Progress of Theoretical Physics, Vol. 54, No. 2, August 1975

Perturbation Expansion for the Anderson Hamiltonian. IV

Kosaku YAMADA*)

Institute for Solid State Physics, University of Tokyo
Roppongi, Tokyo

(Received December 10, 1974)

In the paper III of this series, the relation connecting the imaginary part of the d-electron self-energy with the odd part of the susceptibility has been derived in an indirect way. In this paper, a direct derivation of this relation is given by the diagrammatic method.

1] Recently, by investigating the general terms in the perturbation expansions of thermodynamical quantities, some relations among them for the symmetric Anderson Hamiltonian[1] ($\varepsilon_d = -U/2$) have been found.[2)~4)] (Hereafter, 3) and 4) are cited as II and III, respectively.) The exact relations can be written in the following compact form for the expansion of the d-electron self-energy:

$$\Sigma^R(\omega) = -(\tilde{\chi}_{\text{even}}-1)\,\omega - \frac{i\Delta}{2}\tilde{\chi}_{\text{odd}}^2\left(\left(\frac{\omega}{\Delta}\right)^2+\left(\frac{\pi T}{\Delta}\right)^2\right)+\cdots, \tag{1}$$

where $\tilde{\chi}_{\text{even}}$ and $\tilde{\chi}_{\text{odd}}$ are the even and odd parts of the normalized susceptibility at $T=0$, respectively and they are in the relation of

$$\chi = \frac{1}{2}(g\mu_B)^2\frac{1}{\pi\Delta}\left\{\tilde{\chi}_{\text{even}}\left(\frac{U}{\pi\Delta}\right)+\tilde{\chi}_{\text{odd}}\left(\frac{U}{\pi\Delta}\right)\right\},$$

to the usual susceptibility χ. In this paper we use the same notations as those in II and III. Using the relation (1), we can obtain immediately the T-linear specific heat and the scattering t-matrix in the low energy and low temperature regions.[3),4)] In this paper, we present a derivation of the second term of Eq. (1), namely, Eq. (II, 6·2) or Eq. (III, 6·24), which is rewritten as follows:

$$\lim_{\omega\to 0}\frac{d^2\Sigma(\omega)}{d\omega^2} = -\frac{i}{\Delta}\tilde{\chi}_{\text{odd}}^2\cdot\operatorname{sgn}\omega \quad (T=0) \tag{2}$$

and

$$\lim_{\omega\to 0}\Sigma(\omega) = -\frac{i\Delta}{2}\tilde{\chi}_{\text{odd}}^2\left(\frac{\pi T}{\Delta}\right)^2\operatorname{sgn}\omega+\cdots, \tag{3}$$

where $\Sigma(\omega)$ is the self-energy part of the thermal Green's function.

At first, we rewrite χ_{odd}. From Eqs. (II, 2·16) and (III, 4·6), we obtain χ_{odd} at $T=0$ as follows:

---

*) Present address: College of Engineering, Shizuoka University, Hamamatsu, Shizuoka.

$$\chi_{\rm odd}=-\frac{1}{2}(g\mu_B)^2\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{1}{\beta}\int_0^{\beta}d\tau_1\cdots d\tau_{n+2}\langle T_\tau\tilde{n}_\uparrow(\tau_1)H'(\tau_2)\cdots H'(\tau_{n+1})\tilde{n}_\downarrow(\tau_{n+2})\rangle_{\rm conn} \tag{4}$$

$$=-\frac{1}{2}(g\mu_B)^2\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}U^n\frac{1}{\beta}\int_0^{\beta}d\tau_1\cdots d\tau_{n+2}[D_{n+1}(1,2,\cdots n+1)\times D_{n+1}(2,3,\cdots n+2)]_{\rm conn}$$

$$=-\frac{1}{2}(g\mu_B)^2\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}U^n\frac{1}{\beta}\frac{1}{(\pi\Delta)^2}\int_0^{\beta}d\tau_1\cdots d\tau_n[\sum_{ji}D_{ji}^n]^2_{\rm conn}\,. \tag{5}$$

Because of $D_{2n+1}=0$, the terms for even n vanish. Equation (4) shows that the diagrams for $\chi_{\rm odd}$ are obtained from those for the free-energy, adding a cross to both of an up spin closed loop and a down spin closed loop and repeating such a replacement for every pair of up and down spin closed loops.[3] A cross added to a closed loop $\prod_{j=1}^{m}G^\circ(\omega+\sum_{i=1}^{j}x_i)$ changes it into $\sum_{j=1}^{m}G^{\circ 2}(\omega+\sum_{i=1}^{j}x_i)\cdot\prod_{j'=1}^{\prime m}G^\circ\times(\omega+\sum_{i'=1}^{j'}x_{i'})$ as shown in II. The diagram for $\chi_{\rm odd}$ is shown schematically in Fig. 1. With the twice use of the following transformation (II, 4·8),

$$\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\sum_{j=1}^{m}G^{\circ 2}(\omega+\sum_{i=1}^{j}x_i)\prod_{j'=1}^{m}{}'\,G^\circ(\omega+\sum_{i'=1}^{j'}x_{i'})$$

$$=\frac{-1}{\pi\Delta}\sum_{j=1}^{m}\int_{-\infty}^{\infty}d\omega\,\delta(\omega+\sum_{i=1}^{j}x_i)\prod_{j'=1}^{m}{}'\,G^\circ(\omega+\sum_{i'=1}^{j'}x_{i'}), \tag{6}$$

we obtain $\tilde{\chi}_{\rm odd}=\chi_{\rm odd}[(g\mu_B)^2/2\pi\Delta]^{-1}$ as follows:

$$\tilde{\chi}_{\rm odd}(U)=\frac{1}{\pi\Delta}\Gamma_{\uparrow\downarrow}(0,0;0,0), \tag{7}$$

where $\Gamma_{\uparrow\downarrow}(\omega_1\uparrow,\omega_2\downarrow;\ \omega_3\downarrow,\omega_4\uparrow)$ is the vertex part between up and down spin electrons, as shown in Fig. 2. In the above transformation, we have used the following properties:

$$\Gamma'(0,0;\ 0,0)=\Gamma(0,0;\ 0,0),$$

where

$$\Gamma'(\omega_1,\omega_2;\omega_3,\omega_4)=\Gamma(\omega_1,\omega_2;\omega_3,\omega_4)\prod_{i=1}^{4}(1+G^\circ(\omega_i)\Sigma'(\omega_i)). \tag{8}$$

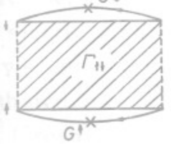

Fig. 1. A diagram of $\chi_{\rm odd}$. The full curve G_σ represents the Green's function $G_\sigma=(1+G_\sigma^\circ\times\Sigma_\sigma')G_\sigma^\circ$.

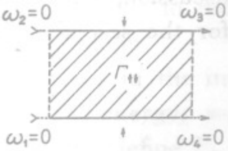

Fig. 2. The vertex part $\Gamma_{\uparrow\downarrow}$.

$\Sigma'(\omega)$ is the self-energy part including the improper one and $\Sigma'(0)=0$.

Equation (7) can be also derived by the use of the determinantal[4] expression in a similar way to the case in III. We consider the two-particle Green's function of d-electrons.

$$
\begin{aligned}
G^{\mathrm{II}}(\omega_1\uparrow, \omega_2\downarrow; \omega_3\downarrow, \omega_4\uparrow) &= \int_0^\beta ds_1 ds_2 ds_3 ds_4 \\
&\quad \cdot e^{i\omega_1 s_1 + i\omega_2 s_2 - i\omega_3 s_3 - i\omega_4 s_4} \langle\langle T_\tau C_{d\uparrow}(s_1) C_{d\downarrow}(s_2) C^+_{d\downarrow}(s_3) C^+_{d\uparrow}(s_4) \rangle\rangle \\
&= \int_0^\beta ds_1 ds_2 ds_3 ds_4 e^{i\omega_1 s_1 + i\omega_2 s_2 - i\omega_3 s_3 - i\omega_4 s_4} \\
&\quad \times \sum_{n=0}^\infty (-1)^n \frac{U^n}{n!} \int_0^\beta d\tau_1 \cdots d\tau_n \{D^n(s_1, s_4; 1, 2, \cdots n) \\
&\quad \times D^n(s_2, s_3; 1, 2, \cdots n)\}_{\mathrm{conn}},
\end{aligned}
$$

where $D^n(s_1, s_4; 1, 2, \cdots, n)$

$$
= \begin{vmatrix} G^\circ(s_1 - s_4) & G^\circ(s_1 - \tau_1) \cdots\cdots & G^\circ(s_1 - \tau_n) \\ G^\circ(\tau_1 - s_4) & 0 & G^\circ{}_{1n} \\ \vdots & \vdots & \vdots \\ G^\circ(\tau_n - s_4) & G^\circ{}_{n1} \cdots\cdots\cdots\cdots & 0 \end{vmatrix},
$$

$$
\begin{aligned}
G^{\mathrm{II}}(\omega_1\uparrow, \omega_2\downarrow; \omega_3\downarrow, \omega_4\uparrow) = \beta[\beta G^\circ(\omega_1) G^\circ(\omega_2) \delta_{\omega_1, \omega_4} \cdot \delta_{\omega_2, \omega_3} \\
+ \sum_{n=1}^\infty (-1)^n \frac{U^n}{n!} \frac{1}{\beta} \int_0^\beta d\tau_1 \cdots d\tau_n \{\sum_{j,i} G^\circ(\omega_1) G^\circ(\omega_4) \\
\times e^{-i\omega_4 \tau_j + i\omega_1 \tau_i} \cdot D^n_{ji} \sum_{j',i'} G^\circ(\omega_2) G^\circ(\omega_3) e^{-i\omega_3 \tau_{j'} + i\omega_2 \tau_{i'}} \\
\times D^n_{j'i'}\}_{\mathrm{conn}}].
\end{aligned}
$$

Putting $\omega_i = 0$ $(i=1, 2, 3, 4)$, we obtain

$$
\Gamma_{\uparrow\downarrow}(\omega_i = 0) = \sum_{n=1}^\infty (-1)^{n+1} \frac{U^n}{n!} \frac{1}{\beta} \int_0^\beta d\tau_1 \cdots d\tau_n [\sum_{ji} D^n_{ji}]^2_{\mathrm{conn}}. \tag{9}
$$

Combining Eqs. (5) and (9), we obtain Eq. (7).

2] Here, we discuss the left-hand term of Eq. (2). Before we start the general discussion, we discuss the second order self-energy as an example. The diagram for the second order self-energy is shown in Fig. 3.

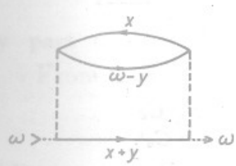

Fig. 3. The 2nd order self-energy part $\Sigma_2(\omega)$.

$$\frac{d^2\Sigma_2(\omega)}{d\omega^2}\bigg|_{\omega\to 0} = -U^2\frac{d^2}{d\omega^2}\left[\int_{-\infty}^{\infty}\frac{dx}{2\pi}\frac{dy}{2\pi}G^\circ(\omega-y)G^\circ(x+y)G^\circ(x)\right]\bigg|_{\omega\to 0}$$

$$= -U^2\int_{-\infty}^{\infty}\frac{dx}{2\pi}\frac{dy}{2\pi}\frac{d}{d\omega}\left[\left(-iG^{\circ 2}(\omega-y)+\frac{2}{i\Delta}\delta(\omega-y)\right)G^\circ(x+y)G^\circ(x)\right]\bigg|_{\omega\to 0}$$

$$= 2U^2\int_{-\infty}^{\infty}\frac{dx}{2\pi}\frac{dy}{2\pi}G^{\circ 3}(y)G^\circ(x+y)G^\circ(x)$$

$$-\frac{U^2}{i\pi\Delta}\int_{-\infty}^{\infty}\frac{dx}{2\pi}\left[-iG^{\circ 2}(x)+\frac{2}{i\Delta}\delta(x+\omega)\right]G^\circ(x)\bigg|_{\omega\to 0}$$

$$= -\left(\frac{U}{\pi\Delta}\right)^2 G^\circ(\omega\to 0) = \frac{i}{\Delta}(\tilde{\chi}^{(1)})^2\,\mathrm{sgn}\,\omega\,. \tag{10}$$

In the above calculation, the finite contribution to $d^2\Sigma(\omega)/d\omega^2|_{\omega\to 0}$ comes from the term reduced by the twice use of the singular δ-function part.

Now, we consider the general order self-energy of d-electron with up spin, $\Sigma_{n\uparrow}(\omega)$. The n-th order self-energy $\Sigma_n(\omega)$ is given by the integral of $2n-1$ product of odd function $G^\circ(x)$. Then, $\Sigma(\omega\to 0)=0$. By the use of the relation

$$\frac{d}{d\omega}G^\circ(x+\omega) = -iG^{\circ 2}(x+\omega)+\frac{2}{i\Delta}\delta(x+\omega),$$

we obtain the following three types of terms produced by differentiating twice the self-energy with respect to ω: (a) The terms which have no δ-function, (b) the terms which have one δ-function, (c) the terms which have two δ-functions. In (a) terms $G^\circ(\omega+x_i)\cdot G^\circ(\omega+x_j)$ in $\Sigma(\omega)$ is replaced with $G^{\circ 3}(x_i)G^\circ(x_j)$ or $G^{\circ 2}(x_i)G^{\circ 2}(x_j)$. In these terms, we have $2n+1$ product of Green's functions. Therefore, their integrals vanish. The terms of (b) type including $G^{\circ 2}(\omega+x_i)\cdot\delta(\omega+x_j)$ become $2n-1$ product, whose integrals vanish also. The finite contribution to $d^2\Sigma(\omega)/d\omega^2|_{\omega=0}$ comes from the terms of (c) type.

Now, we consider what types of (c) terms give finite contributions. For this purpose, we divide the self-energy diagrams into two parts; one of which connects with the incoming line $G^\circ(\omega)$ and the other one connects with the outgoing line $G^\circ(\omega)$. Then, all the interaction lines are included in the above two parts of the self-energy; some electron lines run in the intermediate region between two parts. In this division, we impose the following two conditions: (1) We place in the intermediate region two Green's functions to be differentiated. (2) Furthermore, we change the order of interaction time in order to minimize the number of electron lines which run in the intermediate region.

Now, we consider the differentiation of Green's functions in the intermediate region as shown in Fig. 4. As we consider the proper self-energy, we have at least three Green's functions in the intermediate region. We define P_l and P_r as the left and right part of the divided self-energy. In Fig. 4, we assign the arguments so that ω appears explicitly in only one Green's function. In more general

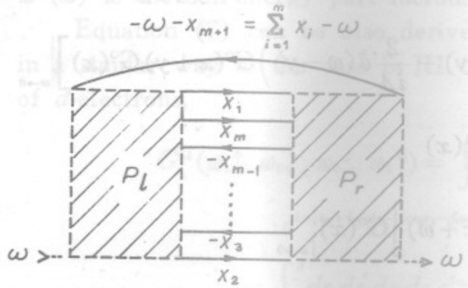

Fig. 4. The divided self-energy part. P_l and P_r represent the left and right side parts, respectively. If we attach $-x_i$ to hole lines, we obtain $\sum_{i=1}^{m+1} x_i = 0$.

diagrams, ω may appear in other lines in the intermediate region. Our problem is to obtain the diagrams which give the finite contribution after the twice use of δ function parts, namely, cut of two lines in the intermediate region. As far as this problem is concerned, the discussion in Eqs. (11)~(12) is general. This is because we discuss there all the cases in which two lines in the intermediate region are cut, as we can select all possible pairs of lines in two steps of differentiation by assigning arbitrarily arguments for lines in Fig. 4.

$$\lim_{\omega\to 0}\int_{-\infty}^{\infty}\frac{dx_1\cdots dx_m}{(2\pi)^m}P_l\cdot P_r\cdot\frac{d^2}{d\omega^2}[G^\circ(x_1)G^\circ(x_2)G^\circ(-x_3)\cdots$$

$$\cdots G^\circ(-x_{m-1})G^\circ(x_m)G^\circ(\sum_{i=1}^{m}x_i-\omega)] \tag{11}$$

$$=\lim_{\omega\to 0}\int_{-\infty}^{\infty}\frac{dx_1\cdots dx_m}{(2\pi)^m}P_l\cdot P_r\frac{d}{d\omega}\Big[(iG^{\circ 2}(\sum_{i=1}^{m}x_i-\omega)$$

$$-\frac{2}{i\Delta}\delta(\sum_{i=1}^{m}x_i-\omega))G^\circ(x_1)\cdots G^\circ(x_m)]$$

$$=\int_{-\infty}^{\infty}\frac{dx_1\cdots dx_m}{(2\pi)^m}P_l\cdot P_r[-2G^\circ(x_1)G^\circ(x_2)\cdots G^{\circ 3}(\sum_{i=1}^{m}x_i)] \tag{11′-a}$$

$$+\lim_{\omega\to 0}\frac{-1}{i\pi\Delta}\int_{-\infty}^{\infty}\frac{dx_2\cdots dx_m}{(2\pi)^{m-1}}P_l\cdot P_r\frac{d}{d\omega}[G^\circ(\omega-\sum_{s=0}^{m}x_i)\cdot G^\circ(x_2)\cdots G^\circ(x_m)] \tag{11′-b}$$

$$=-2\int_{-\infty}^{\infty}\frac{dx_1\cdots dx_m}{(2\pi)^m}G^\circ(x_1)G^\circ(x_2)\cdots G^{\circ 3}(\sum_{i=1}^{m}x_i)P_l\cdot P_r \tag{12-a}$$

$$+\frac{1}{\pi\Delta}\int_{-\infty}^{\infty}\frac{dx_2\cdots dx_m}{(2\pi)^{m-1}}G^\circ(x_2)\cdots G^\circ(x_m)G^{\circ 2}(\sum_{i=2}^{m}x_i)P_l\cdot P_r \tag{12-b}$$

$$+\lim_{\omega\to 0}\frac{1}{(\pi\Delta)^2}\int_{-\infty}^{\infty}\frac{dx_3\cdots dx_m}{(2\pi)^{m-2}}G^\circ(\omega-\sum_{i=3}^{m}x_i)G^\circ(x_3)\cdots G^\circ(x_m)P_l\cdot P_r\,. \tag{12-c}$$

Equations (12-a,b) vanish as said before; Eq. (12-c) is the part reduced by the twice use of δ-function parts. Equation (12-c) is $2n-3$ products of Green's function and its integral vanishes except the case $m=2$. In the case $m\geq 4$, $\sum_{i=3}^{m}x_i=0$ can be satisfied in only the improper self-energy. In the case $m=2$, $G^\circ(\omega-\sum_{i=3}^{m}x_i)$

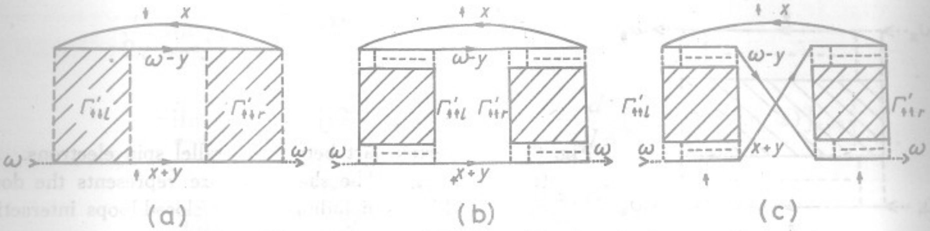

Fig. 5. (a) (b) (c) The diagrams with three lines in the intermediate region.

$=G^\circ(\omega)$ has no integral variables as its argument and is a constant value in the limit $\omega\to 0$. As the result, we have $2n-4$ product of $G^\circ(x)$ whose integral gives a finite contribution. That is, it is the diagram with only three Green's functions in the intermediate region which gives a finite contribution.

Now, we consider the case in which only three Green's functions exist in the intermediate region. It is easily seen that all the such diagrams can be divided into three types of diagrams (a), (b) and (c) as shown in Fig. 5. For convenience we define Γ_l' and Γ_r' as follows:

$$\Gamma_l'(\omega_1,\omega_2;\ \omega_3,\omega_4)\equiv\Gamma(\omega_1,\omega_2;\ \omega_3,\omega_4)\prod_{i=2}^{4}(1+G^\circ(\omega_i)\Sigma'(\omega_i)),$$

$$\Gamma_r'(\omega_1,\omega_2;\ \omega_3,\omega_4)\equiv\Gamma(\omega_1,\omega_2;\ \omega_3,\omega_4)\prod_{i=1}^{3}(1+G^\circ(\omega_i)\Sigma'(\omega_i)).$$

As $\Sigma'(0)=0$, it follows that $\Gamma_l'(\omega_i=0$ for $i=1,2,3,4)=\Gamma_r'(\omega_i=0)=\Gamma(\omega_i=0)$. If we fix three Green's functions to be differentiated in the intermediate region, we have full vertex parts including the above self-energy correction, namely, $\Gamma'_{l,r}$, at the left and right sides. Similar discussion was given by Abrikosov in the calculation of the most divergent terms in the *s-d* model.[5)]

[Case a] *One down spin closed loop and one up spin line in the intermediate region.*

$$\lim_{\omega\to 0}\int_{-\infty}^{\infty}\frac{dx\,dy}{(2\pi)^2}\Gamma'_{\uparrow\downarrow l}(\omega,x;\omega-y,x+y)\Gamma'_{\uparrow\downarrow r}(x+y,\omega-y;x,\omega)$$
$$\times\frac{d^2}{d\omega^2}[-G^\circ{}_\downarrow(x)G^\circ{}_\downarrow(\omega-y)G^\circ{}_\uparrow(x+y)]$$
$$=-\frac{\Gamma_{\uparrow\downarrow}{}^2(\omega_i=0)}{(\pi\Delta)^2}G^\circ(\omega\to 0)=\frac{i}{\Delta}\tilde{\chi}^2_{\rm odd}\,{\rm sgn}\,\omega\,, \tag{13}$$

where we have used Eq. (7). $-$ sign in [ ] comes from a closed loop.

[Case b] *One up spin closed loop and one up spin line in the intermediate region.*

We define $\Gamma_{\uparrow\uparrow}(\omega_1,\omega_2;\ \omega_3,\omega_4)$ as the vertex part between two particles with

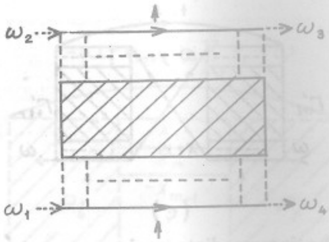

Fig. 6. The vertex part between parallel spin electrons. $\Gamma_{\uparrow\uparrow}(\omega_1, \omega_2; \omega_3, \omega_4)$. The shaded square represents the down spin closed loops including up spin closed loops interacting with them.

parallel spin, as shown in Fig. 6.*)

$$\lim_{\omega\to 0}\int_{-\infty}^{\infty}\frac{dx\,dy}{(2\pi)^2}\Gamma'_{\uparrow\uparrow l}(\omega, x; \omega-y, x+y)\,\Gamma'_{\uparrow\uparrow r}(x+y, \omega-y; x, \omega)$$

$$\times\frac{d^2}{d\omega^2}[-G_\uparrow^\circ(x)\,G_\uparrow^\circ(\omega-y)\,G_\uparrow^\circ(x+y)]=\left[\frac{1}{\pi\varDelta}\Gamma_{\uparrow\uparrow}(\omega_i=0)\right]^2\frac{i}{\varDelta}\operatorname{sgn}\omega\,. \tag{14}$$

[Case c] *Three up spin lines with no closed loops in the intermediate region.*

$$\lim_{\omega\to 0}\int_{-\infty}^{\infty}\frac{dx\,dy}{(2\pi)^2}\Gamma'_{\uparrow\uparrow l}(\omega, x; \omega-y, x+y)\,\Gamma'_{\uparrow\uparrow r}(\omega-y, x+y; x, \omega)$$

$$\times\frac{d^2}{d\omega^2}[G_\uparrow^\circ(x)\,G_\uparrow^\circ(\omega-y)\,G_\uparrow^\circ(x+y)]=-\left[\frac{1}{\pi\varDelta}\Gamma_{\uparrow\uparrow}(\omega_i=0)\right]^2\frac{i}{\varDelta}\operatorname{sgn}\omega\,. \tag{15}$$

From Eqs. (14) and (15), $\Gamma_{\uparrow\uparrow}{}^2$ terms cancel out. We have only Eq. (13). Thus we obtain Eq. (2). It is easily seen that Eq. (11′-b) corresponds to the following differentiation of a vertex part, $\lim_{\omega\to 0} d/d\omega\cdot\Gamma(\omega-\omega_1, \omega_1; \omega_2, \omega-\omega_2)|_{\omega_1=\omega_2=0}$. Therefore, a finite contribution to this quantity comes from the diagrams with two lines in the intermediate region.

3] Now, we consider the T^2-coefficient of $\Sigma(0)$, namely Eq. (3). From Eq. (III,5·1), the T^2-correction for a discrete sum of $F(x_n)$ is given as follows:

$$T\sum_{x_m}F(x_n)=\int_{-\infty}^{\infty}\frac{dx}{2\pi}F(x)+\frac{(\pi T)^2}{6}\frac{1}{2\pi}(F'(0_+)-F'(0_-))+\cdots. \tag{16}$$

We apply this formula to $F(x,\omega)=G^\circ(x)R(x,\omega)$, where $\int_{-\infty}^{\infty}dx/2\pi\cdot F(x,\omega)=\Sigma(\omega, T=0)$. The total T^2-corrections in $\Sigma_{n\uparrow}(\omega\to 0)$ are obtained by taking out the T^2-contributions from each line and collecting these contributions. We obtain the T^2-contribution of a Green's function as follows:

---

*) The vertex defined here does not include the part which comes from the exchange between $G(\omega_3)$ and $G(\omega_4)$. $\Gamma_{\uparrow\uparrow}$ is related to $S_\uparrow$ defined in II by the following relation:

$$\Gamma_{\uparrow\uparrow}(\omega_1, \omega_2; \omega_3, \omega_4)\prod_{i=1}^{4}(1+G^0(\omega_i)\Sigma'(\omega_i))=\sum_n S_\uparrow^{(n)}(\omega_1, \omega_4, \omega_2, \omega_3).$$

$$\frac{1}{2\pi}\lim_{\omega\to0}[F'(0_+)-F'(0_-)]$$

$$=\lim_{\omega\to0}\frac{1}{2\pi}\left\{\frac{d}{dx}[G^\circ(x)R(x,\omega)]_{x=0_+}-\frac{d}{dx}[G^\circ(x)R(x,\omega)]_{x=0_-}\right\}$$

$$=\frac{1}{2\pi i\Delta}\lim_{\omega\to0}\left\{\frac{dR(x,\omega)}{dx}\Big|_{x=0_+}+\frac{dR(x,\omega)}{dx}\Big|_{x=0_-}\right\}, \qquad (17)$$

where, $R(x,0)=R(-x,0)$. In the above transformation we have used the discontinuity of $G^\circ(x)$; that is, we cut one electron line of the self-energy diagram. From the correspondence between the last expression of Eqs. (17) and (11′-b), we can see that the finite contribution comes from only the diagrams of the type of Fig. 5(a) for the same reason as $d^2\Sigma(\omega)/d\omega^2|_{\omega\to0}$.

$$\frac{dR(x,\omega)}{dx}\Big|_{x=\pm0}=\int_{-\infty}^{\infty}\frac{dy}{2\pi}\Gamma_l{}'\Gamma_r{}'\frac{d}{dx}[-G^\circ(x+y)G^\circ(\omega-y)]\Big|_{x=\pm0}$$

$$=i\int_{-\infty}^{\infty}\frac{dy}{2\pi}\Gamma_l{}'\Gamma_r{}'G^{\circ2}(y)G^\circ(\omega-y)-\frac{1}{i\pi\Delta}G^\circ(\omega)\Gamma_l{}'\Gamma_r{}'.$$

$$\frac{1}{\pi i\Delta}\lim_{\omega\to0}\frac{dR(x,\omega)}{dx}\Big|_{x=\pm0}=\frac{\Gamma^2_{\uparrow\downarrow}(\omega_i=0)}{(\pi\Delta)^2}\cdot\frac{1}{i\Delta}\operatorname{sgn}\omega=-\frac{d^2\Sigma(\omega)}{d\omega^2}\Big|_{\omega=0}.$$

In contrast with $d^2\Sigma/d\omega^2$, we can start from any one of three lines in the intermediate region and so we obtain factor three in front of $d^2\Sigma(\omega)/d\omega^2|_{\omega\to0}$, as shown in Fig. 7. Thus we obtain the total T^2-contribution as follows:

$$\Sigma(\omega\to0)=-\frac{(\pi T)^2}{6}\cdot3\cdot\frac{i}{\Delta}\cdot\left(\frac{1}{\pi\Delta}\Gamma_{\uparrow\downarrow}\right)^2\operatorname{sgn}\omega=-\frac{(\pi T)^2}{2}\frac{i}{\Delta}\tilde{\chi}^2_{\rm odd}\cdot\operatorname{sgn}\omega. \qquad (18)$$

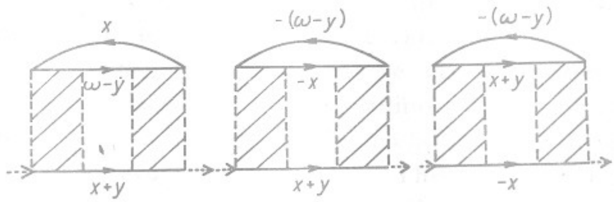

Fig. 7. The three T^2-contributions to $\Sigma(0)$.

Thus, we have derived Eqs. (2) and (3).

4] In the above discussion, only the diagrams with three Green's functions in the intermediate region give finite contributions to ω^2- or T^2-corrections. This fact corresponds to that only the one electron-hole pair excitation is included in the calculation of the collision integral by Nozières.[6] In III, another proof of Eq. (1) is given on the basis of scattering theory.[4),6)] In this paper, we have shown that it can be also derived directly with the use of the diagrammatic method. Using Eq. (1), we obtain the expression (II,6·3) for the resistivity,

$$R=R_0\left[1-\frac{\pi^2}{3}\left(\frac{T}{\Delta}\right)^2(2\tilde{\chi}_{\rm odd}^2+\tilde{\chi}_{\rm even}^2)+\cdots\right].$$

In the *s-d* limit, because of $\tilde{\chi}_{\rm even}=\tilde{\chi}_{\rm odd}=\tilde{\chi}/2$ we obtain

$$R=R_0\left[1-\frac{\pi^2}{4}\left(\frac{T}{\Delta}\right)^2\tilde{\chi}^2+\cdots\right].$$

The study of higher-order expansion coefficients is a future problem. In this case, it is necessary to include multiple electron-hole pair excitations as intermediate states.

Acknowledgements

The author would like to express his sincere thanks to Professor K. Yosida, Professor A. Yoshimori and Dr. A. Sakurai for their valuable discussions and suggestions.

References

1) P. W. Anderson, Phys. Rev. **124** (1961), 41.
2) K. Yosida and K. Yamada, Prog. Theor. Phys. Suppl. No. 46 (1970), 244
3) K. Yamada, Prog. Theor. Phys. **53** (1975), 970.
4) K. Yosida and K. Yamada, Prog. Theor. Phys. **53** (1975), 1286.
5) A. A. Abrikosov, Physics **2** (1965), 5.
6) P. Nozières, J. Low Temp. Phys. **17** (1974), 31.