The Arctangent Approach of Digital PGC Demodulation for Optic Interferometric Sensors

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ABSTRACT

The development of the optic interferometric sensors is partly restricted with the demodulation technique. The Phase Generated Carrier (PGC) modulation scheme is a useful demodulation method for interferometric sensors, for it is simple and accurate. At first, the PGC scheme is demodulated by analog demodulation techniques. Its capability is limited by the electronic devices, and not fit to the large-scale sensors application. In the past few years, a number of digital demodulation processing approaches have been investigated. Another arctangent approach based on digital demodulator exhibits more advantages than the traditional Differentiate and Cross Multiply (DCM) approach, which has a small measurement range and rather complex circuit. In this paper, the arctangent approach on the PGC configuration used in the array of optical fiber interferometer are discussed and emulated. The measurement range, operation complexity, quality of operation, noise performance, and applicability are compared among the variant arctangent approaches and the DCM approach. The variant arctangent approaches and the DCM approach will be influenced by differently factors, such as, the intensity of the light source, the shift of visibility of the interference fringes due to the change of the state of polarization in fiber, the phase delay. The dependence on these influences of these approaches is analyzed in detail. Their effects and removal methods are validated through emulation.

Keywords: PGC, arctangent, demodulation, interferometric, fiber optic,

1. INTRODUCTION

A lot of types of optical fiber sensors are based on the interferometric structure for its high sensitivity. In the interferometer, there are many methods of detecting scheme relative optical phase shift between the signal and reference fibers. Several detection schemes are currently available, such as passive homodyne, active homodyne, true heterodyne, synthetic heterodyne and homodyne demodulation using phase generated carrier (PGC). Among them, the homodyne detection scheme using PGC is capable of high sensitivity, high dynamic range, and good linearity, so it is most widely used now. The traditional PGC scheme uses the Differentiate and Cross Multiply (DCM) approach to recover the time derivative of the phase signal from the interferometric optic signal, which has a small measurement range and rather complex circuit. The output of DCM is correlative with the light power so that it makes the sensors are difficulty to use in array. And also correlative with the initial phase of the integral arithmetic which is used in DCM and is uncertain. The arctangent approach would improve the performance of the interferometric sensors, and is insensitive to the light power and the initial phase.

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2. THEORY

2.1 Interferometric optical signal with PGC homodyne scheme

The classical PGC homodyne scheme was first reported by Dandridge et al. in 1982 [1]. The light intensity of the interferometric optical sensors can be expressed as:

$$I = I_0(1 + V\cos\theta(t)) = I_0 + I_0 V\cos\theta(t) \tag{1}$$

where I_0 is the light intensity, V is the visibility of the interference, and $\theta(t)$ the phase difference between signal and reference arms of the interferometer. After applying a carrier phase modulation, of angular frequency ω_0 and amplitude C to the output, $\theta(t)$ can be expressed as:

$$\theta(t) = C\cos\omega_0 t + \phi(t) \tag{2}$$

where $\phi(t)$ includes both the desired signal and undesired signals. The undesired signals are laser induced phase noise, thermal drifts, and pressure changes sensed by the interferometer. After combining the equation (1) and (2) and expanding into Bessel functions the following is obtained:

$$\begin{aligned} I = I_0 + I_0 V\{[J_0(C) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(C)\cos(2k\omega_0 t)]\cos\psi(t) \\ -2[\sum_{k=0}^{\infty}(-1)^k J_{2k+1}(C)\cos((2k+1)\omega_0 t)]\cos\psi(t)\} \end{aligned} \tag{3}$$

If equation (3) is multiplied by the carrier reference and separately by twice the carrier reference, a pair of signals is produced. This pair, once lowpass filtered at the baseband, can be expressed as:

$$L1 = BJ_1(C)\sin\phi(t) \tag{4}$$

$$L2 = BJ_2(C)\cos\phi(t) \tag{5}$$

2.2 DCM approach

Differentiate and Cross Multiply (DCM) approach was presented also by Dandridge et al. at 1982 to recover the time derivative of the phase signal from the interferometer. The approach was realized in analog circuits that time. It is shown in Fig. 1.

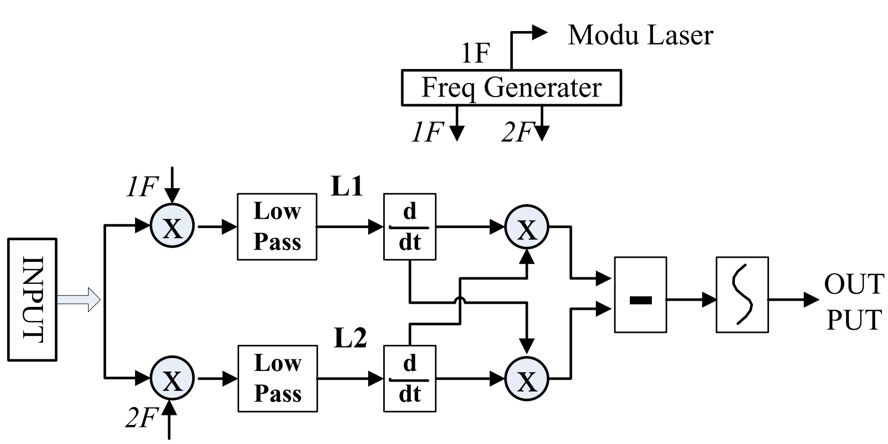

Fig. 1: DCM approach of PGC

The end output of $L1$ and $L2$ after DCM approach can be expressed as:

$$SDCM = B^2 J_1(C) J_2(C) \phi(t) \tag{6}$$

which is sensitive to the light power.

2.3 Arctangent approach

Since the digital signal processing has developed much these years, the arctangent approach is used to solve the Phase Recover problem in PGC. It is shown in Fig. 2. The arctangent function could be realized in PC or DSP and etc., through approximate method. In this paper, a Table-Lookup method is given to decrease the complexity, ensuring the enough precision at the same time. The Table-Lookup only gives the arctangent value between 0 and $\pi/4$. The real value between 0 and 2π could be calculated by the octant Table 1 basing on $L1$, $L2$ and $Diff$ which is defined as:

$$Diff = L1/L2 \tag{7}$$

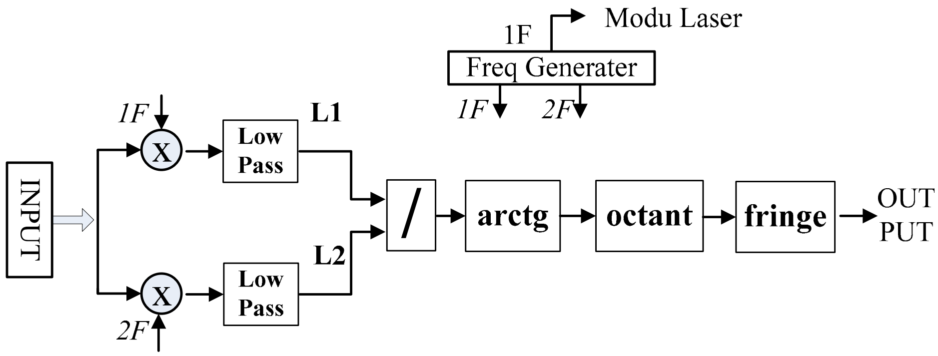

Fig. 2: Arctangent approach of PGC

Sign of *L1*	Sign of *L2*	Value of *Diff*	arctan Φ
+	+	<1	atan(*Diff*)
+	+	>1	π /2- atan(1/*Diff*)
-	+	>1	π /2+ atan(1/*Diff*)
-	+	<1	π - atan(*Diff*)
-	-	<1	π +atan(*Diff*)
-	-	>1	3 π /2- atan(1/*Diff*)
+	-	>1	3 π /2+ atan(1/*Diff*)
+	-	<1	2 π - atan(*Diff*)

Table 1: Octant table

Then a fringe counter should be added to expand the value range when the phase goes more than 2π or less than 0. The actual value range is decided by the sample rate and the frequency of the signal. The difference of the adjacent two values must be less than π, which limits the measurement range of the arctangent approach.

The arctangent function could be realized through many methods, such as FDLIBM developed at SunSoft and Taylor polynomial approximation. They are all too complex to come true for digital signal processing. Here gives a Table-Lookup method to get the arctangent function with less computation and less complexity. It gives 16 sample points between 0 and 1 with equal spacing, and the arctangent values are between 0 and $\pi/4$. They are all remembered into the Table-Lookup of arctangent function. When the requested value is among them 16 points, the arctangent value could be got from the Table directly. If the requested value is not among the 16 points, the nearest two points are picked up to calculate the arctangent value through simple linear fitting method. Such a simple method still could get the enough precision. If the more strict precision is demanded, more than 16 sample points would give the better performance. The only cost is the memory space to remember the value of the more sample points. The demand is easy to meet for the digital signal processing.

3. ANALYSIS

3.1 The inaccuracy of the Table –Lookup arctangent method

The error of the Table-Lookup arctangent method is shown in Fig. 3, the left Fig. is the absolute error and the right is the relative error. Contrastively, the absolute and relative error of the Taylor polynomial approximation (5-rank approximation) is shown in Fig. 4. It can be seen that the Table-Lookup has less error than the 5-rank Taylor polynomial approximation, especially when the phase is near $\pi/4$. The error of the Table-Lookup method is the biggest when the value is lying in the middle of every adjacent two sample points. And the bigger the input value is, the bigger the error is. While the error of Taylor polynomial approximation always grows more when the input value goes up.

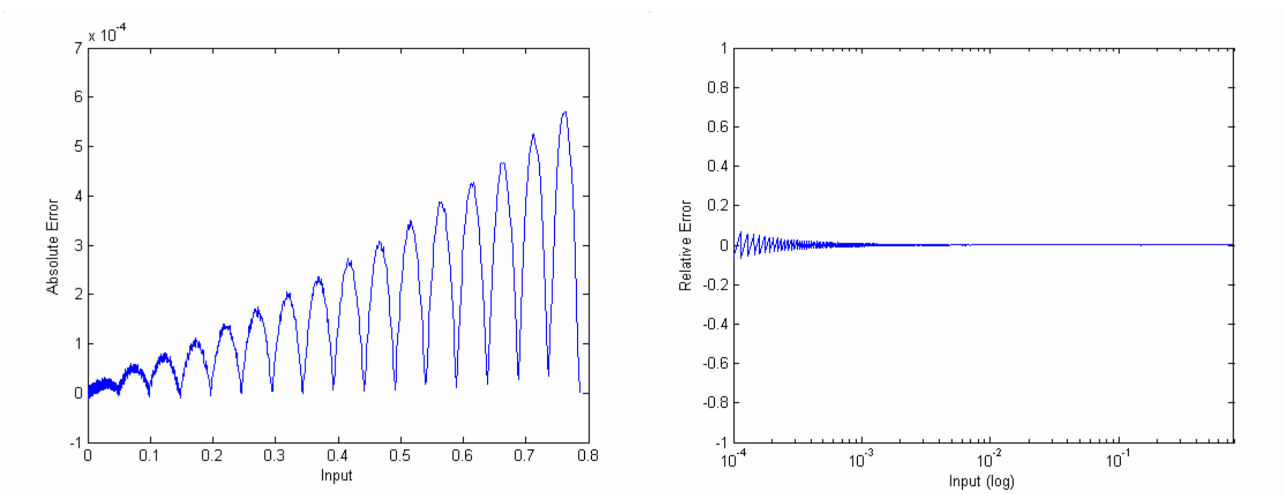

Fig. 3: The error of the Table-Lookup arctangent method;

The left Fig. is the absolute error and the right is the relative error.

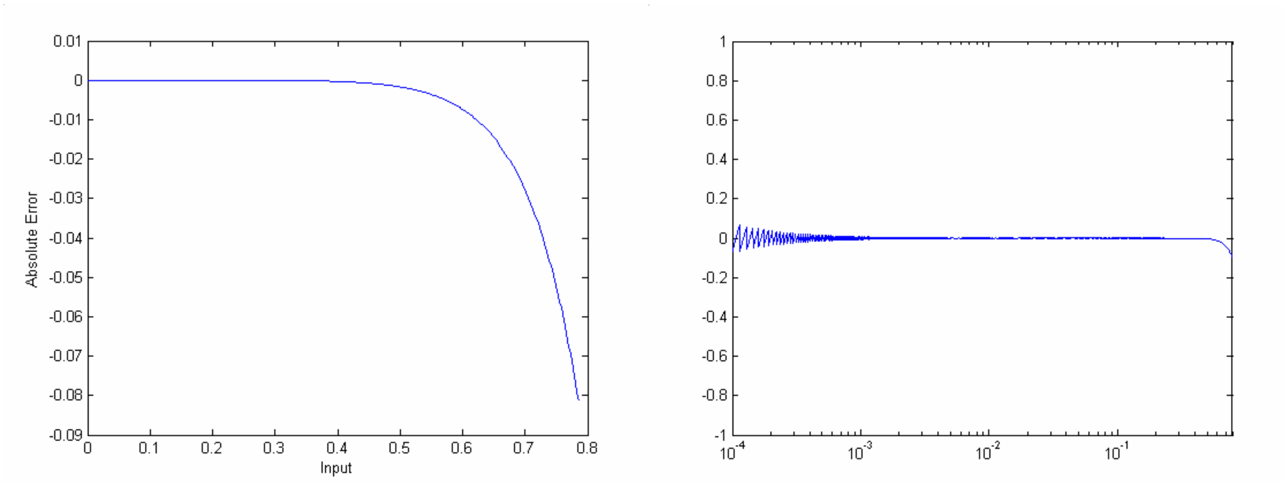

Fig. 4: The error of the 5-rank Taylor polynomial approximation method;

The left Fig. is the absolute error and the right is the relative error.

3.2 measurement range of the two approaches

The measurement range is limited by the ADC sample rate. The actual measurement range is shown in Fig. 5 for the arctangent PGC approach and in Fig. 6 for the traditional DCM PGC approach. The reference of the analysis is : sample rate 112kHz, carrier frequency 11.2kHz, signal frequency 100Hz. The upper limit of the arctangent PGC approach is more than the DCM PGC approach, both the 3dB width (78 radians to 52 radians) and the flatness range (50 radians to 22 radians).

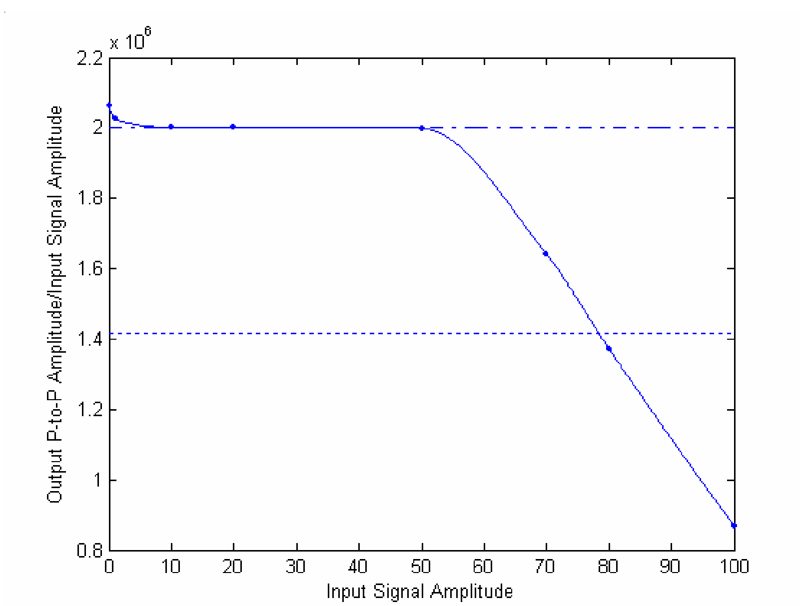

Fig. 5: The measurement range of the arctangent PGC approach

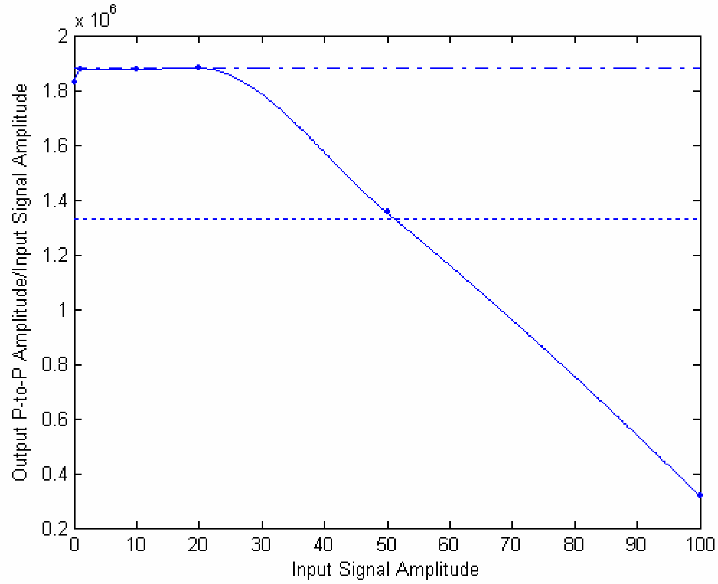

Fig. 6: The measurement range of the DCM PGC approach

3.3 Influence of the light power I_0

The arctangent PGC approach is less influenced by the light power. The Fig. 7 and Fig. 8 separately show the influence of the light power shifting regularly to the DCM approach and the arctangent approach. The demodulation output of the DCM approach is very sensitive to the light power I_0, as shown in equation (6). It needs the auto gain control to reduce the influence of the light power. Although the light power is avoid in the arctangent approach since the light power is divided out. So there is nearly no influence of the light power for the arctangent approach.

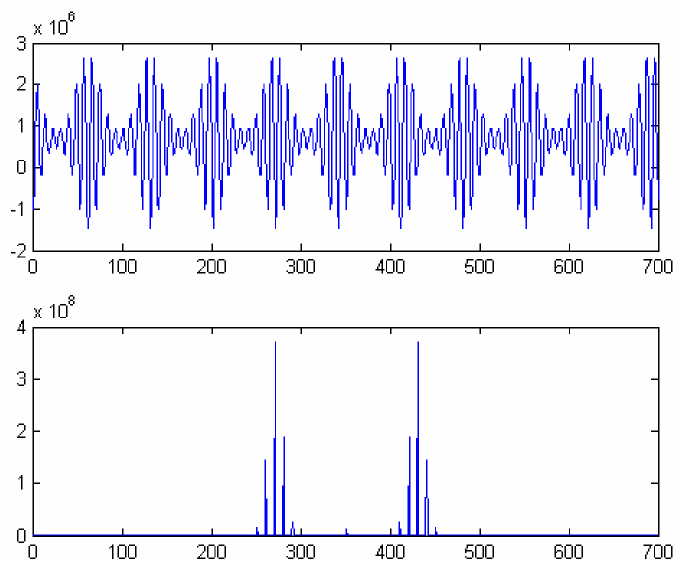

Fig. 7: the influence of the light power to the DCM PGC approach.

The Amplitude changes 3dB.

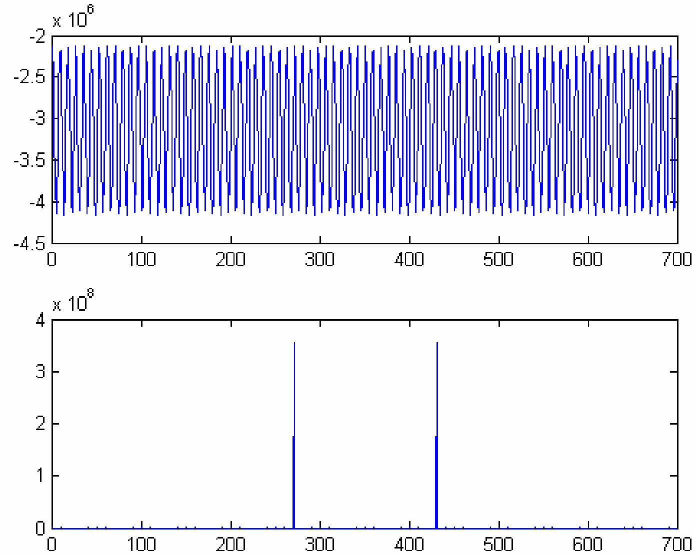

Fig. 8: the influence of the light power to the arctangent PGC approach.

The Amplitude changes 3dB.

Due to the same reason, when there is some noise on the light power, the influence of it is also very small to the arctangent approach while some serious to the DCM approach. They are shown in Fig. 9 and Fig. 10.

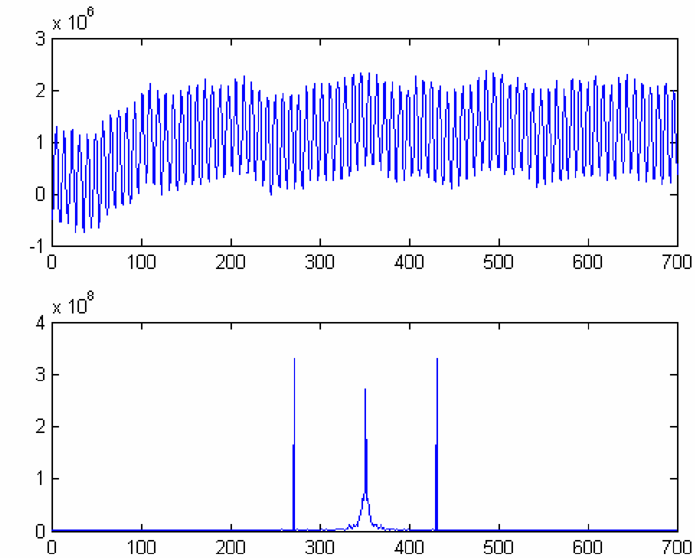

Fig. 9: the influence of the light power to the arctangent PGC approach.

The Noise amplitude is 20dB.

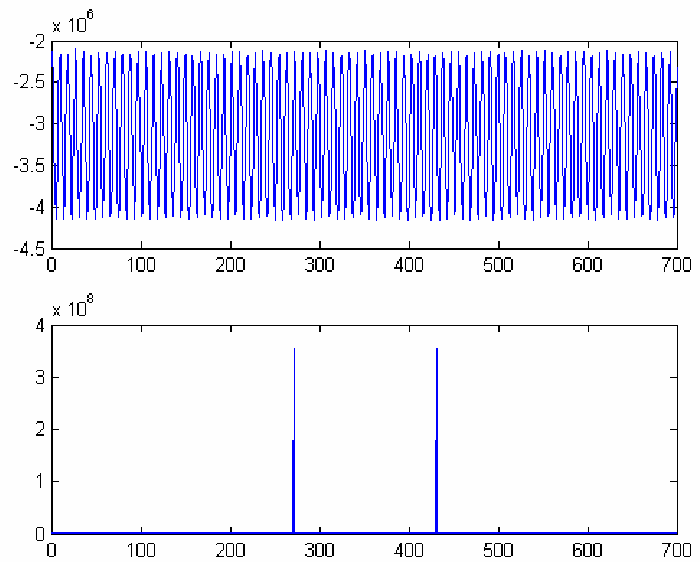

Fig. 10: the influence of the light power to the arctangent PGC approach.

The Noise amplitude is 20dB.

3.4 The impulse response

Since there is a integral operation in the DCM approach, the uncertain initial phase of the integral operation is sure to happen in the demodulation process, especially if there's an impulse input. The impulse response is emulated to confirm this. The input impulse is shown in Fig. 11. The impulse response of the DCM PGC approach is shown in Fig. 12, and the arctangent PGC approach in Fig. 13. The output of the DCM approach jumps when the impulse comes and the jump step is not sure every time so not able to compensate. The arctangent approach is not influenced by the impulse with only losing some signal of high frequency.

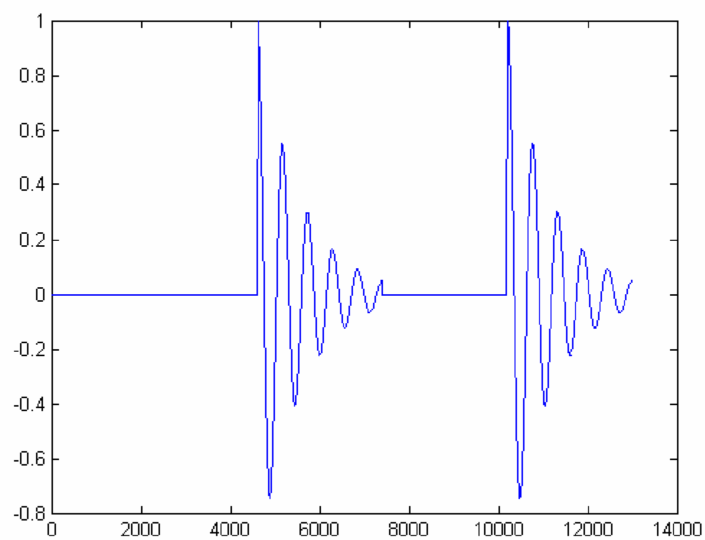

Fig. 11: The input impulse

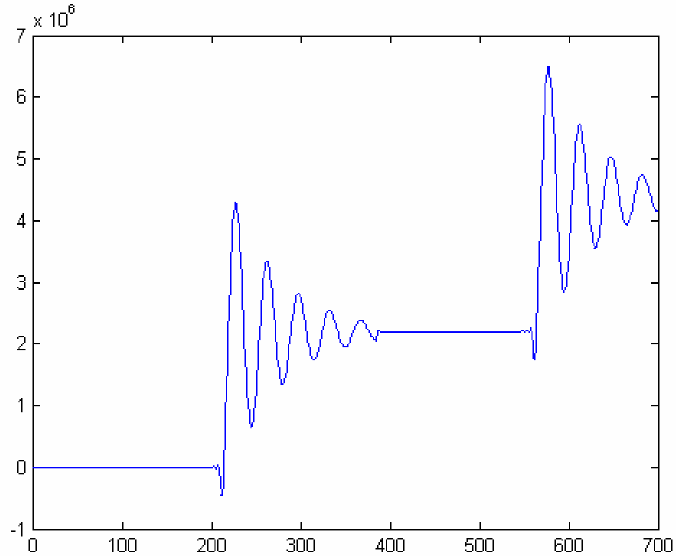

Fig. 12: The impulse response to the DCM approach

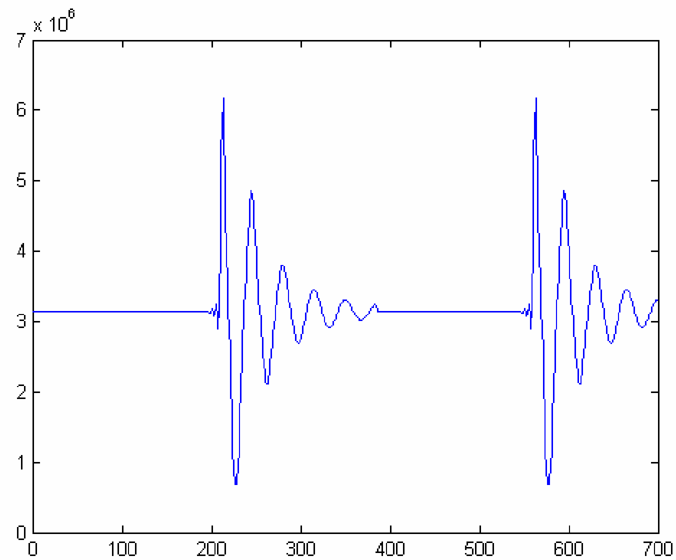

Fig. 13: The impulse response to the arctangent approach

4. CONCLUSION

In this paper, the traditional DCM approach and the arctangent approach for the PGC scheme are all carefully analyzed. The arctangent approach has advantages in calculation complexity, the measurement range, influence of the light power and the impulse response. It has less calculation complexity and more measurement range. Its output does not depend on the light power which makes the interferometer sensor more practical especially for the sensor array. It could reduce the difficulty of sensor array. The most important, it avoids the uncertain phase value of the integral operation in DCM PGC approach and gets the better impulse response.

The Table-Lookup method is presented to do the arctangent operation. It has the less calculation complexity since there is fewer times of multiplication. It has less error than the 5-rank Taylor polynomial approximation, especially when the phase is near $\pi/4$.

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