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CALCULATING SOLAR RADIATION FOR INCLINED SURFACES: PRACTICAL APPROACHES

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Abstract—Various practical approaches to calculating the solar irradiation for an inclined surface are reviewed. All assume that the direct and diffuse irradiation for an inclined surface are available, either as measured or calculated values. The paper distinguishes numerous categories of model on the basis of time scale of applicability (hourly or daily or longer time intervals) and on the basis of the radiant flux that is modelled (direct, diffuse or reflected irradiation). The relative performance of the various models is assessed using the results of several studies. Superior models are identified.

INTRODUCTION

The diverse applications of solar energy lead to a requirement for solar irradiation data for a wide range of surface orientations—far too many for the measurements to be made directly. Consequently, considerable effort has been given to developing methods for estimating the solar irradiation for an inclined surface given data for a horizontal surface. While the latter are usually measured, this is not a requirement. However, where calculated values for the horizontal surface are also used errors will likely be compounded [1].

Calculation of inclined surface irradiance involves separate treatment of the three components of the incident solar radiation: the direct, the diffuse from the sky hemisphere and the reflected from the ground surface within the field of view of the sloping surface.

Much of the detailed information on this topic has been presented by Hay and McKay [2, 3], amongst others, and it will not be repeated here. This paper will merely attempt a summary of the more basic material and follow that with a review of selected approaches, data requirements and accuracy of the modelled values.

DIRECT IRRADIANCE

For short time periods, generally considered to be an hour or less, the direct solar irradiance for an inclined surface is obtained using geometric relationships between the sun's position in the sky and the orientation of the inclined surface. The relevant formulae are

$$S\downarrow_s = S\cos i \qquad (1)$$

where

$$\cos i = \cos\alpha\cos z + \sin\alpha\sin z\cos(a-b) \qquad (2)$$

$$\cos z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos h \qquad (3)$$

$$\cos a = \frac{\cos z\sin\phi - \sin\delta}{\sin z\cos\phi} \qquad (4)$$

and

$S\downarrow_s$ = direct solar irradiance for an inclined surface
S = solar radiant intensity at normal incidence
i = angle of incidence between sun and normal to the surface
α = slope angle
z = solar zenith angle
a = azimuth angle of the sun
b = azimuth angle of the slope
ϕ = latitude
δ = solar declination
h = hour angle of the sun.

These equations may be rewritten and simplified to accommodate specific situations such as vertical equator-facing surfaces [4].

DIFFUSE IRRADIANCE

Early work assumed that the diffuse radiance was equal over the entire sky hemisphere (the assumption of isotropy), resulting in the so-called "isotropic model". Kondratyev [4] and others have provided the mathematical derivation of the isotropic model, which results in the following equation for the diffuse irradiation on an inclined surface ($E_{ds}\downarrow$) given the equivalent value for the horizontal surface ($E_d\downarrow$)

$$E_{ds}\downarrow = 0.5(1+\cos\alpha)E_d\downarrow. \quad (5)$$

However, the highly variable nature of the distribution of solar radiance over the sky hemisphere (Fig. 1) presents a considerable challenge to those wishing to achieve a more realistic estimate of this quantity for the portion of the sky hemisphere within the field of view of the inclined surface. Observational data have demonstrated the strong variability in both time and instantaneous distribution over the sky hemisphere. For the most straightforward situation under cloudless skies the reader is referred to the work of Steven [5], McArthur and Hay [6], Hooper and Brunger [7] and Valko [8], amongst others. Such studies show the strong forward scattering in the form of the "circumsolar radiation" (i.e. brighter sky around the sun) and the effects of increased scattering as a result of the longer pathlengths near the horizon leading to "horizon brightening" [Fig. 1(a)]. This strong anisotropy in the diffuse radiance was recognised by Temps and Coulson [9] and resulted in a model that accommodated increased radiance near the sun and, to a lesser extent, near the horizon under cloudless conditions.

The second most straightforward case is for radiance distributions associated with heavily overcast skies. Due to the low intensities and apparently simple distribution [Fig. 1(b)] few studies of such situations have been undertaken, though Valko [8] and McArthur and Hay [6] provide examples. The isotropic assumption and associated model are usually considered appropriate in such situations.

Under partly cloudy conditions the radiance distribution is complex and strongly anisotropic [Fig 1(c)]. There are substantial variations in the radiance over both the sky hemisphere and with time. This presents a challenge to those making direct measurements as well as to modellers. The reader is referred to Hay and McKay [2, 3] for further discussion of such challenges and attempts to address them. The modelling approaches will be reviewed later in this paper.

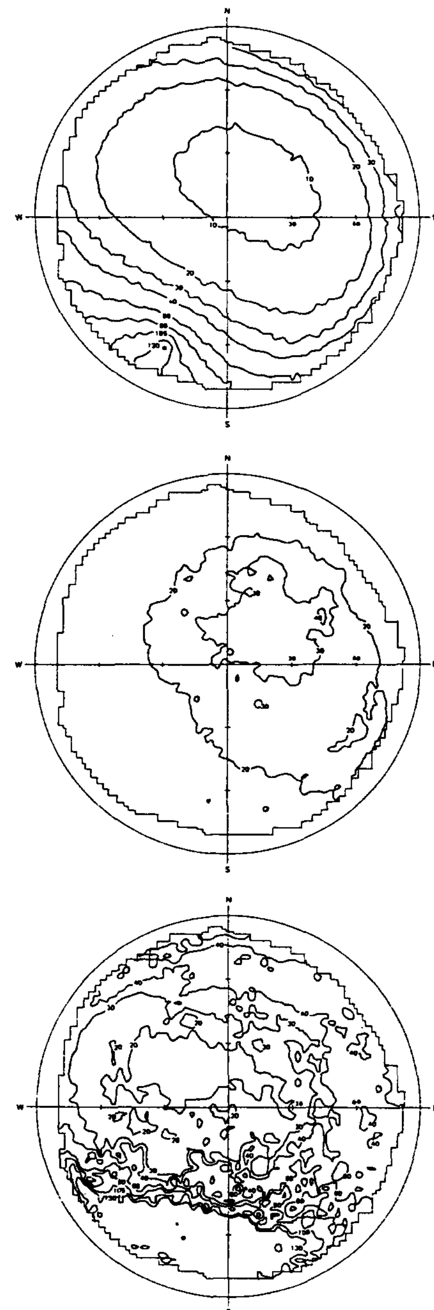

Fig. 1. Diffuse radiance distributions for, from top to bottom: (a) clear sky conditions; (b) overcast sky conditions; and (c) partly cloudy sky conditions. (From McArthur and Hay [6].)

REFLECTED IRRADIANCE FOR AN INCLINED SURFACE

Calculation of the global irradiation for an inclined surface also includes consideration of the fluxes associated with reflection from the adjacent surfaces which can be "seen" by the given inclined surface. Often the computations are simplified by assuming the reflection is isotropic, resulting in

$$E_{rs}\downarrow = 0.5\alpha_s E_g\downarrow(1-\cos\alpha) \quad (6)$$

where

$E_{rs}\downarrow$ = solar radiation reflected onto the inclined surface
α_s = surface albedo
$E_g\downarrow$ = global solar irradiation for a horizontal surface.

Hay [10] has shown that, when the adjacent surface is not horizontal but has a slope of α^*, the last term in eq. (6) should be written $\cos(\alpha+\alpha^*)$, rather than $\cos\alpha$. Directional reflectance models have been developed [9, 11, 12] but there is little evidence of their widespread applicability. Hay and McKay [2] present preliminary results based on data from Vancouver, Canada (see Table 1 and Fig. 2).

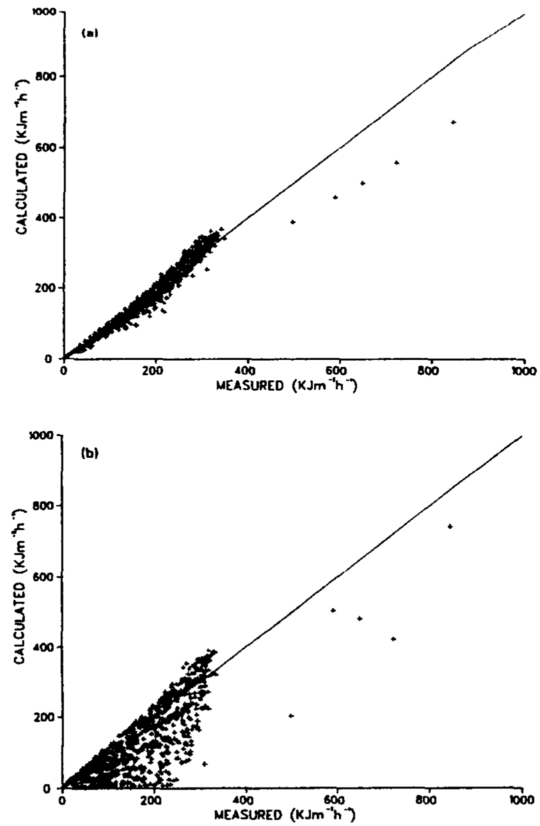

Fig. 2. Comparison between measured and calculated hourly values of reflected radiation incident on a vertical south-facing surface at Vancouver, Canada based on (a) isotropic reflection [eq. (6)] and (b) the anisotropic reflection model of Temps and Coulson [9]. (From Hay and McKay [2].)

MODELS FOR DETERMINING SLOPE IRRADIATION

Following Hay and McKay [2, 3] the models may be grouped into four general categories according to the time scale at which they operate and the radiative flux they attempt to model.

1. *Diffuse irradiance models applicable to time integrals of an hour or less*

In recognition of the typically strong anisotropy that occurs for all but the most heavily overcast skies, many workers have attempted to develop models that take at least some of the more persistent patterns into account. Some attempts are very simple, appealing to computational directness. For example, Bugler [13] accommodated the anisotropy by increasing the direct radiation by 5%, although the original formulation of Bugler should be revised to account for the fact that it treats a portion of the diffuse radiation as both isotropic and circumsolar. Bright sunshine data can provide an indication of the amount of direct and circumsolar radiation, with both these being treated as one component. Cohen and Zerpa [14] also provide an empirical correction to the isotropic model by way of the ratio of the observed global irradiation to the corresponding extraterrestrial value. Ineichen [12] assumed that a portion of the diffuse radiation, equal to at least 6% of the direct radiance, is circumsolar and that the portion increases linearly with the optical air mass. The remainder of the diffuse irradiation is treated as being isotropic.

Hay [15] and Hay and Davies [16] develop an anisotropic slope radiation model based on the following reasoning: when no direct radiation is observed over an hour long period (i.e. direct transmission is zero) skies may be assumed overcast and the isotropic model [eq. (5)] is appropriate for that hour; in the absence of an atmosphere all the radiation is direct (i.e. direct transmission equals 1) and thus, in this

Table 1. Validation of two ground reflectance models for 1 year of data for a vertical south-facing slope at Vancouver, Canada. (After Hay and McKay [2])

	Isotropic reflection [eq. (6)]	Anisotropic reflection (Temps and Coulson [9])
Observed mean ($MJ\ m^{-2}\ h^{-1}$)	0.123	0.123
Calculated mean ($MJ\ m^{-2}\ h^{-1}$)	0.117	0.091
Correlation coefficient	0.991	0.857
Standard error ($MJ\ m^{-2}\ h^{-1}$)	0.013	0.051

limiting case, all the radiation can be treated according to eq. (1). Where the direct transmission lies somewhere between these extremes it is assumed that the hourly integrated direct radiation transmission defines the portion of the diffuse radiation to be treated as isotropic and the portion to be treated as circumsolar (i.e. direct). The direct transmission is used to define an "anisotropy index" (κ) such that:

$$\kappa = S/S_0 \qquad (7)$$

where S_0 is the solar constant.

Under such assumptions the diffuse radiation from the sky hemisphere intercepted by the slope is given by

$$E_{ds}\downarrow = E_d\downarrow[\kappa \cos i/\cos z + 0.5(1.0-\kappa)(1.0+\cos \alpha)]. \qquad (8)$$

Hay and McKay [2] present a more critical evaluation of this model, concluding that while the simplifying assumptions are seldom met, the departures have an insignificant effect on the performance of the anisotropic model. Klucher [17] used a similar approach to that proposed by Hay [10, 15] except that he argued for the use of Temps and Coulson's cloudless sky irradiance distribution to describe clear sky conditions and the ratio $E_d\downarrow/E_g\downarrow$ or $(E_d\downarrow/E_g\downarrow)^2$ rather than κ. Josefsson (personal communication) has suggested an empirical correction to the Hay algorithm to incorporate the effects of horizon darkening under overcast skies and horizon brightening for cloudless skies.

Subsequently Skartveit and Olseth [18] modified the Hay algorithm to include sky radiance anisotropy for overcast as well as cloudless skies. They assumed that under the former condition 30% of the horizontal diffuse irradiation may be treated as collimated radiation from the zenith with the remainder being an isotropic radiance from the sky dome.

In the model presented by Gueymard [19] the conversion factor for the inclined surface irradiance is expressed as a function of the solar zenith angle, and angle of incidence of the direct solar beam (with respect to the slope normal), slope angle and cloud opacity. Separate parameterizations of the sky radiance patterns are provided for clear and cloudy skies. These are subsequently combined in a linear manner according to the observed cloud opacity or the ratio of the diffuse and global irradiation for the horizontal surface.

Perez *et al.* [20] and Perez and Stewart [21] developed a model that attempts to replicate both circumsolar and horizon brightening by superimposing both a disc and a horizontal band with increased radiance upon the isotropic radiance field. The appropriate enhancement factors were evaluated empirically and expressed as a function of the diffuse and global irradiances and of the solar zenith angle. Perez *et al.* [22] have subsequently simplified the model and extended its empirical base.

2. *Direct irradiance models applicable to time integrals of a day or longer*

The major challenge when calculating the irradiance for daily (or longer) time intervals is to integrate the non-linear radiant intensity and geometric relationships between the sun and the normal to the slope in such a way that the model is valid over the longer time scales. This involves accommodating such factors as the diurnal distribution of the direct radiation, diurnal variations in the atmospheric transmittance and the double sunrises and sunsets which occur on some poleward-facing slopes in the summer months.

Following the work of Jones [23] and Revfeim [24] one approach is to assume that the ratio of the daily integrals of the direct radiation for the inclined and horizontal surfaces is the same at the earth's surface as it is at the top of the atmosphere. Hay [15] adopted this approach and developed a model that requires only bright sunshine and surface albedo data. Revfeim [24] went on to recognise variations in the atmospheric transmittance associated with diurnal variations in the solar zenith angle and he recommended the use of a weighting function defined by the expression $\cos(\pi h/2H)$ where H is the half day length. Bremer [25] used a weighting function of $\cos z$.

Klein and Theilacker [26] approached the problem by way of empirical relationships between hourly and daily irradiation. This provided the ratio of the daily integrals of the direct irradiances for the inclined and horizontal surfaces, with the effects of atmospheric attenuation being incorporated implicitly in the model. On the other hand, Page [27] incorporates the effects of atmospheric attenuation by way of a standard direct radiation curve representative of a tropical atmosphere.

3. *Sky diffuse irradiation models applicable to time integrals of a day or longer*

Four distinct approaches may be followed when daily data are used to estimate daily or longer term values of the diffuse irradiation for an inclined surface.

(a) *One calculation per day*. Models in this category either exclude dependencies on solar position (e.g. the isotropic model) or assume for the conversion factor a value representative of the entire day.

(b) *Hourly calculations.* Models in this grouping essentially follow the approach used by Liu and Jordan [28], namely estimate hourly irradiances from the daily total and then sum the values provided by the hourly slope model in order to determine the daily slope irradiation. This is the procedure used by Gueymard [19]. Jain *et al.* [30] also provide a method for obtaining monthly average instantaneous global and diffuse radiation from the respective daily values. An important feature of this method is that it is able to incorporate morning–afternoon asymmetries in the instantaneous radiation.

(c) *Two calculations on the daily partition.* Algorithms in this category assume that the diffuse slope factor may be linearly interpolated between the corresponding values for that part of the day when only diffuse radiation is received by the slope and the parts of the day when there is at least the potential for both direct and diffuse radiation to be incident upon the slope. Effective values of both the solar zenith angle and the angle of incidence of the direct radiation on the slope must be determined for each of these portions of the day. In addition, the model must partition the daily diffuse irradiation into the amounts occurring in each part of the day [i.e. period(s) of diffuse and (potential) direct and period(s) of diffuse only]. The effective solar angles during the period(s) of potential direct radiation may be determined using any of the daily direct irradiance models described in Section 2, above. The partitioning of the daily diffuse radiation requires knowledge of the assumed diurnal distribution of the diffuse irradiance and the relationships described by Collares-Pereira and Rabl [30] can be used for this purpose.

(d) *Use of "representative" estimates for each partition.* Another possible approach is to determine "representative" hourly irradiation estimates for each of the two partitions identified above and then apply any of the hourly diffuse models described in Section 1, above. Such an approach has been evaluated by Hay and McKay [3]. The challenge is to provide the models with representative hourly values consistent with the effective angles associated with the two partitions.

4. *Global irradiance models applicable to time integrals of a day or longer*

Algorithms in this category are either developed explicitly for such calculations or they are a combination of one of the daily direct irradiance models and a daily or hourly sky diffuse model, where the latter is used with a partitioning algorithm allowing it to be applied to daily data. An example of the latter is the model proposed by Gueymard [19]. On the other hand, the McFarland model employs empirical coefficients derived using least-squares regressions between the monthly mean global irradiation for the horizontal and inclined surfaces and a solar position parameter (latitude minus mid-month solar declination) and a clearness parameter (fraction of the extraterrestrial solar irradiation). The McFarland model cannot be extended to poleward facing surfaces since no regressions were performed for such slope orientations.

MODEL PERFORMANCE

Hay and McKay [3] present an evaluation of a wide range of models for 27 data sets from throughout the world. An example of the graphical output in the validation process is given here as Fig. 3. For hourly calculations of slope irradiation the diffuse model of Perez is clearly superior. Despite the use of empirical coefficients it has widespread geographical applicability and maintains its superiority through a wide range of surface orientations and climatic conditions. For the daily calculations of the direct irradiance the algorithm developed by Page showed smaller errors than other models. The model can be made very efficient by integrating the polynomial expression for the standard direct radiation curve [3], thereby avoiding the need to make iterative calculations at small time steps during the day. The validation studies also showed that an algorithm developed by Gueymard was very effective at determining the diffuse irradiance on the basis of daily calculations. The isotropic model was ineffective. When the global irradiation is calculated using daily rather than hourly methods large errors occur in both the short- and long-term estimates. A combination of the approaches proposed by Gueymard and Revfeim yield the best results for daily calculations.

Perez *et al.* [22] have also undertaken comparisons of various models. Their results are reproduced in Tables 2 and 3. They confirm the superiority of the Perez model. Gopinathan [31] evaluated the isotropic model of Liu and Jordan and the Hay anisotropic model using data for locations in Lesotho. He concluded that there was no significant difference in the performance of the two models. On the other hand, Reindl *et al.* [32], using data from Albany and San Antonio, U.S.A., showed that the isotropic model was poor at estimating slope irradiance while a series of anisotropic models (Hay, Perez and some derivatives of these) showed similar, superior performance. Hay [1] has also shown the superiority of anisotropic

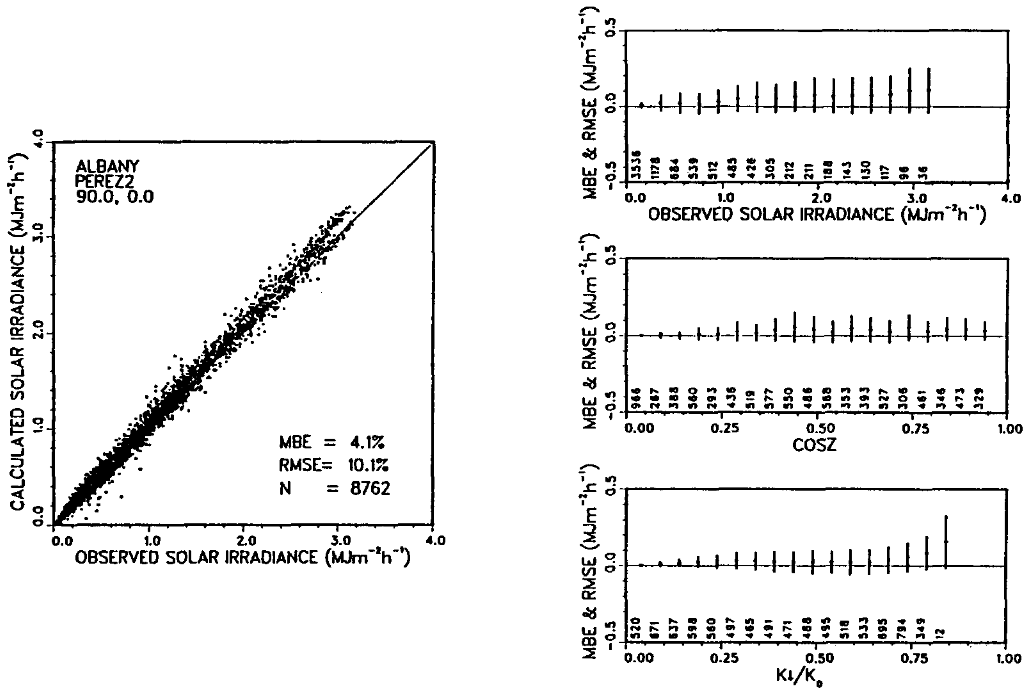

Fig. 3. Example of graphical output for the evaluation of the Perez model for estimating the hourly diffuse irradiation on an inclined surface. (From Hay and McKay [3].)

Table 2. Results of validation for selected hourly diffuse irradiance models. Data for Trappes and Carpentras. Errors are for a composite of five slopes. (After Perez *et al.* [23])

Model	Root mean square error ($kJ\ m^{-2}\ h^{-1}$)	Mean bias error ($kJ\ m^{-2}\ h^{-1}$)
Isotropic	150.4	72.5
Hay	99.3	41.2
Klucher	131.3	61.6
Perez	51.1	11.2

Table 3. Results of validation for selected hourly diffuse irradiance models. Data for Albany, U.S.A. Errors are for a composite of five slopes. (After Perez *et al.* [23])

Model	Root mean square error ($kJ\ m^{-2}\ h^{-1}$)	Mean bias error ($kJ\ m^{-2}\ h^{-1}$)
Isotropic	125.5	46.2
Hay	87.2	23.3
Klucher	120.1	53.6
Perez	49.1	11.8

models and of those that calculate the irradiation on an hourly as opposed to a daily basis.

Gopinathan [31] has compared the performance of the isotropic and Hay models for estimating the global irradiation for an inclined surface. He used data for two equator-facing surfaces inclined at 29 and 19° from the horizontal at Maseru, Lesotho, Southern Africa. Table 4 summarizes the results. Gopinathan concluded that, in the case of Lesotho, the isotropic and Hay models are of comparable accuracy and either one can be used to estimate the monthly mean daily global radiation for inclined surfaces. He went on to use the isotropic model to estimate the global radiation on surfaces of other orientation and deduced that a surface inclined at the latitude of the location collects the maximum energy on a year round basis. Optimum tilt angles in Lesotho are latitude minus 10° in summer and latitude plus 20° in winter.

Hay [1] has verified a variety of hourly and daily (or longer) models using data from both Vancouver and Toronto, Canada and for a wide variety of slope orientations. Evaluations were performed for the Hay and isotropic diffuse models, for the weighted version of the Revfeim [33] model and for that of Liu and Jordan [28]. Substantial improvement in the diffuse

Table 4. Validation statistics for two models used to estimate global irradiation for inclined surfaces at Maseru, Lesotho. (After Gopinathan [34])

	Slope 19°		Slope 29°	
	Isotropic	Hay	Isotropic	Hay
Mean bias error (kJ m^{-2} day^{-1})	−193	226	−240	244
Root mean square error (kJ m^{-2} day^{-1})	373	492	244	672

estimates were obtained using the Hay model rather than assuming isotropy for the diffuse radiance. Both the direct radiance models had difficulty accommodating the diurnal characteristics of the irradiance and consequently modelling errors were substantial for slopes not directly facing the equator. For equator-facing slopes a saving in data requirements and computation effort can be achieved with little additional error by using a daily direct irradiance model.

CONCLUSIONS

A wide variety of methods exist for determining the irradiation for an inclined surface given values for a horizontal surface. In addition to approaches that incorporate the anisotropic distribution of diffuse radiance over the sky hemisphere, there are models which use daily (or longer) time scales and procedures for estimating the reflected radiation. There is a need to provide further guidance as to which approaches are superior and effective. Evidence is strong regarding the superiority of anisotropic models (e.g. those of Perez and Hay) for estimating the diffuse irradiation and the better performance of hourly algorithms over their daily equivalents for calculation of the direct irradiation for the slope.

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