varepsilon_F\tau_0)^{-1}$, where ε_F and τ_0 are the Fermi energy and the relaxation time of an electron, respectively. It has recently been shown[8-10] that quantum corrections with respect to $(\varepsilon_F\tau_0)^{-1}$ are also important in interacting electron systems; the interplay between the interaction and disorder introduces another logarithmic term in resistance.

In this paper we investigate such effects of localization on the superconducting transition temperature T_c. We will examine the quantum corrections to the theory of dirty superconductors by Anderson[11] and Gorkov.[12] One of the characteristic features of their theory[11,12] is that T_c is not affected by static and non-magnetic disorder. In their theory the interplay between the interaction and disorder has not been taken into account. We will see, however, that this interplay, which is due to quantum corrections, introduces important effects in two-dimension: The Coulomb repulsive interaction is enhanced and the density of states of electrons are depressed in disordered systems.

In §2, we evaluate a Cooper pair propagator by using the one-electron eigenfunctions which are diagonal in a disordered system. In §3, the effects of the electron-electron interaction on a Cooper pair in disordered systems are examined, and T_c in the mean-field theory is obtained. It is shown that T_c is reduced as the sheet resistance is increased even if the BCS coupling constant is fixed. In §4, recent experiments by Kobayashi *et al.*[13,14] on granular films of tin are discussed in the present context. In our preliminary report,[15] we have already proposed that localization depresses T_c, although the analysis there was not complete. This paper reports our complete results. We take units $\hbar=k_B=1$.

§2. Formalism

For calculating the superconducting transition temperature T_c in the mean-field theory, we start with the BCS Hamiltonian

$$H=H_0+g_{BCS}\int dr[\Delta^+\psi_\uparrow(r)\psi_\downarrow(r)+\Delta\psi_\downarrow^+(r)\psi_\uparrow^+(r)], \quad (1)$$

where $\psi_\sigma(r)$ is the electron field operator with spin σ and g_{BCS} is the BCS attractive interaction parameter ($g_{BCS}<0$). Here, Δ is the superconducting order parameter defined by

$$\Delta=\int \mathrm{d}\boldsymbol{r}\ \mathrm{Tr}\ \mathrm{e}^{-H/T}\psi_{\uparrow}(\boldsymbol{r})\psi_{\downarrow}(\boldsymbol{r})/\mathrm{Tr}\ \mathrm{e}^{-H/T}. \quad (2)$$

The Hamiltonian H_0 may be written as

$$H_0=H_{KE}+H_{\mathrm{imp}}+H_{\mathrm{int}}, \quad (3)$$

where H_{KE} means the Hamiltonian of Bloch electrons and H_{imp} the impurity potentials. The Hamiltonian H_{int} includes all kinds of electron-electron interaction except the BCS term given in eq. (1). As we will see in the next section, the details of the electron-electron interaction such as the momentum transferred by the interaction become important in disordered systems.

Assuming the second order phase transition and using eq. (2), we obtain the following equation for T_c in the mean-field theory,

$$\begin{aligned}|g_{\mathrm{BCS}}|^{-1}=&\int \mathrm{d}\boldsymbol{r}\int \mathrm{d}\boldsymbol{r}'\int_0^{T^{-1}}\mathrm{d}u\\ &\times\langle Tu\psi_{\downarrow}^{+}(\boldsymbol{r},u)\psi_{\uparrow}^{+}(\boldsymbol{r},u)\\ &\times\psi_{\uparrow}(\boldsymbol{r}',0)\psi_{\downarrow}(\boldsymbol{r}',0)\rangle_{H_0},\end{aligned} \quad (4)$$

where u is the imaginary time, Tu is the time ordering operator, and the bracket means the thermal average in the space of H_0.

Following the theory of dirty superconductors by Anderson,[11] we introduce the eigenfunctions in the disordered system which are diagonal in the space of $(H_{KE}+H_{\mathrm{imp}})$ as[16]

$$\psi_\sigma(\boldsymbol{r})=\sum_\alpha \phi_\alpha(\boldsymbol{r})c_{\alpha\sigma}, \quad (5)$$

where $\phi_\alpha(\boldsymbol{r})$ is the eigenfunction of state α and $c_{\alpha\sigma}$ is the annihilation operator of the state with spin σ. Introducing eq. (5) into eq. (4), we have

$$\begin{aligned}|g_{\mathrm{BCS}}|^{-1}=&\sum_{\alpha,\beta}\int_0^{T^{-1}}\mathrm{d}u\\ &\times\langle Tu\ c_{\alpha\downarrow}^{+}(u)c_{\alpha^*\uparrow}^{+}(u)c_{\beta^*\uparrow}(0)c_{\beta\downarrow}(0)\rangle_{H_0},\end{aligned} \quad (6)$$

where α^* denotes the time-reversed state of α and thus the states α and α^* are degenerate.

Let us first neglect H_{int} in eq. (3). In this case, eq. (6) is evaluated as follows,

$$|g_{\mathrm{BCS}}|^{-1}=T\sum_n\sum_\alpha G_\alpha(\varepsilon_n)G_\alpha(-\varepsilon_n). \quad (7)$$

Here $G_\alpha(\varepsilon_n)$ is the thermodynamic Green's function of state α:

$$G_\alpha(\varepsilon_n)=(\mathrm{i}\varepsilon_n-\xi_\alpha)^{-1}, \quad (8)$$

where $\varepsilon_n=2\pi T(n+\frac{1}{2})$ with n being integer, and ξ_α is the energy of state α relative to the Fermi energy ε_{F}. Replacing the summation over α by the integral over ξ_α in eq. (7), we find

$$|g_{\mathrm{BCS}}|^{-1}=T\sum_n \pi N(0)/|\varepsilon_n|, \quad (9)$$

where $N(0)$ is the density of states of electrons per spin. If $N(0)$ is assumed to be independent of disorder, T_c is unchanged. This result is called as the Anderson's theorem of dirty superconductors.[11] However, it has recently been shown[8-10] that the electron-electron interaction strongly modifies the one-electron states in disordered systems. In the next section, we consider the effects of the electron-electron interaction on a Cooper pair in disordered systems.

§3. Effects of Electron-Electron Interaction

We assume that the interaction is instantaneous and our Hamiltonian is written as

$$\begin{aligned}H_{\mathrm{int}}=&\sum_{\sigma,\sigma'}\int \mathrm{d}\boldsymbol{r}\int \mathrm{d}\boldsymbol{r}'\sum_{\alpha,\beta,\gamma,\delta}\phi_\alpha^*(\boldsymbol{r}')\phi_\beta^*(\boldsymbol{r})v(\boldsymbol{r}-\boldsymbol{r}')\\ &\times\phi_\gamma(\boldsymbol{r})\phi_\delta(\boldsymbol{r}')c_{\alpha\sigma}^{+}c_{\beta\sigma'}^{+}c_{\gamma\sigma'}c_{\delta\sigma},\end{aligned} \quad (10)$$

where α, β, γ, and δ mean the disordered eigenstates, and $v(\boldsymbol{r}-\boldsymbol{r}')$ the interaction function. The first order effects of the interaction on a Cooper pair are given in Fig. 1, where the solid lines are the electron Green's functions and the wavy lines the interaction. In the figure, the processes (a) and (b) denote the Fock corrections, (c) and (d) the Hartree corrections, and (e) the first order vertex correction.

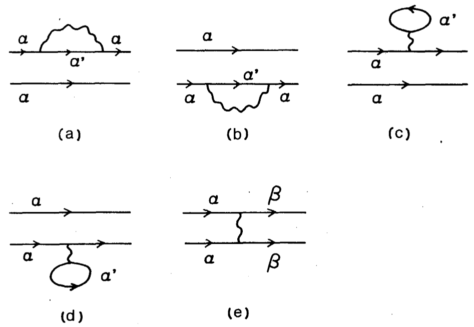

Fig. 1. Corrections of a Cooper pair in the first order of the electron-electron interaction in disordered systems. The processes (a) and (b) are the Fock corrections, (c) and (d) are the Hartree corrections, and (e) is the vertex correction. The wavy lines denote the interaction.

Let us first study the Hartree-Fock (H-F) corrections, Fig. 1(a)~(d). The contribution to the right hand side (*r.h.s.*) in eq. (6) is written as $R_{\rm HF}$ and is given by

$$R_{\rm HF}=-T^2\sum_{n,n'}\sum_{\alpha,\alpha'}\int \mathrm{d}\boldsymbol{r}\int \mathrm{d}\boldsymbol{r}'\, V(\boldsymbol{r}-\boldsymbol{r}')G_\alpha(\varepsilon_n)G_\alpha(-\varepsilon_n)[G_\alpha(\varepsilon_n)+G_\alpha(-\varepsilon_n)]G_{\alpha'}(\varepsilon_{n'}), \tag{11}$$

$$V(\boldsymbol{r}-\boldsymbol{r}')=\phi_\alpha^*(\boldsymbol{r}')\phi_{\alpha'}^*(\boldsymbol{r})v(\boldsymbol{r}-\boldsymbol{r}')\phi_{\alpha'}(\boldsymbol{r}')\phi_\alpha(\boldsymbol{r})-2\phi_\alpha^*(\boldsymbol{r}')\phi_{\alpha'}^*(\boldsymbol{r})v(\boldsymbol{r}-\boldsymbol{r}')\phi_\alpha(\boldsymbol{r}')\phi_{\alpha'}(\boldsymbol{r}), \tag{12}$$

where the factor 2 in the Hartree term is due to the spin. Using eq. (8), we rewrite eq. (11) as

$$R_{\rm HF}=T\sum_n \frac{1}{2\mathrm{i}\varepsilon_n}\sum_\alpha [\delta G_\alpha(\varepsilon_n)-\delta G_\alpha(-\varepsilon_n)], \tag{13}$$

$$\sum_\alpha \delta G_\alpha(\varepsilon_n)=\sum_\alpha T\sum_{n'}\sum_{\alpha'}\int \mathrm{d}\boldsymbol{r}\int \mathrm{d}\boldsymbol{r}'\, V(\boldsymbol{r}-\boldsymbol{r}')G_\alpha^2(\varepsilon_n)G_{\alpha'}(\varepsilon_{n'}). \tag{14}$$

Therefore, the H-F corrections to the Cooper pair is expressed by the correction to the one-electron states. As is easily seen, $\sum_\alpha \delta G_\alpha(\varepsilon_n)$ given by eq. (14) is a first order correction of the trace of the Green's function due to interactions, i.e.

$$I(\varepsilon_n)\equiv\sum_\alpha\left\langle \alpha\left|\frac{1}{\mathrm{i}\varepsilon_n-H}\right|\alpha\right\rangle =\mathrm{Tr}\,\frac{1}{\mathrm{i}\varepsilon_n-H}, \tag{15}$$

where $|\alpha\rangle$ is the eigenstate of the disordered system and H is the total Hamiltonian. Since $I(\varepsilon_n)$ represents the density of states and is independent of the base function, we can evaluate this in terms of the ordinary momentum representations. Such evaluations of the modification of the density of states have been performed by Altshuler *et al.*[8,10)]

Let us introduce electron-hole (e-h) and electron-electron (e-e) vertex functions in Fig. 2 as the broken and the double broken lines, respectively.[9)] Here, the dotted lines and crosses represent impurity potentials and averaging procedure over the distribution of impurities; for simplicity, disorder is expressed as impurities. In the case of $\varepsilon_n(\varepsilon_n-\omega_l)<0$, $|\omega_l|\tau_0\ll 1$, and $Dq^2\tau_0\ll 1$, both of the vertex functions are of diffusion type,

$$\Gamma(q,\omega_l)=\tau_0^{-1}(Dq^2+|\omega_l|)^{-1}, \tag{16}$$

where $D=\varepsilon_F\tau_0/m$ is the diffusion constant with m being the effective mass of an electron. In the presence of finite inelastic scattering rate, τ_ε^{-1}, eq. (16) corresponding to the e-e vertex function has correction. However, in the temperature region where $T\tau_\varepsilon\gg 1$ is satisfied, the correction can be ignored, and we assume in the following that this condition is satisfied.

The H-F corrections in the order of $(\varepsilon_F\tau_0)^{-1}$ of an electron with energy ε_n and momentum k are given in Fig. 3.[17)] In the figure, the processes (a) and (b) are the Fock corrections, and (c) and (d) are the Hartree corrections. The characteristic values of momentum Q and energy ω_l transferred by the process (a) are

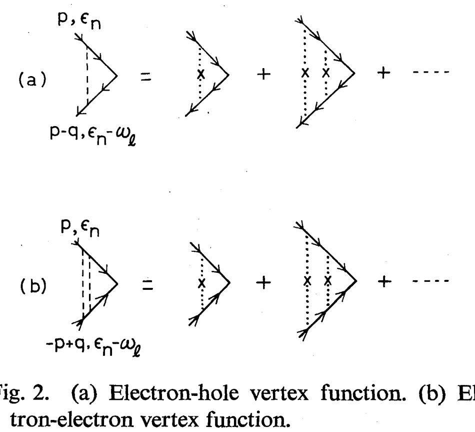

Fig. 2. (a) Electron-hole vertex function. (b) Electron-electron vertex function.

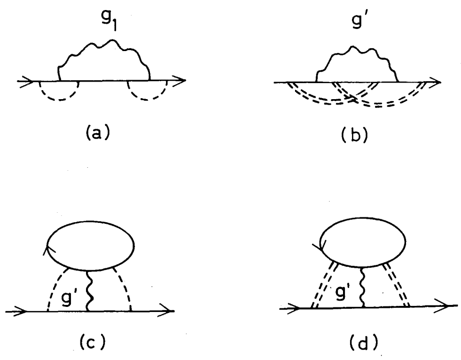

Fig. 3. Self-energy corrections in the order of $(\varepsilon_F\tau_0)^{-1}$: The processes (a) and (b) are the Fock corrections, and (c) and (d) are the Hartree corrections. The wavy lines denote the interaction g_1 and g'.

small, i.e. $|Q| \ll 2k_F$ and $|\omega_l| \ll \tau_0^{-1}$, where k_F is the Fermi momentum, and this interaction is denoted by g_1. On the other hand, the value of Q transferred by the processes (b), (c), and (d) can be large, i.e. $|Q| \lesssim 2k_F$ although $|\omega_l| \ll \tau_0^{-1}$. The strength of these interactions is assumed to be equal, for simplicity, and are denoted by g'.[18] Introducing these corrections into eq. (15), we find[19]

$$R_{HF} = T \sum_n \frac{\pi}{|\varepsilon_n|} \delta N(\varepsilon_n), \tag{17}$$

$$\frac{\delta N(\varepsilon_n)}{N(0)} = \frac{-(g_1 - 3g')N(0)}{2\pi\varepsilon_F\tau_0} \times \left[\ln\frac{1}{2\pi T\tau_0} - \psi\left(\frac{1}{2} + \frac{|\varepsilon_n|}{2\pi T}\right)\right], \tag{18}$$

where $\psi(z)$ is the di-gamma function. We note that the interaction g_1 will be strongly repulsive ($g_1 > 0$) due to the Coulomb interaction, whereas g' will be attractive ($g' < 0$) because the phonon-mediated interaction is dominant at large momentum transfer in superconducting materials; i.e. g' will be close to the BCS coupling constant $g_{BCS}(g_{BCS} < 0)$. Therefore, we expect quite generally that the density of states is depressed. As regards the energy dependence of the density of states, we may introduce the following approximation for $|\varepsilon_n| \gg 2\pi T$,

$$\frac{\delta N(\varepsilon_n)}{N(0)} = -\frac{(g_1 - 3g')N(0)}{2\pi\varepsilon_F\tau_0} \ln\frac{1}{|\varepsilon_n|\tau_0}. \tag{19}$$

Using eq. (19), we obtain

$$R_{HF} = \frac{-(g_1 - 3g')N(0)}{4\pi\varepsilon_F\tau_0}\left(\ln\frac{1}{T\tau_0}\right)^2. \tag{20}$$

Here, we have neglected the constant terms.

Next, we consider the first order vertex correction given in Fig. 1(e). The contribution to the *r.h.s.* in eq. (6) is written as R_v and is expressed as

$$R_v = -T^2 \sum_{n,n'} \sum_{\alpha,\beta} \int d\boldsymbol{r} \int d\boldsymbol{r}' \times \phi_\alpha^*(\boldsymbol{r})\phi_\alpha^*(\boldsymbol{r}')v(\boldsymbol{r}-\boldsymbol{r}')\phi_\beta(\boldsymbol{r})\phi_\beta(\boldsymbol{r}') \times G_\alpha(\varepsilon_n)G_\alpha(-\varepsilon_n)G_\beta(\varepsilon_{n'})G_\beta(-\varepsilon_{n'}). \tag{21}$$

By introducing the real-space Green's function defined by

$$\sum_\alpha G_\alpha(\varepsilon_n)\phi_\alpha^*(\boldsymbol{r})\phi_\alpha(\boldsymbol{r}') = G(\varepsilon_n : \boldsymbol{r}, \boldsymbol{r}'), \tag{22}$$

eq. (21) is rewritten as

$$R_v = T^2 \sum_{n,n'} \sum_{\alpha,\beta} \frac{1}{4\varepsilon_n\varepsilon_{n'}} \int d\boldsymbol{r} \int d\boldsymbol{r}'\, v(\boldsymbol{r}-\boldsymbol{r}') \times [G(-\varepsilon_n : \boldsymbol{r}, \boldsymbol{r}') - G(\varepsilon_n : \boldsymbol{r}, \boldsymbol{r}')] \times [G(-\varepsilon_{n'} : \boldsymbol{r}, \boldsymbol{r}') - G(\varepsilon_{n'} : \boldsymbol{r}, \boldsymbol{r}')], \tag{23}$$

where the relation $\phi_\alpha^*(\boldsymbol{r}) = \phi_\alpha(\boldsymbol{r})$ was used. Introducing the Fourier amplitudes of the interaction $v(\boldsymbol{r}-\boldsymbol{r}')$ and the Green's function $G(\varepsilon_n : \boldsymbol{r}, \boldsymbol{r}')$, we rewrite eq. (23) as

$$R_v = T^2 \sum_{n,n'} \sum_{k,k'} \sum_Q \frac{v(Q)}{4\varepsilon_n\varepsilon_{n'}} \times [G(-\varepsilon_n : k, k') - G(\varepsilon_n : k, k')] \times [G(-\varepsilon_{n'} : -k-Q, -k'-Q) - G(\varepsilon_{n'} : -k-Q, -k'-Q)]. \tag{24}$$

We examine the singular contributions to R_v in the order of $(\varepsilon_F\tau_0)^{-1}$. These are given in Fig. 4 and are expressed as

$$R_v = -T^2 \sum_{n,n'} \sum_k \sum_q \frac{2}{\varepsilon_n\varepsilon_{n'}} \times [g_1\Gamma(q, \varepsilon_n + \varepsilon_{n'})G_k(\varepsilon_n)G_{-k-q}(-\varepsilon_{n'}) + g'\Gamma(q, \varepsilon_n + \varepsilon_{n'})G_k(\varepsilon_n)G_{k+q}(-\varepsilon_{n'})], \tag{25}$$

with $\varepsilon_n > 0$ and $\varepsilon_{n'} > 0$. Here, the interaction $v(Q)$ was replaced by g_1 and g' depending on the value of Q. We note that since the time-reversal symmetry relation has been used in eq. (23), there is no difference between e-e and e-h diffusion processes in Fig. 4. Taking the summations over k, k', and q, we obtain

$$R_v = -T^2 \sum_{n,n'} \frac{2\pi N(0)}{\varepsilon_n\varepsilon_{n'}} \cdot \frac{(g_1 + g')N(0)}{\varepsilon_F\tau_0} \times \ln\frac{1}{\tau_0(\varepsilon_n + \varepsilon_{n'})}, \tag{26}$$

where $\varepsilon_n > 0$, $\varepsilon_{n'} > 0$, and $\tau_0^{-1} \gg (\varepsilon_n + \varepsilon_{n'})$. We take the summations over ε_n and $\varepsilon_{n'}$, and retain only the most dominant contribution. Then, we have

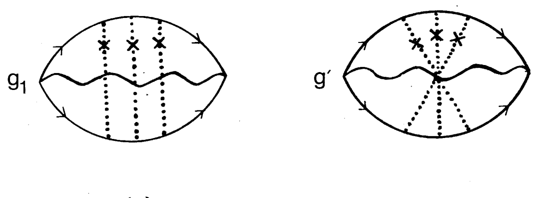

Fig. 4. Diagrams which contribute to the vertex correction of a Cooper pair.

$$R_v = -\frac{(g_1+g')N(0)}{6\pi\varepsilon_F\tau_0}\left(\ln\frac{1}{T\tau_0}\right)^3. \tag{27}$$

Although both g_1 and g' contribute, the Coulomb interaction g_1 will overcome the attractive interaction g'. Using eqs. (9), (20), and (27), we have the equation for T_c

$$\ln\frac{T_c}{T_{c0}} = -\frac{(g_1-3g')N(0)}{4\pi\varepsilon_F\tau_0}\left(\ln\frac{1}{T_c\tau_0}\right)^2 - \frac{(g_1+g')N(0)}{6\pi\varepsilon_F\tau_0}\left(\ln\frac{1}{T_c\tau_0}\right)^3, \tag{28}$$

where T_{c0} is the transition temperature in the pure system. The first term in the *r.h.s.* in eq. (28) is due to the correction of the density of states and the second term is due to the vertex correction. In terms of the superconducting coherence length of the pure system and the mean free path of an electron, $\xi_0 = v_F/1.75\pi T_{c0}$ and $l = v_F\tau_0$, respectively, v_F being the Fermi velocity, we may express eq. (28) as

$$\ln\frac{T_c}{T_{c0}} = -\frac{(g_1-3g')N(0)}{4\pi\varepsilon_F\tau_0}\left[\ln\left(5.40\frac{\xi_0}{l}\frac{T_{c0}}{T_c}\right)\right]^2 - \frac{(g_1+g')N(0)}{6\pi\varepsilon_F\tau_0}\left[\ln\left(5.40\frac{\xi_0}{l}\frac{T_{c0}}{T_c}\right)\right]^3. \tag{29}$$

Thus, the corrections due to disorder are appreciable in real thin films where $\xi_0 \gg l$.

In the usual superconducting materials, the value of $|g'|N(0)$, which is close to $|g_{BCS}|N(0)$, is much smaller than one. On the other hand, the value of $g_1N(0)$ can be of the order of unity. Therefore, ignoring g', we show in Fig. 5 T_c, given by eq. (28), as a function of $\lambda_s = g_1N(0)/2\pi\varepsilon_F\tau_0$ for several values of $\tau_0 T_{c0}$. We see in the figure that T_c is reduced as λ_s is increased. By noting that the sheet resistance is

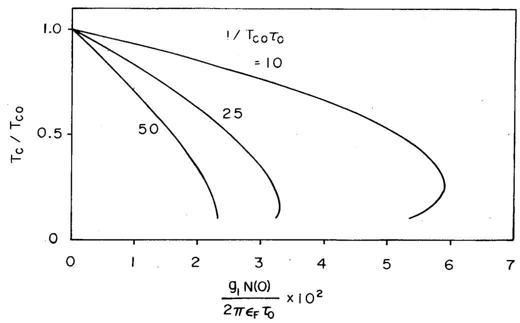

Fig. 5. Numerical results of T_c as a function of the parameter $g_1N(0)/2\pi\varepsilon_F\tau_0$ for several choices of $\tau_0 T_{c0}$ in the case of $g'=0$.

given by $R_\square = \pi/e^2\varepsilon_F\tau_0$, we conclude that T_c is reduced roughly in proportion to $R_\square$. We also see that T_c has two values for certain choices of λ_s and $\tau_0 T_{c0}$ suggesting the reentrance to the normal state at low temperatures. The origin of the reentrance lies in the existence of the logarithmic terms in eq. (28). In three-dimensional bulk systems, the similar reduction of T_c for highly resistive materials also appears. However, the reentrance is not expected because no logarithmic terms exist.

§4. Discussion

We have calculated the superconducting transition temperature in the presence of localization effects within the mean field approximation, and ignored the fluctuation effects which are important in thin films.[20] The calculated transition temperature will, however, be a measure of onset of short range order. Therefore, pronounced changes in the observed resistance may be discussed by use of the results obtained in this paper.

Kobayashi *et al.*[13,14] have observed a rapid rise of resistance below 2 K in superconducting granular films of tin. Such rise of resistance has also been predicted by Hebard and Vandenberg[21] in superconducting granular films of lead. Simanek[22] and Efetov[23] have introduced a phenomenological model for granular superconductors, in which localized superconducting grains couple among themselves and the phase transition is described as the onset of the coherent ordering of Josephson phase. In the model, the Coulomb interaction associated with charge fluctuations is taken into account; it suppresses the phase ordering at low temperatures. Such a phenomenological model has been extended to two-dimensional granular films.[24] In the present theory, the Coulomb interaction suppresses superconductivity and contributes to the reentrance. Therefore, this theory, which was derived from the metallic limit, has common physical aspects as the phenomenological model. It is worth while mentioning that the present theory predicts the reentrance only in two-dimensional systems but not in three-dimension.

In a typical granular film of tin[13,14] which exhibits the reentrance, the sheet resistance $R_\square$ is 6.71×10^3 ohm at room temperature. By using the relation $R_\square = \pi/e^2\varepsilon_F\tau_0$ with $e^2/\pi =$

7.54×10^{-5} ohm^{-1}, we have $\varepsilon_F\tau_0=2.0$. Therefore, our perturbational treatments with respect to $(\varepsilon_F\tau_0)^{-1}$ may be applicable to this case, and the observed reentrant phenomena will be explained by the present mechanism.

In conclusion, we have studied the superconducting transition temperature T_c in two-dimensional disordered systems in the mean-field theory: T_c is reduced by localization. This is because the Coulomb repulsive interaction is enhanced and the density of states of electrons is depressed in dirty systems. The present theory may be an extension of Anderson's theory of dirty superconductors.[11] The diagramatical representation of our results, which may be an extension of Gorkov's theory,[12] will be given in a separate paper.[25]

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17) In ref. 8, only the processes, Fig. 1(a) and (c) are taken into account.
18) In ref. 9, dimensionless coupling constant $g_i N(0)$ is denoted as g_i.
19) If dynamical screened Coulomb interaction is assumed, the process corresponding to the g_1-process results in the more singular contribution $\delta N(\varepsilon_n)/N(0)\propto-(\ln(1/2\pi T\tau_0))^2$ at $\varepsilon_n=0$. Since the qualitative features of T_c are not different, we employ the simple model of the effective constant coupling (eq. (10)) in this paper.
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