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Research Paper

Modified Fredkin gates in logic design

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A modified Fredkin gate is proposed as a basic building block for low-energy computing. Essentially the modified Fredkin gate is a simple crossover switch, but it has the potential to be implemented optically. It therefore appears to be a good choice as a building block for future computers. This paper shows that the modified Fredkin gate can be used to implement multi-valued logic, threshold logic and array logic. The result is that the same basic architecture can be used to implement many different logic design techniques, so that comparisons can be made between them.

1. Introduction

The history of logic design has been dominated by Boolean logic mainly because of the ease with which it can be implemented in electronics. Whether electrical switches, diodes or transistors are used, a switch can be constructed so that there are two stable states, open and closed, or on and off. This has meant that alternative logical methods have been somewhat overlooked. Examples of alternative logic design solutions include threshold logic and multi-valued logic which, despite having a strong case in their favour, always seem to fall prey to the argument that they cannot be implemented as easily as Boolean logic. Comparisons of the logical power tend to favour these alternatives, but the circuitry needed to construct them is always more complex than the equivalent Boolean implementation, and it is difficult to imagine the computing industry suddenly changing to one of these alternatives.

There are currently two factors which might alter this [1]. The first is that predictions of the future of computers show that they cannot go on increasing in power at their present rate. At some point in the future it will no longer be feasible to reduce their size and increase their speed. At that point some of the alternatives may be explored. However, the second factor is the increasing interest in optical computing. Some researchers have already started to point out some of the weaknesses of current electronic computing and are advocating a shift of investment towards optical systems. At present much of the research into optical computing seems to be directed at finding equivalent Boolean logic using methods such as shadow casting [2], even though some of these methods have the potential for implementing threshold logic or multi-valued logic. It would seem that if a strong enough case for some of the alternative logics were to be made now, it could be possible for them to get in 'on the ground floor'.

In this paper, one particular gate is proposed which has the potential to be implemented opti-

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cally. It has the advantage that if necessary it can perform conventional Boolean logic, but it can also perform alternative logics such as conservative logic, multi-valued logic and threshold logic. Since the same gate is used regardless of the logic, it is possible to make a comparison of the various logics, so that hopefully it can be shown that for specific applications Boolean logic may not always be the best solution.

2. The modified Fredkin gate

Figure 1 shows the proposed gate.

The gate has four inputs and four outputs, and performs as follows.

$P = A$

$Q = B$

$R = C$ if $A < B$ $\quad$ $R = D$ if $A \geqslant B$

$S = D$ if $A < B$ $\quad$ $S = C$ if $A \geqslant B$

This gate is called the modified Fredkin gate because it has four inputs and four outputs, whereas a conventional Fredkin gate only has three. However, it can be made to perform as a conventional Fredkin gate if input B is set to a permanent logic 1. Since it is possible to implement any Boolean logic function using Fredkin gates [3], then clearly it is also possible using modified Fredkin gates.

Furthermore, many attempts to construct Fredkin gates using optical or electro-optical technology include some thresholding, either in the material itself [4] or using a separate comparator [5], which implies that there is actually a fourth input which has to be supplied. The modified Fredkin gate makes this explicit.

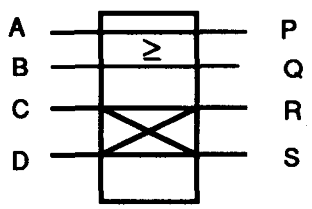

Fig. 1. The modified Fredkin gate.

At first sight it would appear that researchers are not looking into the construction of optical modified Fredkin gates. However, the literature abounds in the latest developments in photonic switching, where there is an immediate demand from the increased use of fibre-optics in telecommunications [6]. At present, circuits of 10s of photonic switches are available, but it is anticipated that this number will increase dramatically. If we examine a photonic switch, then it turns out to be nothing more than a crossover switch, which is exactly what a Fredkin gate is. We could therefore be in the fortunate position of developing a theoretical logic gate in parallel with a photonic switch, which turn out to be one and the same thing.

The main reason that Fredkin suggested his gate was that there is a theoretical physical limit to computing [3]. This is not, as one might imagine, that miniaturisation or increasing the speed of the circuits will hit a barrier, but that the energy dissipation will be so great that it will be impossible for a computer to operate without melting. He suggests the use of conservative logic, as originally applied to bubble logic [7], as one way of limiting the energy dissipation. He also argues that the logic has to be reversible so that no information is lost, thereby also saving on energy dissipation. These arguments will not be disputed in this paper, but are just repeated as support for the use of the modified Fredkin gate, which would have the same properties of being conservative and reversible.

3. Design of Boolean logic circuits

As yet, there is no established way of designing a circuit using Fredkin gates. Fredkin suggested simply designing a logic circuit using conventional logic gates and then substituting Fredkin gates. The circuits lead to very inefficient use of Fredkin gates, and a requirement of many constant inputs. There are also a large number of redundant outputs. An alternative is to use a binary decision diagram [8]. This can be modified to include the redundant outputs and so make more efficient use of the Fredkin gates.

Fredkin does not allow fan-out or feedback in his conservative logic systems. This is because they would imply an increase in energy dissipation (fan-out) or the collapse of reversibility (feedback). However, if energy dissipation is not our main concern, then we might relax these conditions. One advantage that this gives is that more efficient use of the gates can be made, requiring fewer gates and fewer constant inputs, with a consequent reduction in the number of redundant outputs. It has been shown that for an n-input logic function, the total number of inputs required is at most $n+2$, and since the number of outputs always equals the number of inputs, the number of outputs is also a maximum of $n+2$ [9].

4. Multi-valued logic

The addition of the fourth input and output allows multi-valued logic to be implemented. Fredkin originally proposed having a gate input and two data inputs. Depending on the value of the gate input, the data inputs either passed through unchecked or they crossed over. These data inputs could easily be multi-valued, but the gate input can only be binary. The additional input allows the gate to operate using multi-valued signals by comparing the size of the two multi-valued gate inputs. It is possible to show that these gates can implement any multi-valued logic function by emulating the T-gate [10].

4.1. T-gates

In multi-valued logic, if the radix is m, then a T-gate is an m-way switch [11]. It is therefore clear that any multi-valued logic function can be

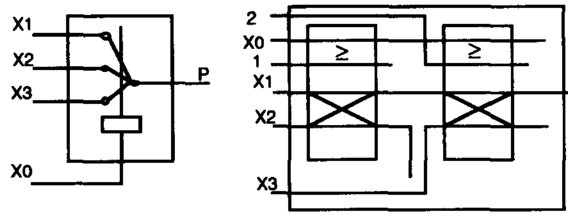

Fig. 2. T-gate implementation using modified Fredkin gates.

implemented using T-gates [12]. Figure 2 shows how a ternary T-gate can be implemented using the modified Fredkin gate.

4.2. Literals

Another approach to the implementation of multi-valued logic is the literal [13]. A literal is defined as

$$^{a}X^{b} = m - 1 \text{ if } a \leqslant X \leqslant b \text{ and } 0 \text{ otherwise}$$

Essentially, the multi-valued inputs are converted to binary inputs, which can then be processed using conventional Boolean logic. Figure 3 shows the functional description of a literal and shows how one can implement a literal using the modified Fredkin gate.

5. Threshold logic

Threshold logic was inspired by the McCulloch-Pitts neuron, and uses weights on the binary inputs and a threshold on the binary output to perform logic. The output is defined as follows:

$$y = 1 \text{ if } \sum_{i=1}^{n} w_i x_i \geqslant t \text{ and } 0 \text{ otherwise}$$

It has been shown that it is possible to construct a threshold logic device using conventional Boolean logic using the digital summation threshold logic device (DSTL) [12]. Figure 4 shows a DSTL, which is made up of an array of cells, each of which performs the AND and OR operation on its two inputs.

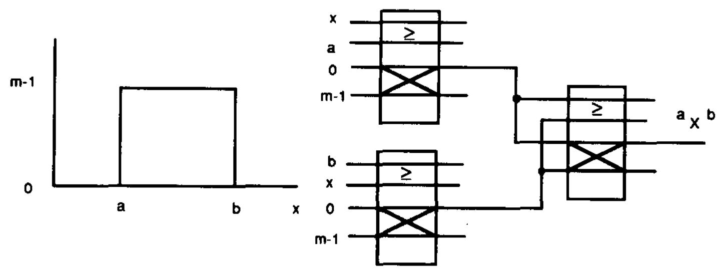

Fig. 3. The literal $^aX^b$ and its implementation using the modified Fredkin gate.

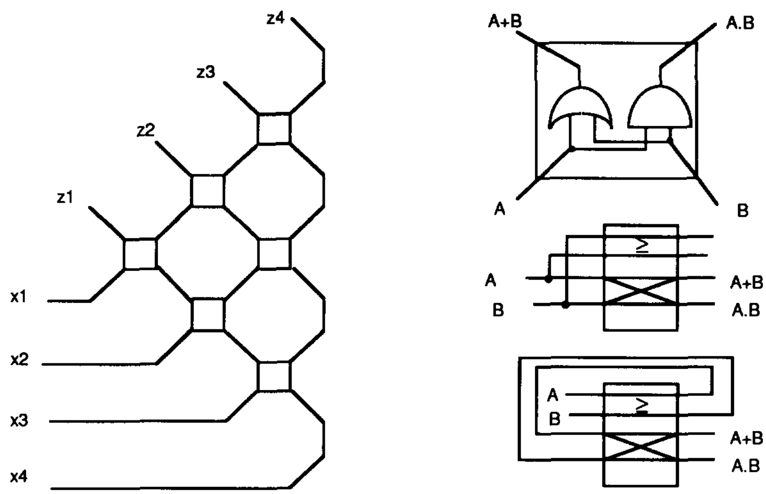

Fig. 4. DSTL, an individual cell from the DSTL and implementation using a modified Fredkin gate.

Figure 4 also shows a single cell, and the equivalent cell made from a modified Fredkin gate. In this figure, a cell is produced using fan-out and an alternative cell using feedback [14].

The cell shown in Fig. 4 performs the AND and OR functions necessary for the DSTL if the signals are binary. An alternative interpretation of the function of this cell is to consider the outputs as the maximum and minimum of the two inputs. Clearly then, if the inputs are multi-valued, these cells still perform the maximum and minimum functions. If the DSTL structure was tried with multi-valued inputs it would be found to be a sorter, arranging the outputs so that the maximum input appears in the first output (z_1 in Fig. 4), and the minimum input appears in the last output (z_4 in Fig. 4). This is not threshold logic, but it could still be a useful circuit. One area where this might find a home is in fuzzy logic.

6. Array logic

Array logic has often been suggested as a compact way of constructing logical functions. One proposal uses crossbar switches [15], which turn out to be nothing more than the Fredkin gate again. It is therefore yet another potential application of these gates, demonstrating their universality. Figure 5 shows an example of an

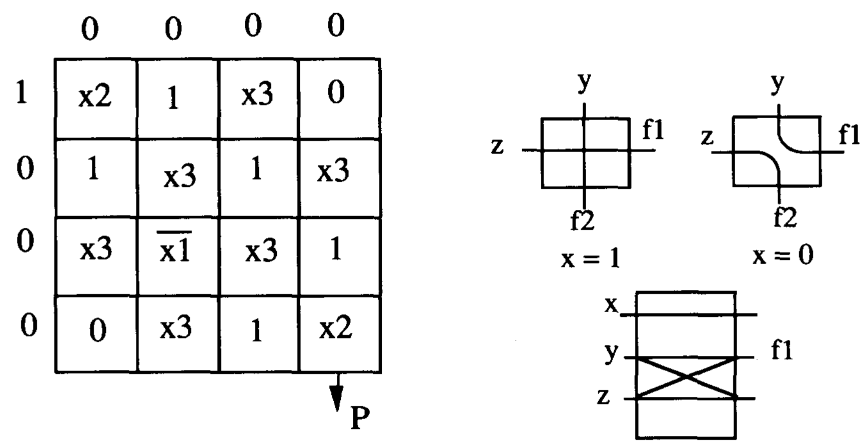

Fig. 5. Array implementation of the function $P = x_1 \overline{x_2} + x_3$ and how an individual cell is built using a Fredkin gate.

array which has been set up to implement the function $P = x_1 \overline{x_2} + x_3$.

7. Conclusions

The conclusions that can be drawn from these examples is that the modified Fredkin gate provides a medium in which conventional Boolean logic can be implemented. There seems to be a strong case for attempting to construct these gates using optical devices such as the photonic switches that are currently being developed in telecommunications. As a consequence, many of the alternative logics can also be implemented where there is a good reason for doing so.

The examples given in this paper are just a selection of some of the alternative logics that have been proposed over the years. All of them are amenable to implementation using the modified Fredkin gate. More alternatives will be explored to see if they too can be implemented, and therefore strengthen the case for this gate.

References

[1] A.D. McCaulay, *Optical Computer Architectures*, Wiley, New York, 1991.

[2] D.G. Feitelson, *Optical Computing*, MIT Press, Cambridge, MA, 1988.

[3] E. Fredkin and T. Toffoli, Conservative logic, *Int. J. Theor. Phys.*, 21 (1982) 219–253.

[4] J. Shamir, H.J. Caulfield, W. Micelli and R. J. Seymour, Optical computing and the Fredkin gates, *Appl. Optics*, 25 (1986) 1604–1607.

[5] A.F. Brenner, Digital optical logic using optically switched directional couplers, *Electron. Lett.*, 26 (1990) 1037–1038.

[6] B.E.A. Saleh and M.C. Teich, *Fundamentals of Photonics*, John Wiley, New York, 1991.

[7] A.D. Friedman and P.R. Menon, *Theory and Design of Switching Circuits*, Computer Science Press, Rockville, MD, 1975.

[8] S.B. Akers, Binary decision trees, *IEEE Trans. Comp.*, C-27 (1978) 509–516.

[9] P.D. Picton, *Conservative Logic*, Systems Architecture Group Report, SAG/1992/RR25/PP, 1992.

[10] P.D. Picton, Opto-electronic multi-valued conservative logic, *Int. J. Optical Comput.*, 2 (1991) 19–29.

[11] C.Y. Lee and W.H. Chen, Several valued switching circuits, *Trans. AIEE*, 25 (1957) 278–283.

[12] S.L. Hurst, *The Logical Processing of Digital Signals*, Crane–Russak, New York, 1978.

[13] J.E. Birk and D.E. Farmer, Design of multivalued switching circuits using principally binary components, *Proc. Int. Symp. Multiple-Valued Logic*, 1974, pp. 115–133.

[14] P.D. Picton, Conservative threshold logic, *Proc. Int. Conf. Information Engineering*, 1991, pp. 120–127.

[15] S.N. Kukreja and I.-N. Chen, Combinational and sequential cellular structures, *IEEE Trans. Comp.*, C-22 (1973) 813–823.