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Physical Theory of the Electric Wave-Filter

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NOTE: The electric wave-filter, an invention of Dr. Campbell, is one of the most important of present day circuit developments, being indispensable in many branches of electrical communication. It makes possible the separation of a broad band of frequencies into narrow bands in any desired manner, and as will be gathered from the present article, it effects the separation much more sharply than do tuned circuits. As the communication art develops, the need will arise to transmit a growing number of telephone and telegraph messages on a given pair of line wires and a growing number of radio messages through the ether, and the filter will prove increasingly useful in coping with this situation. The filter stands beside the vacuum tube as one of the two devices making carrier telegraphy and telephony practicable, being used in standard carrier equipment to separate the various carrier frequencies. It is a part of every telephone repeater set, cutting out and preventing the amplification of extreme line frequencies for which the line is not accurately balanced by its balancing network. It is being applied to certain types of composited lines for the separation of the d.c. Morse channels from the telephone channel. It is finding many applications to radio of which multiplex radio is an illustration. The filter is also being put to numerous uses in the research laboratory.

The present paper is the first of a series on the electric wave-filter to be contributed to the Technical Journal by various authors. Being an introductory paper the author has chosen to discuss his subject from a physical rather than mathematical point of view, the fundamental characteristics of filters being deduced by purely physical reasoning and the derivation of formulas being left to a mathematical appendix.—*Editor.*

THE purpose of this paper is to present an elementary, physical explanation of the wave-filter as a device for separating sinusoidal electrical currents of different frequencies. The discussion will be general, and will not involve assumptions as to the detailed construction of the wave-filter; but in order to secure a certain numerical concreteness, curves for some simple wave-filters will be included. The formulas employed in calculating these curves are special cases of the general formulas for the wave-filters which are, in conclusion, deduced by the method employed in the physical theory.

All the physical facts which are to be presented in this paper, together with many others, are implicitly contained in the compact formulas of the appendix. Although only comparatively few words of explanation are required to derive these formulas, they will not be presented at the start, since the path of least resistance is to rely implicitly upon formulas for results, and ignore the troublesome question as to the physical explanation of the wave-filter. In order to examine directly the nature of the wave-filter in itself, as a physical structure, we proceed as though these formulas did not exist.

It is intended that the present paper shall serve as an introduction to important papers by others in which such subjects as transients on wave-filters, specialized types of wave-filters, and the practical design of the most efficient types of wave-filters will be discussed.[1]

Definition of Wave-Filter

A wave-filter is a device for separating waves characterized by a difference in frequency. Thus, the wave-filter differentiates between certain states of motion and not between certain kinds of matter, as does the ordinary filter. One form of wave-filter which is well known is the color screen which passes only certain bands of light frequencies; diffraction gratings and Lippmann color photographs also filter light. Wave-filters might be constructed and employed for separating air waves, water waves, or waves in solids. This paper will consider only the filtering of electric waves; the same principles apply in every case, however.

In its usual form the electric wave-filter transmits currents of all frequencies lying within one or more specified ranges, and excludes currents of all other frequencies, but does not absorb the energy of these excluded frequencies. Hence, a combination of two or more wave-filters may be employed where it is desired to separate a broad band of frequencies, so that each of several receiving devices is sup-

[1] I take pleasure in acknowledging my indebtedness to Mr. O. J. Zobel for specific suggestions, and for the light thrown on the whole subject of wave-filters by his introduction of substitutions which change the propagation constant without changing the iterative impedance.

plied with its assigned narrower range of frequencies. Thus, for instance, with three wave-filters the band of frequencies necessary for ordinary telephony might be transmitted to one receiving device, all lower frequencies transmitted to a second device, and all higher frequencies transmitted to a third device—separation being made without serious loss of energy in any one of the three bands.

By means of wave-filters interference between different circuits or channels of communication in telephony and telegraphy, both wire and radio, can be reduced provided they operate at different frequencies. The method is furthermore applicable, at least theoretically, to the reduction of interference between power and communication circuits. The same is true of the simultaneous use of the ether, the earth return, and of expensive pieces of apparatus employed for several power or communication purposes. In all cases the principle involved is the same as that of confining the transmission in each circuit or channel to those frequencies which serve a useful purpose therein and excluding or suppressing the transmission of all other frequencies. In the future, as the utility of electrical applications becomes more widely and completely appreciated, there will be an imperative necessity for more and more completely superposing the varied applications of electricity; it will then be necessary, to avoid interference, to make the utmost use of every method of separating frequencies including balancing, tuning, and the use of wave-filters.

Definition of Artificial Line

The wave-filter problem in this paper is discussed as a phase of the artificial line problem, and it is desirable to start with a somewhat generalized definition of the artificial line. The definition will, however, not include all wave-filters or all artificial lines, since a perfectly general definition is not called for here. Even if an artificial line is to be, under certain wave conditions, an imitation of, or a substitute for, an actual line connecting distant points, hardly any limitation is thereby imposed upon the structure of the device; an actual line need not be uniform but may vary abruptly or gradually along its length and may include two, three, four or more transmission conductors of which one may be the earth. Having indicated that wave-filters partake of somewhat this same generality of structure, the present paper is restricted to wave-filters coming under the somewhat generalized artificial line specified by the following definition:

An artificial line is a chain of networks connected together in sequence through two pairs of terminals, the networks being identical but other-

wise unrestricted. This generalized artificial line possesses the well-known sectional artificial line structure but it need not be an imitation of, or a substitute for, any known, real, transmission line connecting together distant points. The general artificial line is shown by Fig. 1 where N, N, . . . are the identical unrestricted networks which may contain resistance, self-inductance, mutual inductance, and capacity.

In discussing this type of structure as a wave-filter, the point of view of an artificial line is adopted for the reason that it is advantageous to regard the distribution of alternating currents as being dependent upon both propagation and terminal conditions, which are to be separately considered. In this way the attenuation, or

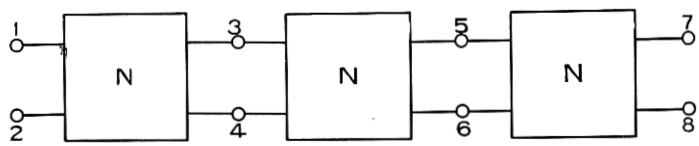

Fig. 1—Generalized Artificial Line as Considered in the Present Paper, where N, N, . . . are Identical Arbitrary Electrical Networks

falling off, of the current from section to section may be most directly studied. Terminal effects are not to be ignored, but are allowed for, after the desired attenuation effects have been secured, possibly by an increase in the number of sections to be employed.

The fundamental property of this generalized artificial line, which includes uniform lines as a special case, is the mode in which the wave motion changes from one section to the next, and may be stated as follows:

Wave Propagation Theorem

Upon an infinite artificial line a steady forced sinusoidal disturbance falls off exponentially from one section to the next, while the phase changes by a constant amount. Reversing the direction of propagation does not alter either the attenuation or phase change. When complex quantities are employed the exponential includes the phase change.[2] This theorem is proved, without mathematical equations, by observing

[2] This theorem is not new, but it is ordinarily derived by means of differential or difference equations whereas it may be derived from the most elementary general considerations, thus avoiding all necessity of using differential or difference equations, as illustrated in my paper "On Loaded Lines in Telephonic Transmission" (*Phil. Mag.*, vol. 5, pp. 313–331, 1903). In that discussion, as well as in this present one, it is tacitly assumed that the line is either an actual line with resistance, or the limit of such a line as the resistance vanishes, so that the amplitude of the wave never increases towards the far end of an infinite line.

that the percentage reduction in amplitude and the change in phase, in passing from the end of one section to the corresponding point of the next section, do not depend upon either the absolute amplitude or phase; they depend, instead, only upon the magnitudes, angles and interconnections of the impedances between the two points and of the impedances beyond the second point. These impedances are, since the line is assumed to be periodic and infinite, identically the same for corresponding points between all sections of the line, and, therefore, the relative changes in the wave will be identical at corresponding points in all sections. This proves the exponential falling off of the disturbance and the constancy of phase change; the ordinary reciprocal property shows that the wave will fall off identically whichever be the direction of propagation. By the superposition property it follows that the steady state on any finite portion of a periodic recurrent structure must be the sum of two equally attenuated disturbances, one propagated in each direction.

The fundamental wave propagation theorem may be generalized for any periodic recurrent structure irrespective of the number and kind of connections between periodic sections, provided the disturbance is such as to remain similar to itself at corresponding points of each of these connections.

Equivalent Generalized Artificial Line

Since, at a given frequency, any network employed solely to connect a pair of input terminals with a pair of output terminals may be replaced by either three star-connected impedances or three delta-connected impedances, the general artificial line of Fig. 1 may be

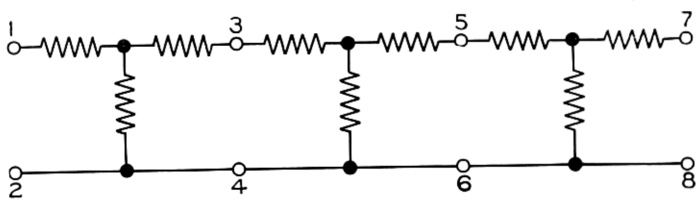

Fig. 2—Equivalent Artificial Line Obtained by Substituting Star Impedances

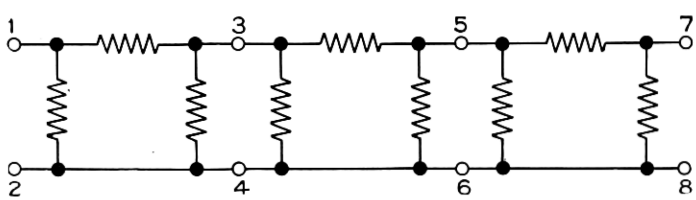

Fig. 3—Equivalent Artificial Line Obtained by Substituting Delta Impedances

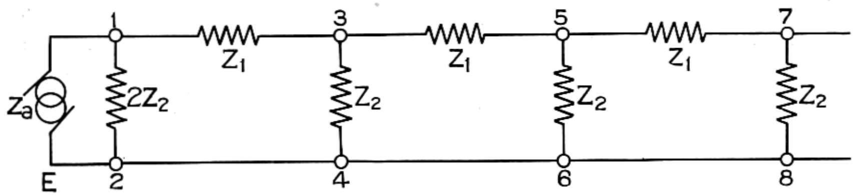

Fig. 4—Equivalent Ladder Artificial Line

replaced by the equivalent artificial line of either Fig. 2 or Fig. 3. By combining the series impedances in Fig. 2 and the parallel impedances in Fig. 3, the equivalent line in Fig. 4 is obtainable. The two ways of arriving at Fig. 4 give different values for the series and shunt impedances Z_1, Z_2, and different terminations for the line, but the propagation of the wave is the same in both cases, since the assumed substitutions are rigorously exact. While Fig. 4 may be considered as the generalized artificial line equivalent to Fig. 1, this requires including in Z_1 and Z_2 impedances which cannot always be physically realized by means of two entirely independent networks, one of which gives Z_1 and the other Z_2. This restriction is of no importance when we are discussing the behavior of the generalized artificial line at a single frequency; accordingly, the ladder artificial line is suitable for this part of the discussion. When we come to the more specific correlation of the behavior of the generalized artificial line at different frequencies, it will be found more convenient to replace the ladder artificial line by the lattice artificial line, which avoids the necessity of considering any impedances which are not individually physically realizable.

The equivalence between Figs. 1 and 4 is implicitly based upon the assumption that it is immaterial, for artificial line uses, what absolute potentials the terminals 1, 2; 3, 4; 5, 6; etc. have—this leaves us at liberty to connect 2, 4, 6, etc., together, so long as we maintain unchanged the differences in potential between 1 and 2, 3 and 4, etc. Instead of connecting 2 and 4 we might equally well connect 2 and 3, and then Z_1 would connect 1 and 4 as in Fig. 5; with these

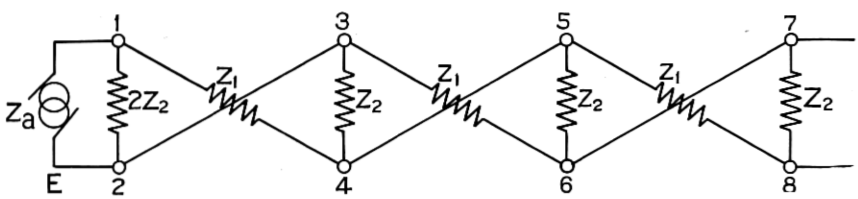

Fig. 5—Equivalent Artificial Line with Crossed Impedances

cross-connections the propagation still remains unchanged. We have again obtained Fig. 4 with no circuit difference except the interchange of terminals 3 and 7 with terminals 4 and 8; or, if this is ignored, a reversal in the sign of the current at alternate pairs of terminals. This shows that the reversal of the current in alternate sections of Fig. 4 may not be of primary significance, since networks which are essentially equivalent have reversed currents.

In order to deal, at the start, with only the simpler terminal conditions, we may consider the line to begin with only one-half of the series impedance Z_1, or only one-half of the bridged admittance $1/Z_2$. These mid-points are called the mid-series and mid-shunt points; knowing the results of termination at either of these points, the effect of termination at any other point may be readily determined. For Fig. 4 termination at mid-shunt has been chosen so that each section of the line adds a complete symmetrical mesh to the network.

An alternator, introducing an impedance Z_a, is shown as the source of the steady-state sinusoidal current in Fig. 4. Assume that the impedance Z_a is variable at pleasure, and that it is gradually adjusted to make the total impedance in the generator circuit vanish,—in this case no e.m.f. will be required to maintain the forced steady-state which becomes a free oscillation. If, in addition, it is assumed that the line has an infinite number of sections, this required value of Z_a will be the negative of the mid-shunt iterative impedance[3] of the artificial line, which will be designated as K_2. The first shunt on the line now includes $-K_2$ in parallel with $2Z_2$ so that its total impedance is, say, $Z' = -2Z_2K_2/(2Z_2-K_2)$. The infinite line with its first shunt given the special value Z' is thus capable of free oscillation.

It is possible to simplify this infinite oscillating circuit by cutting off any part of it which has the same free period as the whole circuit. The entire infinite line beyond the second shunt 3, 4 certainly has this same free period, provided its first shunt also has the impedance Z'. Conceive the shunt Z_2 at 3, 4 as replaced by the four impedances $2Z_2$, $2Z_2$, $+K_2$ and $-K_2$ all in parallel; the first and last, which together make the Z' required by the infinite line, leave $2Z_2$ and

[3] The "iterative impedance" of an artificial line is the impedance which repeats itself when one or more sections of the artificial line are inserted between this impedance and the point of measurement. It is thus the impedance of an infinite length of any actual artificial line, regardless of the termination of the remote end of the line. In general, its value is different for the two directions of propagation, but not when the line is symmetrical, as at mid-series and mid-shunt. The values at these points are denoted by K_1 and K_2. "Iterative impedance" is employed because it is a convenient term which is distinctive and describes the most essential property of this impedance; it seems to be more appropriate than "characteristic impedance," "surge impedance" and the other synonyms in use.

$+K_2$ in parallel, which have the impedance $Z'' = +2Z_2K_2/(2Z_2+K_2)$. Removing Z' together with the infinite line on the right there remains on the left a closed circuit made up of the three impedances Z_1, Z' and Z'' in series.

After the division, the infinite line on the right will continue, without modification, to oscillate freely, since it is an exact duplicate of the original oscillating line, and so must maintain the free oscillation already started. Since it oscillates freely by itself, it had originally no reaction upon the simple circuit from which it was separated; this simple circuit on the left must thus also continue its own free oscillations without change in period or phase.

We might continue and subdivide the entire infinite line into identical simple circuits but it is sufficient to consider this one detached circuit, which is shown separately in two ways by Fig. 6, since from

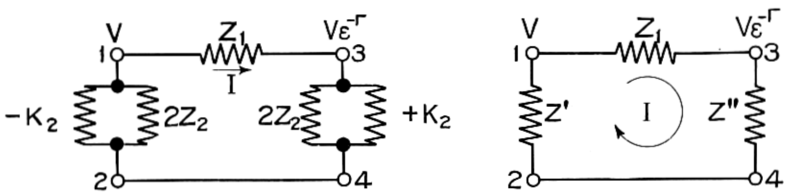

Fig. 6—Equivalent Section of Fig. 4 Terminated for Free Oscillation

its free oscillations the mathematical formulas for the steady-state propagation in the artificial line may be derived. This is deferred, however, until after the physical discussion is completed, so as to leave no room for doubt that the essentials of the physical theory are really deduced without the aid of mathematical formulas.

The generalized artificial line, if made up entirely of pure resistances, will attenuate all frequencies alike, and the entire wave will be in the same phase; this remains true, whatever be the impedance of the individual branch of the network, provided the ratio of the impedances of all branches is a constant independent of the frequency. This is precisely the condition to be avoided in a wave-filter; branches must not be similar but dissimilar as regards the variation of impedance with frequency. This calls for inductance and capacity with negligible resistance, so that there is an opportunity for the positive reactance of one branch to react upon the negative reactance of another branch, in different proportions at different frequencies. Assuming the unit network N of Fig. 1 to be made up of a finite number of pure reactances, the equivalent impedances Z_1 and Z_2 of Figs. 4 and 5 must also be pure reactances. Under this assumption